Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?
Author(s): Martin D. D. Evans and Karen K. Lewis
Source: *The Journal of Finance*, Vol. 50, No. 1 (Mar., 1995), pp. 225-253
Published by: Wiley for the American Finance Association
Stable URL: http://www.jstor.org/stable/2329244
Accessed: 08/10/2013 16:26

THE JOURNAL OF FINANCE • VOL. L, NO. 1 • MARCH 1995

# Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?

MARTIN D. D. EVANS and KAREN K. LEWIS*

**ABSTRACT**

Recent empirical studies suggest that nominal interest rates and expected inflation do not move together one-for-one in the long run, a finding at odds with many theoretical models. This article shows that these results can be deceptive when the process followed by inflation shifts infrequently. We characterize the shifts in inflation by a Markov switching model. Based upon this model's forecasts, we re-examine the long-run relationship between nominal interest rates and inflation. Interestingly, we are unable to reject the hypothesis that in the long run nominal interest rates reflect expected inflation one-for-one.

THE EX ANTE REAL interest rate affects all intertemporal savings and investment decisions in the economy. As such, the behavior of the ex ante real rate plays a central role in the dynamics of asset prices over time. Understanding the ex ante real interest rate and its relationship with other variables such as inflation is therefore a central issue in the study of financial markets.

Our aim in this article is to explain some puzzling aspects of the postwar data revealed by recent research on the long-run behavior of the ex ante real interest rate. Rose (1988), King and Watson (1992), Mishkin (1992), and Crowder and Hoffman (1992) present evidence that real rates are subject to permanent disturbances. These findings are puzzling because, as Rose (1988) points out, they seem to directly contradict the first-order conditions of standard intertemporal models: when real interest rates are subject to permanent disturbances, these models predict that consumption growth rates should also be affected by permanent disturbances, a hypothesis easily rejected in the data.

In this article, we provide an explanation for the apparently puzzling evidence suggesting that real interest rates are affected by permanent shocks. In particular, we consider information about the long-run behavior of the ex ante real interest rate inherent in the long-run behavior of nominal interest rates and inflation through the Fisher identity (Fisher (1930)).

* Evans is from New York University, and Lewis is from the University of Pennsylvania and the National Bureau of Economic Research. We are grateful for useful comments from Bob Barsky, John Campbell, Bob Cumby, Bill Crowder, Frank Diebold, Pierre Perron, and seminar participants at Columbia University, New York University, the Wharton Macro lunch seminar, Ohio State University, the Philadelphia Federal Reserve, and the National Bureau of Economic Research Summer Institute. Any errors are our own.

The long-run behavior of the ex ante real rate is linked directly to the long-run relationship between inflation and nominal interest rates. The Fisher identity defines the ex ante real rate as the difference between the nominal rate and expected inflation. Thus, for the ex ante real interest rate to be affected only by transitory disturbances, any permanent shocks to the nominal interest rate and expected inflation must cancel out through the identity. This is an important observation because recent research has found that both realized inflation and nominal interest rates are affected by permanent shocks. Since permanent shocks to rationally expected inflation match the permanent shocks to realized inflation, these findings imply that ex ante real rates will also be subject to permanent shocks unless nominal interest rates and realized inflation move one-for-one in the long run.

We begin our analysis by examining the long-run Fisher relationship. Using recently developed time-series methods, we estimate the long-run relationship between nominal interest rates and inflation. Based on these estimates, we find that inflation does not move one-for-one with the nominal rate in the long run. These results complement the earlier findings of permanent shocks in real rates in the literature.

To explain this result, we focus on the behavior of inflation. We argue that rational anticipations of infrequent shifts in the inflation process during the postwar sample induce significant small-sample biases in estimates of the long-run Fisher relationship. These small-sample biases create the appearance of permanent shocks to ex ante real rates even when none are truly present. Intuitively, when infrequent shifts in the process of inflation are rationally anticipated, forecast errors may be serially correlated over periods when the shifts do not materialize.[1] Over a longer period that includes many shifts in the inflation process, forecast errors will be serially uncorrelated. However, since the postwar data sample does not include a sufficient number of shifts in the inflation process, the small sample serial correlation in forecast errors induces biases in the estimates of the long-run Fisher relation.

In support of this argument, the paper next presents evidence that the process of inflation has shifted during the postwar period. We first estimate a Markov switching process that appears to characterize the data well. We also find that rational inflationary expectations based upon this Markov model are broadly consistent with Livingston survey forecasts. We then subtract these inflation forecasts from the nominal rate and find that the ex ante real rate based upon the Markov model systematically deviated from the ex post real rate (the nominal rate minus realized inflation) for some periods in the sample. This evidence supports the idea that deviations between ex ante and ex post real interest rates can appear highly persistent in small samples.

[1] The effect of discrete process shifts on forecast errors was first pointed out by Rogoff (1980) and has been subsequently referred to as the "peso problem." "Peso problem" behavior in interest rates and exchange rates is suggested by the evidence in Lewis (1991, 1989) and Evans and Lewis (1994).

We then consider whether expected shifts in inflation not realized over small samples can create the illusion that the ex post real rate has permanent disturbances, even though the ex ante real rate is affected only by transitory disturbances. We use our Markov model as a data-generating process to reexamine the long-run relationship between inflation and the nominal interest rate. In Monte Carlo experiments with sample sizes of 25 years, we show that estimates from cointegrating regressions would tend to reject too often the hypothesis that inflation and nominal interest rates move together one-for-one in the long run.[2] Indeed, when inferences are based on the Monte Carlo empirical distribution, our estimates imply that the long-run Fisher relationship cannot be rejected. Thus, the ex ante real rate appears to be stationary after adjusting for the potential effects of rationally anticipated shifts in inflation.

The structure of the article is as follows. Section I examines the long-run Fisher relation using standard-time-series methods. Section II provides evidence for shifts in the inflation process and shows how these shifts may be characterized by a Markov switching process. Section III uses this switching process as a data-generating process to reexamine the results in Section I. Section IV provides concluding remarks.

## I. The Real Interest Rate without Expected Shifts in the Inflation Process

### *A. Inflation and the Ex Ante Real Interest Rate*

The relationship between the ex ante real rate and inflation has been the subject of a number of articles. Mishkin (1981), Fama and Gibbons (1982), Huizinga and Mishkin (1984, 1986) and others, present evidence of significant negative correlations between ex ante real rates and inflation. In contrast to our analysis below, these articles treat real rates and inflation as stationary processes subject to only temporary disturbances.[3]

Theoretical explanations for this negative correlation fall into three groups. First, Tobin (1969) and Mundell (1976) argue that higher inflation leads to a portfolio shift out of nominal assets and into real assets, which, in turn, pushes down the return on these assets. Second, Fama (1981) and Fama and Gibbons (1982) argue that higher real rates represent greater productivity in the economy. Therefore, higher real rates are correlated with output disturbances. However, increases in output push up money demand. If the increase

[2] The Monte Carlo experiments also allow us to examine how many years of data would be required before the small sample bias in estimates of the long-run Fisher relationship disappear. Our experiments indicate that approximately 100 years of data would be required.

[3] A parallel literature examines the relationship between stock returns and inflation in the short run, finding a similar negative relationship. For example, see Fama and Schwert (1977) and Lintner (1975). Consistent with our long-run analysis below, Boudoukh and Richardson (1993) suggest that expected stock returns and inflation move closely together in the long run. Examining the effects of the inflation shifts we find below upon the long run relationship between stock returns and inflation may be an interesting issue for future research.

in money demand is not accommodated by higher money supply, then these output shocks will push down inflation. Hence, higher real rates will be correlated with lower inflation. General equilibrium models, like those of Danthine and Donaldson (1986) and Stulz (1986), provide a third explanation. Here a liquidity premium drives a wedge between real assets and nominal assets adjusted for inflation. Higher inflation raises the liquidity premium and therefore lowers real returns, generating the negative correlation between inflation and real rates.[4]

More recent studies find evidence of permanent disturbances in either nominal interest rates or inflation. This evidence has introduced a new dimension into the empirical study of real rates. In particular, the appearance of permanent disturbances in nominal interest rates but not inflation led Rose (1988) to conclude that ex ante real rates are also affected by permanent shocks. He argues that such a finding is inconsistent with standard models of intertemporal asset pricing.

To see why, consider the first-order condition of an infinitely lived consumer maximizing expected utility with a time-separable utility function (see Hansen and Hodrick (1983)):

$$1 = E\left[(1 + \rho^i(t))\beta\frac{u'(c(t+1)}{u'(c(t))}\middle|t\right], \qquad 0 < \beta < 1. \tag{1}$$

Here $\rho^i(t)$ is the (net) return (measured in terms of consumption goods) on any asset $i$ at time $t$, $u'(.)$ is marginal utility, $c(t)$ is consumption at $t$, and $E[.\,|\,t]$ denotes expectations conditional on information available at $t$. For the case where the asset is a bond paying off one unit of the consumption good in all states at time $t + 1$, equation (1) becomes

$$1 + r(t) = E\left[\beta\frac{u'(c(t+1)}{u'(c(t))}\middle|t\right]^{-1} \tag{2}$$

where $r(t)$ is the ex ante real interest rate. Thus, the real interest rate depends upon the expected intertemporal marginal rate of substitution in consumption.

Suppose now that the real rates are affected by permanent disturbances. The condition in equation (2) implies that this behavior of real rates must derive from permanent disturbances to the intertemporal marginal rate of substitution in consumption. For given utility functions, this hypothesis is directly testable. For example, Rose (1988) considers the constant relative risk aversion utility function where $u'(c) = c^{-\gamma}$, where $\gamma$ is the parameter of relative risk aversion. Permanent disturbances to the real rate therefore imply that there must be permanent disturbances to $(c(t+1)/c(t))^{-\gamma}$, a function of the consumption growth rate. However, the hypothesis of permanent shocks to consumption growth rates can be rejected in the data. Thus,

[4] Marshall (1992) considers the empirical significance of these three explanations.

from this perspective, the implication that real rates contain permanent shocks seems inconsistent with the behavior of consumption.

A number of recent studies including Mishkin (1992), Crowder and Hoffman (1992), and King and Watson (1992) find evidence that both inflation and the nominal interest rate contain permanent disturbances. These findings raise the possibility that permanent shocks to inflation have permanent effects upon real interest rates. To see why, consider the Fisher equation

$$r(t) = R(t) - E[\pi(t)|t] \tag{3}$$

where $R(t)$ is the yield on a nominal discount bond purchased at time $t$ and $\pi(t)$ is the inflation rate over the maturity of the bond.[5] According to equation (3), if both nominal interest rates and expected inflation are affected by a common set of permanent shocks, the ex ante real rate will also be affected by these same shocks *unless* nominal rates and expected inflation respond to them one-for-one. Without a one-for-one response, the real rate will share the same permanent shocks as expected inflation. This possibility appears inconsistent with superneutrality insofar as permanent shocks to expected inflation originate from long-run changes in the rate of monetary growth.[6]

Below, we begin our analysis by reexamining standard findings in the literature that suggest the real interest rate has permanent disturbances. A conventional interpretation of our results is that ex ante real rates are subject to permanent shocks, which are shared by (expected) inflation. With this view, we would be left with trying to explain the inconsistencies noted above. However, we will later provide an alternative interpretation of the results in which real rates are, in fact, only affected by transitory disturbances.

### *B. Permanent Disturbances and Implications from the Fisher Equation Identity*

To corroborate results found elsewhere in the literature, Panel A of Table I reports unit root tests for nominal interest rates and inflation to test whether the hypothesis of permanent shocks can be rejected.[7] We use interest rates

[5] Shome, Smith, and Pinkerton (1988), Evans and Wachtel (1992), and others show how a generalized version of Fisher's equation can be derived from the first-order condition in equation (1). In addition to the variables shown in equation (3), these generalizations include an inflation risk premium for holding nominal bonds relative to real bonds and a term to account for the difference between expected inflation and the expected depreciation of money. Equation (3) may be viewed as incorporating these additional terms through a generalized Fisher equation, although Evans and Wachtel (1992) find them quantitatively unimportant. Since theory implies that these terms should be stationary, our long-run results below are not affected by them.

[6] Money is superneutral if changes in the long-run rate of monetary growth have no effect on real economic variables. Although there are theoretical reasons why superneutrality may not strictly hold, Danthine, Donaldson, and Smith (1987) argue that it is a reasonable approximation.

[7] Mishkin (1993) studies nominal interest rates and inflation rates over different maturities and finds that they contain unit roots when based upon standard tests. Other studies that find a unit root in inflation include Evans and Wachtel (1993), and Ball and Cecchetti (1990).

**Table I**

## Unit Root and Cointegration Tests

This table reports the results of unit root and cointegration tests on inflation and nominal interest rates. $\pi^k$ and $R^k$ are the $k$ month inflation and nominal interest rates and $E[\pi \mid L]$ is the Livingston survey forecast of inflation. Unit roots tests are shown in the columns headed by $\tau$, $t(\lambda)$ and $t^*(\lambda)$. Under $t$, we report augmented Dickey Fuller test statistics that allow for a constant, and time trend. Monthly tests include 6 lags of the first difference of the variable being tested in the regression, biannual tests include 1 lag: 5 and 1 percent critical values are $-3.43$ and $-3.99$ respectively. Under $t(\lambda)$, we show Zivot and Andrews' minimum $t$-statistics for a unit root allowing for shifts in the mean: 5 and 1 percent critical values are $-4.80$ and $-5.80$ respectively. Perron and Vogelsang's minimum $t$-statistics allowing for shifts in the mean are shown under $t^*(\lambda)$: 5 and 1 percent critical values are $-3.61$ and $-4.18$ respectively. In all cases, statistics less than the critical values indicate that the null hypothesis of a unit root can be rejected. Under the column headed $\lambda_{max}$ we show Johansen's maximum eigenvalue test for the number of stochastic trends in $[\pi^k(t), R^k(t)]$: 5 and 10 percent critical values are 14.595 and 12.783 respectively. Statistics greater than the critical value indicate that the null of no cointegration (or equivalently 2 stochastic trends) can be rejected. The parameter estimates and standard errors (shown in parenthesis) in Panel B correct for the finite sample bias present in cointegrating regressions using the method in Stock and Watson (1989). The standard errors are also corrected for the presence of conditional heteroskedasticity and moving average errors.

Panel A. Unit Root Tests

| | Variable (1) | $\tau$ (2) | $t(\lambda)$ (3) | $t^*(\lambda)$ (4) |
|---|---|---|---|---|
| Monthly data | $\pi^1$ | $-3.313$ | $-4.552$ | $-3.133$ |
| | $\pi^3$ | $-2.991$ | $-3.385$ | $-2.980$ |
| | $R^1$ | $-1.270$ | $-1.765$ | $-1.763$ |
| | $R^3$ | $-1.339$ | $-1.848$ | $-1.847$ |
| Biannual data | $E[\pi^6 \mid L]$ | $-1.958$ | $-5.959$ | $-5.959$ |
| | $\pi^6$ | $-1.517$ | $-4.174$ | $-4.174$ |

Panel B. Long-Run Fisher Equation Estimates

$$R(t) = \alpha_0 + \alpha_1 \pi(t) + \Sigma_{i=-6}^{6} a_i \, \Delta\pi(t-i) + v(t+k)$$

| Maturity in months (1) | Cointegration Tests $\lambda_{max}$ (2) | Parameter Estimates $\alpha_1$ (3) | Parameter Estimates $\Sigma_i a_i$ (4) | $t$-Statistics $H_0$: $\alpha_1 = 1$ (5) |
|---|---|---|---|---|
| 1 | 32.674 | 0.693 | $-0.484$ | 3.381 |
| | | (0.089) | (1.035) | |
| 3 | 38.818 | 0.739 | $-0.848$ | 3.006 |
| | | (0.087) | (1.279) | |

Panel C. Unit Root Tests on Ex Post Real Rates

| | Variable (1) | $\tau$ (2) | $t(\lambda)$ (3) | $t^*(\lambda)$ (4) |
|---|---|---|---|---|
| Monthly data | $R^1 - \pi^1$ | $-3.313$ | $-6.161$ | $-4.887$ |
| | $R^3 - \pi^3$ | $-3.144$ | $-6.301$ | $-5.288$ |

sampled monthly from the McCullough data set from January 1947 to February 1987. The inflation rates are calculated from the consumer price index (CPI)-X series published by the Bureau of Labor Statistics constructed by the Congressional Budget Office. This series corrects for a bias in weighting mortgage payments too heavily in the construction of the CPI prior to 1983 and has been used in other studies of real interest rates, such as Huizinga and Mishkin (1986). To allow for comparison with other studies, we examine both the one-month and one-quarter maturities. Since we will be interested in measures of expectations, we also report unit root tests based upon biannual observations using the Livingston survey of inflation, denoted $E[\pi \mid L]$.

Column 2 in the table reports the augmented Dickey Fuller (ADF) test statistics. The hypothesis of a unit root is not rejected at the 5 percent marginal significance level for any of the variables. Columns 3 and 4 present the minimum $t$-statistics developed in Zivot and Andrews (1992) and Perron and Vogelsang (1992), respectively. These statistics examine whether a structural shift in the mean of the variables can make the process appear to contain a unit root.[8] For the inflation and interest rate variables, the results of these tests are quite similar to the ADF results, and we cannot reject a unit root. While the tests using the Livingston survey data now appear to reject a unit root allowing for a shift, these statistics may be unreliable, since splitting the sample of biannual observations (as required by the test) significantly reduces the degrees of freedom.

The results in Panel A of Table I confirm the typical finding that inflation and nominal interest rates appear to contain permanent disturbances, generating long-run stochastic trends in these variables. We therefore begin our analysis by directly testing the implicit long-run relationship between the nominal rate and inflation, using recent techniques for time-series processes affected by permanent shocks. We will later demonstrate that the inferences based upon these methods can be deceiving in the present case.

It will prove convenient to use the Fisher equation in equation (3) to make inferences about the behavior of the unobserved ex ante real rate. The presence of the unobservable inflationary expectation in equation (3) implies that the ex ante real rate is unobservable as well. To address this problem, the standard approach in the literature is to assume that economic forecasts are unbiased so that $\pi(t) = E[\pi(t) \mid t] + \varepsilon(t)$, where $\varepsilon(t)$ is a forecast residual uncorrelated with all current information. Thus, substituting the actual

[8] This issue is raised by Perron (1989). He finds that for several macroeconomic series, the null hypothesis of a unit root process with drift and an exogenous break point could be rejected in favor of the alternative of a stationary process about a deterministic trend with an exogenous change in trend function. Both statistics test the null hypothesis of a unit root against the alternative that the process is trend stationary with a break in the trend occurring at an unknown point in time. The Perron and Vogelsang statistic, in addition, allows the a priori imposition of a one-sided change in the mean of the series.

for the expected inflation, equation (3) may be written as

$$R(t) = \pi(t) + r(t) - \varepsilon(t). \tag{3'}$$

In equation (3′), only the nominal rate, $R(t)$, and the inflation rate, $\pi(t)$, are observable. The difference between these two variables is identically the ex post real rate comprised of the ex ante real rate, $r$, and the inflation forecast residual, $\varepsilon(t)$.

Since inflation and nominal interest rates contain permanent disturbances, we can use recent time series techniques to infer whether the ex post real rate contains shocks with the same degree of persistence as those variables. In particular, the relationship between the unit root components in these two variables can be found by regressing one of the variables on the other through a cointegrating regression.[9] The intuition may be described by using the example of the following cointegrating regression of $R(t)$ on $\pi(t)$,

$$R(t) = \alpha_0 + \alpha_1 \pi(t) + v(t). \tag{4}$$

Parameter estimates from cointegrating regressions have different interpretations than do ordinary regressions on stationary variables.[10] For a set of variables with unit roots, these regressions provide parameter estimates of the particular linear combination of the variables that is stationary, if one exists. The cointegrating regression of $R(t)$ on $\pi(t)$ provides an estimate of $\alpha_1$ such that $R(t) - \alpha_1\pi(t)$ is a stationary variable. We will denote such a variable $I(0)$ as in the literature.

Comparing the identity in equation (3′) with the cointegration regression in equation (4) reveals that we should find $\alpha_1 = 1$ if the ex post real rate is $I(0)$ stationary since, by construction, $R(t) - \alpha_1\pi(t)$ is an $I(0)$ process. Therefore, the ex post real rate may be written in terms of the cointegrating regression (4) as:

$$r(t) - \varepsilon(t) \equiv R(t) - \pi(t) = -(1 - \alpha_1)\pi(t) + I(0) \text{ process}. \tag{4'}$$

When $\alpha_1$ equals unity, the ex post real rate is simply a stationary process. On the other hand, if $\alpha_1 \neq 1$, then the ex post real rate contains the same unit root component as inflation. Note that under standard rational expectations assumptions, forecast errors follow a stationary process. Hence, conventional assumptions about expectations would imply that if $\alpha_1 \neq 1$, then the ex ante real rate is subject to permanent disturbances.

### *C. The Empirical Results*

Panel B of Table I reports results from estimating the cointegrating regression in equation (4). For comparison with other studies, we report the

[9] This relationship holds only if the variables with unit roots are cointegrated. Below we present evidence that the variables are indeed cointegrated.

[10] Barsky (1987) provides an interpretation of equation (4) when inflation and nominal interest rates are stationary. For more discussion of cointegrating regressions, see Stock (1987) or the survey by Campbell and Perron (1991).

results for both one month and one quarter maturities.[11] We first test for cointegration between the nominal interest rates and inflation. The second column, labeled $\lambda_{\max}$, presents the maximum eigenvalue tests developed in Johansen (1991) and Johansen and Juselius (1990) for the hypothesis that the variables are not cointegrated. These statistics strongly reject this hypothesis.[12]

The third column labeled $\alpha_1$ reports the estimates and the standard errors of the cointegrating regression in equation (4). To obtain parameter estimates and standard errors that correct for the problem of finite sample bias present in cointegrating equations, we use the method developed in Stock and Watson (1989).[13] As these estimates show, the hypothesis that $\alpha_1 = 1$ is strongly rejected. The fourth column reports the sum of the coefficients on the leads and lags of the first differences of inflation, used in the Stock-Watson correction.

Since the estimates of $\alpha_1$ differs from one, equation (4′) shows that the ex post real interest rate shares the same permanent disturbances as the nominal interest rate and inflation. Another way to examine this effect is to test for a unit root in the ex post real rate. The statistics in Panel C of Table I show that we cannot reject a unit root in the ex post real rate at conventional significance levels using the ADF test. This finding accords with the results of Mishkin (1992) and King and Watson (1992).[14] In Panel C of Table I we also show the Zivot-Andrews and Perron-Vogelsang statistics testing for unit roots allowing for a shift in the mean. As these statistics show, the hypothesis is rejected at the 1 percent marginal significance level.[15]

It is also interesting to compare our estimates of $\alpha_1$ with the predicted effects of taxes on nominal interest income. Crowder and Hoffman (1992) point out that pre-tax nominal rates will not move one-for-one with inflation in the long run if post-tax real rates are unaffected by permanent changes in inflation. In particular, they examine the tax-adjusted Fisher equation

$$R(t) = [1/(1 - \tau)]\pi(t) + [1/(1 - \tau)]r(t) - \varepsilon(t) \tag{5}$$

where $\tau$ is the average marginal tax rate. Here a 1 percent increase in $\pi(t)$ raises pre-tax nominal rates by $1/(1 - \tau)$ so that the post-tax real return, $R(t)(1 - \tau) - \pi(t) = r(t)$, remains unaffected. Comparing equation (5) with

[11] Evans and Lewis (1992) report similar results for maturities of 6 and 9 months and for one and two years.

[12] Similar results are reported by Crowder and Hoffman (1992).

[13] An explanation of the Stock-Watson method and how it is implemented in the current situation is given in the Appendix in Evans and Lewis (1992). We use this method instead of the maximum likelihood-based Johansen method because of its tractability in allowing for conditional heteroskedasticity and moving average components in the residuals.

[14] Mishkin notes, however, that the critical levels applied to these tests can be incorrect if the real rate is affected by moving average error terms. On the basis of Monte Carlo experiments, he concludes that the Fisher effect may hold in the long run, even though conventional unit root tests applied to the real rate cannot reject.

[15] This finding is similar to those in Garcia and Perron (1993) who argue that the ex post real rate may be characterized as a random process with a changing mean rather than a unit root.

the cointegrating regression in equation (4), we should expect to find $\alpha_1 = [1/(1-\tau)] > 1$, for $\tau > 0$. Clearly, since the estimates of $\alpha_1$ in Panel B of Table I are less than one, tax effects alone cannot explain our results.

A conventional interpretation of these results is that ex ante real rates are subject to the permanent shocks shared by (expected) inflation. We now offer an alternative explanation. We will argue that standard cointegrating regression estimates of $\alpha_1$ differ from one because rational expectations of infrequent shifts in the inflation process induce small sample biases that persist even in fairly long samples. In particular, these small sample biases can generate estimates of $\alpha_1$ less than one, even though the tax-adjusted Fisher equation holds over a range of marginal tax rates. We will also show that the real rate appears stationary after allowing for anticipated shifts in the inflation process. Thus, we will simultaneously address how anticipated infrequent shifts in the inflation process affect the cointegrating relationship between inflation and the nominal rate and the stationarity of the ex post real rate after allowing for shifts in the mean.

## II. Switches in the Inflation Process

In this section, we examine the potential for small sample problems in the cointegrating regression estimates in Table I arising from rationally anticipated, but infrequent, shifts in the inflation process. We first present evidence that the inflation process shifted over the sample period and then estimate an inflation model that incorporates these shifts. The model is then used to reassess our results in Table I.

### *A. Are There Structural Breaks in the Inflation Process?*

Two approaches are used to investigate whether the inflation rate over the post-war period experienced structural breaks. We informally examine the Brown, Durbin, and Evans (1975) CUSUM statistics and then formally test for structural instability using methods developed by Perron (1991) and Hansen (1991).

The CUSUM statistics are calculated as the cumulated sum of recursive residuals from a rolling regression AR(3) model for the first difference of quarterly inflation. This model allows for permanent shocks to the level of inflation and appears to capture all the serial correlation in the first difference when estimated over the whole sample. If there are no structural breaks in the inflation process, the CUSUM statistics plotted in Figure 1 should remain fairly stable. The figure shows that this is not the case. The statistic shifts upward sharply with realized inflation around the first oil price shock in 1974. Between 1974 and 1985 the CUSUM statistics appear relatively more volatile. The statistic is characterized by a strong upward swing in 1981 and a sharp downward jump in 1983. This graphical evidence suggests that inflation shifted upward in 1974 and perhaps in 1981, became more variable from 1974 through about 1985, and shifted down around 1983.

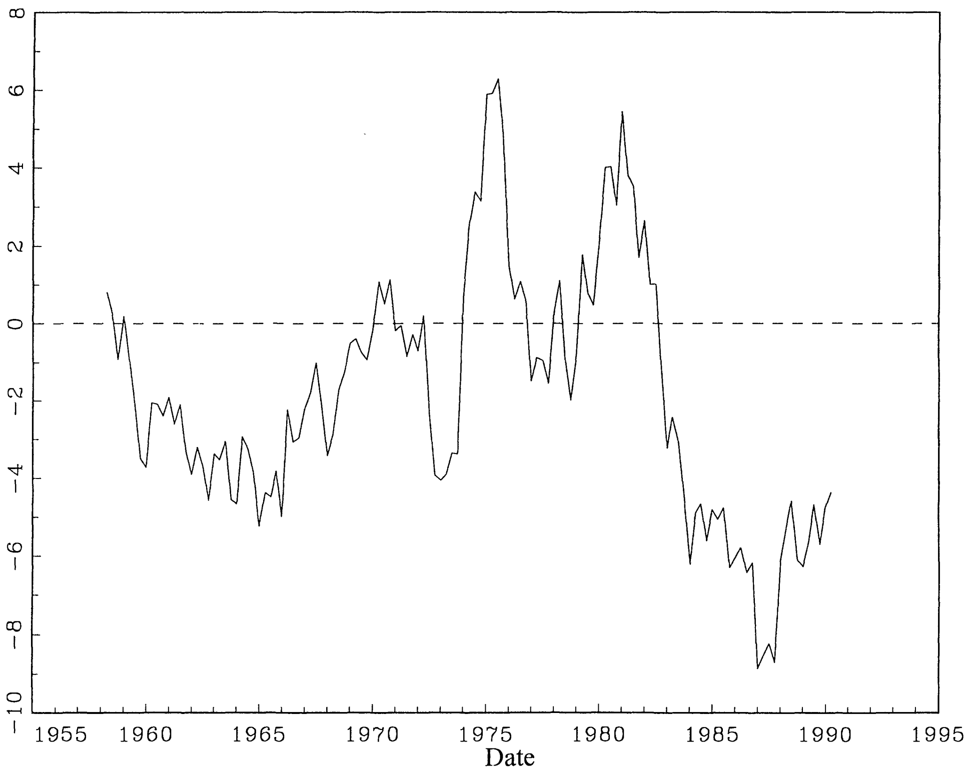


**Figure 1. CUSUM test for structural instability in quarterly inflation.** This figure plots the cumulated sum of recursive residuals from a rolling regression Ar(3) model for the first difference of quarterly inflation.

To test parameter constancy more formally, we begin by testing the null hypothesis of no change in the deterministic trend function of inflation using the Perron (1991) procedure, a test that is valid whether or not inflation is stationary. Table II shows that the null hypothesis is not rejected at the 5 percent significance level.[16] We next use the $L$-statistic proposed in Hansen (1991) to test for parameter stability in the first difference of inflation against the null hypothesis that the parameters follow a martingale process. The test statistics are based on the estimates of the AR(3) model for the first difference of quarterly inflation (estimated over the whole sample) shown in Table II. The lower portion of the table presents the $L$-statistics for the constancy of different combinations of parameters. These statistics reveal that the null hypothesis of no structural instability is rejected in the variance and the autoregressive parameters at the 5 percent significance level.

In summary, the inflation process appears to have undergone structural

[16] However, Perron (1991) notes that this test will have low power if the time series tested has unit roots, as Table I suggests may be true for inflation.

**Table II**

**Tests for Structural Instability in Quarterly Inflation**

This table reports the results of structural instability tests on the process for quarterly inflation, $\pi(t)$. The Perron $QD$ statistics test for structural change in the trend function for quarterly inflation $N_t$, parameterized as $N_t = \gamma_0 + \gamma_1 t$. The Hansen $L$ statistics test for structural changes in an AR(3) model for the first difference of inflation, $\Delta\pi(t)$. * and ** indicate a rejection of the hypothesis of parameter stability at the 5 and 1 percent significance levels, respectively. The estimated AR(3) model and the residual autocorrelations are shown at the bottom of the table, where $\rho_i$ denotes the autocorrelation at lag $i$. Time periods are measured in quarters. d.f. signifies degrees of freedom.

| | |
|---|---|
| *QD* statistics | |
| Constant alone (1 d.f.) | 0.236 |
| Constant and slope (2 d.f.) | 0.124 |
| *L* statistics | |
| Constant alone (1 d.f.) | 0.074 |
| Variance alone (1 d.f.) | 0.959** |
| Constant and variance (2 d.f.) | 1.043* |
| Constant, variance, and AR parameters (5 d.f.) | 1.649* |

Model estimates

$$\Delta\pi(t) = \underset{(0.161)}{0.095} - \underset{(0.083)}{0.673}\ \Delta\pi(t-1) - \underset{(0.088)}{0.458}\ \Delta\pi(t-2) - \underset{(0.041)}{0.255}\ \Delta\pi(t-3) + \varepsilon_t$$

Residual correlations

| $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_6$ |
|---|---|---|---|
| 0.017 | 0.070 | 0.064 | −0.195 |

shifts during the sample period. Below, we will show that these shifts can be characterized as a Markov switching process.

### *B. A Markov Model of Inflation Regimes*

To this point our findings suggest that a specification of the inflation process should allow for both the presence of permanent shocks and for discrete shifts in the process. In addition, the graphical evidence from the CUSUM statistics suggests that an upward shift in the process takes place around 1974, followed by higher variability in the process.

To capture these features, we estimate a Markov switching process for quarterly inflation. We model inflation comprising two independent components, a nonstationary component that follows a random walk, and a serially uncorrelated stationary component.[17] Unlike other structural time-series models, our model allows the innovation variances to the two components to

[17] We also estimated models where the stationary component was serially correlated. These models did not characterize the data any better than the simple specification presented here.

vary across states:

$$\begin{aligned} \pi(t) &= \mu(t) + \xi(t) & \xi(t) &\sim N\big(0, \sigma_\xi^2(s(t))\big) \\ \mu(t) &= \mu(t-1) + \zeta(t) & \zeta(t) &\sim N\big(0, \lambda(s(t))^2 \sigma_\xi^2(s(t))\big) \end{aligned} \tag{6}$$

The dynamics of inflation vary according to changes in the state variable, $s_t$, which follows an independent first-order Markov process and takes the values of one and zero. Changes in the state alter the variance of the temporary inflation component, $\sigma_\xi^2(s(t))$, and the innovation variance to the permanent component, $\sigma_\zeta^2(s(t)) \equiv \lambda(s(t))^2\sigma_\xi^2(s(t))$. The parameter $\lambda$ is defined as the ratio of innovation standard deviations. Since equation (6) implies that $\Delta\pi(t) = \zeta(t) + \xi(t) - \xi(t-1)$, an increase in $\lambda$ across states reduces the degree of mean reversion in the first difference of inflation. An increase in $\lambda$ can therefore be considered an increase in the persistence of inflation.

Table III presents the maximum likelihood estimates of the model in equation (6) with some of the specification tests.[18] The table shows that the parameters are generally precisely estimated. The probabilities of remaining in each state from one quarter to the next, $\Pr(s(t) \mid s(t-1))$ are above 93 percent. One state has both a higher variance of the transitory disturbance and a higher ratio of the permanent to the transitory disturbance. Specifically, in state $s(t) = 0$, $\sigma_\xi^2(0) = 3.077$, which is greater than its counterpart in state $s(t) = 1$, $\sigma_\xi^2(1) = 1.162$. Also, in State 0, $\lambda(0) = 0.574$, which is greater than in State 1, $\lambda(1) = 0.301$. Thus, the change in inflation in State 0 exhibits both greater variance and appears to have greater persistence. Though both states have a unit root component, we will refer to State 0 as the "high persistence" state below for expositional simplicity.

The lower panel of Table III reports the results of some specification tests. The first two rows report tests for the hypothesis of no serial correlation in the residuals in each state. These chi-squared statistics are Lagrange Multiplier (LM) tests of a zero restriction on the first-order autocorrelation of the residuals. As the table shows, this restriction is not rejected at the 5 percent level for either state, although the hypothesis is rejected at the 10 percent level for State 1. The next two rows report statistics that test for conditional heteroskedasticity in the residuals in each state. The reported statistics are LM tests for the hypothesis that the first-order ARCH coefficient is zero. This restriction is not rejected at standard confidence levels.

As Engel and Hamilton (1990) and others have noted, testing whether a Markov switching model includes an appropriate number of states poses some knotty econometric problems.[19] Rather than directly test whether speci-

[18] The model is estimated using an extension of Hamilton's (1989) filtering algorithm that accounts for the fact that neither inflation component is directly observed. This algorithm is discussed in detail in Evans (1993) and Kim (1994).

[19] The information matrix for the model is singular under the null hypothesis of a single state, and the standard regularity conditions for establishing asymptotically valid tests for different numbers of states do not hold.

**Table III**

**Estimates of Markov Model for Quarterly Inflation**

This table reports the maximum likelihood estimates of the Markov switching model for quarterly inflation, $\pi(t)$. Inflation comprises a permanent component, $\mu(t)$, and temporary component, $\xi(t)$. $s(t)$ is a state variable that follows an independent first-order Markov process and takes the values of zero and one. The transition probabilities for this process are shown as $\Pr(1\,|\,1)$ and $\Pr(0\,|\,0)$. Changes in the state alter the innovation variances for both inflation components. Asymptotic standard errors are reported in parenthesis below the parameter estimates.

Panel A. Model Estimates (Quarterly Data)

| | |
|---|---|
| $\pi(t) = \mu(t) + \xi(t)$ | $\xi(t) \sim N(0, \sigma_\xi^2(s(t)))$ |
| $\mu(t) = \mu(t-1) + \zeta(t)$ | $\zeta(t) \sim N(0, \lambda(s(t))^2\sigma_\xi^2(s(t)))$ |
| $\lambda(1) =$ 0.301 (0.108) | $\lambda(0) =$ 0.578 (0.188) |
| $\sigma_\xi^2(1) =$ 1.162 (0.364) | $\sigma_\xi^2(0) =$ 3.077 (1.000) |
| $\Pr(1\,|\,1) =$ 0.936 (0.053) | $\Pr(0\,|\,0) =$ 0.961 (0.027) |

Panel B. Specification Tests

| | Statistic | Significance |
|---|---|---|
| LM test for first-order serial correlation in state $s(t) = 1$: | 3.569 | 0.060 |
| LM test for first-order serial correlation in state $s(t) = 0$: | 0.139 | 0.709 |
| LM test for first-order ARCH in state $s(t) = 1$: | 0.042 | 0.839 |
| LM test for first-order ARCH in state $s(t) = 0$: | 2.450 | 0.118 |
| Test for no change in persistence, $\lambda(1) = \lambda(0)$: | 5.557 | 0.020 |
| Test for Markov independence, $\Pr(1\,|\,1) = 1 - \Pr(0\,|\,0)$: | 759.070 | < 0.001 |

fications with one or three states are preferable to our two state model, we consider two related hypotheses that can be tested without running into these problems. First we test the null hypothesis of no difference in the ratio parameter across states (i.e., $\lambda(1) = \lambda(0)$). As Table III shows, the hypothesis is rejected at the 5 percent level. This finding supports the idea that the persistence of inflation changes during the sample period even after we allow for conditional heteroskedasticity. Second, we test whether the distribution of $s(t)$ is independent of $s(t-1)$ so that both the persistence and variance of inflation vary independently from period to period. Again, Table III shows that this hypothesis is strongly rejected in the data. Taken together, these tests indicate that our switching model provides a good characterization of inflation.

Figure 2 shows some of the implications of the model estimates in Table III. The top panel depicts the probability of being in the high persistence state from the 1950s until the late 1980s. As the figure shows, the probability of being in this state is high in the late 1950s, but then falls in the early 1960s. From 1961 until 1974, the probability never exceeds about 30 percent. The inflation process appears to change sharply in 1974 with the first oil price

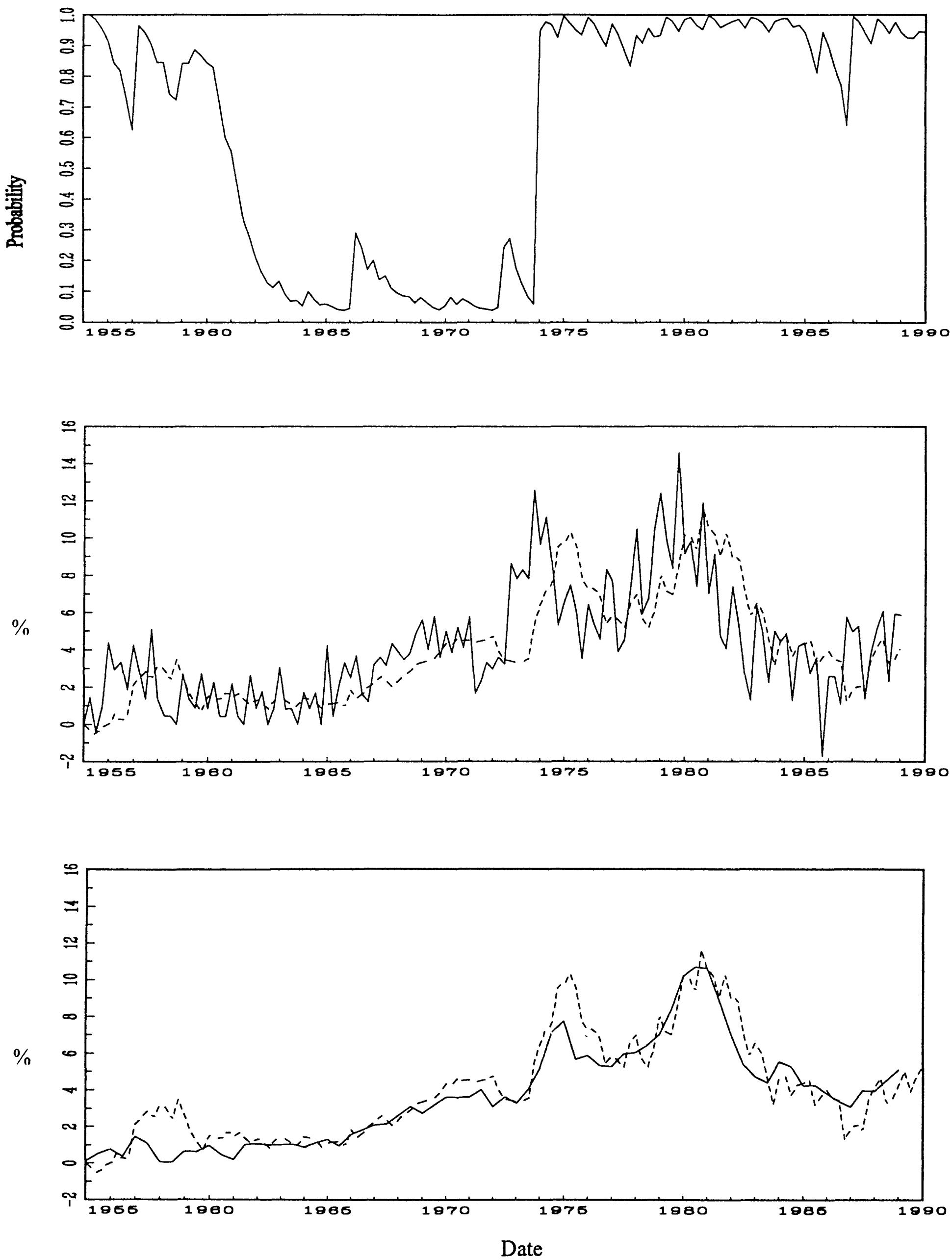


**Figure 2. Markov model estimates.** The *top panel* of the figure shows the probability of being in the high persistence state, $s(t) = 1$. The *middle panel* plots the Markov forecasts of quarterly inflation one year ahead as ——, and the actual rate of inflation as ---. The *bottom panel* shows the Markov forecasts as ---, and the biannual forecasts of the six-month inflation rate from the Livingston survey as ——.

shock, and the probability of the high-persistence state increases dramatically to above 90 percent. The probability remains high thereafter, only dipping to below 65 percent in 1987. Interestingly, the sharp movements in the probability of the high persistence state correspond roughly to the points of structural change in inflation suggested by the CUSUM statistics in Figure 1.

The middle panel of Figure 2 plots the Markov forecasts of quarterly inflation one year ahead together with realized inflation. During the runup in inflation from 1967 to 1974, the inflation forecasts persistently lag behind realized inflation. On the other hand, when inflation turns down beginning in 1980, the large probability of remaining in the high persistence state keeps inflation forecasts above realized inflation. The figure suggests that the forecast residual, identified by the vertical difference between realized inflation and its forecast, is serially correlated, even though forecasts are rational by construction with respect to the sample.

To see if the model captures the general pattern of survey expectations, the bottom panel of Figure 2 compares the Markov forecasts with biannual forecasts of the six-month inflation rate from the Livingston survey. As the figure shows, the Markov forecasts follow the general movements in survey expectations, though the Markov forecasts exhibit greater variation.

The Markov model estimates can also be used, together with nominal interest rates, to construct an estimate of the implied ex ante real interest rate. We calculate the ex ante real rate expected one year ahead by subtracting the Markov forecasts shown in the middle panel of Figure 2 from the one-year forward rate on a three-month bond. These expected ex ante real rates and the ex post real interest rates based upon actual inflation are plotted in Figure 3. As the figure shows, the ex ante real interest rate does not systematically diverge from the ex post real rate until the early 1970s. The sharp decline in the ex post real rate in 1973–1974 does not appear in the ex ante real rate suggesting that the oil price shock to inflation was largely unanticipated by individuals basing their forecasts on anticipated shifts in the inflation process. There is also a wide divergence between the rates in the late 1970s when the ex post real rates again turned sharply negative while the ex ante rate remained positive, reaching 2 percent on occasion. During the early 1980s both the ex post and ex ante real interest rates rose sharply, although the ex ante real rates implied by the model did not rise as much.

### *C. A Monte Carlo Investigation of Rational Forecast Residuals*

We now evaluate whether expectations of shifts in the inflation process in postwar data could have incorrectly led to a rejection of the hypothesis that the Fisher equation holds in the long run, as found in Table I. In particular, we want to examine whether small sample serial correlation in inflation

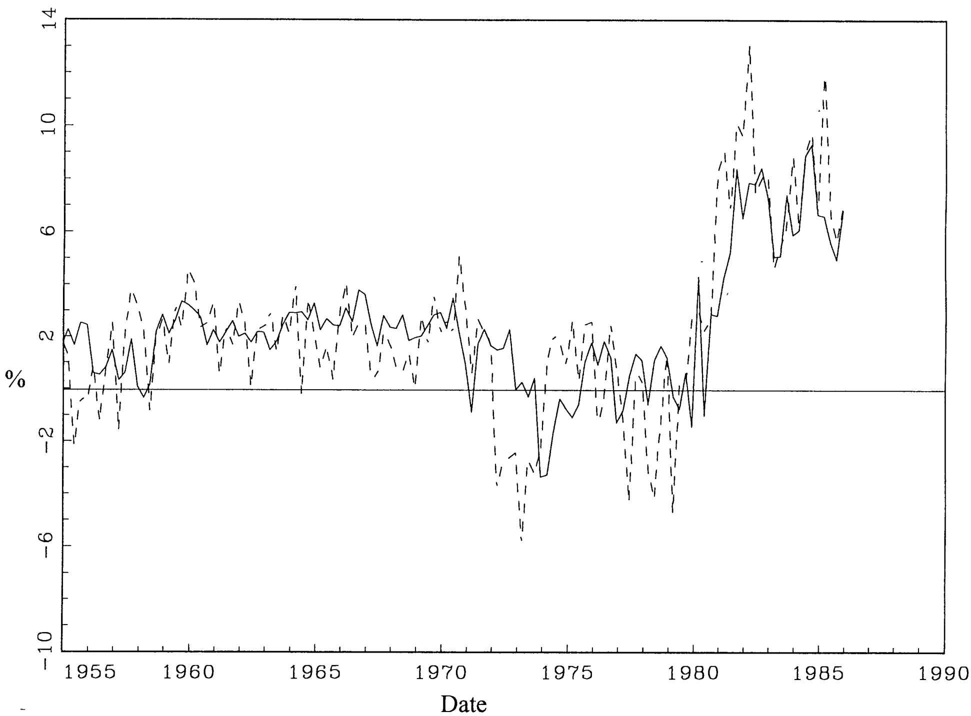


**Figure 3. Expected and actual three-month real rate forecasts one year ahead.** The expected real rate (shown as ——) is calculated as the difference between the one year forward rate on a three month nominal bond and the Markov forecasts of expected The actual rate (shown as ---) is the difference between the forward rate and actual inflation.

forecast errors, introduced by shifts in the inflation process, could introduce a bias into the cointegrating regressions in Table I.

To see the origins of the small sample correlation in the forecast errors, consider the following decomposition of inflation between $t$ and $t + 1$:

$$\begin{aligned}\pi(t) &= E[\pi(t)|s(t+1)=0,\Omega(t)] \\ &\quad + \Delta E[\pi(t)|s(t+1),\Omega(t)]s(t+1) + u(t+1) \\ \Delta E[\pi(t)|s(t+1),\Omega(t)] &\equiv E[\pi(t)|s(t+1)=1,\Omega(t)] \\ &\quad - E[\pi(t)|s(t+1)=0,\Omega(t)], \qquad (8)\end{aligned}$$

where $E[\pi(t) \mid s(t+1), \Omega(t)]$ are the rational forecasts of inflation between $t$ and $t + 1$ conditional on current information $\Omega(t)$, and the future state, $s(t + 1)$.[20] Since rational forecasts are equal to true mathematical expectations, taking conditional expectations of both sides of equation (8) gives $E[u(t+1) \mid s(t+1), \Omega(t)] = 0$. Thus the within-regime error terms, $u(t + 1)$, are uncorrelated with $s(t + 1)$ and the elements of $\Omega(t)$.

[20] Note that equation (8) is an identity and is not based upon any particular inflation process.

The inflation forecast errors are found by subtracting the rate of inflation expected by individuals at time $t$, who do not know the future state $s(t+1)$, from actual inflation in equation (8). Expected inflation is

$$\begin{aligned} E[\pi(t)|\Omega(t)] &= E[\pi(t)|s(t+1)=1,\Omega(t)]\Pr(s(t+1)=1|\Omega(t)) \\ &\quad + E[\pi(t)|s(t+1)=0,\Omega(t)]\Pr(s(t+1)=0|\Omega(t)). \end{aligned} \tag{9}$$

Subtracting equation (9) from (8) and simplifying, we obtain

$$\begin{aligned} \pi(t) - E[\pi(t)|\Omega(t)] &= u(t+1) + \Delta E[\pi(t)|s(t+1),\Omega(t)] \\ &\quad \times \{s(t+1) - E[s(t+1)|\Omega(t)]\}. \end{aligned} \tag{10}$$

The forecast errors in equation (10) are comprised of two terms. The first term on the right, the within-regime error, $u(t+1)$, has mean zero and is uncorrelated with information known at time $t$, $\Omega(t)$. The second term depends upon the difference between the conditional forecasts of future inflation across future states and the error in forecasting $s(t+1)$. In a large sample where there are many changes in $s(t)$, $\{s(t+1) - E[s(t+1)|\Omega(t)]\}$ will be serially uncorrelated and will have an unconditional mean of zero. In this case, the inflation forecast errors will be serially uncorrelated even when $\Delta E[\pi(t)|s(t+1),\Omega(t)] \neq 0$. By contrast, in a small sample where there are few changes in $s(t)$, $\{s(t+1) - E[s(t+1)|\Omega(t)]\}$ may display a good deal of sample serial correlation. When future states affect forecasts so that $\Delta E[\pi(t)|s(t+1),\Omega(t)] \neq 0$, equation (10) shows that this will introduce a small sample serial correlation into the inflation forecast errors.

To examine whether small sample serial correlation could introduce significant bias into our estimates of the long-run Fisher relationship, we conduct a series of Monte Carlo experiments based upon the Markov switching model of inflation described above. We begin with experiments that examine the cointegrating relationship between actual inflation and expected inflation. The first cointegrating regression considered is:

$$E[\pi(t)|\Omega(t),M] = \gamma_0 + \gamma_1\pi(t,M) + \eta(t) \tag{11}$$

where $E[\pi(t)|\Omega(t),M]$ is the expected inflation rate, and $\pi(t,M)$ is the realized inflation process, both based upon the estimated Markov model. The estimates of $\gamma_1$ from this regression allow us to examine whether small sample serial correlation in inflation forecast errors from the Markov models could introduce a bias into the cointegrating regressions in Table I.[21]

In the experiments, we use the estimated Markov model to generate samples for realized and expected inflation of 25 years, as in our data set, and also for samples of 50 years and 100 years. By comparing the results from these simulated samples we can examine how long the span of a data set

[21] Note that the regression in equation (11) is equivalent to equation (4) if the real rate is constant. Below we conduct experiments that allow for variations in the real rate in order to reevaluate the results in Table I.

must be before the small-sample effects vanish.[22] For each sample, we then estimate the cointegrating regression in equation (11) using the Stock-Watson method. The empirical distributions of the estimates for $\gamma_1$ are produced by replicating this procedure 1000 times. The Appendix contains a description of these Monte Carlo experiments.

In the first set of experiments we assume individuals observe the history of states and inflation when making their forecasts. In these experiments, which we refer to as Experiment A, individuals immediately recognize any shift in inflation. They focus upon the role of anticipated future shifts in the inflation process, the so-called "peso problem."[23]

Table IV shows the coefficients $\gamma_1$ at different percentiles of these Monte Carlo distributions. The top left-hand panel of Table IV shows the cumulative distribution function for the $\gamma_1$ coefficients generated by Experiment A based upon quarterly inflation. The panel shows that the probability of finding estimates of $\gamma_1$ less than one is quite high in a sample size of 25 years. Furthermore, as the sample size increases to 100 years, the probability declines.

We next conduct a set of experiments (Experiment B) assuming that individuals do not immediately know the state, but learn about the change over time. Forecasts of inflation are therefore conditioned on just the past history of inflation. This set of experiments incorporates forecast errors based upon both a "peso problem" from anticipated future changes in the process and learning about the current inflation process.

As before, the cointegrating regression in equation (11) is repeatedly estimated to generate the empirical distribution of $\gamma_1$. The right-hand section of Panel A of Table IV depicts the cumulative distributions for these coefficient estimates. Again the probability of observing an estimate of $\gamma_1$ less than one is quite high in data samples of 25 years. Furthermore, this probability declines with increasing sample sizes.

A second way to examine whether the forecast residuals from the Markov model introduce a bias in the cointegrating regression is to run these regressions the other way around:

$$\pi(t, M) = \psi_0 + \psi_1 E[\pi(t)|t, M] + \omega(t). \tag{12}$$

As above, the empirical distributions for $\psi_1$ are calculated by repeatedly estimating equation (12) using data generated by Experiments A and B. The cointegrating regression in equation (11) provides estimates of $\gamma_1$ such that $[1, -\gamma_1][E[\pi(t) \mid t, M], \pi(t, M)]'$ is stationary, which implies that $[1, -(1/\gamma_1)][\pi(t, M), E[\pi(t) \mid t, M]]'$ is also stationary. We therefore expect the estimates of $\psi_1$ from the cointegrating regression in equation (12) to equal $(1/\gamma_1)$.

[22] This issue mirrors the discussion of the low power of serial correlation tests in Fama and French (1988) and Poterba and Summers (1988), who argue that serial correlation tests often fail to reject the null when it is false.

[23] See footnote 1 for references on "peso problems."

**Table IV**

## Monte Carlo Distributions for the Cointegrating Relationship between Actual and Expected Inflation

This table reports the empirical distribution for the estimated cointegrating coefficients in regressions of expected inflation on actual inflation (Panel A) and actual inflation on expected inflation (Panel B). $\pi(t)$ is the actual rate of inflation and $E[\pi(t)\,|\,t, M]$ is expected inflation, both based upon the estimated Markov model in Table III. The empirical distributions for $\gamma_1$ in Panel A and $\psi_1$ in Panel B are derived from Monte Carlo experiments that simulate the data for samples of 25, 50, and 100 years and for forecast horizons of $k$ quarters. Experiment A assumes that the current state is known when forecasts of inflation are made. Experiment B assumes that the state is not known.

Panel A. $E[\pi(t)\,|\,t, M] = \gamma_0 + \gamma_1\,\pi(t) + \eta(t)$,

| | Monte Carlo Experiment A (No Learning) | | | | | | Monte Carlo Experiment B (Learning) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample Size | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. |
| $k$ = | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 2 |
| 5% | 0.795 | 0.905 | 0.953 | 0.805 | 0.910 | 0.957 | 0.892 | 0.945 | 0.975 | 0.899 | 0.947 | 0.975 |
| 10% | 0.854 | 0.923 | 0.965 | 0.856 | 0.929 | 0.966 | 0.918 | 0.955 | 0.980 | 0.922 | 0.958 | 0.980 |
| 25% | 0.921 | 0.962 | 0.980 | 0.924 | 0.964 | 0.981 | 0.948 | 0.974 | 0.988 | 0.952 | 0.974 | 0.987 |
| 50% | 0.971 | 0.986 | 0.993 | 0.975 | 0.989 | 0.994 | 0.973 | 0.985 | 0.993 | 0.973 | 0.985 | 0.993 |
| 75% | 1.013 | 1.008 | 1.003 | 1.020 | 1.010 | 1.004 | 0.987 | 0.994 | 0.998 | 0.987 | 0.995 | 0.999 |
| 90% | 1.050 | 1.029 | 1.014 | 1.058 | 1.032 | 1.015 | 0.997 | 1.000 | 1.001 | 0.999 | 1.003 | 1.004 |
| 95% | 1.079 | 1.041 | 1.021 | 1.092 | 1.044 | 1.022 | 1.003 | 1.002 | 1.003 | 1.007 | 1.007 | 1.007 |

Panel B. $\pi(t) = \psi_0 + \psi_1 E[\pi(t)\,|\,t, M] + w(t)$,

| | Monte Carlo Experiment A (No Learning) | | | | | | Monte Carlo Experiment B (Learning) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample Size | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. | 25 Yrs. | 50 Yrs. | 100 Yrs. |
| $k$ = | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 2 |
| 5% | 0.834 | 0.925 | 0.965 | 0.840 | 0.921 | 0.965 | 0.985 | 0.990 | 0.994 | 0.982 | 0.987 | 0.990 |
| 10% | 0.890 | 0.949 | 0.977 | 0.893 | 0.947 | 0.977 | 0.990 | 0.993 | 0.995 | 0.987 | 0.989 | 0.992 |
| 25% | 0.950 | 0.977 | 0.989 | 0.953 | 0.977 | 0.989 | 0.997 | 0.997 | 0.998 | 0.994 | 0.995 | 0.996 |
| 50% | 1.003 | 1.001 | 1.000 | 1.001 | 1.000 | 1.000 | 1.003 | 1.001 | 1.000 | 1.004 | 1.001 | 0.999 |
| 75% | 1.048 | 1.021 | 1.011 | 1.048 | 1.022 | 1.011 | 1.007 | 1.004 | 1.002 | 1.010 | 1.006 | 1.004 |
| 90% | 1.101 | 1.048 | 1.021 | 1.099 | 1.046 | 1.022 | 1.015 | 1.008 | 1.005 | 1.016 | 1.010 | 1.007 |
| 95% | 1.162 | 1.067 | 1.031 | 1.157 | 1.068 | 1.030 | 1.024 | 1.012 | 1.006 | 1.026 | 1.014 | 1.009 |

Panel B of Table IV presents the empirical distributions of $\psi_1$. In the case of Experiment A, assuming individuals immediately recognize any change in regime, the cumulative distribution of $\psi_1$ in 25-year samples implies that the probability of observing a coefficient less than one remains quite high. Again, this bias disappears with larger sample sizes.

These Monte Carlo experiments show that rationally anticipated inflation shifts can introduce small sample biases in cointegrating regressions of actual and expected inflation in data samples of 25 years. This result suggests that similar small sample biases will be present in cointegrating regressions of nominal interest rates and inflation, an issue we examine next.

## III. A Reevaluation

We now draw on the results of the previous section to reevaluate the long-run relationship between inflation and the nominal interest rate when individuals rationally anticipate a shift in the inflation process. This reevaluation clearly demonstrates the implications of our Monte Carlo experiments for estimates of the long-run relationship between nominal interest rates and realized inflation. Assuming that the Markov models accurately represent the entire inflation process, we can substitute equation (11) into the Fisher equation (3) to obtain

$$R(t) - \pi(t) = r(t) + \gamma_0 - (1 - \gamma_1)\pi(t) + \eta(t). \tag{13}$$

The Monte Carlo experiments shown in Table IV indicate that in standard sample sizes, $\gamma_1$ is biased downward and would be less than one with high probability. Thus, even if the ex ante real rate follows a stationary $I(0)$ process, equation (13) shows that the ex post real rate will quite likely appear to share a unit root component in common with realized inflation.

We can examine the validity of this explanation by comparing the estimates of $\alpha_1$ obtained from the cointegrating regression

$$R(t) = \alpha_0 + \alpha_1\pi(t) + v(t), \tag{4}$$

against an empirical distribution for $\alpha_1$ generated by Monte Carlo experiments that assume inflation follows the estimated Markov processes. These experiments are similar to those described above, except that they generate nominal interest rates from the Markov forecasts of inflation using the Fisher identity. For this purpose we assume that the ex ante real rate follows a stationary $I(0)$ process. The process is parameterized so that once the ex ante real rates are combined with the Markov forecasts, they match the behavior of the nominal interest rate in the data. Specifically, the ex ante real rates are generated by subtracting the Markov forecasts of inflation from the actual nominal rates: $R(t) - E[\pi \mid \Omega(t), M] = r(t \mid M)$. An autoregressive model was fit to these estimated real rates. The ex ante real rates used in the experiments are then generated from this model. The Appendix provides a complete description of these experiments.

Panel A of Table V presents the results of these experiments. As a benchmark, column 2 reports coefficient estimates for $\alpha_1$ in the cointegrating regression (4). The $p$-values based upon standard asymptotic inference for the hypothesis that these coefficients differ from one are shown in column 4. These estimates are comparable to the results found in Table I.[24] The hypothesis that the coefficients equal one is strongly rejected with $p$-values less than 5 percent.

As described in equation (5), tax effects can in principle make $\alpha_1$ differ from one. In fact, this parameter should be $1/(1-\tau)$, where $\tau$ is the appropriate marginal tax rate. However, determining this marginal tax rate has proven quite difficult to researchers. While Darby (1975) and Feldstein (1976) argue that the relevant rate of return to a firm's decision to invest or a consumer's decision to save is the real after-tax interest rate, Feldstein and Summers (1978) and Shiller (1980) illustrate that the effective tax rate on interest payments can vary tremendously for different individuals and firms. Mishkin (1981) and Gandolfi (1982) argue further that it is easy to find cases where the effective tax rate on interest payments ranges from zero on up to the top marginal tax rate. Mishkin claims that it is extremely difficult to know the appropriate tax rate on interest payments for the overall economy and that the tax rate for firms at the margin may very well equal zero.

Nevertheless, to get an idea of how taxes might affect the parameter estimates of $\alpha_1$, we follow Mishkin (1981) as well as Crowder and Hoffman (1992) and consider a case where the effective marginal tax rate is 0.3. Column 5 of Table V reports the $p$-values for the hypothesis that $\alpha_1$ is consistent with this estimate. As the table shows, the presence of taxes makes our estimates even less likely to be consistent with a long-run Fisher equation.

Columns 6 through 8 report diagnostics from Monte Carlo Experiment A in which individuals immediately recognize any shift in the inflation process. Column 6 shows the median value of the empirical distribution of $\alpha_1$ to be less than one, indicating the downward bias in these estimates. Columns 7 and 8 show the probabilities of observing the estimated coefficient in column 2 under the hypothesis that the ex ante real rate is truly stationary and unrelated to inflation but individuals anticipate shifts in the inflation process. These $p$-values are calculated for the hypothesis that $\alpha_1 = 1$, and $\alpha_1 = 1/(1-\tau)$. Neither hypothesis can be rejected at standard marginal significance levels.

Columns 9 through 11 report diagnostics from Monte Carlo Experiment B assuming individuals are learning about the current inflation process as well as anticipating future shifts in this process. Again the median values of $\alpha_1$ in column 9 are all less than one. Column 10 reports the probability of observing the $\alpha_1$ estimates in column 2 when the ex ante real rate is truly stationary. Again, the hypothesis that the real rate is stationary is not rejected at

[24] Note that these are based upon quarterly inflation and therefore do not match exactly the same estimates based upon monthly inflation in Table I.

**Table V**

**Cointegrating Regressions with Actual Inflation and Livingston Survey Forecasts**

This table reports estimates of the cointegrating relationship between nominal interest rates and inflation together with diagnostic statistics based on Monte Carlo experiments using the Markov switching model for inflation. $R(t)$ is the nominal interest rate and $\pi(t)$ is the rate of inflation. The coefficients shown in columns 2 and 3 are corrected for finite sample bias with Stock-Watson procedure. 3 leads and lags of the first difference regressor are included in the quarterly regression and 1 lead and lag in the bi-annual regressions. Asymptotic standard errors are reported in parenthesis below the estimates. The asymptotic $p$-values in columns 4 and 5 are calculated from the Wald statistics of the null hypothesis $\alpha = 1$. The statistics allow for MA(3) serially correlated errors in the quarterly data and MA(2) serially correlated errors in the bi-annual data. The Monte Carlo $p$-values in columns 7, 8, 10, and 11 report the probability that the ex ante real rate is stationary. Experiment A assumes that the current state is known when forecasts of inflation are made. Experiment B assumes that the state is not known. $\tau$ is the tax rate used in each experiment.

Panel A. $R(t) = \alpha_0 + \alpha_1 \pi(t) + \Sigma_\iota a_\iota \Delta\pi(t - i) + v(t + n)$

| | | | Asymptotic | | Monte Carlo Experiment A | | | Monte Carlo Experiment B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Maturity in quarters (1) | $\alpha_1$ (2) | $\Sigma_\iota a_\iota$ (3) | $P$-values (%) $\tau = 0$ (4) | $H_0$: $\alpha_1 = 1$ $\tau = 0.3$ (5) | Median (6) | $P$-values (%) $\tau = 0$ (7) | $H_)$: $\alpha_1 = 1$ $\tau = 0.3$ (8) | Median (9) | $P$-values (%) $\tau = 0$ (10) | $H_0$: $\alpha_1 = 1$ $\tau = 0.3$ (11) |
| (Quarterly data) | | | | | | | | | | |
| 1 | 0.775 (0.103) | −2.926 (0.860) | 2.91 | 0.00 | 0.975 | 33.60 | 19.20 | 0.977 | 30.80 | 16.50 |
| 2 | 0.762 (0.106) | −3.584 (0.940) | 2.48 | 0.00 | 0.971 | 31.30 | 16.50 | 0.978 | 28.00 | 14.30 |
| (Bi-annual data: Livingston) | | | | | | | | | | |
| 2 | 0.927 (0.696) | −1.141 (0.413) | 44.66 | 47.09 | | | | | | |

Panel B. $\pi(t) = \beta_0 + \beta_1 R(t) + \Sigma_\iota b_\iota \Delta R(t - i) + w(t + n)$

| | | | Asymptotic | | Monte Carlo Experiment A | | | Monte Carlo Experiment B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Maturity in quarters (1) | $\beta_1$ (2) | $\Sigma_\iota b_\iota$ (3) | $P$-values (%) $\tau = 0$ (4) | $H_0$: $\beta_1 = 1$ $\tau = 0.3$ (5) | Median (6) | $P$-values (%) $\tau = 0$ (7) | $H_0$: $\beta_1 = 1$ $\tau = 0.3$ (8) | Median (9) | $P$-values (%) $\tau = 0$ (10) | $H_0$: $\beta_1 = 1$ $\tau = 0.3$ (11) |
| Quarterly data | | | | | | | | | | |
| 1 | 0.693 (0.102) | 02.650 (0.581) | 0.24 | 94.52 | 0.429 | 79.10 | 97.90 | 0.435 | 82.00 | 98.70 |
| 2 | 0.636 (0.104) | 2.970 (0.550) | 0.05 | 53.83 | 0.488 | 66.80 | 92.80 | 0.489 | 69.00 | 96.00 |
| Biannual data: Livingston | | | | | | | | | | |
| 2 | 0.803 (0.103) | 0.402 (0.269) | 5.60 | 31.73 | | | | | | |

standard levels of significance. This result also holds when we allow for tax effects as in column 11.

The long-run relationship between inflation and the nominal rate can also be examined with the cointegrating regression of inflation on the nominal interest rate

$$\pi(t) = \beta_0 + \beta_1 R(t) + w(t). \tag{14}$$

This equation puts the Fama (1975) regression into a cointegrating framework. As with $\alpha_1$ in equation (4), the coefficient $\beta_1$ must be equal to one (in the absence of tax effects) if the ex post real rate does not contain permanent disturbances.

Panel B of Table V examines the long-run relationship between inflation and nominal interest rates in the context of the cointegrating regression in equation (14). Column 2 shows that the estimates of $\beta_1$ calculated with the Stock-Watson procedure are all less than one. The asymptotic $p$-values for the hypothesis that these coefficients are equal to one are reported in column 4. These $p$-values show that the hypothesis is rejected at standard significance levels. Column 5 reports the asymptotic $p$-value for the hypothesis that the $\beta_1$ coefficient is equal to the tax-adjusted coefficient $(1 - \tau)$. Since the estimates are less than one, it is not surprising that the hypothesis that $\beta_1 = (1 - \tau) = 0.7$ cannot be rejected. However, this result is also inconsistent with the evidence for the tax effects in Panel A. Thus, the hypothesis that the tax-adjusted Fisher equation holds in the long run is rejected. Under standard assumptions about expectations, these results imply that the ex ante real rate is subject to permanent shocks if the marginal tax rate is zero.

Alternatively, we can interpret these results allowing for the effects of anticipated switches in inflation. In particular, if the Fisher identity is used to substitute for expected inflation in equation (12), we obtain

$$\pi(t) = \psi_0 - \psi_1 r(t) + \psi_1 R(t) + \omega(t). \tag{15}$$

The results of the Monte Carlo experiments shown in Table IV indicate that $\psi_1$ is biased downward and would be less than one with high probability in the available sample sizes.

Columns 7, 8, 10, and 11 report the probability of observing the estimates of $\beta_1$ when inflation shifts between the processes estimated by the Markov model. As before, these calculations assume that the ex ante real rate follows a stationary $I(0)$ process. If the marginal tax rate is zero, the $p$-values based upon Experiment A are all above 50 percent. Thus, one would not reject the hypothesis that the ex ante real rate is stationary at standard confidence levels. The $p$-values are somewhat smaller for Experiment B, in which individuals learn about any switch in inflation over time. The hypothesis that the ex ante real rate is stationary would not be rejected at the 95 percent confidence levels. When tax effects are considered, the $p$-values are somewhat higher in both experiments.

In the last row of each panel we provide estimates of the cointegrating relationship between nominal interest rates and the Livingston survey data.

When the nominal rate is regressed upon the inflation forecast, we cannot reject the hypothesis that the estimated coefficient on the Livingston forecasts is equal to one at high confidence levels. On the other hand, when the inflation forecast is regressed on the nominal rate, we can reject the hypothesis that the coefficient on the nominal rate is equal to one at the 10 percent level. The asymptotic $p$-values are higher when we allow for tax effects.

Overall, the evidence in Table V shows that in typical sample sizes based upon postwar data, cointegrating coefficients in the Fisher identity differ from one. Under standard assumptions, these result would necessarily imply that the ex ante real rate contains unit roots.[25] However, the table also shows we are very likely to find these estimates when the ex ante real rate does not contain unit roots but individuals rationally anticipate shifts in the inflation process. In fact, if the number of shifts in the postwar data sample is representative of the future inflation process, our estimates are consistent with the view that the ex ante real rate is stationary.

## V. Concluding Remarks

In this article, we reexamine the long-run relationship between inflation and nominal interest rates using recent time-series techniques. Based upon these estimates, we show that nominal rates move less than one-for-one with inflation so that there appear to be permanent movements in ex post real interest rates. A conventional interpretation of this finding is that ex ante real rates are subject to permanent shocks which are shared by (expected) inflation. However, this interpretation is hard to square with existing theoretical models.

We propose that the apparent permanent component in ex post real interest rates could arise when people incorporate anticipated shifts in the inflation process into their expectations. We examine the stability of the inflation process and estimate a Markov switching model of inflation. This model appears to capture structural shifts in the inflation process and to characterize the behavior of both inflation and Livingston survey data quite well. Based upon forecasts from the Markov model, we reexamine the long-run relationship between nominal interest rates and inflation. When incorporating anticipated shifts in inflation in this way, we are unable to reject the hypothesis that long-term movements in nominal interest rates reflect one-for-one long-run movements in expected inflation.

## Appendix: Monte Carlo Experiments of Markov Switching Model

This Appendix details the Monte Carlo experiments described in Section II.*C* and Section III.

[25] This conclusion holds whether or not the effective marginal tax rate is zero. Based upon quarterly data, the asymptotic $p$-values in columns 4 and 5 are less than 5 percent in either Panel A or B, whichever tax rate we choose.

*Section II.C*

The Monte Carlo experiments in this section are used to evaluate the cointegrating regressions, equations (11) and (12), in the text. These regressions involve series of expected inflation and actual inflation based upon the estimated Markov model for inflation. As described in the text, two sets of Monte Carlo experiments are conducted based upon different assumptions about the information available to individuals when they forecast inflation.

In the first set, Experiment A, we assume that individuals observe the past history of states and inflation when forecasting. Actual inflation is generated from the estimated model in Table III, and the Markov forecasts at horizons of one and two quarters are calculated assuming the current state is known and the transition probabilities are known. Expected inflation is therefore calculated as

$$E[\pi(t)|\Omega(t)] = E[\pi(t)|s(t+1)=1,\Omega(t)]\Pr(s(t+1)=1|s(t)) + E[\pi(t)|s(t+1)=0,\Omega(t)]\Pr(s(t+1)=0|s(t)). \quad \text{(A1)}$$

where $E[\pi(t)\,|\,s(t+1),\Omega(t)]$ is the forecast of future inflation conditioned on the future state obtained from the extended version of Hamilton's filter (see Evans (1993) and Kim (1994)). After generating the actual and expected series, the cointegrating regressions in equations (11) and (12) are estimated with the Stock-Watson procedure. The coefficient estimates are saved. This procedure is repeated 1000 times, generating an empirical distribution of coefficients. These experiments are conducted for sample lengths of 25, 50, and 100 years.

In the second set of experiments, Experiment B, we assume that individuals only observe the history of inflation. Their forecasts of inflation are therefore calculated as

$$E[\pi(t)|\Omega(t)] = E[\pi(t)|s(t+1)=1,\Omega(t)]\Pr(s(t+1)=1|\Omega(t)) + E[\pi(t)|s(t+1)=0,\Omega(t)]\Pr(s(t+1)=0|\Omega(t)). \quad \text{(A2)}$$

where $\Pr(s(t+1)\,|\,\Omega(t)) = \Pr(s(t+1)\,|\,s(t))\Pr(s(t)\,|\,\Omega(t))$ and $\Pr(s(t)\,|\,\Omega(t))$ is the estimated state from the extended filter. Again actual and expected inflation are generated based on the model estimates in Table III, and the cointegrating regressions in equations (11) and (12) are estimated with the Stock-Watson procedure. This procedure is repeated 1000 times to generate empirical distributions for the coefficients.

*Section IV*

The Monte Carlo experiments in this section are used to evaluate the Fisher equation estimates in equations (4) and (14). These regressions require series on the actual and expected inflation processes from the Markov process, as generated in Section III.*C*, but also require the ex ante real rate to produce a nominal interest rate series. The ex ante real rates are generated by subtracting the actual nominal rates from the Markov forecasts of inflation: $R(t) - E[\pi \mid t, M] = r(t, M)$. Then an autoregressive model of the ex ante real rate process is estimated. The ex ante real rates are then generated from this model. Given these generated ex ante real rates, the nominal rates are constructed as: $r(t, M) + E[\pi(t) \mid t, M] = R(t, M)$. By construction, these nominal interest rates match the behavior of the nominal interest rates in the data.

The Monte Carlo experiments follow the two forms described above for Experiments A and B. For each set of generated nominal interest rates and inflation, equations (4) and (14) are estimated using the Stock-Watson method. The empirical distributions of the coefficients are calculated. Since the ex ante real rate is stationary and the Fisher equation holds by construction, these empirical distributions are used to calculate the $p$-values in Table V.

## REFERENCES


Ball, L., and S. Cecchetti, 1990, Inflation and uncertainty at short and long horizons, *Brookings Papers on Economic Activity* 21, 215–254.

Barsky, R. B., 1987, The Fisher hypothesis and the forecastability and persistence of inflation, *Journal of Monetary Economics* 19, 3–24.

Boudoukh, J., and M. Richardson, 1993, Stock returns and inflation: A long horizon perspective, *American Economic Review* 83, 1346–1355.

Brown, R. L., J. Durbin, and J. M. Evans, 1975, Techniques for testing the constancy of regression relationships over time, *Journal of the Royal Statistical Society Series B* 37, 149–163.

Campbell, J. Y., and P. Perron, 1991, Pitfalls and opportunities: What macroeconomists should know about unit roots, in O. Blanchard and S. Fischer, Eds.: *NBER Macroeconomics Annual* (MIT Press: Cambridge), pp. 141–201.

Crowder, W., and D. Hoffman, 1992, Is there a long run relationship between nominal interest rates and inflation: The Fisher equation revisited, Working paper, Arizona State University.

Danthine, J. P., and J. Donaldson, 1986, Inflation and asset prices in an exchange economy, *Econometrica* 54, 585–606.

———, and L. Smith, 1987, On the superneutrality of money in a stochastic dynamic macroeconomic model, *Journal of Monetary Economics* 20, 475–499.

Darby, M. R., 1975, The financial and tax effects of monetary policy on interest rates, *Economic Inquiry* 12, 266–276.

Engel, C., and J. D. Hamilton, 1990, Long swings in the dollar: Are they in the data and do the markets know it? *American Economic Review* 80, 689–713.

Evans, M. D. D., 1993, Estimating general markov switching models, Working paper, Stern School of Business, New York University.

———, and K. K. Lewis, 1992, Do expected shifts in inflation policy affect real rates? Working Paper Series No. 4134, National Bureau of Economic Research.

———, 1994, Do stationary risk premia explain it all? Evidence from the term structure, *Journal of Monetary Economics* 31, 285–318.

Evans, M. D. D., and P. Wachtel, 1992, Interpreting the movements in short term interest rates, *Journal of Business* 65, 395–429.

———, 1993, Inflation regimes and the sources of inflation uncertainty, *Journal of Money, Credit, and Banking* 25, 475–511.

Fama, E. F., 1975, Short-term interest rates as predictors of inflation, *American Economic Review* 65, 269–282.

———, 1981, Stock returns, real activity, inflation, and money, *American Economic Review* 71, 545–565.

———, and K. R. French, 1988, Permanent and temporary components of stock prices, *Journal of Political Economy* 96, 246–273.

Fama, E. F., and M. R. Gibbons, 1982, Inflation, real returns and capital investment, *Journal of Monetary Economics* 9, 297–323.

Fama, E. F., and G. W. Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics* 5, 115–146.

Feldstein, M., 1976, Inflation, income taxes, and the rate of interest: A theoretical analysis, *American Economic Review* 66, 809–820.

———, and L. Summers, 1978, Inflation, tax rules and the long-term interest rate, *Brookings Papers on Economic Activity* 1, 61–100.

Fisher, I., 1930, *The Theory of Interest* (Macmillan, New York).

Garcia, R., and P. Perron, 1993, An analysis of the real interest rate under regime shifts, Working paper, Princeton University.

Gandolfi, A. E., 1982, Inflation, taxation and interest rates, *Journal of Finance* 37, 797–807.

Hamilton, J. D., 1989, A new approach to the econometric analysis of nonstationary time series and the business cycle, *Econometrica* 57, 357–384.

Hansen, L. P., and R. Hodrick, 1983, Risk averse speculation in forward foreign exchange markets: An econometric analysis of linear models, in J. Frenkel, Ed: *Exchange Rates and International Macroeconomics* (University of Chicago Press, Chicago) pp. 113–152.

Hansen, B. E., 1991, Testing for parameter instability in linear models. Working paper, University of Rochester.

Huizinga, J., and F. S. Mishkin, 1984, Inflation and real interest rates on assets with different risk characteristics, *Journal of Finance* 39, 699–712.

———, 1986, Monetary policy regime shifts and the unusual behavior of real interest rates, *Carnegie-Rochester Conference Series on Public Policy*, 24, 231–274.

Johansen, S., 1991, Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models, *Econometrica* 6, 1551–1580.

———, and K. Juselius, 1990, Maximum likelihood estimation and inferences on cointegrating with applications to the demand for money, *Oxford Bulletin of Economics and Statistics* 52, 169–209.

Kim, C. ,1994, Dynamic linear model with Markov-switching, *Journal of Econometrics* 60, 1–22.

King, R., and M. Watson, 1992, Testing long run neutrality, Working paper, Northwestern University.

Lewis, K. K., 1989, Changing beliefs and systematic forecast errors, *American Economic Review* 79, 621–636.

———, 1991, Was there a peso problem in the U.S. term structure of interest rates: 1979–1982?, *International Economic Review* 32, 159–173.

Lintner, J., 1975, Inflation and security returns, *Journal of Finance* 30, 259–280.

Marshall, D. A., 1992, Inflation and asset returns in a monetary economy, *Journal of Finance* 47, 1325–1342.

Mishkin, F., 1981, The real interest rate: An empirical investigation, *Carnegie-Rochester Conference Series on Public Policy* 15, 151–200.

———, 1992, Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates, *Journal of Monetary Economics* 30, 195–215.

Mundell, R. 1976, Inflation and real interest, *Journal of Political Economy* 71, 280–283.

Perron, P., 1989, The great crash, the oil price shock, and the unit root hypothesis, *Econometrica* 57, 1361–1401.

———, 1991, A test for changes in a polynomial trend function for a dynamic time series, Working paper, Princeton University.

———, and T. Vogelsang, 1992, Nonstationarity and level shifts with an application to purchasing power parity, *Journal of Business and Economic Statistics* 10, 301–320.

Poterba, J., and L. Summers, 1988, Mean reversion on stock prices: Evidence and implications, *Journal of Financial Economics* 22, 27–59.

Rogoff, K. S., 1980, Essays on expectations and exchange rate volatility, Doctoral dissertation, Massachusetts Institute of Technology.

Rose, A. K., 1988, Is the real interest rate stable? *Journal of Finance* 43, 1095–1112.

Shiller, Robert, 1980, Can the fed control real interest rates? in S. Fisher, Ed.: *Rational Expectations and Economic Policy* (University of Chicago Press: Chicago), pp. 117–156.

Shome, D. K., S. D. Smith, and J. M. Pinkerton. 1988, The purchasing power of money and nominal interest rates: A re-examination, *Journal of Finance* 43, 1113–1125.

Stock, J., (1987), Asymptotic properties of least squares estimators of cointegrating vectors, *Econometrica* 55, 1035–1056.

———, and M. Watson, 1989, A simple MLE of cointegrating vectors in higher order integrated systems, Technical Working Paper No. 83, National Bureau of Economic Research.

Stulz, R. M., 1986, Asset pricing and expected inflation, *Journal of Finance* 41, 209–223.

Tobin, James, 1969, A general equilibrium approach to monetary theory, *Journal of Money, Credit, and Banking* 1, 15–29.

Zivot, E., and D. W. K. Andrews, 1992, Further evidence on the great crash, the oil price shock, and the unit root hypothesis, *Journal of Business and Economic Statistics* 10, 251–270.