If market participants expect a future discrete change in asset fundamentals, then rational forecast errors may be correlated with current information and have a mean different from zero in finite samples. This statement may seem inconsistent with the standard assumption that forecast errors are orthogonal to current information and have a mean of zero. By contrast, this article describes how this phenomenon may be rational using the example of the Mexican peso market in which it was first noted. It then illustrates how the peso problem applies more generally to a wide range of asset prices.

Asset prices are determined by expectations about the paths of future economic variables. Therefore, anticipated discrete changes in the distribution of these variables directly affect asset price behaviour. The 'peso problem' focuses upon how asset prices behave when market traders have expectations about infrequent discrete shifts in economic determinants. With these expectations, the discrete switches can induce behaviour in asset prices that apparently contradicts conventional rational expectations assumptions. The fundamental shifts are rare events and typically occur infrequently, even in relatively large samples. As such, the term 'peso problem' is interchangeably with the small-sample inference problems arising from these expected events.

The specific currency reference used in the term 'peso problem' may seem at odds with its general potential effects on asset prices. The origins of the term therefore deserve further explanation. The phenomenon is called the 'peso problem' because it was first noted in the Mexican peso market. The original source of the term is unknown, though some economists have attributed it to Milton Friedman. The empirical phenomenon was originally mentioned in writing in the dissertation by Rogoff (1977; 1980) and in publication form by Krasker (1980). Based upon evidence from the Mexican peso futures market from June 1974 to June 1976, Rogoff used the relationship between futures contracts and spot contracts to test market efficiency under rational expectations and risk neutrality. He found that the implications of market efficiency were rejected, but that the behaviour of futures contracts could be explained by the market's persistent belief that the Mexican peso might be devalued. Consistent with this explanation, the peso was devalued in August 1976.

#### The peso problem in the Mexican currency crisis

To illustrate the effects upon asset prices during this period, consider the relationship between the spot and forward rate of a contract for future delivery. If we define  $S_{t+1}$  as the logarithm of the future spot rate (dollars per peso) at date t+1 and  $F_t$  as the logarithm of the forward rate contracted at date t for delivery at date t+1, the relationship between the two variables may be written

$$S_{t+1} - F_t = r_t + u_{t+1} \tag{1}$$

where  $r_t$  is the risk premium, the forecast error on the spot rate is  $u_{t+1} \equiv S_{t+1} - E_t S_{t+1}$ , and  $E_t$  is the expectations operator conditional on information available at time t. Through covered interest parity, the difference between the spot and forward rate also equals the return on holding peso deposits over the same period and converting the proceeds back into dollars at date t+1. In order to focus on the effect of expectations, the analysis

below will ignore the risk premium effect. This assumption is not necessary, however, and much of the literature described below includes models of the risk premium term,  $r_t$ .

From April 1954 to August 1976, the spot peso exchange rate was fixed at 0.08 dollars per peso. During this period, which covered over 20 years, the exchange rate was constant. If we use the notation above, therefore,  $S_{t+1}$  was equal to a constant, call it  $S^0$ . Nevertheless, futures and forward contracts sold at a discount for much of the early 1970s. For example, the year ahead contract on June 1975 and June 1976 futures contracts sold at a discount of 2.6 and 2.7 per cent respectively. Similarly, Mexican peso deposit rates traded higher than dollar deposit rates over this period, implying a forward rate in (1) that was less than the *ex post* spot rate. Therefore, the *ex post* rate of return on holding Mexican peso accounts,  $S^0 - F_t$ , was systematically positive. Under risk neutrality, this behaviour contradicts the assumption of rational expectations since it implies that the market's forecast errors,  $S_{t+1} - E_t S_{t+1}$ , were biased and serially correlated.

At the end of this period, on 31 August 1976, the authorities allowed the Mexican peso to float. Subsequently, the peso fell to 0.05 dollars per peso, implying a devaluation of about 46 per cent. If we define the logarithm of the spot rate associated with this level as  $S^1$ , the implied forecast error over this event was  $S^1 - F_t \cong -46$  per cent. If one takes account of this large negative observation together with the many small positive observations over the early 1970s the implication is an average forecast error close to zero, which explains the apparent Mexican peso paradox.

Examining how traders with rational expectations would have formed their forecasts helps to define the peso market phenomenon further. Lizondo (1983) postulated that the expected future peso exchange rate could be written as:

$$E_t S_{t+1} = (1 - p_t) S^0 + p_t S^1$$
 (2)

where  $p_t$  is the market's assessed probability that the authorities will devalue the peso to  $S^1$  during the next period. Therefore, as long as the peso remains fixed at  $S^0$ , the forecast error is

$$u_{t+1} = S^0 - E_t S_{t+1} = p_t (S^0 - S^1)$$
(3)

Since the Mexican spot rate over the early period was greater than the devalued August 1976 rate, the initial spot rate  $S^0$  was greater than the anticipated rate if devaluation were to occur,  $S^1$ . As such, ex post forecast errors were systematically positive. The ex post bias observed in forecast errors depended upon both the probability of the devaluation,  $p_t$ , and the expected size of the fall in the exchange rate,  $S^0 - S^1$ . On the other hand, for the period when the devaluation occurred, the forecast error was a large negative number,  $(1 - p_t)(S^1 - S^0)$ .

In a sample with many observations of similar devaluations, forecast errors would be persistently positive with infrequent large negative observations. The frequent small positive forecast errors and the infrequent large negative forecast errors will tend to cancel each other out. Over a sufficiently large sample with enough of the rare events, the forecast errors would roughly sum to zero, as implied by rational expectations. However, the market would appear to make systematic forecast errors between the episodes of discrete changes, even though the forecasts will be unbiased in sufficiently large samples. Even in large samples, therefore, rational forecast errors with a 'peso problem' may be serially correlated.

## The peso problem in general asset prices

Although first noted in the period of the fixed Mexican peso rate, this phenomenon can be found in any forward-looking asset price when market traders anticipate a discrete change in the distribution of its economic determinants. A simple example serves to illustrate the peso problem in general. Suppose that agents rationally anticipate a switch in the process of an economic variable from its current process,  $R^0$ , to an alternative,  $R^1$ . In this case, rational forecasts of asset prices that depend upon this variable include forecasts of the price conditional upon each regime process. Denote the general asset price as  $S_t$  to preserve the same notation as above. Then the expected future value of the asset price is:

$$E_t S_{t+1} = (1 - p_t) E_t (S_{t+1} | R^0) + p_t E_t (S_{t+1} | R^1)$$
 (2')

where  $p_t$  is the market's assessed probability conditional upon time t information that the process will switch to process 1; and where  $E_t(S_{t+1}|R^i)$  for i=0,1 is the expected value conditioned upon time t information and upon process i generating the asset's determining variables.

A few examples of peso problem studies serve to illustrate the breadth of its application in diverse settings. Salant and Henderson (1978) considered the effects upon the price of gold from the market's assessed probability that governments might sell their gold holdings in large discrete amounts. In this case, the spot rate  $S_t$  represents the price of gold,  $E_t(S_{t+1}|R^i)$  are the expected future gold prices conditional upon i = 0, 1, no government sales or government sales, respectively, and  $p_t$  is the market's assessed probability that the government will sell gold. Flood and Garber (1980) examined the price level effects resulting from anticipated monetary reforms in hyperinflationera Germany. In this case, the spot rate represents the price level,  $E_t(S_{t+1}|R^i)$ are the expected future price levels conditional upon no reform and reform, alternatively, and  $p_t$  is the market's assessed probability that the reform will take place. Lewis (1991) evaluated the term structure of US interest rates following the 1979 change in Federal Reserve operating procedures to determine whether the market believed a shift in policy to lower interest rates was possible. In this case,  $S_t$  represents the interest rate,  $E_t(S_{t+1}|R^1)$  is the expected future interest rates conditional upon on shift to lower rates, and  $p_t$ is the marker's assessed probability that this shift will take place. Bates (1991) used option prices to estimate the market's beliefs that the US stock market might crash before October 1987. In this case,  $S_t$  represents the stock price,  $E_t(S_{t+1}|R^i)$  is the expected future stock prices conditional upon no crash or crash, respectively, and  $p_t$  is the market's assessed probability that the crash will occur. Bekaert, Hodrick and Marshall (2001) analysed international term structure returns using expectations of discrete shifts in shortterm interest rate regimes. In this case,  $S_t$  is the excess return of long bonds over short-term bonds, and R' refer to different short-term interest rate regimes. Ang, Gu and Hochberg (2007) examine the effects upon long-horizon initial public offering (IPO) returns based upon uncertainty about which performance regime determines a given initial listing. In this case, S<sub>t</sub> refers to the abnormal returns and  $R^i$  dictate whether they follow under- or overperformance.

In general, when traders believe a future shift may occur in determinants of asset prices, expectations will have the form given in (2'), as the above examples demonstrate. Now suppose that no change in regime occurs in the sample. Define  $(S_{t+1}|R^0)$  as observations drawn from the current regime process. Then, the forecast errors become:

$$u_{t+1} = (S_{t+1}|R^0) - E_t S_{t+1}$$

$$= [(S_{t+1}|R^0) - E_t (S_{t+1}|R^0)]$$

$$+ p_t [E_t (S_{t+1}|R^0) - E_t (S_{t+1}|R^1)]$$
(3')

As long as the process does not change, the first term represents the forecast error conditioned on the current regime and therefore has mean zero. By contrast, the second term captures the effect of an expected switch to process  $R^1$  that does not materialize in the sample. If the expected price conditioned on process  $R^0$  is on average greater, say, than the price conditioned on regime  $R^1$ , the mean of the forecast errors within the sample will tend to be positive. Note that, for the Mexican peso example, the conditional expectations are simply constants where  $E_t(S_{t+1}|R^i) = S^i$ , for i = 0, 1, so that eqs (2) and (3) are equivalent to (2') and (3') in this case. In general, however, the expectation conditional upon each regime varies over time as new information arrives to the market.

The example in (3') illustrates the peso problem effects upon realized returns when no switches occur in the sample. Of course, the forecast error will include this event when the switch occurs. If the switches do not occur with sufficient frequency in the sample, however, forecast errors may continue to appear to be biased. Moreover, even with sufficient occurrences of these shifts, the forecast errors may be serially correlated since they weight the difference between the two expected processes, given by the second term on the right-hand side of eq. (3'). When the probabilities or the differences in expectations under the two regimes are serially correlated, these components of the forecast errors are serially correlated as well. In this case, the difference between the spot rate and the forward rate as in (1) will be serially correlated even in the absence of risk premia. This explanation is consistent with the observation in Rogoff (1977) that Mexican peso futures prices before the devaluation did not follow a martingale as they should have by the efficient markets hypothesis.

### The peso problem and Bayesian learning

The simple intuition of the Mexican foreign exchange devaluation example casts the peso problem as a problem arising from anticipated *future* shifts in fundamentals. More generally, the peso problem phenomenon has also come to encompass the asset price implications due to uncertainty about *past* discrete changes. To see why the asset price behaviour is similar, consider a simple example. Suppose that market participants believe that the regime may have shifted in some past time period,  $\tau < t$ . Given priors about the probability of a change, they will then update their assessed probabilities of living in a new regime as new information arrives. If they learn through Bayesian inference, the forecast errors will depend upon expectations conditioned on each regime process and upon the updated probabilities of being in each regime.

The form of these forecast errors is isomorphic to equation (3'). To illustrate, suppose that in fact the process changed at time  $\tau$ . In this case, the current regime  $R^0$  is the new regime, and the alternative regime  $R^1$  is the old regime. The probability  $p_t$  represents the market's assessed probability that no change took place. Over time, as the market learns the truth, the probability of no change goes to zero and the second component in the forecast error (3') vanishes. Clearly, these forecast errors converge to mean-zero, white-noise levels even though they may appear biased during the learning

process. Similar results hold when the market does not know the parameters of the new distribution but learns them over time. For example, Lewis (1989) relates the US dollar foreign exchange rate behaviour in the early 1980s to the market's uncertainty about whether a past shift to tighter monetary policy took place. Similarly, Timmermann (1993) shows how the learning can help explain the excess volatility in stock markets.

Despite the similarity of expectations based on learning about past discrete changes and on anticipating future discrete changes, their implications for forecast error behaviour in sufficiently large samples can be somewhat different. A once-and-for-all shift in the asset process with subsequent learning will induce forecast errors that are biased and serially correlated over the learning period. However, as the market learns, the probability of the old regime continuing will go to zero and the effect from the second term on the right-hand side of (3) will vanish. Thus, with sufficient observations, forecast errors following learning will behave according to the standard rational expectations assumptions; that is, they will be mean zero and serially uncorrelated. By contrast, with sufficient observations of the discrete shifts in processes, forecast errors arising from anticipated future discrete events will remain serially correlated in general but will be unbiased.

# Empirical approaches to the peso problem in asset prices

As this description makes clear, the peso problem is inherently a problem of identifying a low probability event in a given sample. Many researchers simply acknowledge that this small sample problem may be an issue in their results. Other researchers examine the potential for peso problems to explain anomalous asset price behaviour by using different approaches to identify the peso problem in sample.

These approaches can be grouped into three main groups. The first group uses a calibrated asset pricing model to consider whether a peso problem explanation can explain a given empirical regularity. For example, Rietz (1988) uses this approach to consider whether the equity premium can be explained by rare adverse events. More recently, Barro (2006) examines the plausibility of this explanation using data over the 20th century.

The second group identifies the peso problem by using dates of known discrete changes in fundamentals to empirically back out expectations from asset prices. This group of studies focuses upon easily observable shifts in fundamentals. Examples include exchange rate realignments (Bertola and Svensson, 1993; Campa and Chiang, 1996; Campa, Chiang, and Refalo, 2002; Mundaca, 2004) and announced shifts in monetary policy targeting (Lewis, 1991; Hallwood, MacDonald and Marsh, 2000).

The third group analyses the peso problem by directly estimating regime-switching models of fundamentals to explain anomalous behaviour in their asset prices. This approach has the advantage that the fundamentals process can be estimated from the available data and does not require the researcher to take a stand on the timing of the events. As a result, the analysis can be conducted in a wide range of applications where the dating of events is not known a priori. Many different asset prices have been studied using this approach, including floating spot exchange rates (Engel and Hamilton, 1990; Kaminsky, 1993), the equity premium (Cecchetti, Lam and Mark, 1993), the real interest rate (Evans and Lewis, 1995a), the foreign exchange risk premium (Evans and Lewis, 1995b), the term premium (Bekaert, Hodrick and Marshall, 2001), and IPO abnormal returns (Ang, Gu and Hochberg, 2006).

## **Summary**

In summary, as long as agents anticipate occasional discrete changes in the process of economic variables that affect asset prices, and these changes occur infrequently, asset prices contain the potential for the peso problem. If so, then forecast errors will be serially correlated. Furthermore, unless the sample contains many observations of the discrete shifts, forecast errors will appear biased when observed *ex post* even though traders may have rational expectations. Despite this problem, empirical financial studies frequently measure the risk premium as the predictable component of the realized spot rate less the forward rate, described in (1). Therefore, if the 'peso problem' is present in the sample researchers may incorrectly attribute asset price behaviour to anomalies rather than to the market's rational forecasts of discrete events.

Karen K. Lewis

#### See also

```
< xref = B000314> Bayesian statistics;
< xref = xyyyyyy> covered interest parity;
< xref = F000308> financial market anomalies;
< xref = F000313> finite sample econometrics;
< xref = xyyyyyy> international finance;
< xref = xyyyyyy> rational expectations;
< xref = xyyyyyy> regime-switching models.
```

# **Bibliography**

Ang, A., Gu, L. and Hochberg, Y. 2007. Is IPO underperformance a peso problem? *Journal of Financial and Quantitative Analysis* (forthcoming).

Barro, R. 2006. Rare disasters and asset markets in the 20th century. *Quarterly Journal of Economics* 121, 823–66.

Bates, D.S. 1991. The Crash of 87: was it expected? The evidence from options markets. *Journal of Finance* 46, 1009–44.

Bekaert, G., Hodrick, R.J. and Marshall, D.A. 2001. Peso problem explanations for term structure anomalies. *Journal of Monetary Economics* 48, 241–70.

Bertola, G. and Svensson, L.E.O. 1993. Stochastic devaluation risk and the empirical fit of target-zone models. *Review of Economic Studies* 60, 689–712.

Campa, J.M. and Chang, P.H.K. 1996. Arbitrage-based tests of target-zone credibility: Evidence from ERM cross-rate options. *American Economic Review* 86, 726–40.

Campa, J.M., Chang, P.H.K. and Refalo, J.F. 2002. An options-based analysis of emerging market exchange rate expectations: Brazil's real plan, 1994–1999. *Journal of Development Economics* 69, 227–53.

Cecchetti, S.G., Lam, P. and Mark, N.C. 1993. The equity premium and the risk-free rate: Matching the moments. *Journal of Monetary Economics* 31, 21–45.

Engel, C. and Hamilton, J.D. 1990. Long swings in the dollar: are they in the data and do markets know it? *American Economic Review* 80, 689–713.

Evans, M.D.D. and Lewis, K.K. 1995a. Do expected shifts in inflation affect estimates of the long-run Fisher relation? *Journal of Finance* 50, 225–53.

Evans, M.D.D. and Lewis, K.K. 1995b. Do long-term swings in the dollar affect estimates of the risk premia? *Review of Financial Studies* 8, 709–42.

- Flood, R. and Garber, P. 1980. An economic theory of monetary reform. *Journal of Political Economy* 88, 24–58.
- Hallwood, C.P. MacDonald, R. and Marsh, I.W. 2000. Realignment expectations and the US dollar, 1890–1897: was there a 'Peso Problem'? *Journal of Monetary Economics* 46, 605–20.
- Kaminsky, G. 1993. Is there a peso problem? Evidence from the dollar/pound exchange rate: 1976–1987. *American Economic Review* 83, 450–72.
- Krasker, W.S. 1980. The peso problem in testing the efficiency of forward exchange markets. *Journal of Monetary Economics* 6, 269–76.
- Lewis, K.K. 1989. Changing beliefs and systematic forecast errors. *American Economic Reviews* 79, 621–36.
- Lewis, K.K. 1991. Was there a peso problem in the U.S. interest rate term structure: 1979–1982. *International Economic Review* 32, 159–73.
- Lizondo, J.S. 1983. The efficiency of the Mexican peso market. *Journal of International Economics* 14, 69–84.
- Mundaca, G. 2004. Modeling probabilities of devaluations. Economica 71, 13-37.
- Rietz, T.A. 1988. The equity risk premium: A solution. *Journal of Monetary Economics* 22, 117–31.
- Rogoff, K. 1977. Rational expectations in the foreign exchange market revisited. Unpublished manuscript, Massachusetts Institute of Technology.
- Rogoff, K. 1980. Tests of the martingale model for foreign exchange futures markets. In Essays on expectations and exchange rate volatility. Ph.D. dissertation, Massachusetts Institute of Technology.
- Salant, S. and Henderson, D. 1978. Market anticipations of government policies and the price of gold. *Journal of Political Economy* 86, 627–48.
- Timmermann, A.G. 1993. How learning in financial markets generates excess volatility and predictability in stock prices. *Quarterly Journal of Economics* 108, 1135–45.

#### **Index terms**

efficient markets hypothesis foreign exchange risk premium Friedman, M. German hyperinflation learning martingales peso problem rational expectations regime-switching models risk neutrality stock price volatility term premium white noise

#### Index terms not found:

German hyperinflation stock price volatility