Online Appendix for "Governing Multiple Firms"

A Proofs

A.1 Proofs of Section 1

Proof of Proposition 1. Let $x^*(v,\theta)$ be an equilibrium strategy for type-(v,θ). If the equilibrium involves mixed strategies, then $x^*(v,\theta)$ is a set. We start by proving that there is a unique $\overline{x} > 0$ such that $x_i^*(\underline{v}, L) = x_i^*(\underline{v}, 0) = x_i^*(\overline{v}, L) = \overline{x}$. We argue six points:

1. If $x_i' \in x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$ then $x_i' > 0$. By choosing $x_i = 0$, type-$\underline{v}$ receives a payoff of $\underline{v}$. However, note that there is $x_i'' > 0$ s.t. $x_i'' \in x_i^*(\overline{v}, L)$. Therefore, $p_i(x_i'') > \underline{v}$ with positive probability. By choosing x_i'', type-$\underline{v}$ increases her revenue and obtains an expected payoff strictly greater than $\underline{v}$. Therefore, $0 \notin x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$.

2. If $x_i' \in x_i^*(\overline{v}, 0)$ then $x_i' \notin x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$. Suppose not. Since $x_i' \in x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$, with positive probability $p_i(x_i) < \overline{v}$. Based on point 1, it must be $x_i' > 0$. Since $x_i' > 0$, type-$(\overline{v}, 0)$ will deviate to $x_i = 0$, which generates a strictly higher payoff of $\overline{v}$.

3. If $x_i' \in x_i^*(\overline{v}, 0)$ then $x_i' \notin x_i^*(\overline{v}, L)$. Suppose not. Based on point 2, $x_i' \in x_i^*(\overline{v}, 0)$ implies $x_i' \notin x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$. Therefore, $p_i(x_i') = \overline{v}$ w.p. 1, and type-$(\overline{v}, L)$ can satisfy her liquidity need by choosing x_i'. She chooses $x_i'' \neq x_i'$ only if $p_i(x_i'') = \overline{v}$ w.p. 1. Thus, there is no $x_i'' \in x_i^*(\overline{v}, L)$ s.t. $x_i'' \in x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$. Therefore, $p_i(x_i''') = \underline{v}$ $\forall$ $x_i''' \in x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$ w.p. 1, and so type-$(\underline{v}, \theta)$ receives a payoff of $\underline{v}$. However, type-$(\underline{v}, 0)$ can always choose x_i' and secure a payoff strictly larger than $\underline{v}$, since $p_i(x_i') = \overline{v}$ w.p. 1. We conclude, if $x_i' \in x_i^*(\overline{v}, 0)$, then $x_i' \notin x_i^*(\overline{v}, L) \cup x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$.

4. $x_i^*(\overline{v}, L) = x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$. Suppose on the contrary there is $x_i' \in x_i^*(\overline{v}, L)$ s.t. $x_i' \notin x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$. The contradiction follows from the same arguments as in point 3. Suppose on the contrary there is $x_i' \in x_i^*(\underline{v}, L) \cup x_i^*(\underline{v}, 0)$ s.t. $x_i' \notin x_i^*(\overline{v}, L)$. Based on point 2, it must be $x_i' \notin x_i^*(\overline{v}, 0)$, and so $p_i(x_i') = \underline{v}$ w.p. 1. Moreover, note that if $x_i'' \in x_i^*(\overline{v}, L)$ then $x_i'' > 0$ and $p_i(x_i'') > \underline{v}$ w.p. 1. However, type-$(\underline{v}, 0)$ can always choose x_i'' and secure a payoff strictly larger than $\underline{v}$, a contradiction.

5. If $x' \in x_i^*(\overline{v},0)$ then $x' < x'' \ \forall \ x'' \in x_i^*(\overline{v},L) \cup x_i^*(\underline{v},L) \cup x_i^*(\underline{v},0)$. Suppose on the contrary there are $x' \in x_i^*(\overline{v},0)$ and $x'' \in x_i^*(\overline{v},L) \cup x_i^*(\underline{v},L) \cup x_i^*(\underline{v},0)$ s.t. $x' \geq x''$. Based on the previous points, $x' \notin x_i^*(\overline{v},L) \cup x_i^*(\underline{v},L) \cup x_i^*(\underline{v},0)$, and so $x' > x''$. Moreover, since $x' \notin x_i^*(\underline{v},L) \cup x_i^*(\underline{v},0)$, w.p. 1 $p_i(x') = \overline{v}$. However, type-$(\overline{v},L)$ has a profitable deviation to x': she receives a payoff of $\overline{v}$ and also satisfies her liquidity need. Indeed, since $x' > x''$ and $x'' \in x_i^*(\overline{v},L)$, then if the investor can satisfy her liquidity need by choosing x'', she can do so by choosing x'.

6. $x_i^*(\overline{v},L)$ is a singleton (types-$(\overline{v},L)$, $(\underline{v},L)$, and $(\underline{v},0)$). Suppose on the contrary there are $x' < x''$ where $x', x'' \in x^*(\overline{v},L)$. Since $\theta = L$ it must be $0 < x'$. Based on point 3, $x', x'' \in x_i^*(\underline{v},L) \cup x_i^*(\underline{v},0)$, and so $p_i(x') \in (\underline{v},\overline{v})$ and $p_i(x'') \in (\underline{v},\overline{v})$. Since type-$(\overline{v},L)$ must be indifferent between x' and x'', then

$$\begin{aligned} x''p_i(x'') + (1-x'')\overline{v} &= x'p_i(x') + (1-x')\overline{v} \Leftrightarrow \\ (x''-x')\left(p_i(x'') - \overline{v}\right) &= x'\left(p_i(x') - p_i(x'')\right). \end{aligned}$$

This implies $p_i(x') < p_i(x'')$. Since $x' < x''$, type-$\underline{v}$ strictly prefers x'' over x'. This implies that $x' \in x_i^*(\overline{v},L) \backslash x_i^*(\underline{v},L)$, a contradiction.

Given the claims above, Bayes' rule implies $p_i(\overline{x}) = \overline{p}_{so}(\tau)$. We prove that in any equilibrium that survives the Grossman and Perry (1986) refinement, $\overline{x} = \overline{x}_{so}(\tau)$. Suppose on the contrary that $\overline{x} > \frac{L}{\overline{p}_{so}(\tau)}$. Since the price function is non-increasing, there is $\varepsilon > 0$ such that $(\overline{x} - \varepsilon)\, p_i(\overline{x} - \varepsilon) \geq L/n$. This implies that type $(\overline{v},L)$ will strictly prefer deviating to $\overline{x} - \varepsilon$, a contradiction. We conclude $\overline{x} \leq \overline{x}_{so}(\tau)$. Suppose on the contrary that $\overline{x} < \overline{x}_{so}(\tau)$. This implies that the investor does not raise L in equilibrium by selling $\overline{x}$. Consider a deviation where all types other than $(\overline{v},0)$ deviate from $\overline{x}$ to $\overline{x}_{so}(\tau)$. Given the deviation, the market maker will set $p\left(\overline{x}_{so}(\tau)\right) = \overline{p}_{so}(\tau)$. Therefore, all types who deviate raise strictly more revenue, and so are strictly better off. Since $\overline{p}_{so}(\tau) < \overline{v}$ type, $(\overline{v},0)$'s equilibrium payoff is still strictly higher than selling $\overline{x}_{so}(\tau)$ claims of the firm. Therefore, an equilibrium with $\overline{x} < \overline{x}_{so}(\tau)$ violates the Grossman and Perry (1986) refinement.

Next, note that in equilibrium it must be $x_i^*(\overline{v},0) > 0 \Rightarrow p_i(x_i^*(\overline{v},0)) = \overline{v}$. Since $x_i^*(\overline{v},0) = 0$, the price function given by (4) is consistent with (3) and is non-increasing. Note that (3) is incentive compatible given (4). First, the equilibrium payoff of type-$(\overline{v},0)$ is $\overline{v}$. Since $x_i > 0 \Rightarrow$

$p_i^*(x_i) < \overline{v}$, type $(\overline{v}, 0)$ has no profitable deviation. Second, since $\overline{p}_{so}(\tau)\overline{x}_{so}(\tau) \leq L/n$ and $p_i^*(x_i)$ is flat on $(0, \overline{x}_{so}]$, deviating to $(0, \overline{x}_{so}]$ generates revenue strictly lower than L, and so is suboptimal if $\theta = L$. Moreover, since $x_i > \overline{x}_{so}(\tau) \Rightarrow p_i^*(x_i) = \underline{v}$, the investor has no optimal deviation to $x_i > \overline{x}_{so}(\tau)$, regardless of firm value. Last, it is easy to see that $x_i = \overline{x}_{so}(\tau)$ is optimal for type-$(\underline{v}, 0)$. Before we conclude, we note that Lemma 3 proves that the equilibrium that is given by Proposition 1 satisfies the Grossman and Perry (1986) refinement. ■

Lemma 3 *The equilibrium that is given by Proposition 1 satisfies the Grossman and Perry (1986) refinement.*

Proof. It is sufficient to show that for any i, there is no off-equilibrium strategy $\hat{x}_i$ and a non-empty subset of investor types Λ, that satisfy the following:

1. If the market maker observes $\hat{x}_i$, it believes that $(v, \theta) \in \Lambda$ and sets the price accordingly. Denote this price by $\hat{p}_i$.

2. If $(v, \theta) \in \Lambda$, the investor's payoff from deviating to $\hat{x}_i$, if deviation leads to a price of $\hat{p}_i$, is strictly higher than her equilibrium payoff.

3. If $(v, \theta) \notin \Lambda$, the investor's payoff from deviating to $\hat{x}_i$, if deviation leads to a price of $\hat{p}_i$, is weakly lower than her equilibrium payoff.

First, note that type-$(\overline{v}, 0)$ receives the highest possible payoff of $\overline{v}$, so $(\overline{v}, 0) \notin \Lambda$. Second, it cannot be that $\Lambda = \{(\underline{v}, L), (\underline{v}, 0)\}$, since then $\hat{p}_i = \underline{v}$, implying that these types do not gain from a deviation to $\hat{x}_i$. Third, it cannot be that $\Lambda = \{(\overline{v}, L), (\underline{v}, L), (\underline{v}, 0)\}$, since then $\hat{p}_i$ will be the same price as in equilibrium. This means that $\hat{x}_i > \overline{x}$, otherwise, by construction, the liquidity need will not be met w.p. 1. This also implies that if $\overline{x} = 1$, any deviation to $\hat{x}_i < 1$ is suboptimal since more revenue is raised by following the equilibrium strategy. In turn, $\hat{x}_i > \overline{x}$ implies that type-$(\overline{v}, L)$ is strictly worse off from a deviation. This shows that both $(\underline{v}, L)$ and $(\underline{v}, 0)$ cannot be in $\hat{K}$. Fourth, suppose that $(\overline{v}, L) \in \hat{K}$ and either $(\underline{v}, L)$ and $(\underline{v}, 0)$ is in Λ, but not both. Note that, since $(\overline{v}, L) \in \hat{K}$ and $\hat{x}_i\hat{p}_i \geq L$ w.p. 1, a deviation is feasible for both $(\underline{v}, L)$ and $(\underline{v}, 0)$. Thus, if it is strictly optimal for type-$(\underline{v}, L)$ to deviate, it must be strictly optimal for type-$(\underline{v}, 0)$, and vice versa. Finally, suppose type $(\overline{v}, L)$ deviates to x' but types-$(\underline{v}, 0)$ and $(\underline{v}, L)$ do not find it optimal to deviate. The market maker must set

$p_i(x') = \overline{v}$, and deviation is optimal for type $(\overline{v}, L)$ only if $x'\overline{v} \geq L$. Type-$\underline{v}$ does not deviate only if

$$x'\overline{v} + (1 - x')\,\underline{v} \leq \overline{x}p_i(\overline{x}) + (1 - \overline{x})\,\underline{v},$$

which requires $x' < \overline{x}$. Moreover, the above condition implies

$$(\overline{x} - x')\,\underline{v} \leq \overline{x}p_i(\overline{x}) - x'\overline{v}$$

Since $x' < \overline{x}$ we have $\overline{x}p_i(\overline{x}) > x'\overline{v} \geq L$. This observation, however, contradicts $\overline{x}p_i(\overline{x}) \leq L$. ■

Proof of Proposition 2. Suppose $L/n \leq \underline{v}\,(1 - \tau)$. The investor can raise at least L by selling only bad firms, even if she receives the lowest possible price of $\underline{v}$. Since the investor is never forced to sell a good firm, she sells a positive amount $x_i' > 0$ from a good firm only if $p\,(x_i') = \overline{v}$, i.e. she does not sell x_i' from a bad firm. We first argue that, in any equilibrium, $x_i > 0 \Rightarrow p\,(x_i) < \overline{v}$. Suppose on the contrary there is $x_i' > 0$ s.t. $p\,(x_i') = \overline{v}$, and let x_i' be the highest quantity with this property. The investor chooses not to sell x_i' from a bad firm only if there is x_i'' that she chooses with strictly positive probability, where

$$x_i''p_i\,(x_i'') + (1 - x_i'')\,\underline{v} \geq x_i'p_i\,(x_i') + (1 - x_i')\,\underline{v}. \tag{24}$$

The above inequality requires $p_i\,(x_i'') > \underline{v}$. Moreover, since she sells x_i'' from a bad firm with positive probability, we have $p_i\,(x_i'') < \overline{v}$. Given this price, she will never sell x_i'' from a good firm, contradicting the requirement that $p_i\,(x_i'') > \underline{v}$. Therefore, she sells x_i' from a bad firm with strictly positive probability, which contradicts $p\,(x_i') = \overline{v}$. We conclude that in any equilibrium $x_i > 0 \Rightarrow p\,(x_i) < \overline{v}$, and so $v_i = \overline{v} \Rightarrow x_i = 0$. These conditions also imply $x_i > 0 \Rightarrow p\,(x_i) = \underline{v}$. Note that the condition on $\underset{\sim}{\widetilde{x}}$ simply requires that in expectation (i.e. when the investor plays mixed strategies) she sells enough of the bad firms to meet her liquidity needs, given by the realization of θ. Last, $p^*\,(0)$ follows from Bayes' rule and the observation that $v_i = \overline{v} \Rightarrow x_i = 0$. This completes part (i).

Next, suppose $\underline{v}\,(1 - \tau) < L/n$. We proceed by proving the following claims.

1. In any equilibrium there is a unique $\overline{x} > 0$ s.t. $x_i^*(\overline{v}, L) = x_i^*(\underline{v}, 0) = \overline{x}$. To prove this, suppose that in equilibrium, the investor is selling x_i'' and x_i' of a good firm when $\theta = L$

with strictly positive probability. Without loss of generality, suppose $x_i'' > x_i' \geq 0$. Since she must be indifferent between x_i'' and x_i',

$$x_i'' p\left(x_i''\right) - x_i' p\left(x_i'\right) = \overline{v}\left(x_i'' - x_i'\right) > 0. \tag{25}$$

This condition implies that x_i'' generates strictly higher revenue than x_i'. Therefore, she never sells x_i' of a bad firm. Indeed, she can always sell x_i'' and achieve both higher revenue and a higher payoff:

$$\begin{aligned} x_i'' p\left(x_i''\right) + \left(1 - x_i''\right) \underline{v} &> x_i' p\left(x_i'\right) + \left(1 - x_i'\right) \underline{v} \\ &\Rightarrow x_i'' p\left(x_i''\right) - x_i' p\left(x_i'\right) > \underline{v}\left(x_i'' - x_i'\right) > 0. \end{aligned}$$

Since x_i' is played with positive probability, but only when $v_i = \overline{v}$, then $p\left(x_i'\right) = \overline{v}$. Combined with condition (25), this implies that $p\left(x_i''\right) = \overline{v}$. Recall that $p_i\left(x_i^*(\overline{v}, 0)\right) = \overline{v}$. Therefore, it cannot be that the investor sells $x_i^*\left(\overline{v}, \theta\right)$ of a bad firm. This in turn implies $p_i\left(x_i\left(\underline{v}, \theta\right)\right) = \underline{v}$, for $\theta \in \{0, L\}$ and so her payoff from selling a bad firm is always $\underline{v}$ in equilibrium. This creates a contradiction, since when $\theta = 0$, she can strictly increase her payoff by selling $x_i'' > 0$ of a bad firm and obtaining a payoff $x_i''\overline{v} + \left(1 - x_i''\right) \underline{v} > \underline{v}$. We conclude that, in any equilibrium, there is a unique $\overline{x}$ such that the investor sells $\overline{x}$ of each good firm when $\theta = L$.

Since $\underline{v}\left(1 - \tau\right) < L/n$, it must be $\overline{x} > 0$. We denote $p_i\left(\overline{x}\right) = \overline{p}$. Since the investor sells $\overline{x}$ of a good firm with positive probability, $\overline{p} > \underline{v}$. We argue that, in any equilibrium, if $\theta = 0$ then she sells $\overline{x}$ of every bad firm. Suppose she sells a different quantity. Recall that $p_i\left(x_i^*(\overline{v}, 0)\right) = \overline{v}$ implies that she does not sell $x_i^*(\overline{v}, 0)$ of a bad firm in equilibrium. Since $x_i^*(\underline{v}, 0) \neq \overline{x}$ and $x_i^*(\underline{v}, 0) \neq x_i^*(\overline{v}, 0)$, we must have $p_i\left(x_i^*(\underline{v}, 0)\right) = \underline{v}$, which yields a payoff of $\underline{v}$. This creates a contradiction since she has strict incentives to deviate and sell $\overline{x}$ of a bad firm, thereby obtaining a payoff of strictly more than $\underline{v}$. Note that this implies that $\overline{p} < \overline{v}$.

2. In any equilibrium, either $x_i^*(\underline{v}, L) = \overline{x}$ w.p. 1, or $x_i^*(\underline{v}, L) = 1$ w.p. 1, where $\overline{x}$ is defined as in Claim 1. To prove this, note that the investor cannot sell $x_i^*(\overline{v}, 0)$ of a bad firm in equilibrium. Therefore, if $x_i^*(\underline{v}, L) \neq \overline{x}$, then $p_i\left(x_i^*(\underline{v}, L)\right) = \underline{v}$. Suppose $x_i^*(\underline{v}, L) \neq \overline{x}$ and $x_i^*(\underline{v}, L) < 1$. Then, she can always deviate to fully selling a bad firm, and not selling

some good firms, to keep revenue constant. Her payoff from selling a bad firm is no lower (since she previously received $\underline{v}$ for each bad firm), but by not selling some good firms, for which she previously received $\overline{xp} + (1-\overline{x})\,\overline{v} < \overline{v}$, she increases her payoff. Therefore, $x_i^*(\underline{v}, L) \in \{\overline{x}, 1\}$. Suppose the investor chooses $x_i^*(\underline{v}, L) = 1$ with probability strictly between zero and one. Therefore, $p\,(1) = \underline{v} < \overline{p}$, and the investor chooses $x_i^*(\underline{v}, L) = 1$ with strictly positive probability only if $\overline{xp} < \min\{L/n, \tau\overline{xp} + (1-\tau)\,\underline{v}\}$. That is, it must be that by selling $\overline{x}$ from all firms, she cannot raise revenue of at least L, and by selling $\overline{x}$ of all good firms and 1 of all bad firms, she can raise strictly more. This, however, implies the investor cannot be indifferent between 1 and $\overline{x}$, thereby proving that either $x_i^*(\underline{v}, L) = \overline{x}$ w.p. 1, or $x_i^*(\underline{v}, L) = 1$ w.p. 1, as required.

3. If in equilibrium $x_i^*(\underline{v}, L) = 1$ and $\overline{x} < 1$ then $L/n < \underline{v}$ and $\overline{x} = \overline{x}_{co}\,(\tau)$, as given by (7). To prove this, since $x_i^*(\underline{v}, L) = 1$ and $v_i = \overline{v} \Rightarrow x_i^* < 1$, $p_i\,(1) = \underline{v}$. Moreover, given claims 1 and 2, and by Bayes' rule, $\overline{p}$ is given by $\overline{p}_{co}\,(\tau)$, as given by (8). Suppose $\theta = L$. Since $\overline{p}_{co}\,(\tau) > \underline{v}$, the investor chooses $x_i^*(\underline{v}, L) = 1$ only if the revenue from selling $\overline{x}$ from all firms is strictly smaller than L and also the revenue from selling $\overline{x}$ of all good firms and 1 from all bad firms, i.e.

$$\overline{xp}_{co}\,(\tau) < \min\{(1-\tau)\underline{v} + \tau\overline{xp}_{co}\,(\tau)\,, L/n\} \Leftrightarrow \overline{xp}_{co}\,(\tau) < \min\{\underline{v}, L/n\}\,.$$

Intuitively, we require $\overline{xp}_{co}\,(\tau) < \underline{v}$, since the investor receives $\overline{xp}_{co}\,(\tau)$ by partially selling $\overline{x}$ of a bad firm for price $\overline{p}_{co}\,(\tau)$, and $\underline{v}$ by fully selling a bad firm for price $\underline{v}$. In equilibrium, she would only fully sell a bad firm if doing so raises more revenue.

We now prove that $(1-\tau)\underline{v} + \tau\overline{xp}_{co}\,(\tau) = L/n$, i.e. fully selling bad firms and selling $\overline{x}$ of good firms raises exactly L. We do so in two steps. We first argue that this strategy cannot raise more than L, i.e.

$$(1-\tau)\underline{v} + \tau\overline{xp}_{co}\,(\tau) \leq L/n. \tag{26}$$

Suppose not. Then, the investor has "slack": she can deviate by selling only $\overline{x}-\varepsilon$ instead of $\overline{x}$ from each good firm, while still meeting her liquidity need. Since prices are non-increasing, $p_i\,(\overline{x}-\varepsilon) \geq \overline{p}_{co}\,(\tau)$, and so for small $\varepsilon > 0$, she still raises at least L. Her payoff is strictly higher since she sells less from the good firms. We next argue that this strategy cannot raise less than L, i.e.

$$(1-\tau)\underline{v}+\tau\overline{x}\overline{p}_{co}(\tau)\geq L/n. \tag{27}$$

Suppose not. If the strategy did not raise L, then it must be that $\underline{v}\leq(1-\tau)\underline{v}+\tau\overline{x}\overline{p}_{co}(\tau)$, i.e. the alternative strategy of fully selling her entire portfolio raises even less revenue. Therefore, $\underline{v}\leq\overline{x}\overline{p}_{co}(\tau)$, which contradicts $\overline{x}\overline{p}_{co}(\tau)<\underline{v}$. Intuitively, if fully selling a firm for $\underline{v}$ raises less revenue than selling $\overline{x}$ of a firm for $\overline{p}_{co}(\tau)$, then the investor would not pursue the strategy of fully selling bad firms. Combining (26) and (27) yields

$$(1-\tau)\underline{v}+\tau\overline{x}\overline{p}_{co}(\tau)=L/n$$

as required. This condition implies $\overline{x}=\overline{x}_{co}(\tau)$, and $\overline{x}_{co}\overline{p}_{co}(\tau)<\underline{v}$ implies $L/n<\underline{v}$ as required.

4. If in equilibrium $x_i^*(\underline{v},L)=\overline{x}$ then $\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}\leq L/n$, $\overline{p}=\overline{p}_{so}(\tau)$ and $\overline{x}=\overline{x}_{so}/n$. To prove this, since prices are monotonic, we must have $\overline{x}\overline{p}\leq L/n$. Otherwise, if $\theta=L$ the investor deviates by selling $\overline{x}-\varepsilon$ instead of $\overline{x}$ from a good firm. For small $\varepsilon>0$, she can raise the same amount of revenue and sell less from the good firms. Note that $x_i^*(\underline{v},L)=\overline{x}\Rightarrow\overline{p}=\overline{p}_{so}(\tau)$. Suppose on the contrary that such an equilibrium exists and

$$L/n<\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}.$$

We argue that there is an optimal deviation to fully selling all bad firms, and selling x' from good firms, for some $x'\in(0,\overline{x}]$. Since $\frac{L/n}{\underline{v}}<\frac{1-\tau}{\beta\tau+1-\tau}<1$, she can always raise at least L by selling all firms. Therefore, it must be $\overline{x}\overline{p}_{so}(\tau)=L/n$. Moreover, $\overline{p}_{so}(\tau)>\underline{v}\Rightarrow\overline{x}<1$. Since $\overline{x}$ is an equilibrium, $xp(x)<L/n$ for any $x<\overline{x}$. Let

$$x'=\frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}_{so}(\tau)}$$

Note that $L/n-(1-\tau)\underline{v}>0$ implies $x'>0$ and $\overline{x}\overline{p}_{so}(\tau)=L/n<\underline{v}$ implies $x'<\overline{x}$. By deviating to fully selling all bad firms and selling only $x'\leq\overline{x}$ from all good firms, the revenue raised is at least L. This deviation generates a higher payoff if and only if

$$x'\tau\overline{p}_{so}(\tau)+(1-x')\tau\overline{v}+(1-\tau)\underline{v}>\overline{x}\overline{p}_{so}(\tau)+(1-\overline{x})(\tau\overline{v}+(1-\tau)\underline{v})$$

Using $\overline{x}\overline{p}_{so}(\tau)=L/n$, $x'=\frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}_{so}(\tau)}$, and $\overline{p}_{so}(\tau)=\underline{v}+\Delta\frac{\beta\tau}{\beta\tau+1-\tau}$, we obtain $L/n<\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$, which implies that this deviation is optimal, a contradiction. We conclude that $L/n\geq\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}$ as required. Intuitively, if the shock were smaller, the investor would retain more of good firms. Note that for the same reasons as in the benchmark, it must be $\overline{x}=\overline{x}_{so}/n$.

Consider part (ii). We show that if $\underline{v}(1-\tau)<L/n<\underline{v}$ then the specified equilibrium indeed exists. First note that $L/n<\underline{v}\Rightarrow\overline{x}_{co}(\tau)<1$. Second, note that the prices in (8) are consistent with the trading strategy given by (7). Moreover, the pricing function in (8) is non-increasing. Third, we show that given the price function in (8), the investor's trading strategy in (7) is indeed optimal. Suppose $\theta=0$. Given (8), the investor's optimal response is $v_i=\overline{v}\Rightarrow x_i=0$ and $v_i=\underline{v}\Rightarrow x_i=\overline{x}_{co}(\tau)$, as prescribed by (7). Suppose $\theta=L$. Given (8), the investor's most profitable deviation involves selling $\overline{x}_{co}$ from each bad firm, and the least amount of a good firm, such that she raises at least L. However, recall that by the construction of $\overline{x}_{co}(\tau)$, $(1-\tau)\underline{v}+\tau\overline{x}_{co}(\tau)\overline{p}_{co}(\tau)=L/n$. Also note that $L/n<\underline{v}\Rightarrow\overline{x}_{co}(\tau)\overline{p}_{co}(\tau)<L/n$. Therefore, the most profitable deviation generates a revenue strictly lower than L, and hence is suboptimal. This concludes part (ii).

Consider part (iii). We show that if $\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}\leq L/n$ then the specified equilibrium indeed exists. The proof is as described by Proposition 1, where $\overline{x}_{so}$ is replaced by $\overline{x}_{so}/n$. The only exception is that we note that as per the proof of Claim 4, the condition $\underline{v}\frac{1-\tau}{\beta\tau+1-\tau}\leq L/n$ guarantees that, if $\theta=L$, the investor has no profitable deviation. The proof that the investor has no profitable deviation when $\theta=0$ is the same as in the proof of part (ii) above.

Finally, part (iii) follows from claims 1-4. ■

Proof of Proposition 3. Consider part (i). From Proposition 1,

$$P_{so}(v_i,\tau)=\begin{cases}\overline{p}_{so}(\tau) & \textit{if } v_i=\underline{v}\\ \beta\overline{p}_{so}(\tau)+(1-\beta)\overline{v} & \textit{if } v_i=\overline{v},\end{cases}$$

and from Proposition 2,

$$P_{co}\left(\underline{v},\tau\right)=\begin{cases}\gamma\left[\underline{v}+\Delta\frac{\tau}{\tau+\gamma(1-\tau)}\right]+\left(1-\gamma\right)\underline{v} & \textit{if } L/n\leq\underline{v}\left(1-\tau\right)\\ \left\{\beta\underline{v}+\left(1-\beta\right)\overline{p}_{co}\left(\tau\right),P_{so}\left(\underline{v},\tau\right)\right\} & \textit{if } \underline{v}\left(1-\tau\right)<L/n<\underline{v}\\ P_{so}\left(\underline{v},\tau\right) & \textit{if } \underline{v}\leq L/n\end{cases}$$

$$P_{co}\left(\overline{v},\tau\right)=\begin{cases}\underline{v}+\Delta\frac{\tau}{\tau+\gamma(1-\tau)} & \textit{if } L/n\leq\underline{v}\left(1-\tau\right)\\ \left\{\beta\overline{p}_{co}\left(\tau\right)+\left(1-\beta\right)\overline{v},P_{so}\left(\overline{v},\tau\right)\right\} & \textit{if } \underline{v}\left(1-\tau\right)<L/n<\underline{v}\\ P_{so}\left(\overline{v},\tau\right) & \textit{if } \underline{v}\leq L/n.\end{cases}$$

where the curly brackets encompass the two possible equilibria (types-(ii) and (iii)) that can exist when $\underline{v}\left(1-\tau\right)<L/n<\underline{v}$.

To prove (9), first suppose $\underline{v}\left(1-\tau\right)<L/n$. It is sufficient to note that

$$\beta\underline{v}+\left(1-\beta\right)\overline{p}_{co}\left(\tau\right)<\overline{p}_{so}\left(\tau\right)$$

and

$$\beta\overline{p}_{co}\left(\tau\right)+\left(1-\beta\right)\overline{v}>\beta\overline{p}_{so}\left(\tau\right)+\left(1-\beta\right)\overline{v}$$

which holds given that $\overline{p}_{co}\left(\tau\right)>\overline{p}_{so}\left(\tau\right)$. Next, suppose $L/n\leq\underline{v}\left(1-\tau\right)$. Note that $\gamma\in\left[0,1-\beta\frac{L/n}{\underline{v}(1-\tau)}\right]$ and

$$\begin{aligned}\overline{p}_{so}\left(\tau\right) &\geq \gamma\left[\underline{v}+\Delta\frac{\tau}{\tau+\gamma\left(1-\tau\right)}\right]+\left(1-\gamma\right)\underline{v}\Leftrightarrow\gamma\leq\frac{\beta\tau}{\beta\tau+\left(1-\beta\right)\left(1-\tau\right)}\\ \beta\overline{p}_{so}\left(\tau\right)+\left(1-\beta\right)\overline{v} &< \underline{v}+\Delta\frac{\tau}{\tau+\gamma\left(1-\tau\right)}\Leftrightarrow\gamma<\frac{\beta\tau}{\beta\tau+\left(1-\beta\right)\left(1-\tau\right)},\end{aligned}$$

as required.

Consider part (ii). From Proposition 1,

$$V_{so}\left(v_i,\tau\right)=\begin{cases}\underline{v}+\frac{\overline{x}_{so}(\tau)}{n}\left(\overline{p}_{so}\left(\tau\right)-\underline{v}\right) & \textit{if } v_i=\underline{v}\\ \overline{v}-\beta\frac{\overline{x}_{so}(\tau)}{n}\left(\overline{v}-\overline{p}_{so}\left(\tau\right)\right) & \textit{if } v_i=\overline{v},\end{cases}\tag{28}$$

and from Proposition 2,

$$
\begin{aligned}
V_{co}\left(\underline{v},\tau\right) &= \begin{cases} \underline{v} & \textit{if } L/n \leq \underline{v}\left(1-\tau\right) \\ \left\{\underline{v}+\left(1-\beta\right)\overline{x}_{co}\left(\tau\right)\left(\overline{p}_{co}\left(\tau\right)-\underline{v}\right), V_{so}\left(\underline{v},\tau\right)\right\} & \textit{if } \underline{v}\left(1-\tau\right) < L/n < \underline{v} \\ V_{so}\left(\underline{v},\tau\right) & \textit{if } \underline{v} \leq L/n \end{cases} \\
V_{co}\left(\overline{v},\tau\right) &= \begin{cases} \overline{v} & \textit{if } L/n \leq \underline{v}\left(1-\tau\right) \\ \left\{\overline{v}-\beta\overline{x}_{co}\left(\tau\right)\left(\overline{v}-\overline{p}_{co}\left(\tau\right)\right), V_{so}\left(\overline{v},\tau\right)\right\} & \textit{if } \underline{v}\left(1-\tau\right) < L/n < \underline{v} \\ V_{so}\left(\overline{v},\tau\right) & \textit{if } \underline{v} \leq L/n \end{cases}
\end{aligned}
\tag{29}
$$

To prove (11), it is sufficient to note that both

$$
\begin{aligned}
\overline{v}-\beta\overline{x}_{co}\left(\tau\right)\left(\overline{v}-\overline{p}_{co}\left(\tau\right)\right) &> V_{so}\left(\overline{v},\tau\right) \\
\underline{v}+\left(1-\beta\right)\overline{x}_{co}\left(\tau\right)\left(\overline{p}_{co}\left(\tau\right)-\underline{v}\right) &< V_{so}\left(\underline{v},\tau\right)
\end{aligned}
$$

hold if and only if

$$
\left(\underline{v}+\frac{L/n-\underline{v}}{\tau}\right)\frac{\left(1-\beta\right)}{\underline{v}\left(\beta\tau+\left(1-\beta\right)\left(1-\tau\right)\right)+\Delta\beta\tau} < \frac{L/n}{\underline{v}\left(\beta\tau+1-\tau\right)+\Delta\beta\tau},
$$

which always holds given that

$$
\begin{aligned}
L/n &< \underline{v} \Rightarrow \underline{v}+\frac{L/n-\underline{v}}{\tau} < L/n \\
\Delta &> 0 \Rightarrow \frac{\left(1-\beta\right)}{\underline{v}\left(\beta\tau+\left(1-\beta\right)\left(1-\tau\right)\right)+\Delta\beta\tau} < \frac{1}{\underline{v}\left(\beta\tau+1-\tau\right)+\Delta\beta\tau}.
\end{aligned}
$$

Next, note that

$$
V_{so}\left(\overline{v},\tau\right)-V_{so}\left(\underline{v},\tau\right)=\phi_{voice}\left(\tau\right),
$$

where $\phi_{voice}(\tau)$ is given by (13) and is decreasing with L/n. Also note that

$$V_{co}(\overline{v},\tau) - V_{co}(\underline{v},\tau) = \begin{cases} \Delta & if\ L/n \leq \underline{v}(1-\tau) \\ \left\{\zeta_{voice}(\tau) + \Delta \frac{L/n - \underline{v}(1-\tau)}{\underline{v}(\frac{1-\beta}{\beta}(1-\tau)+\tau)+\Delta\tau}\beta, \phi_{voice}(\tau)\right\} & if\ \underline{v}(1-\tau) < L/n < \underline{v} \\ \phi_{voice}(\tau) & if\ \underline{v} \leq L/n \end{cases} \tag{30}$$

where $\zeta_{voice}(\tau)$ is given by (16) and is decreasing with L/n. We argue that if $\underline{v}(1-\tau) < L/n < \underline{v}$ then

$$\zeta_{voice}(\tau) + \Delta \frac{L/n - \underline{v}(1-\tau)}{\underline{v}(\frac{1-\beta}{\beta}(1-\tau)+\tau)+\Delta\tau}\beta > \phi_{voice}(\tau).$$

If true, this will prove that $V_{co}(\overline{v},\tau) - V_{co}(\underline{v},\tau)$ is globally decreasing in L/n as well. If $\underline{v}(1-\tau) < L/n < \underline{v}$ then

$$\begin{aligned} \zeta_{voice}(\tau) + \Delta \frac{L/n - \underline{v}(1-\tau)}{\underline{v}(\frac{1-\beta}{\beta}(1-\tau)+\tau)+\Delta\tau} &= \Delta\left[1 - \frac{1-\beta}{\tau}\beta\frac{L/n - \underline{v}(1-\tau)}{\underline{v}((1-\beta)(1-\tau)+\beta\tau)+\beta\Delta\tau}\right] \\ \phi_{voice}(\tau) &= \Delta\left[1 - \beta\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau}\right] \end{aligned}$$

and the required inequality holds if and only if

$$\begin{aligned} \frac{1-\beta}{\tau}\frac{L/n - \underline{v}(1-\tau)}{\underline{v}((1-\beta)(1-\tau)+\beta\tau)+\beta\Delta\tau} &< \frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau} \Leftrightarrow \\ \frac{L/n - \underline{v}(1-\tau)}{\tau L/n} &< \frac{1}{1-\beta}\frac{\underline{v}((1-\beta)(1-\tau)+\beta\tau)+\beta\Delta\tau}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau} \Leftrightarrow \\ \frac{L/n - \underline{v}(1-\tau)}{\tau L/n} &< \frac{\underline{v}((1-\tau)+\frac{\beta}{1-\beta}\tau)+\frac{\beta}{1-\beta}\Delta\tau}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau} \Leftrightarrow \\ \frac{1-\tau}{\tau}\frac{L/n - \underline{v}}{L/n} &< \frac{\beta^2}{1-\beta}\tau\frac{\underline{v}+\Delta}{\underline{v} + (\Delta\beta - \underline{v}(1-\beta))\tau}. \end{aligned}$$

Since $L/n < \underline{v}$, the LHS is negative, but the RHS is positive. Therefore, the inequality holds as required. ■

A.2 Proofs of Section 2

We start with an auxiliary lemma which will be useful for the proofs of this section.

Lemma 4 *Consider an equilibrium under common ownership in which each firm is good w.p.* $\tau^* = F(c^*)$*. Then:*

(i) If $L/n \leq \underline{v}(1-\tau^*)$ *then* $L/n \leq \underline{v}(1 - F((1-\beta)\Delta))$ *and*

$$c^* = \min\left\{\Delta, F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)\right\}. \tag{31}$$

(ii) If $\underline{v}(1-\tau^*) < L/n$ *and the equilibrium is type-(ii), then*

$$\zeta_{voice}(F(c^*)) - \beta(1 - \overline{x}_{co}(F(c^*)))\Delta \leq c^* \leq \zeta_{voice}(F(c^*)), \tag{32}$$

where $\zeta_{voice}(\cdot)$ *is given by (16).*

(iii) If $\underline{v}(1-\tau^*) < L/n$ *and the equilibrium is type-(iii), then*

$$c^* = \phi_{voice}(F(c^*)), \tag{33}$$

where $\phi_{voice}(\cdot)$ *is given by (13).*

Proof. Let $\Pi(\tau^*, \tau)$ be the investor's expected payoff, including the possibility of trade, given that the market makers set prices under the belief there are τ^* monitored firms, and the investor monitors a fraction τ of all firms, where we allow for $\tau \neq \tau^*$. Then, τ^* is an equilibrium only if satisfies $\tau^* \in \arg\max_{\tau \in [0,1]} \Pi(\tau^*, \tau)$. We will slightly abuse notation by defining $\Pi(c^*, c) \equiv \Pi(F(c^*), F(c))$ and require $c^* \in \arg\max_{c \geq 0} \Pi(c^*, c)$.

Consider part (i) and suppose $L/n \leq \underline{v}(1-\tau^*)$. The equilibrium must be type-(i) as in Proposition 2. In this case, the investor never sells a good firm if she does not suffer a shock, and she is indifferent between selling and not selling a bad firm. For any cutoff c, she expects to sell a fraction $x^*(c) \in [0,1]$ of a good firm upon a shock, where

$$x^*(c) = \min\left\{1, \max\left\{0, \frac{L/n - \underline{v}(1 - F(c))}{\underline{v}F(c)}\right\}\right\} \tag{34}$$

is increasing in c. Indeed, with $F(c)$ good firms in her portfolio, she will sell good firms only if she cannot satisfy her liquidity need by selling all bad firms. Since $L/n \leq \underline{v}(1-\tau^*) < \underline{v}$, we can rewrite

$$x^*(c) = \max\left\{0, 1 - \frac{1 - \frac{L/n}{\underline{v}}}{F(c)}\right\}.$$

Therefore,

$$\begin{aligned}
\Pi(c^*, c) &= F(c)(\overline{v} - x^*(c)\beta\Delta) + (1 - F(c))\underline{v} - F(c)E[c_i | c_i < c] && (35)\\
&= \underline{v} + \begin{cases} F(c)(\Delta - E[c_i | c_i < c]) & \text{if } c \le F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) \\ \beta\Delta(1 - \frac{L/n}{\underline{v}}) + F(c)(\Delta(1-\beta) - E[c_i | c_i < c]) & \text{otherwise.} \end{cases} && (36)
\end{aligned}$$

Since $\Pi(c^*, c)$ is independent of c^* as long as $L/n \le \underline{v}(1 - \tau^*)$, we write $\Pi(c)$ instead of $\Pi(c^*, c)$, in this range. That is, we require $c^* \in \arg\max_{c \ge 0} \Pi(c)$ s.t., $L/n \le \underline{v}(1 - F(c^*))$. Note that for any $y > 0$,

$$\begin{aligned}
\frac{\partial}{\partial c}(F(c)y - F(c)E[c_i | c_i < c]) &= f(c)(y - c)\\
\frac{\partial^2}{\partial^2 c}(F(c)y - F(c)E[c_i | c_i < c]) &= f'(c)(y - c) - f(c).
\end{aligned}$$

There are three cases to consider:

1. If $\Delta \le F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$ then $\Delta = \arg\max_{c \ge 0} \Pi(c)$.
2. If $(1-\beta)\Delta \le F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) < \Delta$ then $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) = \arg\max_{c \ge 0} \Pi(c)$.
3. If $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) < (1-\beta)\Delta$ then $(1-\beta)\Delta = \arg\max_{c \ge 0} \Pi(c)$.

Since $L/n \le \underline{v}(1 - \tau^*)$, we also require $x^*(c^*) = 0 \Rightarrow c^* \le F^{-1}(1 - \frac{L/n}{\underline{v}})$. Therefore, if $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) < (1-\beta)\Delta$ an equilibrium of this sort cannot exist. If $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) \ge (1-\beta)\Delta$ then it must be $c^* = \min\left\{\Delta, F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)\right\}$, as required.

Consider parts (ii) and (iii) and suppose $L/n > \underline{v}(1 - \tau^*)$. Let $\overline{x}^* = \overline{x}_{co}(\tau^*)$ and $\overline{p}^* = \overline{p}_{co}(\tau^*)$ if the equilibrium is type-(ii) and let $\overline{x}^* = \overline{x}_{so}(\tau^*)/n$ and $\overline{p}^* = \overline{p}_{so}(\tau^*)$ if the equilibrium is type-(iii).

If $\theta = 0$, the investor obtains a payoff of $\overline{v}$ from a good firm. She can obtain a payoff of $\overline{x}^*\overline{p}^* + (1 - \overline{x}^*)\underline{v}$ from a bad firm by selling $\overline{x}^*$ of each bad firm, which generates the highest payoff. Recall that, from Proposition 2 the investor has no incentives to sell less than $\overline{x}^*$ of a bad firm when $\theta = 0$. Thus, for any $x_1' < \overline{x}^*$,

$$
\begin{aligned}
\overline{x}^* \overline{p}^* + (1 - \overline{x}^*)\, \underline{v} &> x_i' p\,(x_i') + (1 - x_i')\, \underline{v} \\
&\Rightarrow \overline{x}^* \overline{p}^* - x_i' p\,(x_i') > (\overline{x}^* - x_i')\, \underline{v} > 0.
\end{aligned}
$$

Therefore, in any equilibrium, the price function in the range $[0, \overline{x}^*)$ must satisfy this condition. This condition also implies that the investor cannot raise more revenue from a single deviation to selling $x' < \overline{x}^*$ from one particular bad firm. Without loss of generality and to simplify the exposition, hereafter we assume $x_i^*(\overline{v}, 0) = 0$ and $x' \in (0, \overline{x}^*) \Rightarrow p\,(x_i') = \overline{p}^*$. Note that these off-equilibrium prices preserve monotonicity, and $\overline{x}^* \overline{p}^* - x_i' p\,(x_i') > (\overline{x}^* - x_i')\, \underline{v}$. Moreover, note that if the investor found it optimal to monitor τ^* firms under the general price function (i.e. before specializing to $p\,(x_i') = \overline{p}^*$), deviating to sell $x' < \overline{x}^*$ from a given firm cannot be sufficiently beneficial to induce deviation from monitoring τ^* firms under the pricing rule $x' \in (0, \overline{x}^*) \Rightarrow p\,(x_i') = \overline{p}^*$. Indeed, since the pricing rule $x' \in (0, \overline{x}^*) \Rightarrow p\,(x_i') = \overline{p}^*$ is the lowest that satisfies monotonicity, a deviation from monitoring τ^* firms is less beneficial than under the general price function, and hence, suboptimal as well. Intuitively, if she sold less than $\overline{x}^*$ from bad firms, she would receive the same price as if she sold $\overline{x}^*$, and so her incentives to monitor are no different.

Suppose $\theta = L$, and consider a type-(iii) equilibrium. We argue that the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm. We consider two cases.

1. Suppose $L/n \geq \underline{v}$. We first argue $\overline{x}^* \overline{p}^* \geq \underline{v}$. To see why, recall that in this case that $\overline{x}^* = \min\left\{\frac{L/n}{\overline{p}^*}, 1\right\}$. Therefore, either $\overline{x}^* \overline{p}^* = L/n \geq \underline{v}$ or $\overline{x}^* = 1$. Since $\overline{p}^* \geq \underline{v}$, $\overline{x}^* = 1 \Rightarrow \overline{x}^* \overline{p}^* \geq \underline{v}$. Second, since $\overline{x}^* \overline{p}^* \geq \underline{v}$, the investor raises more funds when she chooses $x_i = \overline{x}^*$ rather than $x_i = 1$ (when $\overline{x}^* < 1$). Therefore, she will sell $\overline{x}^*$ from each bad firm and $\frac{L/n - \overline{x}^* \overline{p}^* (1-\tau)}{\overline{p}^* \tau}$ from each good firm. If $\overline{x}^* \overline{p}^* = L/n$ then $\frac{L/n - \overline{x}^* \overline{p}^* (1-\tau)}{\overline{p}^* \tau} = \overline{x}^*$, and if $\overline{x} = 1$ then $\overline{p}^* \leq L/n$, which implies that she has to sell the entire portfolio to raise L. Either way, she will sell $\overline{x}^*$ from each firm in her portfolio.

2. Suppose $L/n < \underline{v}$. Note that $\overline{x}^* = \min\left\{\frac{L/n}{\overline{p}^*}, 1\right\}$ and $\overline{p}^* > \underline{v} > L/n$ imply $\overline{x}^* \overline{p}^* = L/n$, and so $\overline{x}^* \overline{p}^* < \underline{v}$. If the investor deviates from selling $\overline{x}^*$ from each firm, she would deviate to fully selling $\min\left\{\frac{L/n}{\underline{v}}, 1 - \tau\right\}$ bad firms, selling a fraction $\overline{x}^*$ of $(1 - \tau) -$

$\min\left\{\frac{L/n}{\underline{v}}, 1-\tau\right\}$ bad firms, and selling a fraction $\hat{x} = \max\left\{0, \frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}^*}\right\}$ of all good firms. Deviation is not strictly preferred if and only

$$\begin{aligned}
& \tau\left[\hat{x}\overline{p}^* + (1-\hat{x})\,\overline{v}\right] + \left[(1-\tau) - \min\left\{\frac{L/n}{\underline{v}}, (1-\tau)\right\}\right]\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right] + \\
& \min\left\{\frac{L/n}{\underline{v}}, (1-\tau)\right\}\underline{v} \\
\leq\ & \tau\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\overline{v}\right] + (1-\tau)\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right] \Leftrightarrow
\end{aligned}$$

$$\max\left\{0, \frac{L/n-(1-\tau)\,\underline{v}}{\tau\overline{p}^*}\right\} \geq \overline{x}^*\left[1 - \min\left\{\frac{L/n}{\underline{v}}, 1-\tau\right\}\frac{(\overline{p}^* - \underline{v})}{\tau\,(\overline{v}-\overline{p}^*)}\right] \tag{37}$$

If $\tau \leq 1 - \frac{L/n}{\underline{v}}$ then condition (37) holds if and only if $\tau \leq \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*}$, where we used $\overline{p}^* = \overline{p}_{so}(\tau^*)$. Note that

$$1 - \frac{L/n}{\underline{v}} < \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*} \Leftrightarrow \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v} < L/n$$

which must hold if the equilibrium is type-(iii). If $\tau > 1 - \frac{L/n}{\underline{v}}$ then condition (37) holds if and only if $L/n \geq \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v}$, which must hold if the equilibrium is type-(iii). Therefore, condition (37) holds, and deviation is suboptimal.

Combining cases 1 and 2 above, we conclude that the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm, and so her payoff is given by

$$\begin{aligned}
\Pi(c^*, c) &= F(c)\left[\overline{v} - \beta\overline{x}^*(\overline{v} - \overline{p}^*)\right] + (1-F(c))\left[\underline{v} + \overline{x}^*(\overline{p}^* - \underline{v})\right] - F(c)\,E\left[c_i|c_i<c\right] \qquad (38) \\
&= \underline{v} + \Delta\left[F(c) + \overline{x}^*(\tau^* - F(c))\frac{\beta}{\beta\tau^*+1-\tau^*}\right] - F(c)\,E\left[c_i|c_i<c\right]
\end{aligned}$$

and hence,

$$\frac{\partial\Pi(c^*, c)}{\partial c}\frac{1}{f(c)} = \Delta\left[1 - \overline{x}^*\frac{\beta}{\beta\tau^*+1-\tau^*}\right] - c.$$

Substituting $\overline{x}^* = \overline{x}_{so}(\tau^*)$ into the first-order condition $\frac{\partial\Pi(c^*,c)}{\partial c} = 0$, yields the condition $c^* = \phi_{voice}(F(c^*))$, as required. This completes part (iii).

Suppose $\theta = L$, and consider the type-(ii) equilibrium. Recall it must be $L/n < \underline{v}$. Also recall that, by construction,

$$\tau^*\overline{x}^*\overline{p}^* + (1-\tau^*)\,\underline{v} = L/n.$$

Therefore, the investor can raise L by fully selling $(1-\tau^*)\, n$ bad firms and selling a fraction $\overline{x}^*$ of $\tau^* n$ good firms. Also, since $\overline{x}^*\overline{p}^* < \underline{v}$, selling $\overline{x}^*$ from every firm will not raise enough revenue to satisfy the shock. Note that, regardless of firm quality, the investor has strict incentives to sell $\overline{x}^*$ rather 1. Indeed, in the latter case the payoff is $\underline{v}$, the lowest possible. Therefore, regardless of the proportion of good firms (i.e. even if $\tau \neq \tau^*$), she will fully sell exactly $(1-\tau^*)\, n$ firms and a fraction $\overline{x}^*$ of all other firms. The investor will prefer fully selling a bad firm if there are sufficient numbers. Therefore, her expected payoff as a function of choosing cutoff c is:

$$\begin{aligned}\Pi(c^*, c) &= (1-F(c))\left[\begin{array}{c}(1-\beta)\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right] \\ +\beta\left(\begin{array}{c}\underline{v}\min\left\{1, \frac{1-\tau^*}{1-F(c)}\right\} \\ +\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right]\left(1-\min\left\{1, \frac{1-\tau^*}{1-F(c)}\right\}\right)\end{array}\right)\end{array}\right] \\ &\quad +F(c)\left[(1-\beta)\,\overline{v} + \beta\left(\begin{array}{c}\underline{v}\max\left\{0, \frac{F(c)-\tau^*}{F(c)}\right\} \\ +\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\overline{v}\right]\left(1-\max\left\{0, \frac{F(c)-\tau^*}{F(c)}\right\}\right)\end{array}\right)\right] \\ &\quad -F(c)\,E\left[c_i | c_i < c\right]\end{aligned}$$

Using $\overline{x}^* = \overline{x}_{co}(\tau^*)$ and $\overline{p}^* = \overline{p}_{co}(\tau^*)$, we obtain:

$$\begin{aligned}\Pi(c^*, c) &= \underline{v} + F(c)\,\Delta + \overline{x}^*(\tau^* - F(c))\,\beta\Delta\frac{1-\beta+\beta\tau^*}{\beta\tau^* + (1-\beta)(1-\tau^*)} - \beta(1-\overline{x}^*)\,\Delta\max\{F(c)-\tau^*, 0\} \\ &\quad -F(c)\,E\left[c_i | c_i < c\right].\end{aligned}$$

Note that

$$\frac{\partial \Pi(c^*, c)}{\partial c}\frac{1}{f(c)} = \zeta_{voice}(\tau^*) - c - \begin{cases}0 & \textit{if } F(c) < \tau^* \\ \beta(1-\overline{x}^*)\,\Delta & \textit{if } F(c) > \tau^*,\end{cases}$$

which is a strictly decreasing function of c, and $\zeta_{voice}(\cdot)$ is given by (16). Since f is strictly positive, $f(0) > 0$ and $\frac{\partial \Pi(c^*,c)}{\partial c}\frac{1}{f(c)}|_{c=0} > 0$. Therefore, $\Pi(c^*, c)$ is maximized at

$$\arg\max_{c\geq 0} \Pi(\tau^*, c) = \begin{cases} \zeta_{voice}(\tau^*) & \textit{if } F(\zeta_{voice}(\tau^*)) < \tau^* \\ F^{-1}(\tau^*) & \textit{if } F(\zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta) \leq \tau^* \leq F(\zeta_{voice}(\tau^*)) \\ \zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta & \textit{if } \tau^* < F(\zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta). \end{cases}$$

In equilibrium, we require $c^*(\tau^*) = F^{-1}(\tau^*)$. Therefore, τ^* must satisfy

$$\zeta_{voice}(\tau^*) - \beta(1-\overline{x}(\tau^*))\Delta \leq F^{-1}(\tau^*) \leq \zeta_{voice}(\tau^*),$$

as required. ■

Note that, in the proof of Lemma 4 above, and in the proofs of the main results of the voice model below, we seemingly ignore the cost K the investor incurs when she does not satisfy her shock. This is without loss of generality. The reason is the following. The application of the Grossman and Perry (1986) refinement to the trade-only model ensures that, if in equilibrium $\underline{v}+\tau^*\Delta \geq L/n$ (i.e. total portfolio value exceeds L), the investor sells exactly enough to satisfy her liquidity needs. (Note that $\underline{v}+\tau^*\Delta \geq L/n$ is the region in which our main results hold.) Moreover, it also implies that, if in equilibrium $\underline{v}+\tau^*\Delta < L/n$, the investor sells her entire portfolio if she suffers a shock, and fully sells the bad firms and fully retains the good firms if she does not suffer a shock. In this case, the investor incurs the cost K whenever she suffers a shock. She is unable to avoid this by monitoring more: since her monitoring is unobserved by the market makers, it does not affect the prices she receives upon selling, and so will not allow her to meet the liquidity need.

We now move to the proofs of the statements in Section 2.

Proof of Lemma 1. For separate ownership, see the proofs of Proposition 4. For common ownership, recall that in equilibrium, $\tau^* \in \arg\max_{\tau\in[0,1]} \Pi(\tau^*, \tau)$, where $\Pi(\tau^*, \tau)$ is defined in Lemma 4. Since all firms are ex-ante identical, the investor will necessarily monitor the mass of $n\tau^*$ firms with the lowest monitoring costs. That is, the investor will monitor firm i if and only if $\widetilde{c}_i \leq F^{-1}(\tau^*)$, as required. ■

Proof of Proposition 4. We solve for the investor's monitoring threshold. Note that even if the investor chooses $\tau \neq \tau^*$, she still faces prices as given by (4), where the market maker anticipates monitoring probability τ^*. Therefore, as in the proof of Proposition 1, the investor has incentives to follow the trading strategy prescribed by (3). She thus chooses $v_i = \overline{v}$ if and only if

$$\widetilde{c}_i/n \leq V_{so}\left(\overline{v}, \tau^*\right) - V_{so}\left(\underline{v}, \tau^*\right), \tag{39}$$

where $V_{so}\left(v_i, \tau\right)$ is given in the proof of Proposition 3 by (28). This holds if and only if

$$\begin{aligned}
\widetilde{c}_i/n &\leq \overline{v} - \beta\frac{\overline{x}_{so}\left(\tau^*\right)}{n}\left(\overline{v} - \overline{p}_{so}\left(\tau^*\right)\right) - \underline{v} - \frac{\overline{x}_{so}\left(\tau^*\right)}{n}\left(\overline{p}_{so}\left(\tau^*\right) - \underline{v}\right) \Leftrightarrow \\
\widetilde{c}_i/n &\leq \Delta\left[1 - \beta\frac{\overline{x}_{so}\left(\tau^*\right)}{n}\frac{1}{\beta\tau^* + 1 - \tau^*}\right] \Leftrightarrow \\
\widetilde{c}_i/n &\leq \phi_{voice}\left(\tau^*\right)
\end{aligned}$$

Thus, the cutoff in any equilibrium must satisfy $c^*/n = \phi_{voice}\left(\tau^*\right)$. In equilibrium, $\tau^* = F\left(c^*\right)$, and hence, $c^*_{so,voice}$ must solve $c^*/n = \phi_{voice}\left(F(c^*)\right)$, as required. Note that as a function of c^*, $\phi_{voice}\left(F(c^*)\right)$ is strictly positive and bounded from above. Therefore, a strictly positive solution always exists. If $\Delta\beta - \underline{v}\left(1-\beta\right) \leq 0$, then $\phi_{voice}\left(F(c^*)\right)$ is decreasing in c^* and so the solution is unique.[28] Note that since $V_{so}\left(v_i, \tau\right)$ is derived from Proposition 1, the equilibrium is characterized by Proposition 1, where τ is given by $\tau^*_{so,voice}$.

For the comparative statics, note that an equilibrium c^* is defined implicitly as the solution to $g(c^*) \equiv \phi_{voice}(F(c^*)) - c^*/n = 0$. Furthermore, stable equilibria satisfy

$$\frac{\partial\phi_{voice}(F(c^*))}{\partial c^*} < \frac{1}{n},$$

which implies $\frac{\partial g(c^*)}{\partial c^*} < 0$. Therefore, by the implicit function theorem, the response of c^* to a given parameter (except for n and $F(\cdot)$) will have the same sign as the partial derivative of $\phi_{voice}(F(c^*))$ with respect to c^*.

- For L, we have

$$\frac{\partial\phi_{voice}(\tau)}{\partial L} = \begin{cases} -\Delta\beta\frac{1/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} < 0 & if\ \frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} < \frac{1}{\beta\tau+1-\tau} \\ 0 & \text{otherwise,} \end{cases} \tag{40}$$

[28]For the comparative statics we restrict attention to stable equilibria, i.e. ones for which $n\phi_{voice}\left(F(c^*)\right)$ crosses the 45-degree line from above.

and so c^* is weakly decreasing in L.

- For Δ, since $\frac{\partial \phi_{voice}(\tau)}{\partial \Delta} > 0$, c^* is increasing in Δ.

- For $\underline{v}$, we have

$$\frac{\partial \phi_{voice}(\tau)}{\partial \underline{v}} = \begin{cases} \Delta\beta \frac{(1-(1-\beta)\tau)L/n}{(\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau)^2} > 0 & \textit{if} \ \frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} < \frac{1}{\beta\tau+1-\tau} \\ 0 & \text{otherwise,} \end{cases} \tag{41}$$

and so c^* is weakly increasing in $\underline{v}$.

- For n, as n increases, c/n decreases and $\phi_{voice}(\tau)$ increases, both of which increase c^*.

- For β, we have

$$\frac{\partial \phi_{voice}(\tau)}{\partial \beta} = \begin{cases} -\Delta \frac{\underline{v}(1-\tau)L/n}{(\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau)^2} < 0 & \textit{if} \ \frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} < \frac{1}{\beta\tau+1-\tau} \\ -\Delta \frac{1-\tau}{(1-\tau-\beta\tau)^2} < 0 & \text{otherwise,} \end{cases} \tag{42}$$

and so c^* is decreasing in β.

- For $F(\cdot)$, consider two distributions such that $F_G(c) > F_B(c)$ for all c. There are two cases to consider. First, suppose $\Delta\beta - \underline{v}(1-\beta) < 0$. Then, $\phi_{voice}(\tau)$ is decreasing in τ. Consider the equilibrium cutoffs under $F_B(c)$ and $F_G(c)$, denoted c_B^* and c_G^*, respectively. We have

$$\begin{aligned} c_B^*/n = \phi_{voice}\left(F_B(c_B^*)\right) &= \Delta\left[1 - \beta\min\left\{\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}\,(1-\beta))\,F_B(c_B^*)}, \frac{1}{\beta F_B(c_B^*) + 1 - F_B(c_B^*)}\right\}\right] \\ &> \Delta\left[1 - \beta\min\left\{\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}\,(1-\beta))\,F_G(c_B^*)}, \frac{1}{\beta F_G(c_B^*) + 1 - F_G(c_B^*)}\right\}\right] \\ &= \phi_{voice}\left(F_G(c_B^*)\right). \end{aligned}$$

Therefore, since $\phi_{voice}(\tau)$ is decreasing, it must be that $c_G^* < c_B^*$. Thus, the cutoff is higher under distribution $F_B(c)$. However, this also implies that $\tau_B^* < \tau_G^*$, and so governance improves under $F_G(c)$.

Second, suppose $\Delta\beta - \underline{v}(1-\beta) > 0$. Consider the case where both τ_G^* and τ_B^* satisfy

$$\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}\,(1-\beta))\,\tau^*} < \frac{1}{\beta\tau^* + 1 - \tau^*}.$$

Then, we have $\phi_{voice}(\tau)$ increasing in τ over this region. Therefore,

$$\begin{aligned}
c_B^*/n = \phi_{voice}\left(F_B(c_B^*)\right) &= \Delta\left[1 - \beta \min\left\{\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}\,(1-\beta))\,F_B(c_B^*)}, \frac{1}{\beta F_B(c_B^*) + 1 - F_B(c_B^*)}\right\}\right] \\
&< \Delta\left[1 - \beta \min\left\{\frac{L/n}{\underline{v} + (\Delta\beta - \underline{v}\,(1-\beta))\,F_G(c_B^*)}, \frac{1}{\beta F_G(c_B^*) + 1 - F_G(c_B^*)}\right\}\right] \\
&= \phi_{voice}\left(F_G(c_B^*)\right).
\end{aligned}$$

This implies that there exists a $c_G^* > c_B^*$ with $\phi_{voice}\left(F_G(c_G^*)\right) = c_G^*$ and is a stable equilibrium. This in turn implies $\tau_G^* > \tau_B^*$.

■

Proof of Proposition 5. We prove the result in three parts. First, suppose $\underline{v} \leq L/n$. Based on Proposition 2, the equilibrium must be type-(iii). Based on part (iii) of Lemma 4, the monitoring threshold must solve $c^* = \phi_{voice}\left(F\left(c^*\right)\right)$. Note that $\phi_{voice}\left(F\left(c\right)\right)$ is continuous, $\phi_{voice}\left(F\left(0\right)\right) = \Delta\left(1-\beta\right)$ and $\phi_{voice}\left(1\right) = \Delta(1 - \min\{\frac{L/n}{\underline{v}+\Delta}, 1\})$, and hence, by the intermediate value theorem, a solution always exists. Given a threshold that satisfies $c^* = \phi_{voice}\left(F\left(c^*\right)\right)$, by construction there is a type-(iii) equilibrium with this threshold.

Second, suppose $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$. Based on Lemma 4, in any equilibrium the threshold is smaller than Δ. Therefore, $L/n \leq \underline{v}\left(1-\tau^*\right)$, and based on Proposition 2, the equilibrium must be type-(i). Based on part (i) of Lemma 4, and given that $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right) \Rightarrow \Delta \leq 1 - F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$, we have $c^* = \Delta$. By construction, there is a type-(i) equilibrium with such a threshold.

Third, suppose $\underline{v}\left(1 - F\left(\Delta\right)\right) < L/n < \underline{v}$. We first analyze which equilibria are sustainable in this range, and then compare the efficiency of the sustainable equilibria. Starting with the first step, we prove that if $\underline{v}\left(1 - F\left(\Delta\right)\right) < L/n < \underline{v}$, there always exists a type-(ii) equilibrium where the monitoring threshold is given by part (ii) of Lemma 4, i.e. the largest solution of $c^* = \zeta_{voice}\left(F\left(c^*\right)\right)$. In particular, it is sufficient to show that $c^* = \zeta_{voice}\left(F\left(c^*\right)\right)$

has a solution such that $F^{-1}\left(1-\frac{L/n}{\underline{v}}\right) < c^*$ (which is equivalent to $\underline{v}(1-\tau^*) < L/n$). Indeed, when $c^* = F^{-1}\left(1-\frac{L/n}{\underline{v}}\right)$ then $\zeta_{voice}(F(c^*)) = \Delta$. Since $\underline{v}(1-F(\Delta)) < L/n$, then $c^* = F^{-1}\left(1-\frac{L/n}{\underline{v}}\right) \Rightarrow \zeta_{voice}(F(c^*)) > F^{-1}\left(1-\frac{L/n}{\underline{v}}\right)$. Furthermore, when $F(c^*) = 1$ then $\zeta_{voice}(F(c^*)) = \Delta\left[1-\frac{L/n}{\underline{v}+\Delta\tau}\right] < \infty$, since $F(c^*) = \tau^* = 1$. Since $\zeta_{voice}(F(c^*))$ is continuous in c^*, by the intermediate value theorem, a solution strictly greater than $F^{-1}\left(1-\frac{L/n}{\underline{v}}\right)$ always exists. By construction, there is a type-(ii) equilibrium with such a threshold.

We now move to the efficiency comparison of the various equilibria. We first show that, for $\underline{v}(1-F(\Delta)) < L/n < \underline{v}$, any type-(i) equilibrium is less efficient than a type-(ii) equilibrium. Based on Lemma 4, if the equilibrium is type-(i), then $c^* = \min\left\{\Delta, F^{-1}(1-\frac{L/n}{\underline{v}})\right\}$. However, $\underline{v}(1-F(\Delta)) < L/n$ implies $c^* = F^{-1}(1-\frac{L/n}{\underline{v}}) < c^*_{ii}$.

Next, consider type-(iii) equilibria. When $L/n < \underline{v}$, such equilibria exhibit $\overline{x}^*\overline{p}^* = L/n$, where $\overline{p}^* = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$. Therefore, whenever these equilibria exist,

$$\phi_{voice}(\tau) \equiv \Delta\left[1-\beta\frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right].$$

Note that $\zeta_{voice}(\tau) > \phi_{voice}(\tau)$ if and only if

$$\Delta\left[1-\frac{L/n-\underline{v}(1-\tau)}{\underline{v}\left(\frac{1-\beta}{\beta}(1-\tau)+\tau\right)+\Delta\tau}\frac{1-\beta+\beta\tau}{\tau}\right] > \Delta\left[1-\beta\frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right] \Leftrightarrow$$
$$\left(\frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right)L/n < \underline{v}. \tag{43}$$

Also note that

$$1 \geq \frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} \Leftrightarrow \beta \geq \frac{\underline{v}}{\underline{v}+\Delta}.$$

Therefore, if $\beta \geq \frac{\underline{v}}{\underline{v}+\Delta}$, then (43) always holds, which implies that the most efficient equilibrium is type-(ii). In this case, $\underline{y} = \overline{y} = \underline{v}$. In other words, whenever a type-(ii) equilibrium exists (i.e. $\underline{v}(1-F(\Delta)) < L/n < \underline{v}$), it is the most efficient equilibrium.

Suppose $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$. Note that (43) is equivalent to $\Lambda(\tau) < 0$, where

$$\Lambda(\tau) = \tau^2 - \tau\left[\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right]\frac{\underline{v}-L/n}{\underline{v}} - \frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}-L/n}{\underline{v}}.$$

Note that $\min \Lambda(\tau) < 0$. Also, recall $\underline{v}(1-\tau_{ii}^{**}) < L/n$. Therefore, it is sufficient to focus on the region where $\underline{v}(1-\tau) < L/n \Leftrightarrow \frac{\underline{v}-L/n}{\underline{v}} < \tau$. It can be verified that $\Lambda\left(\frac{\underline{v}-L/n}{\underline{v}}\right) < 0$. Therefore, there is $\hat{\tau} > \frac{\underline{v}-L/n}{\underline{v}}$ such that $\Lambda(\tau) \geq 0 \Leftrightarrow \tau \geq \hat{\tau}$ where $\hat{\tau}$ is the largest root of $\Lambda(\tau)$, given by

$$\hat{\tau} = \frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right) + \frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\sqrt{\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right)^2 + 4\frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}}{\underline{v}-L/n}}.$$

Note that a type-(iii) equilibrium requires

$$\underline{v}\frac{1-\tau}{\beta\tau+1-\tau} < L/n \Leftrightarrow \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} < \tau$$

where $\frac{\underline{v}-L/n}{\underline{v}} < \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. Also note that $\tau^* < F(\Delta)$ in both a type-(ii) and type-(iii) equilibrium. Therefore, the relevant range is

$$\tau \in \left[\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}, F(\Delta)\right].$$

This interval is non-empty if and only if

$$\frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta} < L/n \Leftrightarrow \frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta.$$

Note that $\underline{v}(1-F(\Delta)) < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta}$ for all β. Since $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$ if $L/n < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\frac{\underline{v}}{\underline{v}+\Delta}}$, the most efficient equilibrium is type-(ii). This establishes the existence of $\underline{y}$, the threshold below which a type-(ii) equilibrium is most efficient.

Suppose

$$\frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta < \frac{\underline{v}}{\underline{v}+\Delta}. \tag{44}$$

Note that if $\beta < \frac{v}{\underline{v}+\Delta}$ then $\phi_{voice}(\tau)$ is a decreasing function, and so τ_{iii}^{**}, given by the solution of $\tau = F(\phi_{voice}(\tau))$, is unique. Therefore, the equilibrium with τ_{iii}^{**} is most efficient if and only if

$$\max\left\{\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}, \hat{\tau}\right\} < \tau_{iii}^{**}.$$

We argue that $\hat{\tau} \geq \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. To see why, we first prove that

$$\hat{\tau} < x \Leftrightarrow \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} < x\frac{\frac{1-\beta}{\beta}+x\frac{v}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}. \tag{45}$$

To see this, note that

$$\hat{\tau} \geq x \Leftrightarrow \sqrt{\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right)^2+4\frac{1-\beta}{\beta}\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}\frac{v}{\underline{v}-L/n}} \geq 2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow$$

$$\begin{aligned}
2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &< 0 \\
&\text{or} \\
2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &\geq 0 \text{ and} \\
4\frac{1-\beta}{\beta}\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}\frac{v}{\underline{v}-L/n} &\geq \left[2x\frac{v}{\underline{v}-L/n}\right]^2-2\left[2x\frac{v}{\underline{v}-L/n}\right]\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
2x\frac{v}{\underline{v}-L/n}+\frac{1-\beta}{\beta} &< \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \\
&\text{or} \\
2x\frac{v}{\underline{v}-L/n}+\frac{1-\beta}{\beta} &\geq \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \text{ and } \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \geq \frac{x\frac{1-\beta}{\beta}+x^2\frac{v}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}
\end{aligned}$$

Note

$$2x\frac{\underline{v}}{\underline{v}-L/n}+\frac{1-\beta}{\beta}>\frac{x\frac{1-\beta}{\beta}+x^2\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}\Leftrightarrow 2x\frac{\underline{v}}{\underline{v}-L/n}\frac{1-\beta}{\beta}+x^2\frac{\underline{v}}{\underline{v}-L/n}+\left[\frac{1-\beta}{\beta}\right]^2>0,$$

which proves (45). Using (45), we have

$$\hat{\tau}>\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\Leftrightarrow\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}>\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}}.$$

This eventually yields

$$\frac{\beta}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\left(1+\frac{L/n}{\underline{v}-L/n}\beta\right)>-\frac{L/n}{\underline{v}-L/n}\frac{\beta\frac{L/n}{\underline{v}-L/n}(1-\beta)}{\frac{1}{\beta}+\frac{L/n}{\underline{v}-L/n}(1-\beta)},$$

which always holds. Therefore, τ_{iii}^{**} is most efficient only if $\hat{\tau}<\tau_{iii}^{**}$ and $\beta<\frac{\underline{v}}{\underline{v}+\Delta}$, i.e.

$$\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}<\tau_{iii}^{**}\frac{\frac{1-\beta}{\beta}+\tau_{iii}^{**}\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+\tau_{iii}^{**}}.$$

Note that $\lim_{L/n\to\underline{v}}\tau_{iii}^{**}>0=\lim_{L/n\to\underline{v}}\hat{\tau}$. By continuity, there is $\overline{y}\in[\underline{y},\underline{v})$ such that, if $L/n>\underline{y}$, the most efficient equilibrium is type-(iii). This completes the proof. ■

Proof of Corollary 1. Recall from Proposition 5 that

$$c_{co,voice}^{**}=\begin{cases}\Delta & \textit{if } L/n\le\underline{v}\,(1-F(\Delta))\\ \max\{c_{ii}^{**},c_{iii}^{**}\} & \textit{if } \underline{v}\,(1-F(\Delta))<L/n<\underline{v}\\ c_{iii}^{**} & \textit{if } \underline{v}\le L/n.\end{cases}\tag{46}$$

It is straightforward to see that $\phi_{voice}(\cdot)$ and $\zeta_{voice}(\cdot)$ are decreasing in L/n. Our focus on stable equilibria (the RHS of equations $c=\phi_{voice}(F(c))$ and $c=\zeta_{voice}(F(c))$ cross the 45-degree line from above) implies that c_{ii}^{**} and c_{iii}^{**} are decreasing in L/n. Therefore, $\max\{c_{ii}^{**},c_{iii}^{**}\}$ is also decreasing in L/n. Finally, note that $\Delta>\max\{c_{ii}^{**},c_{iii}^{**}\}\ge c_{iii}^{**}$. Therefore, $c_{co,voice}^{**}$ is globally decreasing in L/n.

The second statement follows directly from Proposition 5, the observation that $c^{**}_{co,voice} = c^{**}_{so,voice}$ when $n = 1$ and $c^{**}_{co,voice} = c^{**}_{iii}$, where $c^{**}_{so,voice}$ is the largest solution of $c^*/n = \phi_{voice}\left(F(c^*)\right)$. ■

Proof of Proposition 6. Suppose $L > 0$. Let $c^*_{so}(n, L)$ be a solution of $c^* = n\phi_{voice}\left(F(c^*)\right)$ that constitutes a stable equilibrium (i.e., $n\phi_{voice}\left(F(c^*)\right)$ crosses the 45 degree line from above) under separate ownership. Note that $n\phi_{voice}\left(F(\cdot)\right)$ is strictly increasing in n. Therefore, $c^*_{so}(n, L)$ locally increases in n as well. Since for a given c^* we have $\lim_{n\to\infty} n\phi_{voice}\left(F(c^*)\right) = \infty$, $\lim_{n\to\infty} c^*_{so}(n, L) = \infty$ as well. In addition, $c^*_{so}(1, L) < \Delta$. It follows that there is $\overline{n} > 1$ such that if $n > \overline{n}$ then the smallest solution of $c^* = n\phi_{voice}\left(F(c^*)\right)$ is strictly greater than Δ. Since $c^{**}_{co,voice} \leq \Delta$, if $n > \overline{n}$ then any equilibrium under separate ownership is strictly more efficient than any equilibrium under common ownership. This completes part (i).

Consider part (ii). Since $\phi_{voice}\left(F(c^*)\right) < \Delta$, there is $\underline{n}(L) > 1$ such that the largest solution of $c^* = n\phi_{voice}\left(F(c^*)\right)$, denoted by $\overline{c}^*_{so}(n, L)$, is strictly smaller than Δ if $n < \underline{n}(L)$. Note that $\underline{n}(L)$ satisfies $\overline{c}^*_{so}\left(\underline{n}(L), L\right) = \Delta$. Recall from Lemma 4 part (i) that if $L/n \leq \underline{v}(1 - \tau^*)$ then in *any* equilibrium under common ownership it must be $c^* = \Delta$. Based on Proposition 5, in any equilibrium under common ownership $\tau^* \leq F(\Delta)$. Therefore, if $L/n \leq \underline{v}(1 - F(\Delta))$ then $c^* = \Delta$ in *any* equilibrium under common ownership. Since an equilibrium under common ownership always exists according to Proposition 5, we conclude that if $n < \underline{n}(L)$ and $L \leq \underline{v}(1 - F(\Delta))$ then indeed any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership. Therefore, there exists $L^* \geq \underline{v}(1 - F(\Delta))$ as required. ■

Proof of Proposition 7. Given threshold c and number of firms n, the investor's net payoff under separate and common ownership, respectively, are

$$\begin{aligned}
\Pi_{so,voice}(n, c) &= n\left(\underline{v} + F(c)\Delta\right) - F(c)E\left[c_i | c_i < c\right] \\
\Pi_{co,voice}(n, c) &= n\left(\underline{v} + F(c)\Delta\right) - nF(c)E\left[c_i | c_i < c\right].
\end{aligned}$$

Note that $\Pi_{co,voice}(1, c) = \Pi_{so,voice}(1, c)$ for any fixed c, that $\Pi_{co,voice}(n, c)$ and $\Pi_{so,voice}(n, c)$ have a unique maximum at Δ and $n\Delta$ respectively, and that in any equilibrium $c^*_{so,voice} \leq \Delta$ and $c^*_{co,voice} < n\Delta$. Moreover, under the conditions of part (ii) of Proposition 6, $c^*_{so,voice} < c^*_{co,voice}$

under any equilibrium of common and separate ownership. ■

Proof of Corollary 2. Consider separate ownership. If the market maker expects the investor to monitor w.p. τ^*, then the price of any on-equilibrium trade must be $\overline{p} = \underline{v} + \tau^* \Delta$. Since the investor only raises θ, then $\overline{x}(\tau^*) = n \min\left\{1, \frac{L/n}{\underline{v}+\tau^*\Delta}\right\}$. She will monitor if and only if:

$$\begin{aligned} & (1-\beta) n (\underline{v} + \Delta) + \beta [\overline{x}(\tau^*) \overline{p} + (n - \overline{x}(\tau^*)) (\underline{v} + \Delta)] - c_i \\ \geq \; & (1-\beta) n \underline{v} + \beta [\overline{x}(\tau^*) \overline{p} + (n - \overline{x}(\tau^*)) \underline{v}] \Leftrightarrow \\ c_i \leq \; & \Delta (n - \beta \overline{x}(\tau^*)) . \end{aligned}$$

Therefore, the equilibrium monitoring threshold must be a solution of

$$c^* = \Delta (n - \beta \overline{x}(\tau^*)),$$

which always exists.

Under common ownership, the investor's expected payoff per firm is given by

$$\Pi(c, c^*) = \beta \frac{\overline{x}(\tau^*)}{n} (\underline{v} + \tau^* \Delta) + \left(1 - \beta \frac{\overline{x}(\tau^*)}{n}\right) (\underline{v} + F(c) \Delta) - F(c) E[c_i | c_i < c]$$

The first-order condition implies

$$c^* = (1 - \beta \overline{x}(F(c^*))/n) \Delta,$$

and so per-security monitoring incentives are identical to the separate ownership benchmark. ■

A.3 Proofs of Section 3

Proof of Lemma 2. Suppose in equilibrium under ownership structure $\chi \in \{so, co\}$ the market believes that the manager works w.p. τ^*_χ. From (18), if the manager chooses $v_i = \overline{v}$ his expected utility is $\overline{R} + \omega P_\chi(\overline{v}, \tau^*_\chi) - \widetilde{c}_i$, and if he chooses $v_i = \underline{v}$ his expected utility is $\underline{R} + \omega P_\chi(\underline{v}, \tau^*_\chi)$. Therefore, he chooses $v_i = \overline{v}$ if and only if $\widetilde{c}_i \leq c^* \equiv \overline{R} - \underline{R} +$

$\omega \left[P_\chi\left(\overline{v}, \tau_\chi^*\right) - P_\chi\left(\underline{v}, \tau_\chi^*\right)\right]$. ■

Proof of Proposition 8. In equilibrium, the market and the investor believe the manager follows threshold c^*. Given (3) and (4), the manager expects the price to be $P_{so}\left(v_i, F\left(c^*\right)\right)$ if he chooses v_i. Therefore, he chooses $v_i = \overline{v}$ if and only if

$$\overline{R} + \omega P_{so}\left(\overline{v}, F\left(c^*\right)\right) - \widetilde{c}_i \geq \underline{R} + \omega P_{so}\left(\underline{v}, F\left(c^*\right)\right), \tag{47}$$

where $P_{so}\left(v_i, \tau\right)$ is explicitly given in the proof of Proposition 3. In equilibrium, c^* must solve (20). Using the explicit formulation of $P_{so}\left(v_i, \tau\right)$, it follows that (20) is equivalent to $c^* = \phi_{exit}\left(F\left(c^*\right)\right)$. Note that $\phi_{exit}\left(\tau\right)$ is decreasing in τ and is bounded from above and below. Therefore, a solution always exists and is unique, as required. The equilibrium is characterized by Proposition 1, where τ is given by $\tau^*_{so,exit}$.

For the comparative statics, note that $\phi_{exit}(\tau)$ is strictly decreasing in τ, and so there is a unique equilibrium. Furthermore, the derivative of $\phi_{exit}(\tau)$ with respect to a given parameter has the same sign as the response of c^* to that parameter. Therefore, the threshold increases with Δ, ω, and $\overline{R} - \underline{R}$. We also have

$$\frac{\partial \phi_{exit}(\tau)}{\partial \beta} = -\Delta\omega \frac{1-\tau}{(\beta\tau + 1 - \tau)^2} < 0,$$

implying that the threshold decreases in β.

Finally, consider two distributions $F_G(\cdot) > F_B(\cdot)$ and their respective equilibrium cutoffs c_G^* and c_B^*. Then, we have

$$c_B^* = \phi_{exit}(F_B(c_B^*)) > \phi_{exit}(F_G(c_B^*)). \tag{48}$$

With $\phi_{exit}(\tau)$ decreasing in τ, this implies that $c_G^* < c_B^*$. Furthermore, since

$$c_B^* = \phi_{exit}(F_B(c_B^*)) < \phi_{exit}(F_G(c_G^*)) = c_G^*, \tag{49}$$

we must have $\tau_G^* > \tau_B^*$. ■

Proof of Proposition 9. First, suppose $\underline{v} \leq L/n$. Based on Proposition 2, any equilibrium is type-(iii). Therefore, $c^{**}_{co,exit} = c^{*}_{so,exit}$ in this range. Similar to the proof of Proposition 2 part (iii) and Proposition 8, such an equilibrium indeed exists.

Second, suppose $\underline{v}\left(1-\tau^{**}_{ii,exit}\right) < L/n < \underline{v}$. Based on Proposition 2, any equilibrium is either type-(ii) or type-(iii). Consider a type-(ii) equilibrium. The manager has incentives to choose $v_i = \overline{v}$ if and only if

$$\overline{R} + \omega\left[\beta\overline{p}_{co}\left(\tau^*\right) + \left(1-\beta\right)\overline{v}\right] - \widetilde{c}_i \geq \underline{R} + \omega\left[\beta\underline{v} + \left(1-\beta\right)\overline{p}_{co}\left(\tau^*\right)\right]. \tag{50}$$

Using $\overline{p}_{co}\left(\tau\right) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$, we obtain $v_i = \overline{v} \Leftrightarrow \widetilde{c}_i \leq \zeta_{exit}(\tau^*)$. Therefore, c^* must solve $c^* = \zeta_{exit}\left(F\left(c^*\right)\right)$. Similar to the proof of Proposition 2 part (ii), if $\tau = \tau^{**}_{ii,exit}$ then indeed an equilibrium with these properties indeed exists. By definition of $c^{**}_{ii,exit}$, such an equilibrium is more efficient than any other type-(ii) equilibrium. Moreover, simple algebra shows that $\zeta_{exit}\left(\tau\right) > \phi_{exit}\left(\tau\right)$, and so $c^{**}_{ii,exit} > c^{*}_{so,exit}$. That is, an equilibrium with $\tau = \tau^{**}_{ii,exit}$ is more efficient than any equilibrium with the properties of part (iii). Finally, to show that an equilibrium with $\tau = \tau^{**}_{ii,exit}$ is more efficient than any type-(i) equilibrium, note that based on part (i) of Proposition 2, the threshold of the alternative equilibrium must satisfy $L/n \leq \underline{v}\left(1-\tau^*\right)$. However, since by assumption $\underline{v}\left(1-\tau^{**}_{ii,exit}\right) < L/n$, it follows that $\tau^* < \tau^{**}_{ii,exit}$, that is, the alternative equilibrium must be less efficient.

Third, suppose $L/n \leq \underline{v}\left(1-\tau^{**}_{ii,exit}\right)$. We argue that a type-(i) equilibrium exists. If true, then this equilibrium is more efficient than any type-(ii) or type-(iii) equilibrium. We argue that the following strategies are an equilibrium: the manager's working threshold is $c^{**} = \min\{\overline{R} - \underline{R} + \Delta\omega, F^{-1}(1-\frac{L/n}{\underline{v}})\}$, the investor's trading strategy is

$$x^*\left(v_i,\theta\right) = \begin{cases} 0 & \text{if } v_i = \overline{v} \\ 1 \text{ w.p. } 1-\eta^* \text{ and } 0 \text{ otherwise} & \text{if } v_i = \underline{v} \text{ and } \theta = 0 \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases} \tag{51}$$

where

$$\eta^* = \begin{cases} 0 & \text{if } \overline{R} - \underline{R} + \Delta\omega \leq F^{-1}(1-\frac{L/n}{\underline{v}}) \\ \frac{1}{1-\beta}\frac{1-\frac{L/n}{\underline{v}}}{\frac{\omega\Delta}{\overline{R}-\underline{R}+\omega\Delta-F^{-1}(1-\frac{L/n}{\underline{v}})}-\frac{L/n}{\underline{v}}} & \text{otherwise} \end{cases}, \tag{52}$$

and prices are

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{F(c^{**})}{F(c^{**})+(1-\beta)\eta^*(1-F(c^{**}))} & if \ x_i = 0 \\ \underline{v} & if \ x_i > 0. \end{cases} \tag{53}$$

If the above equilibrium indeed exists, note that it is must be the most efficient equilibrium among all type-(i) equilibria, and hence the most efficient equilibrium. To understand why, first note that if $c^{**} = \overline{R} - \underline{R} + \Delta\omega$ then the equilibrium exhibits maximum governance, and hence, by definition, no other equilibrium is strictly more efficient. However if $c^{**} = F^{-1}(1 - \frac{L/n}{\underline{v}})$ then $\underline{v}(1 - F(c^{**})) = L/n$. Therefore, any other type-(i) equilibrium must satisfy $L/n \leq \underline{v}(1 - \tau^*)$, and hence, $\tau^* \leq F(c^{**})$, which implies that threshold c^{**} is more efficient.

We now prove that the above equilibrium indeed exists. First note that the prices in this equilibrium follow from the investor's trading strategy and the application of Bayes' rule. Second, given these prices, the investor's trading strategy is optimal. Indeed, note that $L/n \leq \underline{v}(1 - F(c^{**}))$, and so the investor can satisfy her liquidity needs by selling only bad firms. Since $x_i > 0 \Rightarrow p_i^* = \underline{v}$, the investor has strict incentives to fully retain good firm, and weak incentives to sell bad firms. The manger works if and only if

$$\begin{aligned} \overline{R} + \omega p_i^*(0) - \widetilde{c}_i &\geq \underline{R} + \omega\left[\beta\underline{v} + (1-\beta)\left(\eta^* p_i^*(0) + (1-\eta^*)\underline{v}\right)\right] \Leftrightarrow \\ \overline{R} - \underline{R} + \omega\left(1 - (1-\beta)\eta^*\right)\left(p_i^*(0) - \underline{v}\right) &\geq \widetilde{c}_i \end{aligned}$$

Using the explicit form of $p_i^*(0)$ as given above, the manager works if and only if

$$\begin{aligned} \overline{R} - \underline{R} + \omega\left(1 - (1-\beta)\eta^*\right)\Delta\frac{F(c^{**})}{F(c^{**}) + (1-\beta)\eta^*(1 - F(c^{**}))} &\geq \widetilde{c}_i \Leftrightarrow \\ H(\eta^*, c^{**}) &\geq \widetilde{c}_i \end{aligned}$$

where

$$H(\eta, c) = \overline{R} - \underline{R} + \omega\Delta\left(1 - \frac{1}{\frac{F(c)}{1-\beta}\frac{1}{\eta} + 1 - F(c)}\right)$$

is a continuous function of η and c, and it strictly decreases in η, when $\eta > 0$. There are two cases to consider. First, if $\overline{R} - \underline{R} + \Delta\omega \leq F^{-1}(1 - \frac{L/n}{\underline{v}})$ then $H(0, c^{**}) = \overline{R} - \underline{R} + \omega\Delta$, as required. Second, suppose $\overline{R} - \underline{R} + \Delta\omega > F^{-1}(1 - \frac{L/n}{\underline{v}})$ and note that $H(1, c) < \zeta_{exit}(F(c))$ for all c. Recall, $L/n \leq \underline{v}\left(1 - \tau_{ii,exit}^{**}\right) \Rightarrow c_{ii,exit}^{**} \leq F^{-1}(1 - \frac{L/n}{\underline{v}})$, where $c_{ii,exit}^{**}$ is the largest solution of $c^{**}_{ii,exit} = \zeta_{exit}\left(F\left(c^{**}_{ii,exit}\right)\right)$. Therefore,

$$\begin{aligned}
\zeta_{exit}\left(F\left(F^{-1}(1-\frac{L/n}{\underline{v}})\right)\right) &< F^{-1}(1-\frac{L/n}{\underline{v}}) \Rightarrow \\
H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right) &< \zeta_{exit}\left(F\left(F^{-1}(1-\frac{L/n}{\underline{v}})\right)\right) \Rightarrow \\
H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right) &< F^{-1}(1-\frac{L/n}{\underline{v}}).
\end{aligned}$$

Also note that

$$H\left(0, F^{-1}(1-\frac{L/n}{\underline{v}})\right) = \overline{R} - \underline{R} + \omega\Delta > F^{-1}(1-\frac{L/n}{\underline{v}}).$$

Therefore,

$$H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right) < F^{-1}(1-\frac{L/n}{\underline{v}}) < H\left(0, F^{-1}(1-\frac{L/n}{\underline{v}})\right),$$

and by the intermediate value theorem, there is $\hat{\eta} \in (0,1)$ such that $H\left(\hat{\eta}, F^{-1}(1-\frac{L/n}{\underline{v}})\right) = F^{-1}(1-\frac{L/n}{\underline{v}})$. Since $H(\eta, c)$ is decreasing in η, $\hat{\eta}$ is uniquely defined. Finally, one can verify that

$$H\left(\frac{1}{1-\beta}\frac{1-\frac{L/n}{\underline{v}}}{\frac{\omega\Delta}{\overline{R}-\underline{R}+\omega\Delta-F^{-1}(1-\frac{L/n}{\underline{v}})}-\frac{L/n}{\underline{v}}}, F^{-1}(1-\frac{L/n}{\underline{v}})\right) = F^{-1}(1-\frac{L/n}{\underline{v}}),$$

implying that $\eta^* \in (0,1)$ as required. ■

Proof of Proposition 10. First, for $L/n \leq \underline{v}(1-\tau^{**}_{ii,exit})$, $c^{**}_{co,exit} = \min\{\overline{R} - \underline{R} + \Delta\omega, F^{-1}\left(1-\frac{L/n}{\underline{v}}\right)\}$, which is decreasing in L/n. Note also at $L/n = \underline{v}(1-\tau^{**}_{ii,exit})$, this implies that $c^{**}_{co,exit} = F^{-1}\left(1-\frac{L/n}{\underline{v}}\right) = F^{-1}(\tau^{**}_{ii,exit}) = c^{**}_{ii,exit}$. Furthermore, $c^{**}_{ii,exit}$ is not dependent on L/n. Altogether, this implies that $c^{**}_{co,exit}$ is continuous and weakly decreasing in L/n for $L/n < \underline{v}$. Note that for $\underline{v} \leq L/n$, $c^{**}_{co,exit} = c^{*}_{so,exit}$, which is constant in L/n. Finally, since $\zeta_{exit}(\tau) > \phi_{exit}(\tau)$ for all τ, $c^{**}_{co,exit} = \zeta(F(c^{**}_{co,exit})) > \phi_{exit}(F(c^{**}_{co,exit}))$. Since $\phi_{exit}(F(c))$ is decreasing in c, it implies that $c^{*}_{so,exit}$ such that $\phi_{exit}(F(c^{*}_{so,exit})) = c^{*}_{so,exit}$ is strictly less than $c^{**}_{co,exit}$. This confirms that $c^{**}_{co,exit}$ is decreasing in L/n, and furthermore that $c^{**}_{co,exit} > c^{*}_{so,exit}$ for $\underline{v} < L/n$. The fact that for $\underline{v} \leq L/n$ we have $c^{**}_{co,exit} = c^{*}_{so,exit}$ trivially shows that cutoff is identical under common and separate ownership for such values of L/n. This complete the proof. ■

While Proposition 9 focuses on the most efficient equilibrium, Proposition 12 compares governance under separate and common ownership when taking into account all equilibria.

Proposition 12 *(Comparison of equilibria, exit): For any $L > 0$, there is a unique $\beta^* (L) \in [0, 1)$ s.t.:*

(i) If $\beta \geq \beta^* (L)$, any equilibrium under common ownership is weakly more efficient than the separate ownership benchmark.

(ii) If $\beta < \beta^* (L)$, there is an equilibrium under common ownership that is strictly less efficient than the separate ownership benchmark. The least efficient equilibrium is type-(i), where the threshold is given by the smallest solution of $c^* = q (F (c^*))$ where

$$q (\tau) = \overline{R} - \underline{R} + \omega\Delta \left(1 - \frac{1}{\frac{\tau}{\gamma^*(\tau)} + 1 - \tau}\right) \tag{54}$$

and $\gamma^* (\tau) = 1 - \beta \frac{L/n}{\underline{v}(1-\tau)}$.

(iii) $\beta^* (L)$ decreases with L, where $\beta^* (0) = 1$ and $L/n \geq \underline{v} \left(1 - F \left(\overline{R} - \underline{R}\right)\right) \Rightarrow \beta^* (L) = 0$.

To prove Proposition 12, we start with an auxiliary lemma.

Lemma 5 *There is an equilibrium under common ownership and exit in which the working threshold satisfies $L/n \leq \underline{v} (1 - F (c^*))$ if and only if $L/n \leq \underline{v} \left(1 - F \left(\underline{c}^*_{exit,co}\right)\right)$, where $\underline{c}^*_{exit,co}$ is the smallest solution of $c^* = q (F (c^*))$ where $q (\tau)$ is given by (73).*

Proof of Lemma 5. Suppose $L/n \leq \underline{v} \left(1 - F \left(\underline{c}^*_{exit,co}\right)\right)$. We argue that there exists an equilibrium where the manager's working threshold is $\underline{c}^*_{exit,co}$ and the investor's selling strategy is

$$x^* (v_i, \theta) = \begin{cases} 0 \text{ w.p. } \eta^* \text{ and } 1 \text{ w.p. } 1 - \eta^* & \textit{if } v_i = \underline{v} \textit{ and } \theta = L \\ 0 & \textit{otherwise}, \end{cases}$$

where

$$\eta^* = 1 - \frac{L/n}{\underline{v}(1 - F(\underline{c}^*_{exit,co}))} \in [0,1].$$

Indeed, following Bayes' rule, prices are given by

$$p(x_i) = \underline{v} + \mathbf{1}_{\{x_i=0\}} \times \Delta \frac{F(\underline{c}^*_{exit,co})}{F(\underline{c}^*_{exit,co}) + (1-\beta+\beta\eta^*)(1 - F(\underline{c}^*_{exit,co}))},$$

and so the manager works if and only if

$$\begin{aligned} \underline{R} + \omega(\beta[\eta^* p_i(0) + (1-\eta^*)\underline{v}] + (1-\beta)p_i(0)) &< \overline{R} + \omega p_i(0) - \widetilde{c}_i \Leftrightarrow \\ q(F(\underline{c}^*_{exit,co})) &> \widetilde{c}_i. \end{aligned}$$

Note that a solution for $c^* = q(F(c^*))$ always exists since $q(F(0)) = \overline{R} - \underline{R} > 0$ and $\lim_{c^*\to\infty} q(F(c^*)) < \overline{R} - \underline{R} + \omega\Delta$. Since $L/n \le \underline{v}(1 - F(\underline{c}^*_{exit,co}))$ and, given prices, the investor is indifferent between selling and retaining a bad firm, the strategy is weakly optimal. Thus this equilibrium exists, as required.

Next, suppose $L/n > \underline{v}(1 - F(\underline{c}^*_{exit,co}))$. We argue that in any equilibrium, $L/n > \underline{v}(1 - F(c^*))$. Suppose on the contrary there is an equilibrium where $L/n \le \underline{v}(1 - F(c^*))$, and let $\tau^* = F(c^*)$. The equilibrium must be type-(i) and the following must hold:

- There is $\eta \in \left[0, 1 - \frac{L/n}{\underline{v}(1-\tau^*)}\right]$ s.t. $x^*(\underline{v}, L) \in \left[\frac{L/n}{\underline{v}} \frac{1}{(1-\eta)(1-\tau^*)}, 1\right]$ w.p. $1-\eta$ and $x^*(\underline{v}, L) = 0$ w.p. η.[29]

- There is $\varphi \in [0,1]$ s.t. $x^*(\underline{v}, 0) > 0$ w.p. $1-\varphi$ and $x^*(\underline{v}, 0) = 0$ w.p. φ.

- $p_i^*(0) = \underline{v} + \Delta \frac{\tau^*}{\tau^* + [(1-\beta)\varphi + \beta\eta](1-\tau^*)}$

- $c^* = \hat{q}(F(c^*), \varphi, \eta)$ where

$$\hat{q}(\tau, \varphi, \eta) = \overline{R} - \underline{R} + \omega\Delta \left(1 - \frac{1}{1 - \tau + \frac{\tau}{(1-\beta)\varphi + \beta\eta}}\right) \tag{55}$$

[29]In fact, the investor can mix between more than two quantities. However, since any positive quantity leads to the same price $\underline{v}$, without loss of generality we can assume that the investor mixes between only two quantities, one of which is zero.

Indeed, if $\theta = L$, the investor sells at least $\frac{L/n}{\underline{v}(1-\tau^*)} \in (0,1]$ from the bad firms in aggregate, to raise at least L. Let η be the probability that she retains a bad firm when $\theta = L$. Note that $\eta \in \left[0, 1 - \frac{L/n}{\underline{v}(1-\tau^*)}\right]$ to ensure that at least L is raised. If $\theta = 0$, she is indifferent between selling and not selling a bad firm , since $p_i(x_i) = \underline{v}$ for $x_i > 0$. Let φ be the probability that she retains a bad firm when $\theta = 0$. Then, by Bayes' rule,

$$p_i(0) = \underline{v} + \Delta \frac{\tau^*}{\tau^* + [(1-\beta)\varphi + \beta\eta](1-\tau^*)}$$

and the manager works if and only if

$$\begin{aligned} \underline{R} + \omega\left(\beta\left[\eta p_i(0) + (1-\eta)\underline{v}\right] + (1-\beta)\left[\varphi p_i(0) + (1-\varphi)\underline{v}\right]\right) &< \overline{R} + \omega p_i(0) - \widetilde{c}_i \Leftrightarrow \\ q(\tau^*, \varphi, \eta) &> \widetilde{c}_i, \end{aligned}$$

as required. Note that

$$q(\tau) \leq \hat{q}(\tau, \varphi, \eta) \Leftrightarrow \eta \leq \frac{1-(1-\beta)\varphi}{\beta} - \frac{L/n}{\underline{v}(1-\tau)}$$

Note that $\frac{1-(1-\beta)\varphi}{\beta} \geq 1$, and hence, $\eta \leq 1 - \frac{L/n}{\underline{v}(1-\tau)} \Rightarrow \eta \leq \frac{1-(1-\beta)\varphi}{\beta} - \frac{L/n}{\underline{v}(1-\tau)}$. This implies that any solution of $c^* = \hat{q}(F(c^*), \varphi, \eta)$ is weakly larger than $\underline{c}^*_{exit,co}$. But since $L/n > \underline{v}\left(1 - F\left(\underline{c}^*_{exit,co}\right)\right)$, then it must be $L/n > \underline{v}(1 - F(c^*))$ as well. ■

The full proof of Proposition 12 now follows.

Proof of Proposition 12. Suppose $L/n \geq \underline{v}(1 - F(\overline{R} - \underline{R}))$. Since the threshold in every equilibrium satisfies $c^* > \overline{R} - \underline{R}$, $L/n \geq \underline{v}(1 - F(\overline{R} - \underline{R}))$ implies that any equilibrium must involve selling good firms. In this case, as in the arguments in the proof of Proposition 9, the cutoff rule under common ownership satisfies either $c^* = \phi_{exit}(F(c^*))$ or $c^* = \zeta_{exit}(F(c^*))$. Since $c^*_{so,exit} = \phi_{exit}\left(F\left(c^*_{so,exit}\right)\right)$ and $\phi_{exit}(\tau) < \zeta_{exit}(\tau)$ for all τ, and since $\phi_{exit}(\tau)$ is decreasing in τ, all equilibria under common ownership are weakly more efficient than under separate ownership, that is, $\beta^*(L) = 0$.

Suppose $L/n < \underline{v}\left(1 - F\left(\overline{R} - \underline{R}\right)\right)$. Let $\underline{c}^*(\beta, L) \equiv \underline{c}^*_{exit,co}$ as a function of β and L, where $\underline{c}^*_{exit,co}$ is defined in Lemma 5. Note that $\underline{c}^*(\beta, L)$ is strictly increasing in β and $\underline{c}^*(0, L) = \overline{R} - \underline{R} < \underline{c}^*(1, L)$. Also note that $c^*_{so,exit}(\beta)$, the unique separate ownership equilibrium cutoff as a function of β, is strictly decreasing in β, where $c^*_{so,exit}(0) > \overline{R} - \underline{R} = c^*_{so,exit}(1)$. Thus, there exists a unique $\lambda(L) \in (0,1)$ such that

$$\underline{c}^*(\beta, L) \geq c^*_{so,exit}(\beta) \Leftrightarrow \beta \geq \lambda(L). \tag{56}$$

Suppose $\beta \geq \lambda(L)$. Consider the following two cases.

1. If $L/n \leq \underline{v}(1 - F(\underline{c}^*(\beta, L)))$ then, based on Lemma 5, an equilibrium with cutoff rule $\underline{c}^*(\beta, L)$ exists. Moreover, based on Lemma 5, $\underline{c}^*(\beta, L)$ is the lowest cutoff among all equilibria that satisfy $L/n \leq \underline{v}(1 - F(c^*))$. Any other equilibrium that satisfies $L/n > \underline{v}(1 - F(c^*))$, implies $F(c^*) > 1 - \frac{L/n}{\underline{v}} \geq F(\underline{c}^*(\beta, L))$, and hence, is strictly more efficient. So, $\underline{c}^*(\beta, L)$ is the cutoff of the least efficient equilibrium under common ownership. Since $\beta \geq \lambda(L) \Rightarrow \underline{c}^*(\beta, L) \geq c^*_{so,exit}(\beta)$, all equilibria under common ownership are more efficient than the separate ownership benchmark.

2. If $L/n > \underline{v}(1 - F(\underline{c}^*(\beta, L)))$ then, based on Lemma 5, any equilibrium under common ownership satisfies $L/n > \underline{v}(1 - F(c^*))$. In this case, the equilibrium cutoff rules are the set $\{c^{**}_{ii,exit}, c^*_{so,exit}(\beta)\}$. According to Proposition 9, $c^{**}_{ii,exit} > c^*_{so,exit}(\beta)$, and hence, the equilibrium cutoff is weakly larger than $c^*_{so,exit}(\beta)$. Therefore, common ownership is weakly more efficient than separate ownership for $\beta \geq \lambda(L)$.

Next, suppose instead that $\beta < \lambda(L)$. We proceed in several steps.

1. We first argue that there is a unique $L^{**} \in \left(0, n\underline{v}\left(1 - F\left(\overline{R} - \underline{R}\right)\right)\right)$ s.t.

$$L \leq n\underline{v}\left(1 - F(c^*_{so,exit}(\lambda(L)))\right) \Leftrightarrow L \leq L^{**}. \tag{57}$$

   To see why, note that $\underline{c}^*(\beta, L)$ weakly increases in L, and so $\lambda(L)$ weakly decreases in L. Since $c^*_{so,exit}(\beta)$ decreases in β, $c^*_{so,exit}(\lambda(L))$ weakly increases in L. Moreover, since $\lim_{L\to 0} \underline{c}^*(\beta, L) = \overline{R} - \underline{R}$ and $\beta < 1 \Rightarrow c^*_{so,exit}(\beta) > \overline{R} - \underline{R}$, $\lim_{L\to 0} \lambda(L) = 1$. Therefore,

$$\lim_{L\to 0} \frac{1 - F(c^*_{so,exit}(\lambda(L)))}{L} = \frac{1 - F(c^*_{so,exit}(1))}{\lim_{L\to 0} L} = \frac{1 - F\left(\overline{R} - \underline{R}\right)}{\lim_{L\to 0} L} > \frac{1}{n\underline{v}}.$$

Note also that $L > 0 \Rightarrow \lambda(L) < 1$, and so $c^*_{so,exit}(\lambda(L)) > \overline{R} - \underline{R}$. Therefore,

$$\begin{aligned}\lim_{L \to n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)} \frac{1-F(c^*_{so,exit}(\lambda(L)))}{L} &= \frac{1-F\left(c^*_{so,exit}\left(\lambda\left(n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)\right)\right)\right)}{n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)} \\ &< \frac{1-F\left(\overline{R}-\underline{R}\right)}{n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)} = \frac{1}{n\underline{v}}.\end{aligned}$$

By the intermediate value theorem, there is a unique $L^{**} \in \left(0, n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)\right)$ s.t. $L \leq n\underline{v}\left(1-F\left(c^*_{so,exit}(\lambda(L))\right)\right) \Leftrightarrow L \leq L^{**}$, as required.

2. We next argue $L \leq n\underline{v}(1-\underline{\tau}^*(\beta, L))$, with $\underline{\tau}^* \equiv F(\underline{c}^*(\beta, L))$, if and only if:

   - $L \in (0, L^{**}]$ and $\beta < \lambda(L)$, or
   - $L \in \left(L^{**}, \left(1-F\left(\overline{R}-\underline{R}\right)\right)n\underline{v}\right)$ and $\beta < \varphi(L)$, where $\varphi(L) \in (0, \lambda(L))$ solves $L = n\underline{v}(1-\underline{\tau}^*(\varphi(L), L))$.

   To see why, recall $\underline{c}^*(\beta, L)$ is increasing in β where $\underline{c}^*(0, L) = \overline{R} - \underline{R}$. Therefore, $1-\underline{\tau}^*(\beta, L)$ is decreasing in β. Since $\underline{c}^*(0, L) = \overline{R} - \underline{R}$ and $L < n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)$, if $\beta = 0$ then $L \leq n\underline{v}(1-\underline{\tau}^*(\beta, L))$ holds. Recall also that $\underline{c}^*(\lambda(L), L) = c^*_{so,exit}(\lambda(L))$ and $L \leq n\underline{v}\left(1-\tau^*_{so,exit}(\lambda(L))\right) \Leftrightarrow L \leq L^{**}$. Therefore, if $L \in (0, L^{**}]$ then $L \leq n\underline{v}(1-\underline{\tau}^*(\beta, L))\ \forall\ \beta < \lambda(L)$. Suppose $L \in \left(L^{**}, n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)\right)$. Then $L > n\underline{v}\left(1-\tau^*_{so,exit}(\lambda(L))\right)$ and so $L \leq n\underline{v}(1-\underline{\tau}^*(\beta, L)) \Leftrightarrow \beta < \varphi(L)$, where $\varphi(L) \in (0, \lambda(L))$ solves $L = n\underline{v}(1-\underline{\tau}^*(\varphi(L), L))$, as required.

3. Let

$$\beta^*(L) = \begin{cases} \lambda(L) & \text{if } L \leq L^{**} \\ \varphi(L) & \text{if } L \in \left(L^{**}, n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)\right) \\ 0 & \text{if } L \geq n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right). \end{cases} \tag{58}$$

   Based on the analysis of the case where $L \geq n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)$, and the steps for the case where $L < n\underline{v}\left(1-F\left(\overline{R}-\underline{R}\right)\right)$, $\beta^*(L)$ is as stated in the proposition. Note that, if $\beta < \beta^*(L)$, we proved that an equilibrium with cutoff $\underline{c}^*(\beta, L)$ exists, and its properties are as stated in the proposition.

4. Consider the properties of $\beta^*(L)$. Recall that we proved $\lim_{L\to 0}\lambda(L) = 1$, and so $\beta^*(0) = 1$. Note also that $L \geq n\underline{v}\left(1 - F\left(\overline{R} - \underline{R}\right)\right) \Rightarrow \beta^*(L) = 0$. We argue that $\beta^*(L)$ decreases with L. To see this, first note that $\lambda(L) = \varphi(L) \Leftrightarrow L = L^{**}$, and $\varphi(L) = 0 \Leftrightarrow L = n\underline{v}\left(1 - F\left(\overline{R} - \underline{R}\right)\right)$. Recall we proved earlier that $\lambda(L)$ weakly decreases in L. Note that $\varphi(L)$ is defined by $L = 1 - F\left(\underline{c}^*\left(\varphi(L), L\right)\right)$. Since $\underline{c}^*(\beta, L)$ increases in β and in L, $1 - F\left(\underline{c}^*(\beta, L)\right)$ decreases in β and L. Therefore, $\frac{1-F(\underline{c}^*(\beta,L))}{L}$ decreases in L, and so $\varphi(L)$ must decrease in L as well.

■

As discussed in the main text, the inferior equilibria under common ownership exist under small shocks, where the price upon selling is $\underline{v}$ and so the investor does not have strict incentives to sell bad firms. In the least efficient equilibrium within this class, she retains bad firms with certainty upon no shock and with the maximum frequency that still allows her to meet her liquidity needs upon a shock. Overall, the unconditional probability of retention for a bad firm is $\gamma^*(\tau)$. This reduces the punishment for shirking, and also the reward for working by lowering the price of a retained firm below $\overline{v}$. This disadvantage is more pronounced, the greater the frequency with which a bad firm is retained. This frequency is greater if β is low, since a bad firm is always retained upon no shock, and if L is low, since a smaller shock allows the investor to retain more bad firms upon a shock.

Proof of Proposition 11. Consider separate ownership. Suppose in equilibrium the manager follows threshold c^*. Prices at $t = 2$ and the investor's trades are as described by Proposition 1, given the threshold c^*. If there is a single investor, market makers learn nothing about v_i from observing x_j. If there are multiple investors with perfectly correlated shocks, then if the market maker observes that some other firms are fully retained, then it infers that a shock did not occur. Thus, if $x_i = \overline{x}_{so}(\tau^*)$, the price falls to $p_i = \underline{v}$ at $t = 2.5$, as the sale is fully revealing of a bad firm. If it observes that all firms are at least partially sold, then it infers that a shock occurred. Thus, if $x_i = \overline{x}_{so}(\tau^*)$, the price increases to the unconditional value of

$\underline{v} + \Delta\tau$. In all other cases, prices do not change. Therefore, the manager shirks if and only if

$$\overline{R} + \omega\left(\beta\left(\underline{v} + \Delta\tau\right) + (1-\beta)\,\overline{v}\right) - \widetilde{c}_i \quad \leq \quad \underline{R} + \omega\left(\beta\left(\underline{v} + \Delta\tau\right) + (1-\beta)\,\underline{v}\right) \Leftrightarrow \tag{59}$$

$$\overline{R} - \underline{R} + \omega\,(1-\beta)\,\Delta \quad \leq \quad \widetilde{c}_i, \tag{60}$$

and $c^* = c^*_{so,exit,obs}$, as required.

Consider common ownership. If $\underline{v} \leq L/n$ then, based on part (iii) of Proposition 2, the equilibrium has the same properties as in the separate ownership case. Therefore, prices at $t = 2.5$ will be the same, and the manager's threshold is $c^*_{so,exit,obs}$. Suppose $\underline{v}\left(1 - F\left(\overline{R} - \underline{R} + \Delta\omega\right)\right) < L/n < \underline{v}$. We argue that there is a type-(ii) equilibrium. In this equilibrium, by observing all other trades, the market can perfectly infer the value of each firm. Therefore, the manager's threshold will be the highest possible, $\overline{R} - \underline{R} + \omega\Delta$. Since $\underline{v}\left(1 - F\left(\overline{R} - \underline{R} + \Delta\omega\right)\right) < L/n$, the investor does not have enough bad firms to satisfy her shock, and a type-(ii) equilibrium indeed exists. Finally, suppose $L/n \leq \underline{v}\left(1 - F\left(\overline{R} - \underline{R} + \Delta\omega\right)\right)$. We argue that there is a type-(i) equilibrium where γ^*, given in the proof of Proposition 9, is zero. Then, prices at $t = 2.5$ are fully revealing of firm value, and so the manager's threshold must be $\overline{R} - \underline{R} + \Delta\omega$. Since $L/n \leq \underline{v}\left(1 - F\left(\overline{R} - \underline{R} + \Delta\omega\right)\right)$ then investor has enough bad firms in equilibrium in order to satisfy her shock. Based on the properties of a type-(ii) equilibrium, the investor is indifferent between selling and retaining a bad firm, to the extent she raises enough revenue. Therefore, playing $\gamma^* = 0$ is weakly optimal, as required. ■

B Common Monitoring Cost: Analysis

This section shows that the model is very similar if there is a common monitoring cost and the investor's private information is instead over whether monitoring has been successful. In each firm i, the investor now chooses $\lambda_i \in [0, 1]$, the probability of $v_i = \overline{v}$. She incurs a cost $C(\lambda_i)$, where $C' > 0$, $C'(0) > 0$, and $C'(1) = \infty$. This monitoring technology is common to all firms. Whether monitoring succeeds is i.i.d. and privately observed by the investor.

The investor chooses $\lambda^* \in (0, 1)$ such that, in equilibrium, each firm has value $\overline{v}$ w.p. λ^* and $\underline{v}$ otherwise. Thus, the equilibrium is the same as in the core model, except that λ^* replaces

τ^*. Under separate ownership, λ^* solves

$$\begin{aligned}
&\max_{\lambda\in[0,1]} \lambda\left(\beta\left[\overline{v}\left(n-\overline{x}\right)+\overline{x}p_i(\overline{x})\right]+(1-\beta)n\overline{v}\right)+(1-\lambda)\left(\underline{v}\left(n-\overline{x}\right)+\overline{x}p_i(\overline{x})\right)-C\left(\lambda\right)\\
\Leftrightarrow\quad &\phi_{voice}\left(\lambda^*\right)=C'\left(\lambda^*\right)/n,
\end{aligned}$$

compared to $\phi_{voice}\left(F(c^*)\right)=c^*/n$ in the core model. Thus, the equilibrium has the same properties, and is exactly the same if $C'\left(\cdot\right)=F^{-1}\left(\cdot\right)$.

Under common ownership, λ is applied to all firms in a symmetric equilibrium (since they are ex ante identical), and so the investor's marginal aggregate monitoring cost is $nC'\left(\lambda\right)$. Under the core model, the investor effectively has an aggregate monitoring cost function given by

$$g\left(c\right)=nF(c)E[c_i|c_i<c]=n\int_0^c rf\left(r\right)dr,$$

The investor's problem can be written in terms of τ instead of c :

$$\begin{aligned}
g\left(\tau\right) &= n\int_0^{F^{-1}(\tau)} rf\left(r\right)dr\\
g'\left(\tau\right) &= n\left(F^{-1}\left(\tau\right)\right)'\left[F^{-1}\left(\tau\right)f\left(F^{-1}\left(\tau\right)\right)\right]\\
&= n\frac{1}{f\left(F^{-1}\left(\tau\right)\right)}\left[F^{-1}\left(\tau\right)f\left(F^{-1}\left(\tau\right)\right)\right]=nF^{-1}\left(\tau\right)
\end{aligned}$$

Thus, as with common ownership, the equilibrium has the same properties as the core model and is identical if the marginal cost of monitoring, $C'\left(\cdot\right)$, equals $F^{-1}\left(\tau\right)$.

C Extensions: Analysis

C.1 Endogenous Information Acquisition

In this section, we consider a variant of the trade-only model in which the investor is initially uninformed about the value of each firm. This extension shows that, contrary to conventional wisdom, common ownership can *increase* the incentives to gather information.

For firm i, she can pay a cost c_i with CDF $F(c_i)$ to learn firm value $v_i\in\{\underline{v},\overline{v}\}$, where c_i is independent of v_i. We refer to paying this cost as "investigating" or "acquiring information". Thus, the expected value of a firm to the investor is $\underline{v}+\tau\Delta$ if she has not investigated, and either $\underline{v}$ or $\overline{v}$ if she has. Having decided which firms to investigate, she observes their values and chooses how much to sell.

As in the baseline model, we let $x_i(v_i, \theta)$ denote the selling strategy for firm i where the firm has expected value $v_i \in \{\underline{v}, \underline{v}+\tau\Delta, \overline{v}\}$ and the investor faces liquidity shock θ. Proposition 13 describes the equilibrium under separate ownership.

Proposition 13 *(Information Acquisition, Separate Ownership): Consider the model with information acquisition and separate ownership. In any equilibrium, the investor investigates if and only if* $c_i \leq c^*_{so,info}$, *where* $c^*_{so,info}$ *is unique and defined by the solution to:*

$$c^* = \beta\left(1-\beta\right)\tau(1-\tau)n\Delta \min\left\{\begin{array}{c} \frac{L/n}{\underline{v}[F(c^*)(1-\beta)(1-\tau)+\beta]+\Delta\beta\tau}, \\ \frac{1}{F(c^*)(1-\beta)(1-\tau)+\beta} \end{array}\right\} \tag{61}$$

The equilibrium can be implemented with selling strategies

$$x_i\left(v_i,\theta\right) = \begin{cases} 0 & \text{if } v_i \in \{\underline{v}+\tau\Delta, \overline{v}\} \text{ and } \theta = 0 \\ \overline{x} \equiv n \times \min\left\{\frac{L/n}{p_i^*(\overline{x})}, 1\right\} & \text{otherwise,} \end{cases}$$

and prices that satisfy

$$p_i^*\left(x_i\right) = \begin{cases} \underline{v} + \Delta\frac{\tau}{F(c^*_{so,info})\tau+1-F(c^*_{so,info})} & \text{if } x_i = 0 \\ \underline{v} + \Delta\frac{\beta\tau}{F(c^*_{so,info})(1-\beta)(1-\tau)+\beta} & \text{if } x_i \in (0,\overline{x}] \\ \underline{v} & \text{if } x_i > \overline{x}. \end{cases}$$

Proof of Proposition 13. Let α^* be the probability that the investor investigates in equilibrium. Since the gross profit from investigation is bounded by $n\Delta < \infty$, in any equilibrium $\alpha^* < 1$. Moreover, since c_i has full support on $(0,\infty)$, in any equilibrium $\alpha^* > 0$. We first argue a set of results regarding the possible equilibrium selling strategies, similar to the proof of Proposition 1.

1. If $x_i' \in x_i(\underline{v}, L) \cup x_i(\underline{v}, 0) \cup x_i(\underline{v}+\tau\Delta, L)$, then $x_i' > 0$. To see this, if $\theta = L$, the investor will sell a positive amount. If $\theta = 0$ and $v_i = \underline{v}$, suppose that $x_i = 0$. Her payoff in this case is $\underline{v}$. Since there exists $x_i'' \in x_i(\underline{v}+\tau\Delta, L)$ with $x_i'' > 0$, it must be that $p_i(x_i'') > \underline{v}$, and so the investor has a profitable deviation, yielding a contradiction.

2. If $x_i' \in x_i(\overline{v}, 0)$, then $x_i' \notin x_i(\underline{v}, L) \cup x_i(\underline{v}, 0) \cup x_i(\underline{v} + \tau\Delta, L)$. To see this, suppose instead that there is an $x_i' \in x_i(\overline{v}, 0)$ and also in $x_i(\underline{v}, L) \cup x_i(\underline{v}, 0) \cup x_i(\underline{v} + \tau\Delta, L)$. Then, from point 1 it must be that $x_i' > 0$. Furthermore, $p_i(x_i') < \underline{v}$. Thus, type-$(\overline{v}, 0)$ can deviate to $x_i'' = 0$ and receive a payoff of $\overline{v}$. This yields a contradiction.

3. There is $\overline{x} > 0$ such that $x_i(\underline{v}, 0) = x_i(\underline{v}, L) = \{\overline{x}\}$. Suppose instead that there exist $x_i' > x_i''$ both in $x_i(\underline{v}, \theta)$. Then, it must be that

$$x_i' p_i(x_i') + (1 - x_i')\underline{v} = x_i'' p_i(x_i'') + (1 - x_i'')\underline{v} \Leftrightarrow$$
$$x_i' p_i(x_i') - x_i'' p_i(x_i'') = (x_i' - x_i'')\underline{v}.$$

Note that both $p_i(x_i') > \underline{v}$ and $p_i(x_i'') > \underline{v}$, otherwise there would be a profitable deviation for type-$\underline{v}$ to $0 < x_i \in x_i(\overline{v}, L)$ where $p_i(x_i) > \underline{v}$. Therefore, both x_i' and x_i'' must be played by one of the types with $v_i \in \{\overline{v}, \underline{v} + \Delta\tau\}$. This implies either that one of the types is mixing between the two strategies, or that one is playing x_i' and another x_i''. The first case requires either

$$x_i' p_i(x_i') - x_i'' p_i(x_i'') = (x_i' - x_i'')(\underline{v} + \Delta\tau)$$

or

$$x_i' p_i(x_i') - x_i'' p_i(x_i'') = (x_i' - x_i'')\overline{v}.$$

However, this contradicts $x_i' p_i(x_i') - x_i'' p_i(x_i'') = (x_i' - x_i'')\underline{v}$. Thus, it requires one of the types to prefer x_i' to x_i'', implying either

$$x_i' p_i(x_i') - x_i'' p_i(x_i'') \geq (x_i' - x_i'')(\underline{v} + \Delta\tau)$$

or

$$x_i' p_i(x_i') - x_i'' p_i(x_i'') \geq (x_i' - x_i'')\overline{v}.$$

However, neither of these can hold when $x_i' p_i(x_i') - x_i'' p_i(x_i'') = (x_i' - x_i'')\underline{v}$, yielding a contradiction. Thus, $x_i(\underline{v}, \theta)$ must be a singleton.

To show $x_i(\underline{v}, 0) = x_i(\underline{v}, L)$, it suffices to show that $x_i' \in x_i(\underline{v}, 0)$ implies either that $x' p_i(x') \geq L/n$ or that $x' p_i(x')$ is the highest revenue that can be obtained in equilibrium. If true this would also imply $\overline{x} > 0$. Suppose not. From point 2, $x_i' \notin x_i(\overline{v}, 0)$. Furthermore, it cannot be that $p_i(x_i') = \underline{v}$, otherwise there would be a profitable deviation. Thus, it must be that $x_i' \in x_i(\underline{v} + \Delta\tau, 0)$. However, then it must be that $p_i(x_i') < \underline{v} + \Delta\tau$, which implies that type-$(\underline{v} + \Delta\tau, 0)$ has a profitable deviation to $x_i = 0$. This yields a contradiction.

4. $x_i(\underline{v} + \Delta\tau, L) = \{\overline{x}\}$. Suppose instead there is $x_i' \in x_i(\underline{v} + \Delta\tau, L)$ such that $x_i' \neq \overline{x}$. Since $x_i' \neq \overline{x}$, based on point 3 we have $p_i(x_i') \geq \underline{v} + \Delta\tau$. Moreover, note that either $x_i' p_i(x_i') \geq L/n$ or x_i' generates the highest revenue that can be obtained in equilibrium. There are two cases:

   (a) Suppose $p_i(\overline{x}) < p_i(x_i')$. If $x_i' \geq \overline{x}$ then type-$(\underline{v}, 0)$ has a profitable deviation to x_i', since she can sell more shares for a higher price. If $x_i' < \overline{x}$ then if type-$(\overline{v}, L)$ plays $\overline{x}$ with positive probability, she has a profitable deviation to x_i' (which satisfies her liquidity needs). If instead type-$(\overline{v}, L)$ plays $\overline{x}$ w.p. zero, then $p_i(\overline{x}) = \underline{v}$, and type-$(\underline{v}, 0)$ has a profitable deviation to x_i', a contradiction.

   (b) Suppose $p_i(\overline{x}) \geq p_i(x_i')$. Then, type-$(\overline{v}, L)$ must play $\overline{x}$ with positive probability. By revealed preference, this means that

$$\overline{x} p_i(\overline{x}) - x_i' p_i(x_i') \geq (\overline{x} - x_i')\, \overline{v}.$$

   Since type $\underline{v}$ also weakly prefers $\overline{x}$ over x_i',

$$\overline{x} p_i(\overline{x}) - x_i' p_i(x_i') \geq (\overline{x} - x_i')\, \underline{v}.$$

   However, type $\underline{v} + \Delta\tau$ weakly prefer x_i' over $\overline{x}$,

$$\overline{x} p_i(\overline{x}) - x_i' p_i(x_i') \leq (\overline{x} - x_i')\, (\underline{v} + \Delta\tau)\,.$$

   The combination of the three conditions implies $\overline{x} - x_i' = 0$, a contradiction.

5. $p_i(\overline{x}) < \underline{v} + \tau\Delta$. Based on points 1-4 and the application of Bayes' rule,

$$p_i(\overline{x}) \leq \max_{\gamma \in [0,1]} \left\{ \underline{v} + \Delta \frac{(1-\beta)(1-\alpha^*)\gamma\tau + \beta\tau}{(1-\beta)\left[\alpha^*(1-\tau) + (1-\alpha^*)\gamma\right] + \beta} \right\}.$$

Indeed, at best, type-$(\overline{v}, L)$ chooses $\overline{x}$ w.p. one, and type-$(\underline{v} + \Delta\tau, 0)$ chooses $\overline{x}$ w.p. γ. Note that since $\tau \in (0, 1)$ and $\alpha^* \in (0, 1)$, for every $\gamma \in [0, 1]$ the RHS is strictly smaller than $\underline{v} + \Delta\tau$.

6. $x_i(\overline{v}, L) = \{\overline{x}\}$. Suppose instead that there is $x_i' \in x_i(\overline{v}, L)$ such that $x_i' \neq \overline{x}$. Based on points 3 and 5, it must be that $p_i(x_i') > \underline{v} + \Delta\tau > p_i(\overline{x})$. Therefore, type-$(\underline{v} + \Delta\tau, L)$ has a profitable deviation to x_i' since it leads to a trading profit and also satisfies her liquidity needs, a contradiction.

7. $\overline{x} \notin x_i(\underline{v} + \Delta\tau, 0)$. Suppose instead that $\overline{x} \in x_i(\underline{v} + \Delta\tau, 0)$ w.p. $\gamma > 0$. Based on point 5, $p_i(\overline{x}) < \underline{v} + \Delta\tau$. Therefore, type-$(\underline{v} + \Delta\tau, 0)$ has strict incentives to choose $x_i = 0$ instead of $\overline{x}$, a contradiction.

8. $x_i(\overline{v}, 0) = \{0\}$. Suppose instead there exists $x_i' \in x_i(\overline{v}, 0)$ with $x_i' > 0$. Then, $p_i(x_i') = \overline{v}$. Therefore, it must be $x_i' \notin x_i(\underline{v} + \Delta\tau, 0)$. Based on points 1-7, if $x_i'' \in x_i(\underline{v} + \Delta\tau, 0)$ then either $x_i'' = 0$ or $p_i(x_i'') = \underline{v} + \Delta\tau$. This implies that the equilibrium payoff of type-$(\underline{v} + \Delta\tau, 0)$ is $\underline{v} + \Delta\tau$. However, in this case, type-$(\underline{v} + \Delta\tau, 0)$ has strict incentives to play x_i' and receive a payoff strictly higher than $\underline{v} + \Delta\tau$, a contradiction.

These points show that in any equilibrium, we have $x_i(\overline{v}, L) \cup x_i(\underline{v} + \Delta\tau, L) \cup x_i(\underline{v}, L) \cup x_i(\underline{v}, 0) = \{\overline{x}\}$, $x_i(\overline{v}, 0) = \{0\}$, and $\overline{x} \notin x_i(\underline{v} + \Delta\tau, 0)$. Now, given this, without loss of generality (for investigation incentives) we consider the case where $x_i(\underline{v} + \Delta\tau, 0) = \{0\}$. Note that the selling strategies in the Proposition satisfy the conditions of the previous part of the proof. Given these selling strategies, the prices satisfy Bayes' rule. The Grossman and Perry (1986) refinement leads to $\overline{x} = n \times \min\left\{\frac{L/n}{p_i^*(\overline{x})}, 1\right\}$.

The investor's expected payoff from investigating is

$$\tau[(1-\beta)n\overline{v} + \beta(\overline{x}p_i^*(\overline{x}) + (n - \overline{x})\overline{v})] + (1-\tau)[\overline{x}p_i^*(\overline{x}) + (n - \overline{x})\underline{v}] - c_i,$$

If instead she does not investigate, her expected payoff is

$$(1-\beta)n(\underline{v} + \Delta\tau) + \beta[\overline{x}p_i^*(\overline{x}) + (n - \overline{x})(\underline{v} + \Delta\tau)].$$

Thus, the investor investigates if and only if

$$c_i \leq (1-\beta)(1-\tau)\overline{x}\left(p_i^*(\overline{x}) - \underline{v}\right).$$

Therefore, in equilibrium there exists c^* such that the investor investigates if and only $c_i \leq c^*$. This implies $\alpha^* = F(c^*)$, and therefore c^* must solve

$$c^* = \beta(1-\beta)\tau(1-\tau)n\Delta \min\left\{\frac{L/n}{\underline{v}\left[F(c^*)(1-\beta)(1-\tau)+\beta\right]+\Delta\beta\tau}, \frac{1}{F(c^*)(1-\beta)(1-\tau)+\beta}\right\}$$

Note that the RHS is decreasing in c^*, and so if a solution exists, it is unique. Furthermore, at $c^* = 0$, the RHS is greater and as $c^* \to \infty$, the LHS is greater. Thus, there exists a unique c^* satisfying this equation. ■

The intuition behind the investigation threshold, (61), is as follows. Up to a point, the greater the investor's number of securities n, the greater the incentives to investigate. This is because, when her stake n is small, a liquidity shock forces her to sell it in its entirety. Thus, if she learns that the firm is bad, she also sells her entire stake, because doing so disguises her sale as being motivated by a shock. A higher n allows her to sell more securities if she learns that the firm is bad, increasing her trading profits, and thus her incentives to gather information. However, after we cross the point at which

$$\frac{L/n}{\underline{v}\left[F(c^*)(1-\beta)(1-\tau)+\beta\right]+\Delta\beta\tau} = \frac{1}{F(c^*)(1-\beta)(1-\tau)+\beta},$$

the investigation threshold is now defined by

$$c^* = \frac{\beta(1-\beta)\tau(1-\tau)\Delta L}{\underline{v}\left[F(c^*)(1-\beta)(1-\tau)+\beta\right]+\Delta\beta\tau}$$

and is independent of n. The investor's stake n is now sufficiently large, compared to the liquidity shock L, that she is no longer forced to sell it in its entirety. As a result, she can only partially sell her stake upon learning that the firm is bad, otherwise she is fully revealed. Further increases in stake size do not increase her investigation incentives, since they do not lead to a higher sale volume upon acquiring negative information. This result is exactly the same as the separate ownership model of Edmans (2009). Thus, the conventional wisdom that information acquisition incentives are increasing in stake size is only true up to a point.

Next, we consider the model under common ownership.

Proposition 14 *(Information Acquisition, Common Ownership): Consider the model with information acquisition and common ownership where* $L/n \leq F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$*. There is an equilibrium in which the investor investigates if and only if* $c_i \leq c^*_{co,info} = F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right)$*. The equilibrium can be implemented with selling strategies*

$$x_i(v_i, \theta) = \begin{cases} 0 & \text{if } v_i \in \{\underline{v} + \tau\Delta, \overline{v}\} \\ 1 & \text{if } v_i = \underline{v}, \end{cases}$$

and prices that satisfy

$$p^*_i(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{F(c^*_{co,info})\tau + 1 - F(c^*_{co,info})} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases}$$

Proof of Proposition 14. Similar to separate ownership, in equilibrium there is c^* such that the investor investigates if and only if $c_i \leq c^*$. Suppose $L/n \leq F(c^*)(1-\tau)\underline{v}$ (which implies $L/n < (1-\tau)\underline{v}$), then the investor can satisfy the shock from selling only bad firms. In this case, the proposed selling strategies are consistent with such an equilibrium given prices, and prices are consistent with the proposed strategies by Bayes' rule.

Suppose the market makers believe that $L/n \leq F(c^*)(1-\tau)\underline{v}$ but the investor chooses c, where we allow for $c \neq c^*$.

1. If $L/n \leq F(c)(1-\tau)\underline{v}$, she chooses the same strategies as under the equilibrium c^*, i.e. sells bad firms and retains all others.

2. If $L/n > F(c)(1-\tau)\underline{v}$ and $\theta = 0$, the investor strictly prefers to retain all firms not identified as bad, and is indifferent between selling and retaining bad firms. If she suffers a shock, she sells all bad firms, $\min\left\{\frac{L/n - F(c)(1-\tau)\underline{v}}{(1-F(c))\underline{v}}, 1\right\}$ from all uninvestigated firms, and $\min\left\{\max\left\{\frac{L/n-(1-F(c)\tau)\underline{v}}{F(c)\tau\underline{v}}, 0\right\}, 1\right\}$ from all good firms. Since $L/n < (1-\tau)\underline{v} \Rightarrow L/n < (1-F(c)\tau)\underline{v}$, the last term is zero, i.e. the investor never has to sell good firms.

Let $\Pi(c^*, c)$ denote the profits from choosing c when the market maker believes the investor follows threshold c^*. Then,

$$\begin{aligned}\frac{1}{n}\Pi(c^*,c) &= -F(c)\mathbb{E}[c_i|c_i<c]+(1-\beta)\left(\underline{v}+\Delta\tau\right)\\ &\quad+\beta\left[\begin{array}{c} F(c)\left(1-\tau\right)\underline{v}+F(c)\tau\overline{v}+ \\ (1-F(c))\left(\begin{array}{c}\min\left\{\frac{L/n-F(c)(1-\tau)\underline{v}}{(1-F(c))\underline{v}},1\right\}\underline{v}\\ +\left(1-\min\left\{\frac{L/n-F(c)(1-\tau)\underline{v}}{(1-F(c))\underline{v}},1\right\}\right)\left(\underline{v}+\Delta\tau\right)\end{array}\right)\end{array}\right].\end{aligned}$$

Thus,

$$\frac{\partial\Pi(c^*,c)}{\partial c}\frac{1}{n}\frac{1}{f(c)}=\begin{cases}-c+\beta(1-\tau)\tau\Delta & \text{if } c\leq F^{-1}\left(\frac{L/n}{\underline{v}(1-\tau)}\right)\\ -c & \text{otherwise}\end{cases}$$

and the first-order condition with respect to c implies

$$c^*=\begin{cases}\beta(1-\tau)\tau\Delta & \text{if } \underline{v}(1-\tau)F\left(\beta(1-\tau)\tau\Delta\right)<L/n\\ F^{-1}\left(\frac{L/n}{\underline{v}(1-\tau)}\right) & \text{otherwise.}\end{cases}$$

However, if $\underline{v}(1-\tau)F\left(\beta(1-\tau)\tau\Delta\right)<L/n$, the equilibrium strategy of selling only bad firms upon a shock would not raise sufficient revenue, yielding a contradiction. Thus, for equilibria in which the investor only sells bad firms, it must be that $L/n\leq F(\beta(1-\tau)\tau\Delta)(1-\tau)\underline{v}$ and $c^*=F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right)$. ■

Intuitively, when the investor investigates a certain measure of firms, by the law of large numbers she knows how many will be bad. Thus, she chooses to investigate just enough firms to reveal just enough bad firms that she can exactly satisfy any liquidity shock by selling them all. She has no incentive to investigate fewer firms, since if she suffers a liquidity shock, she will have to sell some uninvestigated (and thus some good) firms. She has no incentive to investigate more firms. Doing so will uncover additional bad firms, but she does not need to sell these firms as she is already satisfying her liquidity shock, and earns no trading profit by voluntarily selling these firms since selling leads to a price of $\underline{v}$.

This intuition explains the investor's investigation threshold, $c^*_{co,info}=F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right)$, which is increasing in the per-security liquidity shock L/n. When the liquidity shock is large relative to the number of firms, the investor needs to sell a larger stake in each firm to satisfy a shock. This increases the losses from selling uninvestigated firms that are actually good, and thus the incentives to investigate firm value. Interestingly, this contrasts the threshold in the voice model, which is decreasing in L/n.

While Proposition 14 gives conditions under which there exists an equilibrium under which investigation is higher under common ownership, Proposition 15 shows, in *any* equilibrium under common ownership, investigation is higher than in *any* equilibrium under separate ownership, if β is sufficiently large and L is sufficiently small.

Proposition 15 *(Information Acquisition, Comparison of Ownership Structures): There is* $\overline{\beta} < 1$ *such that if* $L/n \leq F(\overline{\beta}\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ *and* $\beta > \overline{\beta}$ *then the investor acquires strictly more information in any equilibrium under common ownership than in any equilibrium under separate ownership.*

Proof of Proposition 15. Based on Proposition 14, if $L/n \leq F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ then there always exists an equilibrium under common ownership in which the investor can satisfy her liquidity needs by selling only bad firms. In this equilibrium, the cutoff is $F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right) > 0$. Based on Proposition 13, in any equilibrium under separate ownership, the cutoff is unique and given by $c^*_{so,info}(\beta)$, where $\lim_{\beta \to 1} c^*_{so,info}(\beta) = 0$. Note that $F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ is increasing in β. Therefore, there exists $\overline{\beta} < 1$ such that if $L/n \leq F(\overline{\beta}\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ and $\beta > \overline{\beta}$ then $F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right) > c^*_{so,info}(\beta)$, as required.

Next, suppose $\beta > \overline{\beta}$ and $L/n \leq F(\overline{\beta}\tau(1-\tau)\Delta)(1-\tau)\underline{v}$, and consider a candidate equilibrium under common ownership in which the investor must sell some uninvestigated firms to satisfy her shock, i.e.

$$F(c^*)(1-\tau)\underline{v} < L/n.$$

Note that

$$L/n \leq F(\overline{\beta}\tau(1-\tau)\Delta)(1-\tau)\underline{v} < (1-\tau)\underline{v} = \min_{c \geq 0}\{(1-\tau F(c))\underline{v}\}.$$

Therefore, if $L/n \leq F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ then good firms are never sold in equilibrium. Hereafter, we maintain this assumption, which implies that in equilibrium there are only two types of firms the investor potentially sells: bad and uninvestigated firms. Let $\hat{n}(c^*)$ denote the mass of firms not identified as a good, a fraction $\hat{\tau}(c^*)$ of which are valued at $\widehat{\overline{v}}$, and a fraction $1-\hat{\tau}(c^*)$ of which are valued at $\underline{v}$, where

$$\begin{aligned}
\hat{n}(c^*) &= n(1-\tau F(c^*)) \\
\hat{\tau}(c^*) &= \frac{1-F(c^*)}{F(c^*)(1-\tau)+1-F(c^*)} = \frac{1-F(c^*)}{1-\tau F(c^*)} \\
\widehat{\overline{v}} &= \underline{v}+\hat{\Delta} \\
\hat{\Delta} &= \tau\Delta.
\end{aligned}$$

Note that

$$\begin{aligned}
F(c^*)(1-\tau)\underline{v} &< L/n < (1-\tau F(c^*))\underline{v} \Leftrightarrow \\
(1-\hat{\tau}(c^*))\underline{v} &< L/\hat{n}(c^*) < \underline{v}
\end{aligned}$$

We therefore apply parts (ii) and (iii) of Proposition 2, where τ is replaced by $\hat{\tau}(c^*)$, $\overline{v}$ by $\widehat{\overline{v}}$, Δ by $\hat{\Delta}$, and n by $\hat{n}(c^*)$. The only exception is $p_i(0)$, which is different (since firms that are identified as good are fully retained) but has no effect on the decision to acquire information. Therefore, there are only two possible equilibria, type (ii) and type (iii), as described in Proposition 2.

We proceed by showing that there are $\varepsilon>0$ and $\hat{\beta}<1$ such that if $\beta>\hat{\beta}$ and an equilibrium of type (ii) or type (iii) exists, then in this equilibrium $c^*(\beta)>\varepsilon$. Since $\lim_{\beta\to1} c^*_{so,info}(\beta)=0$, if this claim holds then the proof is completed.

Consider an equilibrium of type $\sigma\in\{ii,iii\}$, and suppose by the way of contradiction that the statement above fails. Then, there exists a sequence of $\{\beta_k\}$ with $\lim_{k\to\infty}\beta_k=1$ and corresponding equilibria cutoffs $\{c^*_\sigma(\beta_k)\}$ such that $\lim_{k\to\infty} c^*_\sigma(\beta_k)=0$. Therefore, $\lim_{k\to\infty}\hat{\tau}(c^*_\sigma(\beta_k))=1$ and $\lim_{k\to\infty}\hat{n}(c^*_\sigma(\beta_k))=n$. Moreover, let

$$\begin{array}{cc}
\overline{p}^*_{iii}(\beta)=\overline{p}_{so}(\hat{\tau}(c^*(\beta))) & \overline{p}^*_{ii}(\beta)=\overline{p}_{co}(\hat{\tau}(c^*(\beta))) \\
\overline{x}^*_{iii}(\beta)=\frac{\overline{x}_{so}(\hat{\tau}(c^*(\beta)))}{\hat{n}(c^*(\beta))} & \overline{x}^*_{ii}(\beta)=\overline{x}_{co}(\hat{\tau}(c^*(\beta)))
\end{array},$$

and notice that for $\sigma\in\{ii,iii\}$

$$\begin{aligned}
\lim_{k\to\infty}\overline{p}^*_\sigma(\beta_k) &= \underline{v}+\tau\Delta \\
\lim_{k\to\infty}\overline{x}^*_\sigma(\beta_k) &= \frac{L/n}{\underline{v}+\tau\Delta}.
\end{aligned}$$

Also note that since we require $L/n < \underline{v}$, the investor always meets her liquidity needs in equilibrium. Fix β, and to ease the notation, we hereafter omit the argument β from the endogenous variables. To further simplify the notation, we also omit the subscripts σ whenever there is no risk of confusion.

Consider a deviation of the investor to a threshold $c > c^*$. Moreover, suppose that under this deviation the investor fully retains $n\tau F(c)$ firms, fully sells $nh(c)$ firms, and partly sells $n(1 - \tau F(c) - h(c))$ firms, where $h(c)$ is determined by

$$\begin{aligned} (1 - \tau F(c) - h)\, \overline{x}^* \overline{p}^* + h\underline{v} &= L/n \Leftrightarrow \\ h(c) &= \frac{L/n - (1 - \tau F(c))\, \overline{x}^* \overline{p}^*}{\underline{v} - \overline{x}^* \overline{p}^*}. \end{aligned}$$

We argue $h(c) \in (0, F(c)(1 - \tau))$. There are two cases:

1. Consider an equilibrium of type (ii). Recall $L/n < \underline{v}(1 - \tau)$ and that in this type of equilibrium
$$\overline{x}^* \overline{p}^* = \frac{L/n - \underline{v}(1 - \tau) F(c^*)}{1 - F(c^*)}$$
Therefore,
$$\overline{x}^* \overline{p}^* < L/n < (1 - \tau)\, \underline{v} < \underline{v}.$$
Since $(1 - \tau F(c))\, \overline{x}^* \overline{p}^* < L/n$, we have $h(c) > 0$. Also recall that in this type of equilibrium
$$L/n = (1 - F(c^*))\, \overline{x}^* \overline{p}^* + F(c^*)(1 - \tau)\, \underline{v}.$$
Therefore,
$$\begin{aligned} F(c^*) &< F(c) \Rightarrow \\ L/n &< (1 - F(c))\, \overline{x}^* \overline{p}^* + F(c)(1 - \tau)\, \underline{v} \Rightarrow \\ h(c) &< F(c)(1 - \tau), \end{aligned}$$
as required.

2. Consider an equilibrium of type (iii). Recall $L/n < \underline{v}(1 - \tau)$ and that in this type of equilibrium, $\overline{x}^* \overline{p}^* = L/n$. Since $(1 - \tau F(c))\, \overline{x}^* \overline{p}^* < L/n$, we have $h(c) > 0$. Since

$\overline{x}^*\overline{p}^* = L/n < \underline{v}(1-\tau)$ we have

$$L/n = \overline{x}^*\overline{p}^* < (1-F(c))\,\overline{x}^*\overline{p}^* + F(c)(1-\tau)\,\underline{v},$$

and therefore,

$$h(c) < F(c)(1-\tau),$$

as required.

Since $h(c) < F(c)(1-\tau)$, the investor fully retains all $F(c)\,\tau$ good firms, partly sells all $1-F(c)$ uninvestigated firms, fully sells $nh(c)$ bad firms, and the rest of $n(F(c)(1-\tau)-h(c))$ bad firms are partly sold. The profit from this deviation is

$$\begin{aligned}\Pi(c,\beta) &= F(c)\,\tau(\underline{v}+\Delta) + h(c)\,\underline{v} + (F(c)(1-\tau)-h(c))\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right]\\ &\quad + (1-F(c))\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)(\underline{v}+\tau\Delta)\right] - F(c)\,E\left[c_i|c_i<c\right]\end{aligned}$$

where the argument β emphasizes that $\overline{x}^*$ and $\overline{p}^*$ (and also indirectly $h(c)$) depend on β. Therefore,

$$\begin{aligned}\frac{\partial\Pi(c,\beta)}{\partial c}\frac{1}{f(c)} &= \tau\overline{v} + \frac{\tau\overline{x}^*\overline{p}^*}{\underline{v}-\overline{x}^*\overline{p}^*}\underline{v} + \left((1-\tau) - \frac{\tau\overline{x}^*\overline{p}^*}{\underline{v}-\overline{x}^*\overline{p}^*}\right)\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right]\\ &\quad - \left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)(\underline{v}+\tau\Delta)\right] - c\\ &= \tau(\underline{v}+\Delta) - (1-\overline{x}^*)\,\tau\Delta - \frac{\underline{v}\tau}{\underline{v}-\overline{x}^*\overline{p}^*}(1-\overline{x}^*)\,\underline{v} - c\\ &= \tau\overline{x}^*\left(\Delta - \underline{v}\frac{\overline{p}^*-\underline{v}}{\underline{v}-\overline{x}^*\overline{p}^*}\right) - c.\end{aligned}$$

Recall that $\lim_{k\to\infty}\overline{p}^*(\beta_k) = \underline{v}+\tau\Delta$ and $\lim_{k\to\infty}\overline{x}^*(\beta_k) = \frac{L/n}{\underline{v}+\tau\Delta}$. Therefore,

$$\begin{aligned}\lim_{k\to\infty}\frac{\partial\Pi(c,\beta_k)}{\partial c}\frac{1}{f(c)} &= \tau\frac{L/n}{\underline{v}+\tau\Delta}\left(\Delta - \underline{v}\left(\frac{\tau\Delta}{\underline{v}-L/n}\right)\right) - c\\ &= \Delta\tau\frac{L/n}{\underline{v}+\tau\Delta}\,\frac{\underline{v}(1-\tau)-L/n}{\underline{v}-L/n} - c.\end{aligned}$$

Since $L/n < \underline{v}(1-\tau)$,

$$\lim_{k\to\infty}\frac{\partial\Pi(c,\beta_k)}{\partial c}\frac{1}{f(c)}|_{c=0} > 0.$$

Since $\lim_{k\to\infty} c^*(\beta_k) = 0$, for k sufficiently large (that is β sufficiently close to one), the deviation above is a profitable, which creates a contradiction as required. ■

Proposition 16 shows that price informativeness can be higher under *any* common ownership equilibrium than under *any* separate ownership equilibrium, even when information acquisition is endogenous. Moreover, even if information acquisition is lower under common ownership, price informativeness may be higher due to the key force of the core model – common ownership increases adverse selection.

Proposition 16 *(Price informativeness Under Information Acquisition):*

(i) *There is $\beta^* < 1$ such that if $\beta > \beta^*$ and $L/n \leq F(\beta^*\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ then for any equilibrium under common ownership and any equilibrium under separate ownership*

$$P_{co,info}(\overline{v},\tau) > P_{so,info}(\overline{v},\tau) \text{ and } P_{co,info}(\underline{v},\tau) \leq P_{so,info}(\underline{v},\tau). \tag{62}$$

Moreover, there is an equilibrium under common ownership such that the above inequalities are strict with respect to any equilibrium under separate ownership.

(ii) *There is $\beta^{**} < 1$ and $n^{**} < \infty$ such that if $\beta > \beta^{**}$, $n > n^{**}$, and $L/n \leq F(\beta^*\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ then there is an equilibrium under common ownership and an equilibrium under separate ownership such that the investor acquires strictly more information under separate ownership yet the inequalities in (62) hold strictly.*

Proof of Proposition 16. In the proof of Proposition 13 there is an implicit assumption that $x_i(\underline{v}+\Delta\tau, 0) = \{0\}$.[30] To maximize price informativeness under separate ownership (thus, to bias the proof against common ownership having larger price informativeness), we suppose $x_i(\underline{v}+\Delta\tau, 0) = \{\varepsilon\}$, where $0 \leq \varepsilon < \overline{x}$ is arbitrarily small such that no deviation for other types

[30] This assumption has no effect on the analysis of $c^*_{so,info}$, because regardless of whether she retains uninvestigated firms, she obtains their expected value, $\underline{v} + \Delta\tau$.

is profitable and

$$x_i(v_i,\theta) = \begin{cases} 0 & \text{if } v_i = \underline{v} \text{ and } \theta = 0 \\ \varepsilon & \text{if } v_i = \underline{v} + \tau\Delta \text{ and } \theta = 0 \\ \overline{x}_{so} \equiv n \times \min\left\{\frac{L/n}{\overline{p}_{so}}, 1\right\} & \text{otherwise,} \end{cases}$$

$$p^*_{so,info}(x_i) = \begin{cases} \overline{v} & \text{if } x_i = 0 \\ \underline{v} + \tau\Delta & \text{if } x_i \in (0,\varepsilon] \\ \overline{p}_{so} = \underline{v} + \Delta\frac{\beta\tau}{F(c^*_{so,info})(1-\beta)(1-\tau)+\beta} & \text{if } x_i \in (\varepsilon, \overline{x}_{so}] \\ \underline{v} & \text{if } x_i > \overline{x}_{so}. \end{cases}$$

Under this formulation,

$$P_{so,info}(v_i,\tau) = \begin{cases} \beta\overline{p}_{so} + (1-\beta)\left[F(c^*_{so,info})\overline{p}_{so} + \left(1-F(c^*_{so,info})\right)(\underline{v}+\tau\Delta)\right] & \text{if } v_i = \underline{v} \\ \beta\overline{p}_{so} + (1-\beta)\left(F(c^*_{so,info})\overline{v} + \left(1-F(c^*_{so,info})\right)(\underline{v}+\tau\Delta)\right) & \text{if } v_i = \overline{v}. \end{cases}$$

Consider common ownership. There are three cases to consider, one for each type of equilibrium that can emerge under common ownership when $L/n \le F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$.

1. Consider an equilibrium in which the investor can satisfy her liquidity needs by selling only bad firms. In the proof of Proposition 14 there is an implicit assumption that bad firms are always fully sold, even if the investor does not suffer a shock and is indifferent between selling and not selling.[31] To minimize price informativeness under common ownership (thus, to bias the proof against common ownership having larger price informativeness), we suppose that the investor fully retains bad firms if there is no shock. Under this assumption, $x_i(\underline{v},0) = 0$, and similar to Proposition 14, if $L/n \le F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ then prices satisfy

$$p^*_{co,info}(x_i) = \begin{cases} \underline{v} + \Delta\frac{\tau}{F(c^*_{co,info})(\tau+(1-\tau)(1-\beta))+1-F(c^*_{co,info})} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases}$$

[31] This assumption has no effect on the analysis of $c^*_{so,info}$, because regardless of whether she fully sells bad firms, she obtains their fundamental value, $\underline{v}$.

Therefore,

$$P_{co,info}\left(v_i,\tau\right)=\begin{cases}F(c^*_{co,info})\left(\beta\underline{v}+\left(1-\beta\right)p^*_{co,info}\left(0\right)\right)+\left(1-F(c^*_{co,info})\right)p^*_{co,info}\left(0\right) & if\ \ v_i=\underline{v}\\ p^*_{co,info}\left(0\right) & if\ \ v_i=\overline{v}.\end{cases}$$

If $\beta=1$ then

$$\begin{aligned}P_{co,info}\left(\overline{v},\tau\right) &> P_{so,info}\left(\overline{v},\tau\right)\Leftrightarrow\\ p^*_{co,info}\left(0\right) &> \overline{p}_{so}\Leftrightarrow\\ \underline{v}+\Delta\frac{\tau}{F(c^*_{co,info})\tau+1-F(c^*_{co,info})} &> \underline{v}+\Delta\tau\end{aligned}$$

and

$$\begin{aligned}P_{co,info}\left(\underline{v},\tau\right) &< P_{so,info}\left(\underline{v},\tau\right)\Leftrightarrow\\ F(c^*_{co,info})\underline{v}+\left(1-F(c^*_{co,info})\right)p^*_{co,info}\left(0\right) &< \overline{p}_{so}\Leftrightarrow\\ \underline{v}+\Delta\tau\frac{1-F(c^*_{co,info})}{F(c^*_{co,info})\tau+1-F(c^*_{co,info})} &< \underline{v}+\Delta\tau\end{aligned}$$

which always holds.

2. Consider a type-(ii) equilibrium, if it exists. Recall that in this equilibrium

$$x^*_{co}\left(v_i,\theta\right)=\begin{cases}0 & if\ \ v_i=\overline{v},\ or\ v_i=\underline{v}+\tau\Delta\ and\ \theta=0\\ \overline{x}_{co}=\frac{\underline{v}+\frac{L/\hat{n}\left(c^*_{ii}\right)-\underline{v}}{\hat{\tau}\left(c^*_{ii}\right)}}{\overline{p}_{co}}=\\ \frac{L/n-\underline{v}(1-\tau)F\left(c^*_{ii}\right)}{1-F\left(c^*_{ii}\right)}\frac{1}{\overline{p}_{co}}<1 & if\ \ v_i=\underline{v}\ and\ \theta=0,\ or\ v_i=\underline{v}+\tau\Delta\ and\ \theta=L\\ 1 & if\ \ v_i=\underline{v}\ and\ \theta=L,\end{cases} \tag{63}$$

and prices of firm i are:

$$p^*_{co,info}(x_i) = \begin{cases} \underline{v} + \Delta \frac{F\left(c^*_{ii}\right)\tau + (1-\beta)\left(1-F\left(c^*_{ii}\right)\right)\tau}{F\left(c^*_{ii}\right)\tau + (1-\beta)\left(1-F\left(c^*_{ii}\right)\right)} & \text{if } x_i = 0 \\ \overline{p}_{co} = \underline{v} + \Delta \frac{\beta\left(1-F\left(c^*_{ii}\right)\right)\tau}{\beta\left(1-F\left(c^*_{ii}\right)\right) + (1-\beta)F\left(c^*_{ii}\right)(1-\tau)} & \text{if } x_i \in (0, \overline{x}_{co}], \\ \underline{v} & \text{if } x_i > \overline{x}_{co}. \end{cases} \tag{64}$$

Therefore,

$$P_{co,info}(v_i, \tau) = \begin{cases} \begin{array}{l} \beta\left[(1-F(c^*_{ii}))\,\overline{p}_{co} + F(c^*_{ii})\,\underline{v}\right] + \\ (1-\beta)\left[(1-F(c^*_{ii}))\,p^*_{co,info}(0) + F(c^*_{ii})\,\overline{p}_{co}\right] \end{array} & \text{if } v_i = \underline{v} \\ \begin{array}{l} \beta\left[(1-F(c^*_{ii}))\,\overline{p}_{co} + F(c^*_{ii})\,p^*_{co,info}(0)\right] \\ + (1-\beta)\,p^*_{co,info}(0) \end{array} & \text{if } v_i = \overline{v}. \end{cases}$$

If $\beta = 1$ then

$$\begin{aligned} P_{co,info}(\overline{v}, \tau) &> P_{so,info}(\overline{v}, \tau) \Leftrightarrow \\ (1-F(c^*_{ii}))\,\overline{p}_{co} + F(c^*_{ii})\,p^*_{co,info}(0) &> \overline{p}_{so} \Leftrightarrow \\ (1-F(c^*_{ii}))(\underline{v} + \Delta\tau) + F(c^*_{ii})(\underline{v} + \Delta) &> \underline{v} + \Delta\tau \end{aligned}$$

and

$$\begin{aligned} P_{co,info}(\underline{v}, \tau) &< P_{so,info}(\underline{v}, \tau) \Leftrightarrow \\ (1-F(c^*_{ii}))\,\overline{p}_{co} + F(c^*_{ii})\,\underline{v} &< \overline{p}_{so} \Leftrightarrow \\ (1-F(c^*_{ii}))(\underline{v} + \Delta\tau) + F(c^*_{ii})\,\underline{v} &< \underline{v} + \Delta\tau \end{aligned}$$

which always holds.

3. Consider a type-(iii) equilibrium, if it exists. Focusing on the selling strategy that minimizes price informativeness, this equilibrium involves

$$x^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \overline{v}, \text{ or } v_i = \underline{v} + \tau\Delta \text{ and } \theta = 0 \\ \overline{x}_{so} = \frac{L/\hat{n}\left(c^*_{iii}\right)}{\overline{p}_{co}} & \text{otherwise,} \end{cases} \tag{65}$$

and prices

$$p^*_{co,info}(x_i) = \begin{cases} \underline{v} + \Delta \frac{F(c^*_{iii})\tau + (1-\beta)\left(1-F(c^*_{iii})\right)\tau}{F(c^*_{iii})\tau + (1-\beta)\left(1-F(c^*_{iii})\right)} & if \;\; x_i = 0 \\ \overline{p}_{co} = \underline{v} + \Delta \frac{\beta\tau}{F(c^*_{iii})(1-\beta)(1-\tau)+\beta} & if \;\; x_i \in (0, \overline{x}_{so}], \\ \underline{v} & if \;\; x_i > \overline{x}_{so}(\tau). \end{cases} \tag{66}$$

Therefore,

$$P_{co,info}(v_i, \tau) = \begin{cases} \beta\overline{p}_{co} + (1-\beta)\left[F(c^*_{iii})\,\overline{p}_{co} + (1-F(c^*_{iii}))\,p^*_{co,info}(0)\right] & if \;\; v_i = \underline{v} \\ \beta\left[(1-F(c^*_{iii}))\,\overline{p}_{co} + F(c^*_{iii})\,p^*_{co,info}(0)\right] + (1-\beta)\,p^*_{co,info}(0) & if \;\; v_i = \overline{v}. \end{cases}$$

If $\beta = 1$ then

$$\begin{aligned} P_{co,info}(\overline{v}, \tau) &> P_{so,info}(\overline{v}, \tau) \Leftrightarrow \\ (1-F(c^*_{ii}))\,\overline{p}_{co} + F(c^*_{ii})\,p^*_{co,info}(0) &> \overline{p}_{so} \Leftrightarrow \\ (1-F(c^*_{ii}))\,(\underline{v} + \Delta\tau) + F(c^*_{ii})\,(\underline{v} + \Delta) &> \underline{v} + \Delta\tau \end{aligned}$$

and

$$P_{co,info}(\underline{v}, \tau) = P_{so,info}(\underline{v}, \tau) = \underline{v} + \tau\Delta.$$

This concludes part (i).

To show part (ii), consider the equilibrium under common ownership described in Case 1 in the proof of part (i) of this proposition, but now assume $x_i(\underline{v}, 0) = 1$, then

$$p^*_{co,info}(0) = \underline{v} + \Delta \frac{\tau}{F(c^*_{co,info})\tau + 1 - F(c^*_{co,info})}$$

and

$$P_{co,info}(v_i, \tau) = \begin{cases} F(c^*_{co,info})\underline{v} + \left(1 - F(c^*_{co,info})\right) p^*_{co,info}(0) & if \;\; v_i = \underline{v} \\ p^*_{co,info}(0) & if \;\; v_i = \overline{v}. \end{cases}$$

Moreover, consider the equilibrium under separate ownership with $\varepsilon = 0$. In this case,

$$x_i(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \underline{v} \text{ and } \theta = 0 \text{ or, } v_i = \underline{v} + \tau\Delta \text{ and } \theta = 0 \\ \overline{x}_{so} \equiv n \times \min\left\{\frac{L/n}{\overline{p}_{so}}, 1\right\} & \text{otherwise,} \end{cases}$$

$$p^*_{so,info}(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{F(c^*_{so,info})\tau + \left(1 - F(c^*_{so,info})\right)} & \text{if } x_i = 0 \\ \overline{p}_{so} = \underline{v} + \Delta \frac{\beta\tau}{F(c^*_{so,info})(1-\beta)(1-\tau)+\beta} & \text{if } x_i \in (0, \overline{x}_{so}] \\ \underline{v} & \text{if } x_i > \overline{x}_{so}. \end{cases}$$

and

$$P_{so,info}(v_i, \tau) = \begin{cases} \beta\overline{p}_{so} + (1-\beta)\left[F(c^*_{so,info})\overline{p}_{so} + \left(1 - F(c^*_{so,info})\right) p_{so,info}(0)\right] & \textit{if } v_i = \underline{v} \\ \beta\overline{p}_{so} + (1-\beta)\, p_{so,info}(0) & \textit{if } v_i = \overline{v}. \end{cases}$$

This implies that if $c^*_{co,info} = c^*_{so,info} = c^*$ then

$$P_{co,info}(\overline{v}, \tau) > P_{so,info}(\overline{v}, \tau) \Leftrightarrow p^*_{co,info}(0) > p^*_{so,info}(0)$$

which always holds, and

$$\begin{aligned} P_{co,info}(\underline{v}, \tau) &< P_{so,info}(\underline{v}, \tau) \Leftrightarrow \\ F(c^*)\underline{v} + (1 - F(c^*))\, p^*_{co,info}(0) &< \beta\overline{p}_{so} + (1-\beta)\left[F(c^*)\overline{p}_{so} + (1 - F(c^*))\, p_{so,info}(0)\right] \\ F(c^*)\underline{v} + (1 - F(c^*))\, p^*_{co,info}(0) &< F(c^*)\overline{p}_{so} + (1 - F(c^*))\left[(1-\beta)\, p_{so,info}(0) + \beta\overline{p}_{so}\right] \\ \beta(1 - F(c^*))\left(p_{so,info}(0) - \overline{p}_{so}\right) &< F(c^*)\left(\overline{p}_{so} - \underline{v}\right) \\ \frac{1 - F(c^*)}{F(c^*)\tau + (1 - F(c^*))} &< \frac{\beta(1 - F(c^*)) + F(c^*)}{F(c^*)(1-\beta)(1-\tau) + \beta} \\ (1 - F(c^*))\,\tau &> -F(c^*)\tau \end{aligned}$$

which always holds. Thus price informativeness is higher under common ownership, if the investigation cutoff is the same. Suppose n is sufficiently large and $\beta < 1$ is sufficiently close to one such that $L/n \leq F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v}$ and $\varepsilon > c^*_{so,info} - F^{-1}\left(\frac{L/n}{(1-\tau)\underline{v}}\right) > 0$ (note that $c^*_{so,info}$ is invariant to n if $L/n \leq F(\beta\tau(1-\tau)\Delta)(1-\tau)\underline{v})$ for some arbitrarily small $\varepsilon > 0$. Then, the above equilibrium under common ownership has a strictly lower cutoff but strictly higher price informativeness than the above equilibrium under separate ownership. ■

C.2 Single Market Maker

This section considers the case of a single market maker, who observes the order flows of all firms when setting prices. For brevity, we consider the small shock case, since this is where our results are strongest.

Since shocks are i.i.d. under separate ownership, x_j contains no information relevant for the pricing of firm $i \neq j$. Therefore, the analysis of separate ownership does not change. Below we show that our main results continue to hold for both voice and exit, i.e. governance is strictly superior under common ownership than separate ownership when L is small.

Proposition 17 *(Single market maker):*

(i) *Consider the voice model and suppose $L/n \leq \underline{v}\,(1 - F(\Delta))$. In any equilibrium under common ownership, per-security monitoring incentives are strictly higher than under separate ownership.*

(ii) *Consider the exit model and suppose $L/n < \underline{v} + \Delta F(\overline{R} - \underline{R} + \Delta\omega)$. There is an equilibrium under common ownership in which the working threshold is strictly greater than under separate ownership.*

Proof of Proposition 17. Consider part (i). We show that, in any equilibrium under common ownership, the monitoring threshold is weakly greater than Δ. If true, noting that $\frac{c^*_{so,voice}}{n} < \Delta$ completes the proof. We proceed in two steps.

1. We show that there exists an equilibrium with cutoff $c^* = \Delta$. In this equilibrium, the investor's trading strategy is given by:

$$x_i^*\left(v_i, \theta\right) = \begin{cases} 1 & if\ v_i = \underline{v} \\ 0 & if\ v_i = \overline{v} \end{cases} \tag{67}$$

and prices are given by

$$p_i^*\left(\mathbf{x}\right) = \begin{cases} \overline{v} & if\ x_i = 0 \\ \underline{v} & if\ x_i > 0. \end{cases} \tag{68}$$

Consider first the selling decision when $L/n \leq \underline{v}(1-F(\Delta))$. Given a measure τ of monitored firms, and given the pricing function, the investor is willing to sell all bad firms. Upon a shock, she will sell the $1-\tau$ bad firms, and if $L/n > \underline{v}(1-\tau)$, she will also sell $\frac{L/n-\underline{v}(1-\tau)}{\underline{v}}$ shares from good firms. Note that if $\tau = F(\Delta)$, this implies that in equilibrium she will only sell shirking firms since $L/n \leq \underline{v}(1-F(\Delta))$. Thus, if the market maker expects the threshold to be c^*, the investor's payoff from choosing threshold c is

$$\Pi(c^*, c) = \underline{v} + \Delta F(c) \times \begin{cases} 1 & \textit{if } F(c) \leq 1 - \frac{L/n}{\underline{v}} \\ 1 - \beta \frac{L/n - \underline{v}(1-F(c))}{\underline{v}} & \textit{if } F(c) > 1 - \frac{L/n}{\underline{v}} \end{cases} - F(c)\mathbb{E}[c_i | c_i < c].$$

The first-order condition with respect to c implies $c^* = \Delta$ if $F(\Delta) \leq 1 - \frac{L/n}{\underline{v}}$, which holds by assumption. Thus, in such an equilibrium, the optimal cutoff rule is Δ, and the investor sells all shirking firms and keeps all working firms. Bayes' rule shows that the pricing function satisfies the equilibrium condition.

2. Suppose on the contrary that there exists an equilibrium with threshold $\underline{c}^* < \Delta$. We first argue that the investor's expected payoff in this equilibrium is

$$\underline{\Pi}^* = \underline{v} + \Delta F(\underline{c}^*) - F(\underline{c}^*)\mathbb{E}[c_i | c_i < \underline{c}^*].$$

Since the market maker prices the securities fairly, the investor receives the fair value of her portfolio unconditional on her shock. (Her trading profits if she suffers no shock equal her losses if she suffers a shock). Second, we show that the investor has a profitable deviation to $c^* = \Delta$. Indeed, since $L/n \leq \underline{v}(1-F(\Delta))$, the investor never has to sell any of the $nF(\Delta)$ good firms, and so receives $\overline{v}$. Moreover, she can receive at least $\underline{v}$ for every bad firm, and perhaps more. Therefore, the total profit from this deviation is at least $\underline{v} + \Delta F(\Delta) - F(\Delta)\mathbb{E}[c_i | c_i < \Delta]$. Last, note that the function

$$\underline{v} + \Delta F(c) - F(c)\mathbb{E}[c_i | c_i < c]$$

obtains its unique maximum at $c = \Delta$. Therefore, this is an optimal deviation. This complete part (i).

Consider part (ii). We show that there is an equilibrium under common ownership in which the monitoring cutoff is $\overline{R} - \underline{R} + \omega\Delta$. If true, noting that $c^*_{so,voice} < \overline{R} - \underline{R} + \omega\Delta$ completes the proof. Consider the following equilibrium. The monitoring cutoff is $\overline{c}^* = \overline{R} - \underline{R} + \omega\Delta$, and prices and trading strategies are as in the first stage of the proof of part (i). To see that this is an equilibrium, consider manager i's effort decision. Note that, given the proposed trading strategies and prices, a shirking (working) firm always receives a price of $\underline{v}$ $(\overline{v})$. Thus, the manager will work if and only if

$$\overline{R} - \tilde{c}_i + \omega\overline{v} \leq \underline{R} + \omega\underline{v} \Leftrightarrow \overline{R} - \underline{R} + \omega\Delta \leq \tilde{c}_i,$$

as required. Next, consider the pricing function. Given the trading strategies and the expected working cutoff, the prices follow Bayes' rule. Finally, consider the investor's trading decision. Since $L/n < \underline{v} + \Delta F(\overline{R} - \underline{R} + \Delta\omega)$, the investor can satisfy her shock by selling only shirking firms. Since $x_i > 0 \Rightarrow p_i(x_i) = \underline{v}$, she has no incentives to sell any working firms. Therefore, the trading strategy is incentive-compatible. ■

C.3 Discontinuing Relationships

In this section, we apply our model to situations in which the investor decides whether to (partly) discontinue a relationship with the firm. Examples include a bank terminating a lending relationship with a borrower, or a venture capital investor choosing not to invest in a future financing round. We thus distinguish between two concepts. The price $p_i(x_i)$ reflects the impact of (dis)continuation on firm i's reputation. In the core model, this equalled the investor's payoff upon selling. Here, since we consider a discontinuation rather than a sale decision, the investor receives her outside option, $r < \overline{v}$, upon sale.[32] Importantly, unlike in the core model, this reservation payoff is fixed and independent of the impact of sale on the firm's reputation. However, we nevertheless show that common ownership can still improve governance. Note that this extension can be applied to stakeholders other than investors, e.g. a supplier or customer's decision to terminate its relationship with a firm, in which case it also receives a fixed reservation payoff.

We consider two cases based on the magnitude of r. In the first case, $r \in (\underline{v}, \overline{v})$, and so

[32] If $r \geq \overline{v}$, the analysis is trivial since the investor is weakly better off exiting all firms, regardless of their value and her liquidity needs. This behavior will result in identical governance under separate and common ownership, and it particular, it will exhibit the lowest monitoring and working thresholds.

the investor wishes to terminate her relationship with bad firms, but retain it in good firms. In the second case, $r < \underline{v}$, and so the investor exits only if she needs liquidity. As we show below, our main results on the superiority of common ownership continue to hold, except that if $r \in (\underline{v}, \overline{v})$, then governance through exit is independent of the ownership structure.

Proposition 18 *(Discontinuing relationships, high reservation payoff): Suppose $r \in (\underline{v}, \overline{v})$. Then:*

- (i) *Consider the voice model. The investor's per-security monitoring incentives are strictly higher under common ownership than under separate ownership if and only if $L/n < r\left(1 - F\left((1-\beta)(\overline{v} - r)\right)\right)$.*
- (ii) *Consider the exit model. The working threshold in any equilibrium is independent of the ownership structure.*

Proof of Proposition 18. We start by deriving trading strategies given τ, and then endogenize τ under the voice and exit models. Consider separate ownership. Since the payoff to exit is $r \in (\underline{v}, \overline{v})$ per unit, the investor always fully exits bad firms and exits good firms just enough to satisfy her liquidity needs. Therefore,

$$x_{so}^*(v_i, \theta) = \begin{cases} 0 & \textit{if } v_i = \overline{v} \textit{ and } \theta = 0 \\ \overline{x}_{so} = n \min\left\{1, \frac{L/n}{r}\right\} & \textit{if } v_i = \overline{v} \textit{ and } \theta = L \\ n & \textit{if } v_i = \underline{v}. \end{cases}$$

Similarly, in any equilibrium under common ownership,

$$x_{co}^*(v_i, \theta) = \begin{cases} 0 & \textit{if } v_i = \overline{v} \textit{ and } \theta = 0 \\ \overline{x}_{co}(\tau) = \min\left\{1, \max\left\{0, \frac{L/n - r(1-\tau)}{\tau r}\right\}\right\} & \textit{if } v_i = \overline{v} \textit{ and } \theta = L \\ 1 & \textit{if } v_i = \underline{v}. \end{cases}$$

Next, we endogenize τ using the voice model. Consider separate ownership. Given the trading strategy above, the investor's payoff from monitoring is

$$(1-\beta)\overline{v} + \beta\left(\frac{\overline{x}_{so}}{n} r + \left(1 - \frac{\overline{x}_{so}}{n}\right)\overline{v}\right) - \frac{\tilde{c}_i}{n}.$$

Her payoff from not monitoring is r. Thus, she monitors if and only if

$$\frac{\tilde{c}_i}{n} \leq \left(1 - \beta \frac{\overline{x}_{so}}{n}\right) (\overline{v} - r).$$

Therefore, the equilibrium monitoring threshold under separate ownership is unique and given by:

$$\frac{c^{***}_{so,voice}(n)}{n} = \left(1 - \beta \frac{\overline{x}_{so}}{n}\right) (\overline{v} - r).$$

Next, we argue that the equilibrium monitoring threshold under common ownership is unique and given by:

$$c^{***}_{co,voice} = \begin{cases} \overline{v} - r & \textit{if } \frac{L/n}{r} \leq 1 - F(\overline{v} - r) \\ F^{-1}(1 - \frac{L/n}{r}) & \text{if } 1 - F(\overline{v} - r) \leq \frac{L/n}{r} < 1 - F((1-\beta)(\overline{v} - r)) \\ (1-\beta)(\overline{v} - r) & \text{if } 1 - F((1-\beta)(\overline{v} - r)) \leq \frac{L/n}{r}. \end{cases}$$

To see why, note that based on the trading strategy, the investor's payoff from choosing a cutoff rule c^* is given by

$$\Pi(c^*) = F(c^*)[\overline{v} + \beta \overline{x}_{co}(F(c^*))(r - \overline{v})] + (1 - F(c^*))r - F(c^*)\mathbb{E}[c_i | c_i < c^*].$$

Here, her payoff is independent of prices. Her first-order condition is

$$\frac{d\Pi(c^*)}{dc^*}\frac{1}{f(c^*)} = \begin{cases} \overline{v} - r - c^* & \textit{if } c^* \leq F^{-1}(1 - \frac{L/n}{r}) \\ (1-\beta)(\overline{v} - r) - c^* & \textit{if } c^* > F^{-1}(1 - \frac{L/n}{r}). \end{cases} \quad (69)$$

There are three cases to consider:

1. If $\overline{v} - r \leq F^{-1}(1 - \frac{L/n}{r})$, the unique solution is $c^* = \overline{v} - r$.
2. If $(1-\beta)(\overline{v} - r) < F^{-1}(1 - \frac{L/n}{r}) < \overline{v} - r$, the unique solution is $c^* = F^{-1}(1 - \frac{L/n}{r})$.
3. If $F^{-1}(1 - \frac{L/n}{r}) \leq (1-\beta)(\overline{v} - r)$, the unique solution is $c^* = (1-\beta)(\overline{v} - r)$. This implies the cutoff rule is $c^{***}_{co,voice}$.

Next, we prove that $c^{***}_{so,voice}(1) < c^{***}_{co,voice}$ if and only if $L/n < r(1 - F((1-\beta)(\overline{v} - r)))$.

Note that the investor's per-security monitoring incentives under separate ownership are given by $\left(1-\beta\min\left\{1,\frac{L/n}{r}\right\}\right)(\overline{v}-r)$. There are four cases to consider:

1. If $\frac{L/n}{r}\leq 1-F(\overline{v}-r)$ then $\overline{v}-r>\left(1-\beta\min\left\{1,\frac{L/n}{r}\right\}\right)(\overline{v}-r)$.

2. If $1-F(\overline{v}-r)\leq\frac{L/n}{r}<1-F((1-\beta)(\overline{v}-r))$, then

$$F^{-1}(1-\frac{L/n}{r})>\left(1-\beta\frac{L/n}{r}\right)(\overline{v}-r)\Leftrightarrow L/n<r\left(1-F\left[\left(1-\beta\frac{L/n}{r}\right)(\overline{v}-r)\right]\right)$$

   and note that

$$r\left(1-F\left[\left(1-\beta\frac{L/n}{r}\right)(\overline{v}-r)\right]\right)<r\left(1-F\left((1-\beta)(\overline{v}-r)\right)\right)\Leftrightarrow\frac{L/n}{r}<1.$$

   Therefore, the investor's per-security monitoring incentives are strictly higher under common ownership.

3. If $1-F((1-\beta)(\overline{v}-r))\leq\frac{L/n}{r}<1$ then $c^{***}_{co,voice}=(1-\beta)(\overline{v}-r)<\left(1-\beta\frac{L/n}{r}\right)(\overline{v}-r)$ and the investor's per-security monitoring incentives are strictly higher under separate ownership.

4. If $1\leq\frac{L/n}{r}$ then $(1-\beta)(\overline{v}-r)=\left(1-\beta\min\left\{1,\frac{L/n}{r}\right\}\right)(\overline{v}-r)$, and the investor's per-security monitoring incentives are independent of ownership structure.

We finally endogenize τ using the exit model. We first derive the prices, i.e. the firm's reputation post-exit.[33] Under separate ownership, prices follow directly from the application of Bayes' rule to $x^*_{so}(v_i,\theta)$. In particular, if $L/n<r$, prices are

$$p^*_i(x_i)=\begin{cases}\overline{v} & \textit{if }\ x_i\in\{0,\overline{x}_{so}\}\\ \underline{v} & \textit{if }\ x_i=n,\end{cases}$$

[33]Since off-equilibrium prices are irrelevant for the investor's exit decision, we do not specify them to ease the exposition.

and when $L/n \geq r$, they are

$$p_i^*(x_i) = \begin{cases} \overline{v} & \textit{if } x_i = 0 \\ \underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau} & \textit{if } x_i = n. \end{cases}$$

Under common ownership, prices follow directly from the application of Bayes' rule to $x_{co}^*(v_i, \theta)$. If $L/n < r$, prices are

$$p_i^*(x_i) = \begin{cases} \overline{v} & \textit{if } x_i \{0, \overline{x}_{co}(\tau)\} \\ \underline{v} & \text{if } x_i = 1, \end{cases}$$

and when $L/n \geq r$, they are

$$p_i^*(x_i) = \begin{cases} \overline{v} & \textit{if } x_i = 0, \\ \underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau} & \textit{if } x_i = 1. \end{cases}$$

Next, we argue that the working threshold in any exit equilibrium, under any ownership structure, is given by:

$$c_{exit}^{***} = \begin{cases} \overline{R} - \underline{R} + \omega\Delta & \textit{if } L/n < r \\ c_{ii,exit}^{***} \text{ s.t. } c^* = \zeta_{exit,fixed}(F(c^*)) & \textit{if } L/n \geq r. \end{cases}$$

where

$$\zeta_{exit,fixed}(\tau) = \overline{R} - \underline{R} + \Delta\omega\left(1 - \frac{1}{\tau + \frac{1-\tau}{\beta}}\right).$$

To see why, suppose $L/n < r$. Under either ownership structure, the price is $\underline{v}$ if the manager shirks and $\overline{v}$ if he works. Thus, the manager's action is perfectly revealed, and so the optimal cutoff rule satisfies $\overline{R} - \underline{R} + \omega\Delta$ under any ownership structure. Suppose $L/n \geq r$. Under any ownership structure, the price is $\overline{v}$ if $\theta = 0$ and $v_i = \overline{v}$ and $p_i^*(1) = \underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau}$ otherwise. Therefore, the manager works if and only if

$$\begin{aligned} \overline{R} - \tilde{c}_i + \omega(\beta p_i^*(1) + (1-\beta)\overline{v}) &\geq \underline{R} + \omega p_i^*(1) \Leftrightarrow \\ \overline{R} - \underline{R} + \omega(1-\beta)(\overline{v} - p_i^*(1)) &\geq \tilde{c}_i \Leftrightarrow \\ \zeta_{exit,fixed}(\tau) &\geq \tilde{c}_i. \end{aligned}$$

Thus, an optimal cutoff rule must satisfy $c^* = \zeta_{exit,fixed}(F(c^*))$. Noting that $\zeta_{exit,fixed}(\tau)$ is a decreasing function, and hence a solution to $c^* = \zeta_{exit,fixed}(F(c^*))$ is unique, completes the proof. ■

The intuition is as follows. Since the investor's payoff upon exit is independent of how much she exits, she is unconcerned with price impact. As a result, she exits from a bad firm completely, and so it receives the lowest possible price of $\underline{v}$ under any ownership structure. This contrasts the core model where, in some equilibria, the investor only partially sells a bad firm, to disguise the sale as being motivated by a shock and receive a price exceeding $\underline{v}$.

Consider voice. Under the core model, governance was stronger under common ownership through two channels. The first is the lower payoff to cutting and running: under separate ownership, the investor receives a price exceeding $\underline{v}$; under common ownership, adverse selection is more severe and so she receives only $\underline{v}$. This effect no longer operates here, because the investor's payoff is r under cutting and running, irrespective of ownership structure. However, the second channel – the higher payoff to monitoring and intervening – continues to operate. Under separate ownership, an investor is forced to sell a good firm upon a shock, and receives $r < \overline{v}$ on the amount that she sells. Under common ownership, she can satisfy her shock by selling only bad firms, and thus receives the full payoff of $\overline{v}$ from good firms.

However, the analog to this second channel – the higher payoff to working – does not apply in the exit model. Importantly, the manager's payoff depends on the security price p_i, and not the amount received by the investor r. Under separate ownership, if $L/n < r$, a working firm is only partially sold upon a shock. As a result, it still receives the full price of $\overline{v}$, because the market maker knows that the investor would have fully sold the firm if it had shirked. Thus, even though the investor only receives r for the part that she sells, reducing her payoff from monitoring in the exit model, the manager receives a price of $\overline{v}$. As a result, the reward for working and punishment from shirking are already at the maximum level of $\overline{R} - \underline{R} + \omega\Delta$ under separate ownership, and cannot be improved upon by common ownership.

Proposition 19 *(Discontinuing relationships, low reservation payoff): Suppose $r < \underline{v}$. Then, per-security monitoring (working) incentives in any equilibrium of voice (exit) under common ownership are strictly higher than under separate ownership if and only if $L/n < r$, and the same otherwise.*

Proof of Proposition 19. We start by deriving the trading strategies given τ, and then endogenize τ under the voice and exit models. Consider separate ownership. Since $r < \underline{v}$, the investor exits if and only if she needs liquidity. Therefore:

$$x_{so}^{*}(v_i, \theta) = \begin{cases} 0 & \textit{if } \theta = 0 \\ \overline{x}_{so} = n \min\left\{1, \frac{L/n}{r}\right\} & \textit{if } \theta = L. \end{cases}$$

Under common ownership, the trading strategy is given by:

$$x_{co}^{*}(v_i, \theta) = \begin{cases} 0 & \textit{if } \theta = 0 \\ \overline{x}_{co,good}(\tau) = \min\left\{1, \max\left\{0, \frac{L/n - r(1-\tau)}{\tau r}\right\}\right\} & \textit{if } v_i = \overline{v} \textit{ and } \theta = L \\ \overline{x}_{co,bad}(\tau) = \min\left\{1, \frac{L/n}{r(1-\tau)}\right\} & \textit{if } v_i = \underline{v} \text{ and } \theta = L. \end{cases} \tag{70}$$

The investor exits only if she needs liquidity; if so, she exits bad firms first.

Next, we endogenize τ using the voice model. Consider separate ownership. Given the trading strategies, the investor monitors if and only if

$$(1-\beta)\overline{v} + \beta\left(\frac{\overline{x}_{so}}{n}r + \left(1 - \frac{\overline{x}_{so}}{n}\right)\overline{v}\right) - \frac{\tilde{c}_i}{n} > (1-\beta)\underline{v} + \beta\left(\frac{\overline{x}_{so}}{n}r + \left(1 - \frac{\overline{x}_{so}}{n}\right)\underline{v}\right)$$

where the LHS (RHS) is her payoff from monitoring (not monitoring). This inequality yields $\frac{\tilde{c}_i}{n} < \left(1 - \beta\frac{\overline{x}_{so}}{n}\right)\Delta$. Consider common ownership. There are two cases to consider.

1. If $L/n < r$, the investor's expected payoff is given by

$$\begin{aligned} \Pi(c^*) &= -F(c^*)\mathbb{E}[c_i | c_i < c^*] \\ &\quad + \begin{cases} F(c^*)\overline{v} + (1 - F(c^*))[\underline{v} + \beta\overline{x}_{co,bad}(F(c^*))(r - \underline{v})] & \textit{if } c^* \leq F^{-1}(1 - \frac{L/n}{r}) \\ F(c^*)[\overline{v} + \beta\overline{x}_{co,good}(F(c^*))(r - \overline{v})] & \\ \quad +(1 - F(c^*))[(1-\beta)\underline{v} + \beta r] & \textit{if } c^* > F^{-1}(1 - \frac{L/n}{r}) \end{cases} \\ &= -F(c^*)\mathbb{E}[c_i | c_i < c^*] + F(c^*)\overline{v} + (1 - F(c^*))\underline{v} + \beta\frac{L/n}{r}(r - \underline{v}). \end{aligned}$$

   The first-order condition implies $c^* = \Delta$, which is strictly higher than under separate ownership.

2. If $r \leq L/n$, the investor sells her entire portfolio if she suffers a shock, and fully keeps it

otherwise. Her expected payoff is given by

$$\Pi(c^*) = F(c^*)[(1-\beta)\,\overline{v} + \beta r] + (1 - F(c^*))[(1-\beta)\,\underline{v} + \beta r] - F(c^*)\mathbb{E}[c_i | c_i < c^*].$$

The first-order condition implies $c^* = (1-\beta)\,\Delta$, which is the same as under separate ownership.

We finally endogenize τ using the exit model. We first derive the prices. Under separate ownership, prices are $p_i^*(x_i) = \underline{v} + \tau\Delta$. Under common ownership, there are three cases to consider.

1. If $L/n \leq r(1-\tau)$ the investor never exits good firms and prices are:

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta\frac{\tau}{1-\beta+\beta\tau} & \textit{if } x_i = 0 \\ \underline{v} & \text{if } x_i = \overline{x}_{co,bad}(\tau). \end{cases} \tag{71}$$

2. If $r(1-\tau) < L/n < r$, the investor fully exits bad firms and partially exits good firms upon a shock. Therefore,

$$p_i^*(x_i) = \begin{cases} \underline{v} + \tau\Delta & \textit{if } x_i = 0 \\ \overline{v} & \text{if } x_i = \overline{x}_{co,good}(\tau) < 1 \\ \underline{v} & \text{if } x_i = 1. \end{cases} \tag{72}$$

3. If $L/n \geq r$, the investor sells her entire portfolio upon a shock and retains it otherwise. Thus, prices are $p_i^*(x_i) = \underline{v} + \tau\Delta$.

Consider the manager's incentives under separate ownership. In this case, the investor's decision to exit, and thus prices, are independent of the manager's action. Therefore, the working threshold is $\overline{R} - \underline{R}$. Consider common ownership and the three cases in turn:

1. If $L/n \leq r(1-\tau)$, the manager works if and only if

$$\begin{aligned} \overline{R} - \tilde{c}_i + \omega(\underline{v} + \Delta\frac{\tau}{1-\beta+\beta\tau}) &\geq \underline{R} + \omega\left(\beta\underline{v} + (1-\beta)\,(\underline{v} + \Delta\frac{\tau}{1-\beta+\beta\tau})\right) \Leftrightarrow \\ \overline{R} - \underline{R} + \omega\beta\Delta\frac{\tau}{1-\beta+\beta\tau} &\geq \tilde{c}_i. \end{aligned}$$

Thus, an optimal cutoff rule must satisfy

$$c^* = \overline{R} - \underline{R} + \omega\beta\Delta\frac{F(c^*)}{1-\beta+\beta F(c^*)},$$

which is strictly greater than $\overline{R} - \underline{R}$.

2. If $r(1-\tau) < L/n < r$, the manager works if and only if

$$\begin{aligned} \overline{R} - \tilde{c}_i + \omega(\beta\overline{v} + (1-\beta)(\underline{v}+\tau\Delta)) &\geq \underline{R} + \omega(\beta\underline{v} + (1-\beta)(\underline{v}+\tau\Delta)) \Leftrightarrow \\ \overline{R} - \underline{R} + \omega\beta\Delta &\geq \tilde{c}_i, \end{aligned}$$

which is strictly greater than $\overline{R} - \underline{R}$.

3. If $r \leq L/n$, the price is always $\underline{v} + \tau\Delta$ and the working threshold is $\overline{R} - \underline{R}$.

■

The intuition is as follows. Since the reservation payoff is so low, the investor never exits, even from a bad firm, unless she is forced to do so by a shock. Upon a shock, the investor sells the minimum amount possible, regardless of whether the firm is good or bad. As a result, voice is stronger under common ownership for a similar reason to the high-r case of Proposition 18 (and also the core model): under a shock, the investor can sell bad firms more and good firms less, and thus enjoy a higher payoff from monitoring. Unlike in the high-r case, exit is also stronger under common ownership. With high r, exit was already the strongest possible under separate ownership, since the investor fully sold bad firms. Here, she sells a firm to the minimum extent possible, regardless of whether it is good or bad, and so the manager's price incentives to work are zero. Under common ownership, she sells shirking firms more and good firms less, thus creating price incentives to work.

C.4 Heterogeneous Firms

In the core model, all firms have the same distribution of valuations. We now analyze the case in which firms have different valuation distributions, and thus differ in their information asymmetry and the price impact of selling. For brevity, we consider the small shock case, since this is where our results are strongest.

For example, suppose there are $J \geq 1$ classes of firms. The valuation of firm i in class $j \in \{1, ..., J\}$ is $v_j \in \left\{\underline{v}_j, \overline{v}_j\right\}$ where $\Delta_j \equiv \overline{v}_j - \underline{v}_j > 0$. We assume $\underline{v}_{j'} < \overline{v}_{j''}$ for all $j' \in J$ and $j'' \in J$, i.e. the worst good firm is more valuable than the best bad firm. We also index by j the exogenous parameters F, ω, $\overline{R}$, and $\underline{R}$. We maintain the assumption that each firm has its own market maker, and the class to which firm i belongs is common knowledge. All random variables are independent across firms and classes.

The analysis of separate ownership remain unchanged; we now add a subscript j to the benchmarks under separate ownership to denote that they apply to a firm of class j. Under common ownership, we assume the investor owns a mass of $n_j \geq 0$ firms from class j.

Proposition 20 *(Heterogeneous firms):*

(i) *Consider the voice model and suppose $L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j\left(\Delta_j\right)\right)$. In any equilibrium under common ownership, per-security monitoring incentives are strictly higher than under separate ownership.*

(ii) *Consider the voice model and suppose $L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j(\overline{R}_j - \underline{R}_j + \Delta_j \omega_j)\right)$. There is an equilibrium in which the working threshold is strictly greater than under separate ownership.*

Proof of Proposition 20. Consider the voice model under common ownership. We argue that, if $L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j\left(\Delta_j\right)\right)$, then $c_j^* = \Delta_j$ in any equilibrium. If true, noting that $c_{so,voice,j}^* < \Delta_j$ completes the proof of part (i). We first show that such an equilibrium exists. Since $L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j\left(\Delta_j\right)\right)$, under this equilibrium the investor never has to sell a good firm from any class to satisfy her liquidity needs. Since $\underline{v}_{j'} < \overline{v}_{j''}$ for all j' and j'', the investor has weak incentives to sell only bad firms, and only as much as needed in satisfy her liquidity needs. Therefore, and similar to the core model, if firm i belongs to class j, it must be that $x_i > 0 \Rightarrow p_i\left(x_i\right) = \underline{v}_j$. Given this pricing rule, and regardless of her expected liquidity needs, the investor solves

$$\max_{c_1^*, \ldots, c_J^*} \sum_{j=1}^{J} n_j \left(F_j(c_j^*)\overline{v}_j + (1 - F_j(c_j^*))\underline{v}_j - F_j(c_j^*)\mathbb{E}_j[c_i | c_i < c_j^*]\right).$$

The first-order condition with respect to firms from class j implies $c_j^* = \Delta_j$. To see that no other equilibrium exists, first note that in any equilibrium, $c_i^* \leq \Delta_i$. Indeed, the investor's benefit from monitoring a firm of type i cannot exceed Δ_i, and so there is no equilibrium in which $\widetilde{c}_i > \Delta_i$ and the investor monitors. Consider an equilibrium in which $c_j^{**} \leq \Delta_j$ for all j and there is j' such that $c_{j'}^{**} < \Delta_{j'}$. Note that this implies

$$\sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j\left(c_j^{**}\right)\right) \geq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j\left(\Delta_j\right)\right) \geq L,$$

and so the investor never has to sell a bad firm to satisfy her liquidity need. Therefore, in this equilibrium, if firm i belongs to class j', then $x_i > 0 \Rightarrow p_i\left(x_i\right) = \underline{v}_{j'}$.[34] However, this contradicts $c_{j'}^{**} < \Delta_{j'}$, since the investor can benefit from monitoring firms from class j' if $\widetilde{c}_i \in \left(c_{j'}^{**}, \Delta_{j'}\right)$.

Consider the exit model under common ownership. We argue that, if

$$L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j(\overline{R}_j - \underline{R}_j + \Delta_j \omega_j)\right),$$

there is an equilibrium under common ownership with $c_j^{*} = \Delta_j$. If true, noting that $c_{so,exit,j}^{*} < \Delta_j$ completes the proof of part (ii). Since $L \leq \sum_{j=1}^{J} n_j \underline{v}_j \left(1 - F_j(\overline{R}_j - \underline{R}_j + \Delta_j \omega_j)\right)$, under this equilibrium the investor never has to sell a good firm from any class in order to satisfy her liquidity needs. Since $\underline{v}_{j'} < \overline{v}_{j''}$ for all j' and j'', the investor has weak incentives to fully sell all bad firms, and fully retain all good firms, regardless of her liquidity needs. Therefore, and similar to core model, if firm i belongs to class j, it must be that $x_i > 0 \Rightarrow p_i\left(x_i\right) = \underline{v}_j$ and $p_i\left(0\right) = \overline{v}_j$. Given this pricing rule, the manager of firm i who belongs to class j works if and only if

$$\overline{R}_j - \tilde{c}_i + \omega_j \overline{v}_j \geq \underline{R}_j + \omega_j \underline{v}_j \Leftrightarrow \overline{R}_j - \underline{R}_j + \omega_j \Delta_j \geq \tilde{c}_i,$$

as required. ■

The intuition is as follows. Regardless of whether the firms have the same or different payoff distributions, it remains the case that, for a sufficiently small shock, common ownership allows the investor to fully retain good firms upon a shock, thus maximizing the payoff to monitoring under voice and working under exit. As a result, the adverse selection problem upon selling is severe, thus minimizing the payoff to not monitoring under voice and shirking under exit.

[34] The arguments are the same as in the baseline model, and thus omitted for brevity.

C.5 Spin-Offs and Mergers

We consider the case of small or intermediate shocks ($L/n < \underline{v}$) and $\beta = 1$.[35] We analyze both the voice and exit models under two cases: a merged entity, and n divisions after a spin-off. (We will sometimes use "division" and "firm" separately.) The only difference is that, in the latter, each firm can be traded separately. Thus, in the voice model we are assuming that the investor can monitor each division separately even upon merger (if she cannot, then spin-offs provide an additional benefit). In the exit model, under merger, we assume that the managers of each division are compensated based on the security price of the merged entity, and the investor can observe each manager's actions separately. (The results would be the same if she could only observe the aggregate value of the merged entity.)

Under merger, the investor must engage in balanced exit and cannot sell good divisions more and bad divisions less. Thus, in equilibrium, the amount that she sells of firm i is uninformative of its fundamental value. If the market maker believes that the investor follows threshold c^*, the expected value of each firm is $\underline{v} + F(c^*)\Delta$. This implies that, if x_i is on the equilibrium path, the price of firm i must be $p_i(x_i) = \underline{v} + F(c^*)\Delta$. Therefore, the investor sells $\overline{x}_{spin}(F(c^*)) = \frac{L/n}{\underline{v}+F(c^*)\Delta}$, just enough to meet her liquidity need. In particular, suppose the prices of firm i are:

$$p_i^*(x_i) = \begin{cases} \underline{v} + \tau\Delta & \textit{if } \ x_i \leq \overline{x}_{spin}(F(c^*)) \\ \underline{v} & \textit{if } \ x_i > \overline{x}_{spin}(F(c^*)) . \end{cases}$$

Under the exit model, price informativeness is zero since the investor always engaged in balanced exit. Thus, governance is absent upon merger, and so $c^* = \overline{R} - \underline{R}$. This threshold is less than in the core model under common ownership (where $c^* > \overline{R} - \underline{R}$ for $L/n < \underline{v}$), and thus under a spin-off

Consider the voice model and let the market maker's expected threshold be c^*, but the investor chooses $c \neq c^*$. She must sell at least $\overline{x}_{spin}(F(c^*))$ from all firms to meet her liquidity needs. If $c < c^*$, the investor is monitoring less than expected, and thus has incentives to sell as much as she can from all firms. However, since $x_i > \overline{x}_{spin}(F(c^*)) \Rightarrow p_i^*(x_i) = \underline{v}$, she has no incentives to do so as long as $c \geq 0$, and she will choose $x_i(c) = \overline{x}_{spin}(F(c^*))$. If $c > c^*$, the

[35] The results for the exit model extend to all $\beta \in [0, 1]$. For the voice model, a pure-strategy equilibrium for the investor's monitoring decision does not exist if $\beta < 1$.

investor is monitoring more than expected, and thus has incentives to sell as little as she can from all firms, and will choose $x_i(c) = \overline{x}_{spin}(F(c^*))$ as well. Her expected payoff is:

$$\begin{aligned}\Pi(c^*, c) &= \overline{x}_{spin}(F(c^*))(\underline{v} + F(c^*)\Delta) + (1 - \overline{x}_{spin}(F(c^*)))(\underline{v} + F(c)\Delta) \\ &\quad -F(c)E[c_i|c_i < c]\end{aligned}$$

This implies

$$\frac{\partial \Pi(c^*, c)}{\partial c}\frac{1}{f(c)} = -c + \Delta\left[1 - \frac{L/n}{\underline{v} + F(c^*)\Delta}\right],$$

and an equilibrium must solve

$$c^* = \Delta\left[1 - \frac{L/n}{\underline{v} + F(c^*)\Delta}\right].$$

Since the RHS is bounded within $\left[\Delta\left(1 - \frac{L/n}{\underline{v}}\right), \Delta\left(1 - \frac{L/n}{\underline{v}+\Delta}\right)\right] \subset (0, \Delta)$, a solution always exists. Note that this solution is smaller than Δ and that $L/n < \underline{v}$ implies

$$\zeta_{voice}(\tau) > \Delta\left(1 - \frac{L/n}{\underline{v} + \tau\Delta}\right) = \phi_{voice}(\tau).$$

Therefore, the level of monitoring is less than the core model under common ownership, and thus under a spin-off.