Online Appendix for "Corporate Control Activism"

B Supplemental results for Section 3.2

B.1 Should a takeover premium drop after the bidder wins a proxy fight?

The commitment problem implies that if the bidder wins a proxy fight, he would abuse the power of the target board and low-ball the takeover premium. Does the model predict that the takeover offer should drop after the bidder wins a proxy fight? The answer is no, and for two reasons. To see why, suppose the activist is absent ($\alpha = 0$) and consider the following thought experiment: Target shareholders are naive and would elect the bidder to the board in spite of his inability to commit to act in their best interests once elected. Based on Lemma 1, the bidder would abuse the power of the board and offer target shareholders q, the lowest offer they would accept. Therefore, if the first round of negotiations fails, the bidder would run (and win) a proxy fight if and only if

$$q + \Delta - (1 - m)\, q - \kappa \geq qm \Leftrightarrow \Delta \geq \kappa,$$

and make an expected profit of $\Delta - \kappa$. Suppose $\Delta \geq \kappa$ and consider the first round of negotiations.[31] If the bidder is the proposer, he would offer to buy the target for q. The target board and shareholders accept this offer since if they do not, the bidder would run and win a proxy fight, and then offer them the same amount. Therefore, even if the target board or shareholders unexpectedly reject the initial offer q (e.g., a trembling hand), the post proxy fight negotiations would yield the same offer, not a lower one! Intuitively, the anticipation of winning a proxy fight and low-balling the takeover offer incentivizes the bidder to low-ball the takeover offer at the beginning of first round of negotiations. Moreover, notice that the expected takeover premium in this equilibrium is strictly positive. Indeed, when the target board is the proposer he asks for $q + \kappa$, the highest offer that the bidder would accept. The bidder is willing to pay more than q in order to save the cost of running a proxy fight. Therefore, the lack of commitment does not imply that the expected takeover premium should be zero.

[31] If $\Delta < \kappa$ then the bidder never runs a proxy fight even if he can win one. In these cases, the analysis is reduced to the baseline model where $\alpha \leq \frac{\kappa/s}{b}$.

The second reason why a takeover offer is unlikely to drop after a bidder wins a proxy fight is that if a bidder was able to win a proxy fight and target shareholders have rational expectations, then it is likely that the bidder found a way to commit to act in their best interests.

B.2 Increasing the target's standalone value

Consider the baseline model but suppose that the activist is absent (i.e., $\alpha = 0$) and a value of Δ can be created if the bidder's proposal is implemented. The proposal can be successfully implemented either by the incumbent or by the bidder, regardless of the target's ownership structure. In particular, the proposal can be implemented even if the target remains independent after the failure of the second round of negotiations. Either way, the incumbent loses his private benefits of control if the proposal is implemented.

Proposition 7 *Suppose the first round of negotiations fails. If the bidder can increase the standalone value of the target, he runs a proxy fight if and only if*

$$\frac{\kappa/m}{1-m} \leq \frac{\Delta}{1-m} < b, \tag{27}$$

and whenever the bidder runs a proxy fight, he wins.

Proof. If the second round of negotiations succeeded and the target is acquired by the bidder, then the bidder implements his proposal if it has not been implemented yet. Therefore, the post takeover target value is $q + \Delta$. If the second round of negotiations failed and the firm remains independent (that is, its ownership structure did not change), there are two cases. First, if the bidder controls the target board then he implements her proposal if it has not been implemented yet, and the target value is $q + \Delta$. Second, if the incumbent board retains control then he implements the proposal if and only if $b \leq \Delta$, and hence, the target value is $q + \mathbf{1}_{\{b \leq \Delta\}}\Delta$.

Consider the second round of negotiations. There are two cases. First, suppose that either the bidder controls the target board or the incumbent retains control and $b \leq \Delta$. The bidder's proposal is implemented whether or not the bid fails. For this reason, the bidder will not offer more than $q + \Delta$ per share. Moreover, target shareholders will not accept offers lower than $q + \Delta$, since they can always reject the bid and obtain a value of $q + \Delta$ once the proposal is implemented. Therefore, whether or not target is acquired, the bidder's payoff is $m(q+\Delta)$ and the shareholder value is $q+\Delta$. Second, suppose incumbent board retains control and $b > \Delta$. If the negotiations fail, the proposal will not be implemented and the bidder's payoff would be mq. If the bidder acquires the firm, her payoff is $q+\Delta-(1-m)\,\pi_2$, where π_2 is the offer made to target shareholders. Therefore, the bidder is willing to offer up to $q+\frac{\Delta}{1-m}$ per share. The incumbent board and the bidder will reach an agreement if and only if $b \leq \frac{\Delta}{1-m}$. If $\frac{\Delta}{1-m} < b$ then the takeover fails and the shareholder value is q. If $\Delta < b \leq \frac{\Delta}{1-m}$ then the incumbent and the bidder reach an agreement in which $\pi_2 \geq q+b > q+\Delta$. Therefore, target shareholders approve any agreement reached by the bidder and the incumbent, and target is acquired by the bidder. In this case, the expected shareholder value is $q+s\frac{\Delta}{1-m}+(1-s)\,b$.

Consider the proxy fight stage. There are three cases to consider. First, if $b \leq \Delta$ then the bidder's payoff is $m(q+\Delta)$ whether or not she gets the control of the board. Therefore, she has no reason to run and incur the cost of a proxy fight. Second, if $\Delta < b \leq \frac{\Delta}{1-m}$ then the bidder always loses the proxy fight if he decides to start one. The reason is that shareholders know that if they elect the bidder they will get $q+\Delta$ whereas if they reelect the incumbent, the bidder will takeover the target and pay shareholders on average $q+s\frac{\Delta}{1-m}+(1-s)\,b$, which is strictly higher. Anticipating his defeat, the bidder never runs a proxy fight in this region. Third, if $\frac{\Delta}{1-m} < b$ then the shareholder value is $q+\Delta$ if the bidder gets the control of the board, and q otherwise. Therefore, shareholders always elect the bidder if he runs a proxy fight. The bidder's payoff is $m(q+\Delta)-\kappa$ if he runs a proxy fight, and mq otherwise. Therefore the bidder runs a proxy fight only if $\kappa/m \leq \Delta$. Combining this condition with $b > \frac{\Delta}{1-m}$ yields (27).

Finally, consider the first round of negotiations.[32] There are four cases to consider. First, if $b \leq \Delta$ then the target value is $q+\Delta$ whether or not the bidder acquires the target. Second, if $\Delta < b \leq \frac{\Delta}{1-m}$ then the bidder reaches an agreement in the first round of negotiations with the incumbent board in which he pays $q+s\frac{\Delta}{1-m}+(1-s)b$ per share and acquires full control of the target. If $\frac{\Delta}{1-m} < b$ and $\Delta < \kappa/m$ then the target remains independent and the proposal is not implemented. If $\frac{\Delta}{1-m} < b$ and $\Delta \geq \kappa/m$ then the bidder reaches an agreement in the first round of negotiations with the incumbent board in which he pays on average $q+s\left(\Delta+\frac{\kappa}{1-m}\right)+(1-s)\,\Delta$ per share and acquires full control of the target. ■

B.3 The value of commitment

Suppose that before the first round of negotiations starts, the bidder could fully commit to act in the best interests of target shareholders if they elect him to their board. What is the

[32] This part is not discussed in the main text and only provided for completeness.

value of such commitment? If the bidder chooses to commit, he would use the control of the target board to maximize the target shareholder value whenever he is negotiating on their behalf, which happens with probability s. Therefore, shareholders are confident that if they elect the bidder to their board, he will negotiate the "fair price", which is $q+s\frac{\Delta}{1-m}$. Since both the bidder and the activist can "promise" an expected premium of $s\frac{\Delta}{1-m}$, target shareholders reelect the incumbent whenever $b \leq \frac{\Delta}{1-m}$, and are indifferent between electing the bidder or the activist when $\frac{\Delta}{1-m} < b$. Therefore, the bidder and the activist will run a proxy fight only if the other party is not expected to do so. Subject to this constraint, the incentives of the activist to run a proxy fight are the same as in Proposition 1 part (ii). However, unlike part (i) of Proposition 1, here the bidder can win a proxy fight. The bidder's expected profit from running a proxy fight is $\Delta(1-s)-\kappa$, and therefore, the bidder will run (and win) a proxy fight if and only if the activist does not run a proxy fight and

$$\frac{1}{1-m}\frac{\kappa}{1-s} \leq \frac{\Delta}{1-m} < b. \tag{28}$$

It follows that by committing to act in the best interests of target shareholders, the bidder obtains the option to unseat the incumbent via proxy fight if the first round of negotiations fails. The next result, which characterizes the value of this option, follows directly from Proposition 2.

Proposition 8 *The net expected value that the bidder obtains from a commitment to act in the best interests of target shareholders is*

$$R = \int_{(1-m)\min\{b,\frac{\kappa/s}{\alpha},\frac{1}{1-m}\frac{\kappa}{1-s}\}}^{(1-m)\min\{b,\frac{\kappa/s}{\alpha}\}} \left[(1-s)\Delta - s\kappa\right] dF(\Delta), \tag{29}$$

which is decreasing in α and increasing in b.

Proposition 8 has several implications. First, since R is decreasing in α and increasing in b, bidders would have stronger incentives to incur the costs of resolving their commitment problem when the prospective target companies have more severe agency problems and face weaker pressure from activist investors. This observation, however, ignores the possibility that the cost of obtaining a commitment might also depend on factors that affect the benefit R. For example, higher b may be the result of weak investor protection laws, which could make it harder for the bidder to obtain a credible commitment. Moreover, note that R is an upper bound on the value from commitment, since in practice the bidder may only be able to obtain partial rather than full commitment. Second, if $b \leq \frac{1}{1-m}\frac{\kappa}{1-s}$ then $R = 0$ and the analysis is identical to Section 3. There, the bidder's threat of running a proxy fight was not credible because target shareholders would never elect him to the board, while here it is not credible because the benefit from replacing the incumbent is not high enough to compensate for the cost of running (and winning) a proxy fight whenever the incumbent board resists the takeover. Interestingly, if $\frac{\kappa/s}{\alpha} < b \leq \frac{1}{1-m}\frac{\kappa}{1-s}$ then the activist's threat of running a proxy fight is credible even though the bidder's threat is not.[33] Third, the bidder's threat of running a proxy fight is credible if and only if

$$\frac{1}{1-m}\frac{\kappa}{1-s} \leq \frac{\Delta}{1-m} < \min\left\{b, \frac{\kappa/s}{\alpha}\right\}. \tag{30}$$

If $\frac{\Delta}{1-m} \geq \min\{b, \frac{\kappa/s}{\alpha}\}$ then as in Proposition 6 part (i), the takeover goes through even if the bidder can never win a proxy fight. If $\frac{\Delta}{1-m} < \min\{b, \frac{\kappa/s}{\alpha}, \frac{1}{1-m}\frac{\kappa}{1-s}\}$ then as in Proposition 6 part (ii), the target remains independent. The key difference arises when (30) holds.[34] In those cases, the activist's threat of running a proxy fight is not credible, and unless the bidder initiates his own challenge, the target would remain independent. In equilibrium, the bidder reaches an agreement in the first round of negotiations with the incumbent board in which he pays an expected price of $q + s\frac{\Delta+\kappa}{1-m}$ per share. The bidder pays more than $q + s\frac{\Delta}{1-m}$, the "fair price", since the incumbent exploits the fact that the bidder would be willing to pay a higher price up-front in order to avoid the cost of running a proxy fight later on. This explains why κ appears in the integrand in (29).

C Extensions

C.1 Activist's proposals

Proposition 9 *Suppose the activist can make a proposal as described in Section 5.2, then:*

(i) If the first round of negotiations fails, then the bidder never runs a proxy fight, while the activist runs a proxy fight if and only if $\kappa/\alpha \leq \varepsilon < b$ and $\Delta < b$, or $\varepsilon < \kappa/\alpha$ and $\varepsilon + \frac{\kappa/\alpha-\varepsilon}{s} \leq \Delta < b$. If the activist runs a proxy fight, she wins.

[33] The activist's advantage can also stem from having more governance expertise (i.e., lower costs of running a proxy fight) due to their experience in challenging entrenched incumbents of other public companies.

[34] In Proposition 8 we implicitly assume that whenever both conditions (2) and (28) hold, the equilibrium of the subgame in which the activist runs a proxy fight is in play. This selection is conservative in the sense that it gives an upper bound on the bidder's value from commitment. Similar results hold under a different selection.

(ii) The expected shareholder value is $q + v(\alpha, \varepsilon)$ *where*

$$v(\alpha,\varepsilon) = \begin{cases} v(0) + \int_{\min\{b,\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}\}}^{b} [\varepsilon + s(\Delta-\varepsilon)]\, dF(\Delta) & \text{if } \varepsilon < \min\{b, \kappa/\alpha\} \\ \varepsilon + s\int_{\varepsilon}^{\infty} (\Delta-\varepsilon)\, dF(\Delta) & \\ +(1-s)\int_{b}^{\infty} \max\{0, b-\varepsilon\}\, dF(\Delta) & \text{if } \varepsilon \geq \min\{b,\kappa/\alpha\}. \end{cases} \tag{31}$$

(iii) The expected value created by the takeover and the activist's proposal is

$$w(\alpha,\varepsilon) = \begin{cases} \int_{\min\{b,\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}\}}^{\infty} \Delta dF(\Delta) & \text{if } \varepsilon < \min\{b,\kappa/\alpha\} \\ \int_{-\infty}^{\varepsilon} \varepsilon dF(\Delta) + \int_{\varepsilon}^{\infty} \Delta dF(\Delta) & \text{if } \varepsilon \geq \min\{b,\kappa/\alpha\}. \end{cases} \tag{32}$$

(iv) Let $\Pi_B(\alpha,\varepsilon)$ *be the bidder's expected profit. Then:*

- *(a) If* $\varepsilon < \min\{b,\kappa/\alpha\}$ *then* $\alpha > 0 \Rightarrow \Pi_B(\alpha,\varepsilon) \geq \Pi_B(0,\varepsilon)$ *and* $\lim_{b\to\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}} \frac{\partial \Pi_B(0,\varepsilon)}{\partial \varepsilon} > 0$.
- *(b) If* $\varepsilon > b$ *then for all* $\alpha > 0$, $\Pi_B(\alpha,\varepsilon)$ *is strictly decreasing in* ε, $\Pi_B(\alpha,\varepsilon) < \Pi_B(0,\varepsilon)$, *and takeover is less likely when the activist is present than when she is not.*

Proof. Consider the following three cases:

1. First, suppose $\max\{\varepsilon, b\} \leq \Delta$. If the incumbent retains control of the board and the firm remains independent, the incumbent implements the activist's proposal if and only if $\varepsilon \geq b$. Therefore, the reservation value of the incumbent in this case is $q + \max\{\varepsilon, b\}$ per share. Since $\max\{\varepsilon, b\} \leq \Delta$, an agreement in which the bidder pays an expected premium of $s\Delta + (1-s)\max\{\varepsilon, b\}$ is always reached under the control of the incumbent board. On the other hand, if the activist obtains control of the board, she will reach an agreement with the bidder in which the expected takeover premium is $s\Delta + (1-s)\varepsilon$. Therefore, the activist has no incentives to run a proxy fight. Overall, the expected firm value is $q + s\Delta + (1-s)\max\{\varepsilon, b\}$.

2. Second, suppose $\Delta < \varepsilon$ and $b \leq \varepsilon$. Since $\Delta < \varepsilon$ and $b \leq \varepsilon$, if the incumbent retains control of the board, the incumbent is willing to implement the activist's proposal but refuses the sell the firm. Since $\Delta < \varepsilon$, a takeover cannot increase the value of the firm even if shareholders extract all the surplus. Therefore, the activist has no incentives to run a proxy fight, and the value of the firm under the incumbent's control is $q + \varepsilon$.

3. Third, suppose $\max\{\varepsilon,\Delta\}<b$. Since $\max\{\varepsilon,\Delta\}<b$, if the incumbent retains control of the board, the incumbent refuses the sell the firm or implement the activist's proposal. Therefore, under the incumbent's control the firm value is q. Suppose the activist controls the target board. If $\varepsilon>\Delta$ then she would implement the proposal, and if $\varepsilon\leq\Delta$ then she would reach an acquisition agreement in which the bidder pays an expected premium of $s\Delta+(1-s)\,\varepsilon$. Therefore, under the activist's control firm value is $q+\varepsilon+s\max\{0,\Delta-\varepsilon\}$, and shareholders always elect the activist if she decides to run a proxy fight. The activist has incentives to run a proxy fight if and only if

$$\alpha\left[q+\varepsilon+s\max\{0,\Delta-\varepsilon\}\right]-\kappa\geq\alpha q,$$

which holds if and only if $\varepsilon\geq\kappa/\alpha$ or, $\varepsilon<\kappa/\alpha$ and $\Delta\geq\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}$. Part (i) follows from the intersection of this condition with $\max\{\varepsilon,\Delta\}<b$.

Consider the first round of negotiations. All parties involved anticipate the dynamic above if the first round fails. Therefore, if $\max\{\varepsilon,b\}\leq\Delta$ then the bidder pays $q+s\Delta+(1-s)\max\{\varepsilon,b\}$ and takes over the target after the first round of negotiations. If $\Delta<\varepsilon$ and $b\leq\varepsilon$ then the target remains independent and the activist's proposal is implemented. If $\max\{\varepsilon,\Delta\}<b$ then the bidder pays $q+\varepsilon+s\max\{0,\Delta-\varepsilon\}$ if $\varepsilon\geq\kappa/\alpha$ or, $\varepsilon<\kappa/\alpha$ and $\Delta\geq\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}$, and otherwise, the target remains independent but the activist's proposal is not implemented. Integrating over all values of Δ, which is drawn from cdf $F(\cdot)$, firm value is $q+v(\alpha,\varepsilon)$ where

$$v(\alpha,\varepsilon)=\begin{cases}\varepsilon+s\int_{\varepsilon}^{\infty}(\Delta-\varepsilon)\,dF(\Delta) & \textit{if }\ b\leq\varepsilon\\ v(0)+\int_{-\infty}^{b}\left[\varepsilon+s\max\{0,\Delta-\varepsilon\}\right]dF(\Delta) & \textit{if }\ \kappa/\alpha\leq\varepsilon<b\\ v(0)+\int_{\min\{b,\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}\}}^{b}\left[\varepsilon+s(\Delta-\varepsilon)\right]dF(\Delta) & \textit{if }\ \varepsilon<\min\{b,\kappa/\alpha\},\end{cases}$$

which can be rewritten as in the statement. Parts (ii) and (iii) follow directly from the analysis above.

Consider part (iv) and note that $\Pi_B(\alpha,\varepsilon)=w(\alpha,\varepsilon)-v(\alpha,\varepsilon)$. Based on parts (i)-(iii), if $\varepsilon<\min\{b,\kappa/\alpha\}$ then

$$\Pi_B(\alpha,\varepsilon)=(1-s)\left[\int_{b}^{\infty}(\Delta-b)\,dF(\Delta)+\int_{\min\{b,\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}\}}^{b}(\Delta-\varepsilon)\,dF(\Delta)\right].$$

Clearly, $\Pi_B(\alpha,\varepsilon) \geq \Pi_B(0,\varepsilon)$ for $\alpha > 0$ where the inequality is strict if $b > \varepsilon + \frac{\kappa/\alpha-\varepsilon}{s}$. Suppose $b > \varepsilon + \frac{\kappa/\alpha-\varepsilon}{s}$, and note that

$$\frac{\partial \Pi_B(\alpha,\varepsilon)}{\partial \varepsilon} = \left(\frac{1-s}{s}\right)^2 (\kappa/\alpha - \varepsilon) f\left(\varepsilon + \frac{\kappa/\alpha-\varepsilon}{s}\right) - (1-s)\int_{\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}}^{b} dF(\Delta)$$

and $\lim_{b\to\varepsilon+\frac{\kappa/\alpha-\varepsilon}{s}} \frac{\partial \Pi_B(\alpha,\varepsilon)}{\partial \varepsilon} > 0$. This completes part (iv.a). To see part (iv.b), suppose that $\varepsilon > b$. Then,

$$\Pi_B(\alpha,\varepsilon) = (1-s) \times \begin{cases} \int_\varepsilon^\infty (\Delta-\varepsilon)\,dF(\Delta) & \textit{if } \alpha > 0 \\ \int_b^\infty (\Delta-b)\,dF(\Delta) & \textit{if } \alpha = 0. \end{cases}$$

Since

$$\varepsilon > b \Rightarrow \int_\varepsilon^\infty (\Delta-\varepsilon)\,dF(\Delta) < \int_b^\infty (\Delta-b)\,dF(\Delta)$$

and $\int_\varepsilon^\infty (\Delta-\varepsilon)\,dF(\Delta)$ is strictly decreasing in ε, $\Pi_B(\alpha,\varepsilon)$ is strictly decreasing in ε and $\Pi_B(\alpha,\varepsilon) < \Pi_B(0,\varepsilon)$ for all $\alpha > 0$ as required in the statement. Moreover, takeover probability is $1-F(\varepsilon)$ if the activist is present and $1-F(b)$ if the activist is not present, where the former decreases in ε and is larger than the latter since $\varepsilon > b$, concluding the proof. ■

C.2 Limited veto power and tender offers

Proposition 10 *Suppose the bidder can make a tender offer as described in Section 5.3, then:*

(i) If the first round of negotiations fails then the bidder never runs a proxy fight, while the activist runs a proxy fight if and only if

$$\frac{\kappa/s}{\lambda\alpha} \leq \Delta < b, \tag{33}$$

in which case she wins.

(ii) The expected shareholder value is

$$q + \lambda v(\alpha\lambda) + (1-\lambda)\varphi\int_0^\infty \Delta dF(\Delta) \tag{34}$$

where $v(\cdot)$ is given by (5).

Proof. Suppose the second round of negotiations fails. All parties involved expect that with probability $1-\lambda$ the bidder will offer target shareholders $q + \varphi\Delta$ and take over the firm, and with probability λ the target would remain independent. The bidder's expected profit is $(1-\lambda)(1-\varphi)\Delta$, and hence, he will not agree to pay a premium larger than

$$\Delta(1-(1-\lambda)(1-\varphi)).$$

Therefore, if the bidder controls the target board in the second round then he would "reach an agreement" in which the takeover offer is q. Suppose the incumbent controls the target board in the second round. If no agreement is reached with the bidder, the incumbent's expected payoff per share is

$$q+\lambda b+(1-\lambda)\varphi\Delta.$$

Therefore, the incumbent would reject any premium lower than $\lambda b+(1-\lambda)\varphi\Delta$. It follows that an agreement between the bidder and the incumbent is reached if and only if

$$\lambda b+(1-\lambda)\varphi\Delta \leq \Delta(1-(1-\lambda)(1-\varphi)) \Leftrightarrow b \leq \Delta.$$

If $b \leq \Delta$ then the target is taken over by the bidder and the expected premium is

$$s(1-(1-\lambda)(1-\varphi))\Delta+(1-s)[\lambda b+(1-\lambda)\varphi\Delta]=(1-\lambda)\varphi\Delta+\lambda[s\Delta+(1-s)b],$$

and if $b>\Delta$ then no agreement is reached between the incumbent and the bidder. In this case, the target is taken over through tender offer with probability $1-\lambda$, in which case, the premium is $\varphi\Delta$.

A similar analysis follows when the activist controls the board, only then b is replaced by zero everywhere. That is, the bidder always reaches an agreement with the activist, and the expected premium is

$$(1-\lambda)\varphi\Delta+\lambda s\Delta.$$

To conclude, in the second round of negotiations the expected shareholder value conditional on Δ is

$$\Pi_{SH}(\Delta)=\begin{cases} q+(1-\lambda)\varphi\Delta+\mathbf{1}_{\{b\leq\Delta\}}\cdot\lambda[s\Delta+(1-s)b] & \textit{if the incumbent board retains control} \\ q+(1-\lambda)\varphi\Delta+\lambda s\Delta & \textit{if the activist controls the board,} \\ q & \textit{if the bidder controls the board.} \end{cases} \tag{35}$$

Therefore, shareholders are always worse off under the control of the bidder, and prefer the activist over the incumbent if and only if $b > \Delta$. Similar to part (i) of Proposition 1, shareholders never elect the bidder to the board, and the latter never runs a proxy fight. Similar to part (ii) of Proposition 1, the activist runs a proxy fight if and only if $b > \Delta$ and

$$\alpha \left(q + \lambda s\Delta + (1-\lambda)\varphi\Delta\right) - \kappa \geq \alpha \left(q + (1-\lambda)\varphi\Delta\right) \Leftrightarrow \Delta \geq \frac{\kappa/s}{\alpha\lambda}. \tag{36}$$

This concludes part (i) of the proposition.

Consider part (ii). There are two cases. First, suppose that $\Delta \geq \min\{b, \frac{\kappa/s}{\alpha\lambda}\}$. Then, given the arguments above, all parties involved expect that if the first round of negotiations fails, takeover would take place with probability one with an expected price per share of

$$q + (1-\lambda)\varphi\Delta + \lambda \times \begin{cases} s\Delta + (1-s)\,b & \textit{if } \Delta \geq b \\ s\Delta & \textit{if } \Delta < b \text{ and } \Delta \geq \frac{\kappa/s}{\alpha\lambda} \end{cases}. \tag{37}$$

Therefore, similar to Proposition 2, in the first stage the bidder and incumbent reach an agreement where the offer per share is (37). Second, suppose that $\Delta < \min\{b, \frac{\kappa/s}{\alpha\lambda}\}$. Note that if the first round of negotiations fails, then given the arguments above the activist does not run a proxy fight and the second round of negotiations fails as well. Therefore, the payoffs of the incumbent board and the bidder if the first round round of negotiations fails are identical with their respective payoffs if the second round of negotiations fails. Hence, combined with $\Delta < b$, the first round of negotiations fails due to the same reasoning of why the second round of negotiations fails, the latter of which was shown above. Therefore, both rounds of negotiations fail, and the target is taken over through tender offer with probability $1-\lambda$, in which case, the premium is $\varphi\Delta$. Hence, expected shareholder value is

$$q + (1-\lambda)\,\Delta\varphi. \tag{38}$$

The term $q + \lambda v(\alpha\lambda) + (1-\lambda)\varphi\int_0^\infty \Delta dF(\Delta)$ is the integration of (37) and (38) over all values of Δ. ■

C.3 Hidden values

In reality, corporate boards often have private information about the standalone value of the target q, and bidders often have private information about the expected synergy Δ. The baseline model abstracts from these information asymmetries and the resulting adverse selection in order to focus on agency problems as the key friction. This section shows that the asymmetric information can in fact exacerbate the commitment problem of bidders in takeovers and sometimes enhance the ability of the activist to resolve it.

C.3.1 Uncertainty about q

Incumbent boards often justify their resistance to takeovers by claiming that the fundamental value of the target under their control is higher than the proposed takeover offer, even if the offer represents a significant premium relative to the unaffected stock price. Essentially, they claim that based on their private information the target is undervalued by the market as a standalone firm. In this section we solve the baseline model under the assumption that $q \in \{q_L, q_H\}$ is uncertain, $q_H > q_L \geq 0$, and q is privately observed by whoever controls the target board, including the activist and the bidder if they win a proxy fight. We denote the prior by $\tau = \Pr[q = q_H]$. We also assume that the identity of the proposer, the value of the offer, and the counter-party response (i.e., accept or reject) are made public in each round. We focus attention on Perfect Bayesian Equilibria in pure strategies. Therefore, any equilibrium is either pooling or fully separating.

Lemma 2 *Suppose no information about q is revealed in the first round of negotiations, and consider the second round of negotiations.*

(i) *If the bidder controls the target board then:*

- (a) *If $\Delta \geq E[q] - q_L$ then the bidder offers shareholders $E[q]$ and takes over the target with probability one.*
- (b) *If $\Delta < E[q] - q_L$ then the bidder offers shareholders q_H and takes over the firm if and only if $q = q_H$.*

(ii) *If the target board has private benefits of control per share $\beta \in \{0, b\}$ and the bidder makes an offer to the target board then:*

- (a) *If $\Delta \geq \beta + \frac{1-\tau}{\tau}(q_H - q_L)$ then the bidder offers $q_H + \beta$ and the board accepts the offer with probability one.*
- (b) *If $\beta \leq \Delta < \beta + \frac{1-\tau}{\tau}(q_H - q_L)$ then the bidder offers $q_L + \beta$ and the board accepts the offer if and only if $q = q_L$.*

(c) *If $\Delta < \beta$ then the takeover always fails.*

(iii) *If the target board has private benefits of control per share $\beta \in \{0, b\}$ and the target board makes an offer to the bidder then:*

(a) *If $\Delta \geq \beta + (1-\tau)(q_H - q_L)$ then the target board asks for $E[q] + \Delta$ regardless of his type and the bidder accepts the offer.*

(b) *If $\beta \leq \Delta < \beta + (1-\tau)(q_H - q_L)$ then the target board asks for $q_L + \Delta$ if $q = q_L$ and the bidder accepts the offer. If $q = q_H$ the target remains independent.*

(c) *If $\Delta < \beta$ then the takeover always fails.*

Proof. Suppose information about q is not revealed in the first round. The proxy fight stage does not reveal any information about q, since q is only observed by this stage by the incumbent.

Consider part (i) and suppose the bidder controls the target board. There is no information asymmetry between the bidder and the target board (since it is controlled by the bidder), but target shareholders still need to approve the deal. We proceed in four steps. First, we show that the takeover succeeds with a strictly positive probability in any equilibrium. To see why, suppose on the contrary that the takeover always fails. Therefore, no offer $\pi' \in [q_H, q_H + \Delta]$ is on equilibrium path, because otherwise it would be accepted by shareholders. However, since $\Delta > 0$, if $q = q_H$ then the bidder strictly prefers an off-equilibrium offer $\pi'_H \in (q_H, q_H + \Delta)$ over his equilibrium offer since the former would be accepted by shareholders and generate a profit, creating a contradiction.

Second, consider a pooling equilibrium where the takeover always takes place. Shareholders accept the pooling offer only if it is higher than $E[q]$. The bidder has incentives to make the pooling offer when $q = q_L$ only if it is smaller than $q_L + \Delta$. Therefore, a pooling equilibrium exists if and only if $E[q] \leq q_L + \Delta$. In this case, the target is taken over for sure. Notice that the only pooling equilibrium that survives the Grossman and Perry (1986) criterion is the one in which the pooling offer is $E[q]$.

Third, consider a separating equilibrium. There are three sub-cases to consider:

1. The bidder makes different offers depending on q and the takeover always takes place. However, the bidder has incentives to deviate to offering the lower offer even if $q = q_H$. So this equilibrium cannot exist.

2. The takeover takes place if and only if $q = q_L$. Suppose the bidder offers π^* when $q = q_L$. However, the bidder has incentives to deviate by offering π^* also when $q = q_H$. So this equilibrium cannot exist.

3. The takeover takes place if and only if $q = q_H$: if $q = q_L$ the bidder does not take over the firm and if $q = q_H$ the bidder offers π_H and the offer is accepted by shareholders. This is an equilibrium only if $\pi_H = q_H$, because if $\pi_H > q_H$ then whenever $q = q_H$ the bidder is strictly better off by deviating to an offer $\pi' \in (q_H, \pi_H)$ which would be always accepted by the shareholders, and if $\pi_H < q_H$ then shareholders would reject π_H. Therefore, $\pi_H = q_H$. Moreover, this can be an equilibrium only if shareholders reject any offer lower than q_H. However, off-equilibrium beliefs that support this equilibrium and satisfy the Grossman and Perry (1986) criterion exist if and only if $E[q] > q_L + \Delta$.

Fourth, overall, if the off-equilibrium beliefs are required to satisfy the Grossman and Perry (1986) criterion, then the unique outcome is as described in part (i) of the proposition's statement. In this case, target shareholder expected value is $E[q]$.

Consider part (ii). Suppose the target board has private benefit of control per share $\beta \in \{0, b\}$ and the bidder makes an offer to the target board ($\beta = 0$ if the activist controls the board and $\beta = b$ if the incumbent retains control). Since $\beta \geq 0$ shareholders approve any offer that is approved by the target board. If $\Delta < \beta$ then a takeover can never succeed, because otherwise either the bidder or the target board (or both) make negative profit on the equilibrium path. In this case, in equilibrium, the bidder makes an offer strictly smaller than $q_L + \beta$, which is always rejected by the target board. Suppose that $\Delta > \beta$.[35] If the bidder offers $q_H + \beta$ then the takeover succeeds for sure. If the bidder offers $q_L + \beta$ the takeover succeeds with probability $1 - \tau$, only when $q = q_L$. The bidder prefers the higher offer if and only if

$$\begin{aligned} E[q] + \Delta - q_H - \beta &\geq (1 - \tau)(q_L + \Delta - q_L - \beta) \Leftrightarrow \\ \Delta &\geq \beta + \frac{1 - \tau}{\tau}(q_H - q_L). \end{aligned}$$

Note that the bidder does not have any incentive to make any other offer. Hence if $\Delta \geq \beta + \frac{1-\tau}{\tau}(q_H - q_L)$ the offer is pooling and shareholder value is $q_H + \beta$, if $\beta \leq \Delta < \beta + \frac{1-\tau}{\tau}(q_H - q_L)$ the offer is separating and shareholder value is $E[q] + (1 - \tau)\beta$, and if $\Delta < \beta$ the takeover never takes place and shareholder value is $E[q]$.

[35] If $\Delta = \beta$ then the equilibrium can have the properties of parts (ii.b) or (ii.c).

Consider part (iii). Suppose the target board has private benefit of control per share $\beta \in \{0, b\}$ and the target board makes an offer to the bidder. Note that shareholders approve any offer asked by the target board since $\beta \geq 0$. If $\Delta < \beta$ then a takeover can never succeed, because otherwise either the bidder or the target board (or both) make negative profit on the equilibrium path. In this case, in equilibrium, the target board always asks a sufficiently high offer that is rejected by the bidder. This equilibrium can be supported by off-equilibrium beliefs that $q = q_L$ upon observing any off-equilibrium path offer $\pi'' \geq q_L + \beta$, which satisfy the Grossman and Perry (1986) criterion. Suppose $\Delta > \beta$.[36] We proceed in three steps:

1. First, we show that in any equilibrium the takeover succeeds with a strictly positive probability. Suppose not. Then, no offer $\pi' \in [q_L + \beta, q_L + \Delta]$ is on the equilibrium path, because otherwise it would be accepted by the bidder and the shareholders. However, in any such equilibrium if $q = q_L$ then the target board strictly prefers any off-equilibrium offer $\pi' \in (q_L + \beta, q_L + \Delta)$ over the equilibrium offer since the former would be accepted by the bidder and shareholders, creating a contradiction.

2. Second, suppose the target board makes a pooling offer where the takeover always takes place. Then, he must ask the bidder to pay no more than $E[q] + \Delta$. The board has incentives to make this offer when $q = q_H$ only if it is higher than $q_H + \beta$. Therefore, the pooling equilibrium exists if and only if

$$E[q] + \Delta - q_H - \beta \geq 0 \Leftrightarrow \Delta \geq \beta + (1 - \tau)(q_H - q_L).$$

   When it exists, the pooling equilibrium requires that the off-equilibrium beliefs are such that higher offers are rejected by the bidder. Notice, however, that the only pooling equilibrium that survives the Grossman and Perry (1986) criterion is the one in which the pooling offer is $E[q] + \Delta$.

3. Third, suppose the target board makes a separating offer. Then, the takeover cannot take place with probability one, because otherwise the target board making the lower equilibrium offer strictly prefers making the higher equilibrium offer. Moreover, the takeover takes place if and only if $q = q_L$, because otherwise if $q = q_L$ then the target board strictly prefers making the equilibrium offer that is made when $q = q_H$. Therefore, it must be that the target board is asking from the bidder no more than $q_L + \Delta$ when $q = q_L$ and this offer is accepted, and when $q = q_H$ his offer is rejected by the bidder.

[36] If $\Delta = \beta$ then the equilibrium can have the properties of parts (iii.b) or (iii.c).

Moreover, the target board has no incentive to ask for the separating offer when $q = q_H$ if and only if the separating offer is smaller than $q_H + \beta$. In addition, since the bidder accepts any offer equal to or smaller than $q_L + \Delta$ under any off-equilibrium beliefs, the separating offer made when $q = q_L$ is at least $q_L + \Delta$, and since the bidder has to make nonzero profit in equilibrium, it cannot be strictly larger than $q_L + \Delta$. Hence, the separating offer is $q_L + \Delta$. Therefore, the separating equilibrium exists if and only if

$$q_L + \Delta \leq q_H + \beta.$$

This equilibrium, however, survives the Grossman and Perry (1986) criterion if and only if $E[q] + \Delta - q_H - \beta < 0$.

We conclude, if $\Delta \geq \beta + (1-\tau)(q_H - q_L)$ the offer is pooling and shareholder value is $E[q] + \Delta$, if $\beta \leq \Delta < \beta + (1-\tau)(q_H - q_L)$ the offer is separating and shareholder value is $E[q] + (1-\tau)\Delta$, and if $\Delta < \beta$ the takeover never takes place and shareholder value is $E[q]$. ■

Lemma 3 *Suppose the first round of negotiations fails and no information about q is revealed. Then:*

(*i*) *The bidder never runs a proxy fight.*

(*ii*) *If the activist owns α shares of the target, the activist runs a proxy fight if and only if $\Delta \in \Gamma(\alpha)$ where*

$$\Gamma(\alpha) = \left\{ \Delta : \frac{\frac{1}{\tau}\frac{\kappa/\alpha}{s} - \frac{1-\tau}{\tau}\frac{1-s}{s}\begin{bmatrix} (q_H - q_L)\cdot \mathbf{1}_{\left\{\frac{1-\tau}{\tau}(q_H-q_L)\leq\Delta\right\}} \\ -b\cdot\mathbf{1}_{\{b\leq\Delta\}} \end{bmatrix}}{\frac{1-\tau}{\tau}\cdot\mathbf{1}_{\{0\leq\Delta<b\}} + \mathbf{1}_{\{(1-\tau)(q_H-q_L)\leq\Delta<b+(1-\tau)(q_H-q_L)\}}} \leq \Delta < b + \frac{1-\tau}{\tau}(q_H - q_L) \right\} \tag{39}$$

Whenever the activist runs a proxy fight, she wins.

Proof. Suppose no information about q is revealed in the first stage. Based on part (i) of Lemma 2, shareholder value under the bidder's control is $E[q]$. Therefore, electing the bidder to the board is a weakly dominated strategy, and strictly dominated if extraction of value is possible. Based on parts (ii) and (iii) of Lemma 2, the expected shareholder value under the incumbent's control is

$$s\left[\begin{array}{c} E\left[q\right]+\left(1-\tau\right)\Delta\cdot\mathbf{1}_{\{\Delta\geq b\}} \\ +\tau\Delta\cdot\mathbf{1}_{\{\Delta\geq b+(1-\tau)(q_H-q_L)\}} \end{array}\right]+\left(1-s\right)\left[\begin{array}{l} E\left[q\right]+\left(1-\tau\right)b\cdot\mathbf{1}_{\left\{b+\frac{1-\tau}{\tau}(q_H-q_L)>\Delta\geq b\right\}} \\ +\left(\left(1-\tau\right)\left(q_H-q_L\right)+b\right)\cdot\mathbf{1}_{\left\{\Delta\geq b+\frac{1-\tau}{\tau}(q_H-q_L)\right\}} \end{array}\right],$$

and the expected shareholder value under the activist's control, if she chooses to run a proxy fight, is

$$s\left[\begin{array}{c} E\left[q\right]+\left(1-\tau\right)\Delta\cdot\mathbf{1}_{\{\Delta\geq 0\}} \\ +\tau\Delta\cdot\mathbf{1}_{\{\Delta\geq(1-\tau)(q_H-q_L)\}} \end{array}\right]+\left(1-s\right)\left[\begin{array}{c} E\left[q\right] \\ +\left(1-\tau\right)\left(q_H-q_L\right)\cdot\mathbf{1}_{\left\{\Delta\geq\frac{1-\tau}{\tau}(q_H-q_L)\right\}} \end{array}\right].$$

The activist runs a proxy fight if and only if the increase in value under her control is greater than κ/α, which holds if and only if $\Delta\in\Gamma\left(\alpha\right)$. ■

Remark: Based on Lemma 3, note that

$$\begin{aligned}\lim_{b\to\infty}\Gamma\left(\alpha\right) &= \left\{\Delta:\frac{\frac{1}{\tau}\frac{\kappa/\alpha}{s}-\frac{1-\tau}{\tau}\frac{1-s}{s}\left(q_H-q_L\right)\cdot\mathbf{1}_{\left\{\frac{1-\tau}{\tau}(q_H-q_L)\leq\Delta\right\}}}{\frac{1-\tau}{\tau}\cdot\mathbf{1}_{\{0\leq\Delta\}}+\mathbf{1}_{\{(1-\tau)(q_H-q_L)\leq\Delta\}}}\leq\Delta\right\} \\ &= \begin{cases}\left[\min\left\{\frac{\kappa/\alpha}{s}\frac{1}{1-\tau},\left(1-\tau\right)\left(q_H-q_L\right)\right\},\infty\right) & \textit{if } \frac{\kappa/\alpha}{s}\frac{1}{1-\tau}\leq q_H-q_L \\ \left[\min\left\{\frac{\kappa/\alpha}{s},\frac{1-\tau}{\tau}\left(q_H-q_L\right)\right\},\infty\right) & \textit{if } \frac{\kappa/\alpha}{s}\frac{1}{1-\tau}\frac{1}{\frac{1}{\tau}+\frac{1-s}{s}}\leq q_H-q_L<\frac{\kappa/\alpha}{s}\frac{1}{1-\tau} \\ \left[\frac{\kappa/\alpha}{s}-\left(1-\tau\right)\frac{1-s}{s}\left(q_H-q_L\right),\infty\right) & \textit{if } q_H-q_L<\frac{\kappa/\alpha}{s}\frac{1}{1-\tau}\frac{1}{\frac{1}{\tau}+\frac{1-s}{s}}\end{cases}\end{aligned}$$

This demonstrates that if q_H-q_L is large then $\lim_{b\to\infty}\Gamma\left(\alpha\right)\subset\left[\frac{\kappa/\alpha}{s},\infty\right)$ and if q_H-q_L is small then $\left[\frac{\kappa/\alpha}{s},\infty\right)\subset\lim_{b\to\infty}\Gamma\left(\alpha\right)$. Thus, adverse selection can either increase or decrease the incentives of the activist to run a proxy a fight. Finally, note that

$$\lim_{s\to 0}\Gamma\left(\alpha\right)=\begin{cases}\left[\frac{1-\tau}{\tau}\left(q_H-q_L\right),b+\frac{1-\tau}{\tau}\left(q_H-q_L\right)\right) & \textit{if } \frac{\kappa/\alpha}{1-\tau}+b\leq q_H-q_L \\ \left[\min\left\{b,\frac{1-\tau}{\tau}\left(q_H-q_L\right)\right\},b\right) & \textit{if } \frac{\kappa/\alpha}{1-\tau}\leq q_H-q_L<\frac{\kappa/\alpha}{1-\tau}+b \\ \varnothing & \textit{if } q_H-q_L<\frac{\kappa/\alpha}{1-\tau}\end{cases}$$

This demonstrates that unlike the baseline model, here the activist may run a proxy fight even if $\Delta>b$. Intuitively, the activist who is less biased against the takeover can overcome the adverse selection problem while the incumbent cannot.

To conclude, the existence of private information reduces the bidder's credibility even further since it creates adverse selection and additional opportunities for the bidder to abuse the power of the target board once it is given to him. The existence of private information, however, has an ambiguous effect on the activist. On the one hand, private information increases the activist's incentives to run a proxy fight since the activist can extract information rents from the bidder once she gets access to the target's private information. On the other hand, private information creates adverse selection which decreases the probability of reaching an acquisition agreement with the bidder, thereby weakening the activist's incentives to run a proxy fight. The latter effect dominates the former if $q_H - q_L$ is large.

C.3.2 Uncertainty about Δ

In this section we solve the baseline model under the assumption that $\Delta \in \{\Delta_L, \Delta_H\}$ is uncertain, $\Delta_H > \Delta_L > 0$, and Δ is privately observed by the bidder. We denote the prior by $\psi = \Pr[\Delta = \Delta_H]$. We also assume that the identity of the proposer, the value of the offer, and the counter-party response (i.e., accept or reject) are made public in each round. We focus attention on Perfect Bayesian Equilibria in pure strategies. Therefore, any equilibrium is either pooling or fully separating.

Lemma 4 *Suppose the first round of negotiations fails and no information about Δ is revealed in the first stage or in the proxy fight stage. Consider the second round of negotiations. Then:*

- (i) *If the bidder controls the target board then the bidder offers shareholders q and takes over the target with probability one.*
- (ii) *If the target board has private benefit of control per share $\beta \in \{0, b\}$ and the bidder is the proposer then:*
  - (a) *If $\beta \leq \Delta_L$ then the bidder offers $q + \beta$ and the board accepts the offer.*
  - (b) *If $\Delta_L < \beta \leq \Delta_H$ then the bidder offers $q + \beta$ when $\Delta = \Delta_H$ and the board accepts the offer, and when $\Delta = \Delta_L$ the takeover fails.*
  - (c) *If $\Delta_H < \beta$ then the takeover always fails.*
- (iii) *If the target board has private benefit of control per share $\beta \in \{0, b\}$ and the target board is the proposer then:*

(a) *If* $\beta \leq \frac{\Delta_L - \psi\Delta_H}{1-\psi}$ *then the target board asks for* $q + \Delta_L$ *and the bidder accepts the offer with probability one.*

(b) *If* $\frac{\Delta_L - \psi\Delta_H}{1-\psi} < \beta \leq \Delta_H$ *then the target board asks* $q + \Delta_H$ *and the bidder accepts the offer if and only if* $\Delta = \Delta_H$*.*

(c) *If* $\Delta_H < \beta$ *then the takeover always fails.*

Proof. Suppose information about Δ is not revealed in the first round or in the proxy fight stage. There are three cases. First, suppose the bidder controls the target board. There is no information asymmetry between the bidder and the target board, but target shareholders still need to approve the deal. Shareholders will approve any offer higher than q regardless of their beliefs about Δ. Since $\Delta_L \geq 0$, regardless of the realization of Δ the bidder offers shareholders q and the offer is accepted. In this case target shareholder value is q. Notice that this argument holds for any set of shareholder beliefs about Δ.

Second, suppose the target board has private benefit of control per share $\beta \in \{0, b\}$ and the bidder is the proposer. Notice that regardless of his beliefs about Δ, the target board rejects any offer below $q + \beta$ and accepts any offer above $q + \beta$. Therefore, the bidder has incentives to offer $q + \beta$, provided that her expected profit is non-negative. This completes part (ii).[37]

Third, suppose the target board has private benefits of control per share $\beta \in \{0, b\}$ and the target board is the proposer. Since $\beta \geq 0$ shareholders approve any offer that is asked by the target board. If $\Delta_H < \beta$ then a takeover can never succeed, because otherwise either the bidder or the target board (or both) make negative profit on the equilibrium path. In this case, in equilibrium, the target board makes an offer strictly larger than $q + \Delta_H$, which is always rejected by the bidder. Suppose $\Delta_H > \beta$.[38] If the board asks for $q + \Delta_L$ then the takeover succeeds for sure, and the board's expected profit is $\Delta_L - \beta$. If the board asks for $q + \Delta_H$ then the takeover succeeds with probability ψ, only when $\Delta = \Delta_H$, and the board's expected profit is $\psi\left(\Delta_H - \beta\right)$. The board prefers the former over the latter if and only if

$$\Delta_L - \beta \geq \psi\left(\Delta_H - \beta\right) \Leftrightarrow \beta \leq \frac{\Delta_L - \psi\Delta_H}{1-\psi},$$

where $\frac{\Delta_L - \psi\Delta_H}{1-\psi} < \Delta_L < \Delta_H$. Note that the target board does not have any incentive to make any other offer. This completes part (iii). ■

[37] Note that if $\Delta_H = \beta$ then the equilibrium can have the properties of parts (ii.b) or (ii.c).

[38] If $\Delta_H = \beta$ then the equilibrium can have the properties of parts (iii.b) or (iii.c).

Lemma 5 *Suppose the first round of negotiations failed and no information about* Δ *was revealed. Then:*

(i) The bidder never runs a proxy fight, and no information about Δ *is revealed.*

(ii) If the activist owns α *shares of the target, the activist runs a proxy fight if and only if* $b \in \Lambda(\alpha)$ *where*

$$\Lambda(\alpha) = \left\{ b : 0 \leq b \leq \frac{\max\{\psi\Delta_H, \Delta_L\} - \psi\Delta_H \cdot \mathbf{1}_{\left\{\frac{\Delta_L - \psi\Delta_H}{1-\psi} < b \leq \Delta_H\right\}} - \Delta_L \cdot \mathbf{1}_{\left\{b \leq \frac{\Delta_L - \psi\Delta_H}{1-\psi}\right\}} - \frac{\kappa/\alpha}{s}}{\frac{1-s}{s}\left[\mathbf{1}_{\{b \leq \Delta_L\}} + \psi \cdot \mathbf{1}_{\{\Delta_L < b \leq \Delta_H\}}\right]} \right\}$$

Whenever the activist runs a proxy fight, she wins.

Proof. Based on part (i) of Lemma 4, shareholder value under the bidder's control is q regardless of their beliefs about Δ. Therefore, electing the bidder to the board is a weakly dominated strategy, and strictly dominated if extraction of value is possible. This also implies that the bidder's decision not to run a proxy fight is not informative about Δ.

Based on parts (ii) and (iii) of Lemma 4, the expected shareholder value when the target board's private benefits are $\beta \in \{0, b\}$ is

$$\begin{aligned} \Pi_{SH} \quad (\beta) &= q + s\left[\psi\Delta_H \cdot \mathbf{1}_{\left\{\frac{\Delta_L - \psi\Delta_H}{1-\psi} < \beta \leq \Delta_H\right\}} + \Delta_L \cdot \mathbf{1}_{\left\{\beta \leq \frac{\Delta_L - \psi\Delta_H}{1-\psi}\right\}}\right] \\ &+ (1-s)\beta\left[\mathbf{1}_{\{\beta \leq \Delta_L\}} + \psi \cdot \mathbf{1}_{\{\Delta_L < \beta \leq \Delta_H\}}\right]. \end{aligned}$$

Therefore, the activist runs a proxy fight if and only if

$$\Pi_{SH}(0) - \Pi_{SH}(b) \geq \kappa/\alpha,$$

which holds if and only if $b \in \Lambda(\alpha)$. Since $\kappa > 0$, whenever the activist runs a proxy fight, she is elected by shareholders. ■

C.4 Biased Activist

In this section we analyze the baseline model under the assumption that the activist is biased and has private benefits from control. Specifically, we assume that the activist and the bidder can divert a non-trivial amount of corporate resources as private benefits after winning control of the target board, if the target remains independent. We denote the value transfer by $q\eta$, where $\eta \in (0,1)$. In the baseline model, $\eta \to 0$. For simplicity, we assume that the transfer involves no deadweight loss. Moreover, consistent with the common critique that activist investors have a short-term investment horizon (for example, because of their desire to establish reputation, higher alternative cost of capital, or the need to meet interim fund out-flows), we assume that the activist discounts the standalone value of the firm by $1-\gamma \in [0,1]$. Since the proceeds from a takeover are received by shareholders before the standalone value of the firm is realized, this assumption implies that relative to other investors, the activist is biased toward selling the firm.

Lemma 6 *In the second round of negotiations, the target is acquired by the bidder unless the incumbent board retains control and $\Delta < b$, or the activist obtains control and $\Delta < \beta(\alpha,\eta)$ where*

$$\beta(\alpha,\eta) \equiv q[\eta(1-\alpha)/\alpha - \gamma(1-\eta)]. \tag{40}$$

Moreover, the expected shareholder value is given by

$$\Pi_{SH} = \begin{cases} q + \mathbf{1}_{\{b \le \Delta\}} \cdot [s\Delta + (1-s)b] & \text{if the incumbent board retains control,} \\ (1-\eta)\, q & \text{if the bidder controls the board,} \\ (1-\eta)\, q & \text{if the activist controls the board and } \beta(\alpha,\eta) > \Delta, \\ q + s\Delta + (1-s)\, m(\alpha,\eta) & \text{if the activist controls the board and } \beta(\alpha,\eta) \le \Delta. \end{cases} \tag{41}$$

where

$$m(\alpha,\eta) \equiv \max\{-\eta q, \beta(\alpha,\eta)\} \tag{42}$$

Proof. There are three scenarios to consider. In the first scenario, the incumbent board retains control of the target. Then the proof is identical to the first scenario of Lemma 1. In the second scenario, the bidder controls the board. With control, the bidder uses the board's authority to sign on a deal that offers target shareholders the lowest amount they would accept. Moreover, by controlling the target board, the bidder can extract ηq from the target's standalone value. Therefore, in this case, the bidder offers shareholders $(1-\eta)q$, and shareholders, who at this point cannot prevent the bidder from extracting ηq, accept this offer. In the third scenario, the activist controls the target board. If no agreement is reached with the bidder, the activist's payoff is $\alpha q(1-\eta)(1-\gamma) + q\eta$. Abusing her control of the board, the activist extracts $q\eta$ from the target. In addition, each share of the target has a value of $q(1-\eta)(1-\gamma)$, which is the standalone value of the target from the activist's perspective, taking into account the adverse effect of the value extraction and the activist's higher discount rate, $1-\gamma$. The activist agrees to sell the firm if and only if her proceeds from the takeover are higher than $\alpha q(1-\eta)(1-\gamma)+q\eta$, which holds if and only if $q+\beta(\alpha,\eta)\leq\pi_2$. Once the activist has control of the board, shareholders would vote to approve the takeover if and only if the price is higher than $q(1-\eta)$, and the bidder will never offer more than $q+\Delta$. The bidder and the activist reach an acquisition agreement that is acceptable to shareholders if and only if $\beta(\alpha,\eta)\leq\Delta$. If $\beta(\alpha,\eta)>\Delta$ then the firm remains independent, and the long term shareholder value is $q(1-\eta)$. If $\beta(\alpha,\eta)\leq\Delta$ then the firm is sold to the bidder in the second round of negotiations. With probability s the activist offers $\pi_2=q+\Delta$, an offer which is always accepted by the bidder and the target shareholders, and with probability $1-s$ the bidder offers $\pi_2=q+\max\{-\eta q,\beta(\alpha,\eta)\}$, which is always accepted by the activist and the shareholders. ■

Lemma 7 *Suppose the first round of negotiations fails. Then:*

(i) The bidder never runs a proxy fight.

(ii) If the activist owns α shares in the target, the activist runs a proxy fight if and only if

$$\rho(\alpha,\eta)\leq\Delta<b \quad or \quad b+\frac{\kappa/\alpha}{1-s}\leq\beta(\alpha,\eta)\leq\Delta, \tag{43}$$

where

$$\rho(\alpha,\eta)\equiv\max\left\{\beta(\alpha,\eta),-\frac{1-s}{s}m(\alpha,\eta),\frac{\kappa/\alpha-q\gamma}{s}-\frac{1-s}{s}m(\alpha,\eta)\right\}. \tag{44}$$

Whenever the activist runs a proxy fight, she wins.[39]

Proof. Consider part (i). Based on Lemma 6, without the ability to commit not to abuse the power of the board, target shareholders are always worse off if they elect the bidder. Since $\kappa>0$, no party initiates a proxy fight she expects to lose. Hence, in equilibrium the bidder never runs a proxy fight. Consider part (ii). Based on Lemma 6, if $\beta(\alpha,\eta)>\Delta$ and the activist obtains control, shareholder value is $q(1-\eta)$, and therefore, shareholders would support the incumbent. Suppose $\beta(\alpha,\eta)\leq\Delta$. There are two cases to consider. First, if $b\leq\Delta$ then

[39] If $b+\frac{\kappa/L}{1-s}\leq\beta(L,\eta)\leq\Delta$ then shareholders elect the activist to the board not because the incumbent would otherwise block the takeover (as in the baseline model), but rather because the activist can negotiate a higher takeover premium, similar to our analysis in Section 5.1.

under the incumbent's control $\Pi_{SH}(\Delta) = q + s\Delta + (1-s)b$, while under the activist's control $\Pi_{SH}(\Delta) = q + s\Delta + (1-s)\,m(\alpha,\eta)$. Therefore, shareholder support the activist if and only if $b \leq m(\alpha,\eta)$. Since $b \geq 0$, this condition becomes $b \leq \beta(\alpha,\eta) \leq \Delta$. The activist runs a proxy fight only if

$$\alpha\left[q + s\Delta + (1-s)\,\beta(\alpha,\eta)\right] - \kappa \geq \alpha\left[q + s\Delta + (1-s)\,b\right] \Leftrightarrow \beta(\alpha,\eta) \geq b + \tfrac{\kappa/\alpha}{1-s}. \tag{45}$$

Combined, the activist runs a proxy fight if and only if $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha) \leq \Delta$, as required. Second, suppose $\Delta < b$. Under the incumbent's control $\Pi_{SH}(\Delta) = q$, while under the activist's control $\Pi_{SH}(\Delta) = q + s\Delta + (1-s)\,m(\alpha,\eta)$. Therefore, shareholders support the activist only if $-\frac{1-s}{s}m(\alpha,\eta) \leq \Delta$. Combined, the condition becomes

$$\max\left\{\beta(\alpha,\eta), -\tfrac{1-s}{s}m(\alpha,\eta)\right\} \leq \Delta < b. \tag{46}$$

Provided the activist is getting the support from shareholders, she runs a proxy fight if and only if

$$\alpha\left[q + s\Delta + (1-s)\,m(\alpha,\eta)\right] - \kappa \geq \alpha q\,(1-\gamma) \Leftrightarrow \tfrac{\kappa/\alpha - q\gamma}{s} - \tfrac{1-s}{s}m(\alpha,\eta) \leq \Delta \tag{47}$$

Combined, the activist initiates a proxy fight if and only if $\rho(\alpha,\eta) \leq \Delta < b$, as required. ■

Proposition 11 *Suppose the bidder identifies firm i as a target and the activist owns α shares of that firm. Then, the unconditional shareholder value of firm i is $q + \tilde{v}(\alpha)$, where $\tilde{v}(\cdot)$ is given by*

$$\begin{aligned}\tilde{v}(\alpha) &= \int_b^\infty \left[s\Delta + (1-s)b\right] dF(\Delta) + \int_{\min\{b,\rho(\alpha,\eta)\}}^b \left[s\Delta + (1-s)\,m(\alpha,\eta)\right] dF(\Delta) \\ &\quad + \mathbf{1}_{\{b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha,\eta)\}} \int_{\beta(\alpha,\eta)}^\infty (1-s)\left(\beta(\alpha,\eta) - b\right) dF(\Delta),\end{aligned} \tag{48}$$

Proof. Given Lemma 7 and Lemma 6, the proof is similar to the proof of Proposition 2, and for brevity, we only highlight the differences. Based on Lemma 7, if the activist owns α shares of the target and either $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha,\eta) \leq \Delta$ or $\rho(\alpha,\eta) \leq \Delta < b$, then the activist would run a successful proxy fight if the first round of negotiations fails. Based on Lemma 6, all players expect that once the activist obtains control of the board, she will reach a sale agreement in which the bidder pays in expectation $\pi_2'' = q + s\Delta + (1-s)\,m(\alpha,\eta)$ per share. Therefore, similar to the proof of Proposition 2, the incumbent and the bidder reach an agreement in the first round where the offer is π_2''. Note that if $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha,\eta) \leq \Delta$ then $0 < \beta(\alpha,\eta)$ and hence, $m\left(\alpha,\eta\right) = \beta\left(\alpha,\eta\right)$. In all other cases, if $b \leq \Delta$ the incumbent and the bidder reach an agreement in the first round where the offer is $q+s\Delta+(1-s)b$, and if $b > \Delta$ the target remains independent under the incumbent board's control. This explains the term behind $\tilde{v}\left(\alpha\right)$. ■