Why do stocks and consumption imply such different gains from international risk sharing?[☆]

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Abstract

Estimates of the gains to international risk-sharing based upon stock returns tend to find dramatically higher gains than do estimates from consumption-based models. In this paper, I examine the reasons for these differences. Using a common theoretical framework for both approaches, I find that the differences are largely due to the much higher variability of stock returns and its implied intertemporal substitution in marginal utility. Also, contrary to conventional wisdom, the differences in gains from the two approaches do not arise from treating stock returns as exogenous rather than endogenous. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Domestic investors do not appear to hold a sufficient proportion of their wealth in foreign assets to diversify away domestic idiosyncratic risk. This is the

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conclusion of research using both consumption data and stock return data.[1] Since imperfect risk-sharing means that potential welfare gains are being foregone, the observation leads directly to the question: how large are these gains?

On this issue, the literature has been quite divided. Some calculations of risk sharing gains based upon international consumption data suggest that these gains are quite small. For example, Cole and Obstfeld (1991) find that for representative consumers calibrated to US data, the gains are less than 0.5% of permanent consumption for plausible parameter values. Tesar (1995) and van Wincoop (1994) report similarly small gains from international risk sharing.

On the other hand, calculations of the gains from risk-sharing based upon stock returns give much larger estimates. The approach typically constructs combinations of domestic and foreign portfolios that minimize variance and maximize returns and asks whether domestic portfolios are dominated by these portfolios. In papers at least as early as Levy and Sarnat (1970), portfolios with foreign stocks were shown to strictly dominate domestic US portfolios. Using utility functions similar to those used in the general equilibrium literature, I show below that this simple partial equilibrium framework implies welfare gains of at least 20% of permanent consumption and often-times near 100%.

In this paper, I address the question: why are the magnitudes of the gains based upon these two approaches so different?[2] I develop a common unifying framework and then show that the differences can come from three potential avenues: (1) the treatment of stock returns as exogenous or endogenous; (2) the statistical properties of stock returns relative to consumption growth; and (3) the set of preference parameters.[3] Since these three factors are at the core of this investigation, I next discuss the significance of each in turn.

(1) The equity-based approach takes the stock price as exogenous and asks how an investor would choose an optimal portfolio given the mean and variance of this process. Thus, an investor does not take into account the effect that his decision may have on the stock price. On the other hand, the consumption-based approach takes a production process as exogenous and asks how optimal risk-sharing would affect the investor's consumption path. This approach implies that stock prices will change as a result of risk-sharing. This distinction suggests an intuitive reason why the equity-based approach leads to significantly higher gains than the consumption-based approach: the equity-based approach does not incorporate the effect of risk-sharing on the stock price.

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[1] For a recent discussion of these two literatures and their relationship, see Lewis (1999).

[2] A related question is: what is the 'true' risk-sharing gain? In this paper, I examine only the narrower question articulated in the title.

[3] Note that these avenues need not be independent. For example, it is well known that by treating stock returns as endogenous [avenue (1)] with a set of plausible preference parameters [avenue (3)] implies that the statistical properties of stock returns are difficult to reconcile with consumption growth [avenue (2)]. Below, I discuss and focus upon this interdependence.

In this paper, I show that this intuition is not true in general. The reason is simple. When stock prices are endogenous, these prices must adjust to make international investors willing to buy the country's equity. This adjustment in prices leads to a one-time intertemporal substitution of consumption from low growth economies to high growth economies that leaves all countries better off. As a result, the endogenous stock price reaction allows for an extra avenue of welfare gains that are not present when stock prices are treated as exogenous. Therefore, the partial equilibrium nature of the equity approach does not independently explain the difference in welfare gains.

(2) The gains from risk-sharing depend crucially upon the benefits of reducing the variability of the marginal utility over time. In the equity-based approach, this marginal utility depends upon stock returns, while in the consumption-based approach this marginal utility depends upon consumption. Thus, an obvious reason for the difference in measuring gains in the two approaches arises from the greater variability in stock returns relative to consumption.

In an international growing economy, the potential gains from moving to an integrated world capital market depends upon the means as well as the variances.[4] To see why, consider the common consumption approach of assuming that mean consumption growth across countries is equal. This assumption has the effect of making the deterministic growth rate the same so that international capital market integration only reduces the variability of consumption around this common world growth rate. On the other hand, the equity approach focuses upon increasing mean returns while minimizing variance. When the mean stock price returns differ across countries, then the differences between growth rates imply that international capital markets allow domestic investors to move to a different deterministic growth rate in consumption and thereby intertemporally smooth.

Therefore, the difference between the risk-sharing gains may appear to arise from the combination of assumed: (a) common consumption growth across countries; and (b) higher variability of stock returns compared to consumption growth. In this paper, I show that the difference in consumption growth has surprisingly little effect upon risk-sharing gains, while the higher variability of equity returns does.

(3) Both consumption-based and equity-based approaches must specify prefer-ence parameters that govern risk-aversion and intertemporal substitution. However, the low variability of consumption growth relative to equity returns has an

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[4]Whether the underlying process has permanent or transitory disturbances is another important effect. As Obstfeld (1994a) shows, permanent disturbances to idiosyncratic consumption imply that risk-sharing gains are higher. In this paper, I assume that the disturbances to idiosyncratic consumption are permanent (i.e. shocks are permanent and not cointegrated across countries.) Therefore, if the disturbances are not permanent, I am biasing upward my gains from the consumption approach. Since I find that even the estimates based upon permanent shocks to consumption are dramatically smaller than the equity approach gains, transitory shocks to consumption will only deepen the gap between gains from the two approaches.

important effect upon the measured gains from risk-sharing. This observation coupled with plausible preference parameters leads to well-known inconsistencies between consumption-based models and observed financial market data.

To investigate the importance of reconciling preference parameters in the consumption based approach with the observed behavior of stock prices, I conduct two sets of experiments to solve for risk aversion and intertemporal substitutability endogenously. First, I set the means of stock returns and the risk-free rate to equal their values implied by the consumption model. In this case, risk aversion and intertemporal substitution are high. Since higher risk aversion and intertemporal substitutability both increases the value of reducing variability in the future, the risk-sharing gains are quite high, consistent with the equity approach gains. Second, I set the variances of stock returns equal to their values implied by the consumption model. To explain the high variance, risk aversion and intertemporal substitutability must be low. In this case, risk-sharing gains are quite low, even lower than those implied by the standard consumption approach.

The structure of the paper is as follows. In Section 1, I describe the welfare gain function. In Section 2, I use stock returns to provide measures of risk-sharing gains using the equity-based approach. In Section 3, I use consumption data to calculate risk-sharing gains using a standard consumption-based approach. In Section 4, I use stock return data to calculate gains using the consumption-based approach. In Section 5, I use moments of stock return data to back out implied preference parameters and re-examine the consumption-based gains. Concluding remarks follow.

2. The gain function

2.1. The basic framework

To calculate welfare gains, I follow standard practice and calculate the equivalent variation of current utility that brings the investor/consumer up to the same utility level as he would enjoy under optimal risk-sharing.[5] In the consumption-based literature this utility depends upon the consumption level. In the equity-based literature, utility depends upon wealth directly. For now, I simply denote the argument in utility at time t as X_t for generality.

Below, I assume that X_t is log-normally distributed:[6]

$$x_{t+1} = x_t + \mu - \frac{1}{2}\sigma^2 + \varepsilon_t \quad \text{where } \varepsilon_t \sim N(0, \sigma^2) \tag{1}$$

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[5] For example, see Lucas (1987) and Obstfeld (1994a,b).

[6] In simulation experiments, serially correlated consumption and equity returns gave similar results to those found below. I focus upon the analytical solutions in the text, however.

and where both here and below the lower-case letters refer to the natural logarithm of the variable [i.e. $x = \ln(X)$], unless noted otherwise. Furthermore, the optimal path for X_t is denoted as $\underline{X}_t$ while its counterparts to μ and σ are defined as $\underline{\mu}$ and $\underline{\sigma}$, respectively.

Thus, the welfare gain δ is defined by the equation:

$$U(X_t(1 + \delta), \mu, \sigma) = U(\underline{X}_t, \underline{\mu}, \underline{\sigma}) \tag{2}$$

where U is the utility function. Where possible below, I denote the utility conditioned on the time t variable as simply U_t.

Calculating the gains requires specifying a utility function. Constant-relative risk aversion (CRRA) is a standard utility function used in asset pricing as well as calculating welfare gains. However, this utility function assumes that the coefficient of relative risk aversion is the same as the inverse of the intertemporal elasticity of substitution. As shown in Obstfeld (1994a), risk aversion and the inverse of intertemporal substitutability have opposite effects upon welfare gains. Therefore, it is important to use a utility function that does not impose this constraint upon preferences.

For this reason, I use the Epstein and Zin (1989) utility function:[7]

$$U_t = \{X_t^{1-\theta} + \beta[E_t(U_{t+1}^{1-\gamma})]^{(1-\theta)/(1-\gamma)}\}^{1/(1-\theta)} \quad \text{for } \gamma, \theta > 0, \neq 1 \tag{3}$$

The parameter θ can be interpreted as the inverse of the intertemporal elasticity of substitution in consumption. On the other hand, γ is the parameter of relative risk aversion. The standard time-additive utility function results when $\gamma = \theta$.

When X is log-normally distributed, Appendix B shows that time t utility is:

$$U_t = U(X_t, \mu, \sigma) = X_t\left(1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2}\gamma\sigma^2\right)\right]\right)^{-(1/(1-\theta))} \tag{4}$$

Similarly, utility under optimal risk-sharing is given by:

$$U_t = U(\underline{X}_t, \underline{\mu}, \underline{\sigma}) = \underline{X}_t\left(1 - \beta \exp\left[(1 - \theta)\left(\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2\right)\right]\right)^{-(1/(1-\theta))} \tag{4'}$$

where risk-sharing suggests that $\underline{\sigma} < \sigma$. This relationship will be determined endogenously below.

The two parameters, γ and θ, are important in the welfare gain analysis. The role of these two key parameters therefore warrants inspection.

First, note that utility is increasing in the certainty equivalent log consumption growth path, $\mu - \frac{1}{2}\gamma\sigma^2$. Therefore, reductions in the variance of this path will

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[7]While I use the Epstein-Zin function because it provides a more parsimonious representation of reduced-form utility, Obstfeld (1994a,b) uses the Weil (1990) utility function. However, the Weil function is a monotonic transformation of the Epstein-Zin function so that the gain function and all asset pricing relationships are identical using the two utilities.

increase this certainty equivalent according to the parameter of risk aversion, γ. Clearly, then, higher risk aversion γ will lead to greater welfare gains.

Second, note that intertemporal elasticity declines as θ increases. Therefore, the utility value of gains along the CE path in the future declines. For this reason, higher values of θ will lead to lower welfare gains from a reduction in variance.

To solve explicitly for the gain function, substitute Eq. (4) and Eq. (4′) into the gain definition Eq. (2) and solve for δ at an initial time period 0. This implies:[8]

$$\delta = \frac{X_0\left(1 - \beta \exp\left[(1-\theta)\left(\mu - \frac{1}{2}\gamma\sigma^2\right)\right]\right)^{1/(1-\theta)}}{\underline{X}_0\left(1 - \beta \exp\left[(1-\theta)\left(\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2\right)\right]\right)^{1/(1-\theta)}} - 1 \tag{5}$$

Thus, the welfare gains depend upon, first, the current level of the utility determinant relative to the optimal $X_0/\underline{X}_0$, and, second, the relationship between the two certainty equivalent growth paths, $\mu - \frac{1}{2}\gamma\sigma^2$ and $\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2$ evaluated with the intertemporal elasticity of substitution parameter, θ.

2.2. Components of the gain function

Below, I examine the equity-based and consumption-based literature on risk-sharing gains using the gain function Eq. (5). To describe the experiments below, I write this function generally as:

$$\delta = \delta(\mu, \underline{\mu}, \sigma, \underline{\sigma}; \gamma, \theta, \beta; X_0/\underline{X}_0) = \delta(M; \Omega; I) \tag{5′}$$

where M is a moment matrix of means and variances of disturbances along the autarky and optimal growth path; Ω is the set of preference parameters; and $I \equiv X_0/\underline{X}_0$.

I consider two sets of values for the moment matrix M. For the equity-based approach, M corresponds to the moments of stock-returns defined as M_s. On the other hand, for consumption-based calculations, M is comprised of moments of consumption growth rates, defined as M_c.

I evaluate the welfare gains over two ranges of the parameter vector Ω: a set of plausible parameters, denoted Ω^{plaus}, and a set of parameters that match certain moments of asset prices, denoted Ω^{match}.

To determine values for Ω^{plaus}, I consult the literature. Plausible values for risk aversion are considered to be between 1 and 10.[9] On the other hand, θ is typically

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[8]This is the same gain function as used in Obstfeld (1994a) for the case where $X_t = \underline{X}_t$.

[9]Risk aversion coefficients within this range are examined in studies for the welfare gains of international risk-sharing, such as Obstfeld (1994a), Cole and Obstfeld (1991), and Tesar (1995). Mehra and Prescott (1985) consider risk aversion of 10 to be too high.

assumed to be rather high.[10] Finally, β is usually assumed to be less than 1.[11] For Ω^{plaus}, I assume β equals 0.98, following Obstfeld (1994a), and allow θ and γ to vary between 2 and 5.

Since these values for parameters cannot explain asset pricing relationships, I also investigate the set of parameters Ω^{match} which includes values that do match certain relationships. To determine these values, I choose the parameters to equate the means and variances of stock returns and the risk-free rate in the data to their theoretically predicted values.

Finally, the gain function depends upon the variable I which depends upon the ratio of the determinant of utility in autarky to its counterpart under risk-sharing.

2.3. Outline of the remaining analysis

Below, I begin by calculating the gain function for two benchmark cases assuming plausible preference parameters.

In Section 2, I examine the first benchmark case. I show that the equity-based model implies the gain value:

$$\delta = \delta(M_s; \Omega^{\mathrm{plaus}}; 1) \tag{6}$$

That is, for plausible parameters, Ω^{plaus}, the gains depend upon the means and variances of stock returns, M_s. Also, since initial wealth, W_0, is unaffected by risk-sharing, $I = W_0 / \underline{W}_0 = X_0 / \underline{X}_0 = 1$.

In Section 3, I study the second benchmark case. I show that the consumption-based approach implies the gain value:

$$\delta = \delta(M_c; \Omega^{\mathrm{plaus}}; C_0 / \underline{C}_0) \tag{7}$$

where the endogenous determination of stock prices implies that initial autarky consumption does not equal initial consumption under risk-sharing. As described in the introduction, the gains in Eq. (6) are much larger than the gains obtained from Eq. (7).

I then investigate the reasons for these differences. In Section 4, I focus upon the endogenous equity gain function Eq. (7) and ask what assumption can make the gains match those of the exogenous equity gain function Eq. (6). I first relax the common assumption that mean consumption growth rates are common across countries. I next use exogenous equity return moments to counterfactually calculate the endogenous equity welfare gains. Thus, the gain is:

$$\delta = \delta(M_s; \Omega^{\mathrm{plaus}}; W_0 / \underline{W}_0) \tag{8}$$

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[10]For example, Hall (1988) argues that θ is probably not less than 10.

[11]See Kocherlakota (1990), however, for an argument that β can exceed 1.

Finally, in Section 5, I study the effects of preference parameters using the consumption moments. In this case, the gains are the same as in Eq. (7) but with different preference parameters:

$$\delta = \delta(M_c; \Omega^{\text{match}}; C_0 / \underline{C}_0) \tag{9}$$

These experiments in Eqs. (8) and (9) demonstrate the important role played by high variability in marginal utility of consumption, whether in the form of actual equity returns as in Eq. (8) or in risk aversion as in Eq. (9).

3. Equity-based gains using equity returns

3.1. The basic framework

The gains from international diversification in stocks have been noted since at least the 1970s. A standard approach for examining these gains is to calculate the historical means and variances of portfolios that include foreign stocks and determine whether they dominate portfolios of domestic stocks alone. I follow this approach below although clearly this approach ignores the potential for estimation risk to affect portfolio decisions.[12] These more diversified portfolios generate lower variance and/or higher means than do domestic equities alone.

To illustrate, Fig. 1 depicts a combination of mean returns and standard deviations of portfolio combinations that allow for different weights on foreign stocks in the portfolio of a domestic US investor. In particular, I take the returns on the stock market indices from Morgan Stanley International Capital Market Perspectives for the G-7 from 1969 to 1993. I then construct a mutual fund by taking a population-weighted average of the non-US country equities, converting the foreign returns into dollars and then deflating by the US price level.[13] Details are provided in Appendix A.

In Fig. 1, Point A represents the mean and standard deviation (S.D.) of the US stock market over the period, corresponding to a zero weight on foreign stocks. Moving along the curve represents higher weights to the foreign stock. Clearly, the US stock market is dominated by portfolios include foreign stocks. However, the

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[12]Lewis (1999) discusses a recent empirical literature that considers estimation risk in international portfolio allocation and has even questioned the presence of equity home bias. For different evidence on this issue, see Gorman and Jorgensen (1996), Bekaert and Urias (1996), Stambaugh (1997), and Pastor (1999). The question addressed in this paper pertains only to the conventional analysis of welfare gains based upon historical means and variances.

[13]I choose a population-weighted average in order to make the analysis consistent with the consumption-based representative agent framework of the next section and thereby to facilitate the experiments using moments of stock returns below. However, similar results were obtained using, alternatively, a capitalization-weighted average and a simple average of equities.
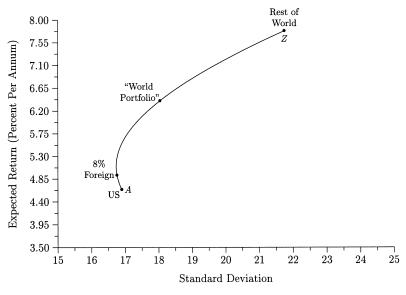
Fig. 1. Risk return tradeoff for US investor.

US aggregate proportion of wealth held in foreign equities appears to be only about 8%, according to Bohn and Tesar (1996). The figure also notes the 'world portfolio' where the shares of stocks equal their shares in the world index.

The portfolios represented by Fig. 1 are therefore combinations of the US stock market and a fixed portfolio of foreign stocks. A fully optimal combination of foreign stocks would be the portfolios providing the lowest variance for any given mean return, the so-called 'efficient frontier.' Since the portfolios of this efficient frontier would imply even higher utility than those given by this more restrictive risk-return tradeoff, the true gains from stock diversification will be even higher than those measured by Fig. 1.

3.2. Calculating welfare gains

With the utility function in Eq. (3), I calculate the gains from moving from the utility of a portfolio of 100% US stocks at point A to the utility at the optimal combination. I follow the standard mean-variance assumption that wealth is the determinant of utility. Thus, if wealth is log-normally distributed, the utility functions at the autarky point A and the optimum are given by Eqs. (4) and (4′), respectively, where $X_t = \underline{X}_t = W_t$, initial US wealth. The means and variances of the autarky and optimal portfolios are different, however, and are given by:

$$w_{t+1} = w_t + \mu_{us} - \frac{1}{2}\sigma_{us}^2 + \zeta_{t+1} \quad \text{where } \zeta \sim N(0, \sigma_{us}^2) \tag{10}$$

$$\underline{w}_{t+1} = \underline{w}_t + \underline{\mu} - \frac{1}{2}\,\underline{\sigma}^2 + \underline{\zeta}_{t+1} \quad \text{where } \underline{\zeta} \sim N(0, \underline{\sigma}^2) \tag{10'}$$

The mean and return of the foreign-allocated wealth portfolio depends upon the mean and variance of the overall portfolio. The evolution of this portfolio can be approximated as:

$$\underline{w}_{t+1} = \underline{w}_t + (1 - \phi)R^{us}_{t+1} + \phi R^{*}_{t+1} \tag{11}$$

where R^{us}_t and R^{*}_t are the returns in the US and foreign equity markets, respectively, and where ϕ is the portfolio share in the foreign equity. Table 1, Panel (A) gives the summary statistics for the US and foreign equity markets.

Then, the optimal choice of the foreign equity share ϕ can be determined by maximizing the utility function Eq. (4′) subject to the constraint that the mean and the variance of wealth in Eq. (11) determines $\underline{\mu}$ and $\underline{\sigma}$ in Eq. (10′). Substituting the mean and variance of Eq. (11) into Eq. ($\overline{10}'$), the parameters of the wealth distribution can be expressed in terms of the portfolio share in the foreign equity, $\underline{\mu}(\phi)$ and $\underline{\sigma}(\phi)$. Thus, the first-order conditions from maximizing Eq. (4′) with respect to the portfolio share ϕ subject to the constraint that $\underline{\mu} = \underline{\mu}(\phi)$ and

Table 1
Equity-based model gains from US diversification using equity data $\delta(M_{\mathrm{s}}; \varOmega^{\mathrm{plaus}}; 1)$

	Mean	S.D.	Correlation	
(A) Summary statistics				
US	4.64	16.90	0.673	
Foreign	7.78	21.75	0.673	
	Foreign share	Mean	S.D.	
(B) Optimal portfolio				
$\gamma = 2$	1.00	7.78	21.75	
$\gamma = 3$	0.74	6.96	19.32	
$\gamma = 4$	0.54	6.34	17.93	
$\gamma = 5$	0.44	6.03	17.42	
	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$
(C) Gains				
$\gamma = 2$	28.83	18.03	12.96	10.02
$\gamma = 3$	26.97	18.20	13.65	10.87
$\gamma = 4$	32.39	26.00	21.75	18.70
$\gamma = 5$	51.80	NA	NA	NA

NA, not available, because diversified utility is not defined: $\beta^{-1} \leq \underline{M}_j^{1-\theta}$ (see Eq. (13) in the text).

$\underline{\sigma} = \underline{\sigma}(\phi)$ is:

$$(\partial \underline{\mu}/\partial \phi) = \frac{1}{2}\gamma(\partial \underline{\sigma}/\partial \phi) \tag{12}$$

Solving Eq. (12) for ϕ determines the optimal portfolio. Note that the portfolio allocation decision depends only upon the degree of risk aversion and no other preference parameters. Details are given in Appendix C.

Panel (B) reports the optimal allocation into the foreign equity as the risk aversion parameter varies from 2 to 5. For γ equal to 2, the investor allocates his portfolio 100% in foreign stocks. However, as risk aversion rises up to γ equal to 5, the investor reduces the variability of his portfolio by moving his foreign allocation share down to 44%. Correspondingly, the mean of his portfolio also declines.

Given these optimal portfolios based upon γ, the gain function can be calculated as in Eq. (5) where $X_0 = \underline{X}_0$ and μ and σ, and $\underline{\mu}$ and $\underline{\sigma}$ are determined by the US stock market and the optimal portfolio of US and foreign stock markets, respectively. Table 1 reports the welfare gains in Panel (C). As noted above, these measures represent the lower bounds for the true gains since the feasible set of portfolios is restricted to linear combinations of the US stock market and a fixed mutual fund of foreign stocks.

Each row of Panel (C) reports the gains for a given level of γ, and therefore a given portfolio allocation. From left to right, as θ increases the welfare gains decrease since the elasticity of substitution decreases and the investor places less utility on future gains in certainty equivalent consumption. For example, for $\gamma = 2$, the gains are 28.8% of current wealth when $\theta = 2$, but only 10% of wealth when $\theta = 5$. For higher risk aversion, the gains generally increase.[14] For example, when $\gamma = 5$ and $\theta = 2$, the gains are about 52% of wealth. For high levels of γ and θ, expected utility is not defined.

To see why utility is not defined as θ and/or γ times the variance σ^2 increases, note that utility in Eq. (4) depends upon the condition that:

$$\beta^{-1} > \exp\left[(1-\theta)\left(\mu - \frac{1}{2}\gamma\sigma^2\right)\right] \tag{13}$$

This is because the inverse of $\beta \exp[(1-\theta)(\mu - \frac{1}{2}\gamma\sigma^2)]$ acts as an overall discount rate for future returns.[15] When Eq. (13) does not hold, as can happen when the certainty equivalent growth rate ($\mu - \frac{1}{2}\gamma\sigma^2$) is negative, utility does not converge as t goes to infinity and this discount rate exceeds unity. This possibility is more likely as γ or σ^2 increase. In the present case, the condition is violated

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[14]Note that welfare gains are strictly increasing in γ for given distribution parameters: $\underline{\mu}$, $\underline{\sigma}$, μ and σ. However, the distribution parameters for the optimal portfolio vary with γ as noted above.

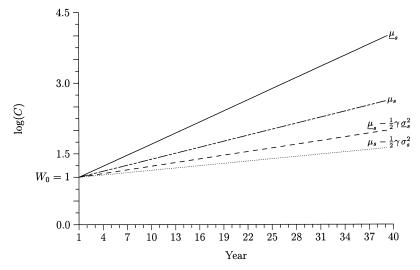[15]See the discussion in Obstfeld (1994a).

Fig. 2. Log consumption profile for US investor facing exogenous stock returns.

because the variability of stock returns is high. In Section 5 below, the condition will be violated in some circumstances because risk aversion is too high.

3.3. Graphical description of the gains from diversification

Fig. 2 illustrates the gains to an individual US investor to moving from the growth path associated with domestic returns alone to the path associated with the optimal portfolio. The top two lines show the difference in growth paths associated with holding the foreign relative to the domestic portfolio in the case where there is no uncertainty. Clearly, the higher returns, labeled $\underline{\mu}_s$, correspond to the investor with the optimal portfolio having a much higher consumption profile than the lower mean US returns, labeled μ_s.

The lower two lines show the certainty equivalent paths in the presence of uncertainty. Both paths are lower than their counterparts in the absence of uncertainty. However, the portfolio with foreign returns also has lower variance than the domestic portfolio.

4. Consumption-based gains using consumption growth

I now provide a simple framework for assessing gains from risk-sharing using the consumption-based approach. Following the literature, I assume that identical representative agents in country j for each of N countries receive their own country's per capita output stream, Y_t^j. For simplicity, I assume that the production

of this output is given as an exogenous endowment process. As before, I calculate welfare gains by solving for and comparing the certainty equivalent consumption paths in the absence of risk-sharing and under perfect risk-sharing. To do so, I first briefly review the closed economy case in the absence of international risk sharing before constructing the diversified equilibrium.

4.1. Autarky

The equilibrium consumption process without access to international markets is trivially given by the endowment process. For countries with identical preferences, the pricing of risk will differ in the N closed economies if the output processes differ across countries. This closed economy equilibrium is well-known and therefore is only briefly summarized here.[16]

Defining s_t as the state of the economy at time t, including realizations of the endowment, the representative agent will maximize utility in Eq. (3) such that his budget constraint holds. Specifically, he will consume each period and buy shares in the domestic equity. This optimization is given by the Bellman equation:

$$V(W^j_t, s_t) = \underset{C^j_t, \kappa^j_t}{\text{Max}}[(C^j_t)^{1-\theta} + \beta E_t(V(W^j_{t+1}, s_{t+1})^{1-\gamma})^{(1-\theta)/(1-\gamma)}]^{(1/(1-\theta))} \quad (14)$$

s.t.

$$C^j_{t+1} + \kappa^j_{t+1} Q^j_{t+1} = (Q^j_{t+1} + Y^j_{t+1})\kappa^j_t \quad (15)$$

where V is the value function based upon utility Eq. (3) which is in turn a function of the wealth of a country j investor at time t, W^j_t, and the state vector at time t, s_t. In particular, $W^j_t \equiv \kappa^j_t Q^j_t$ where κ^j_t are the shares held of stocks paying out the per capita endowment of country j and Q^j_t is the stock price. This maximization implies the first-order condition:

$$\beta^{(\gamma-1)/(\theta-1)} E_t\{(C^j_{t+1}/C^j_t)^{-\theta(\gamma-1)/(\theta-1)}(1 + R^j_{t+1})^{(\gamma-1)/(\theta-1)}\} = 1 \quad (16)$$

where R^j_t is the return on the domestic stock paying domestic per capita endowments, Y^j_t. I assume that the endowment process is log-normally distributed as above. Defining $y^j_t \equiv \ln(Y^j_t)$,

$$y^j_{t+1} = y^j_t + \mu_j - \frac{1}{2}\sigma^2_j + \zeta_{t+1} \quad \text{where } \zeta \sim N(0, \sigma^2_j) \quad (17)$$

In this case, Appendix B shows that the stock price has an analytical solution that depends upon the current level of output as well as the distribution and preference parameters: $Q^j_t = Q^j(Y^j_t; \mu_j, \sigma^2_j; \gamma, \theta, \beta)$. In equilibrium, $\kappa^j_t = 1$ so that each investor holds one share of per capita output.

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[16]See Epstein (1988) and, for the time additive case, Lucas (1982).

4.2. Risk-sharing with open financial markets

Suppose now that international capital markets are opened so that each country can trade in the equities of all other countries. In the new equilibrium, investors in country j hold κ^{ij} shares in stocks of country i output. Instead of Eq. (15), the budget constraint becomes:

$$C_t^j + \sum_{i=1}^{N} \kappa_t^{ij} Q_t^i \leq \sum_{i=1}^{N} \kappa_{t-1}^{ij} (Y_t^i + Q_t^i) \qquad (18)$$

where the maximization in Eq. (14) is now over C_t^j and κ^{ij}, $\forall i = 1, \ldots, N$.

Since all countries have the same homothetic utility function, then each country holds the same portfolio allocation in equilibrium.[17] Therefore, the problem can be rewritten in terms of a world mutual fund paying out the world per capita endowment, defined as $\underline{Y}$. At time t, shares of the mutual fund held by country j and its price are defined as $\underline{\kappa}_t^j$ and $\underline{Q}_t$, respectively. Rewriting the budget constraint Eq. (18) with these definitions implies:

$$C_t^j + \underline{\kappa}_t^j \leq \underline{\kappa}_{t-1}^j (\underline{Y}_t + \underline{Q}_t) \qquad (19)$$

This maximization implies the first-order condition,

$$\beta^{(\gamma-1)/(\theta-1)} E_t \{ (C_{t+1}^j / C_t^j)^{-\theta(\gamma-1)/(\theta-1)} (1 + \underline{R}_{t+1})^{(\gamma-\theta)/(\theta-1)} (1 + \underline{R}_{t+1}^j) \} = 1 \qquad (20)$$

where $\underline{R}_t$ and $\underline{R}_t^j$ are, respectively, the returns on the world mutual fund and the domestic equity at world market prices. The world endowment process is log-normally distributed as above,[18]

$$\underline{y}_{t+1} = \underline{y}_t + \underline{\mu} - \frac{1}{2} \underline{\sigma}^2 + \underline{\zeta}_{t+1} \quad \text{where } \underline{\zeta} \sim N(0, \underline{\sigma}^2) \qquad (21)$$

Appendix B shows that the world mutual fund price, $\underline{Q}$, and the domestic stock price in world markets, $\underline{Q}^j$, have analytical solutions of the form: $\underline{Q}_t^j = \underline{Q}^j(Y_t^j; \mu_j, \sigma_j, \underline{\mu}, \underline{\sigma}; \gamma, \theta, \beta)$ and $\underline{Q}_t = \underline{Q}^j(\underline{Y}_t; \underline{\mu}, \underline{\sigma}; \gamma, \theta, \beta)$ where σ_j is the covariance of the world production process and the domestic production process.

The equilibrium allocation of shares of the world mutual fund depends upon these stock prices. Investors in each country sell off claims to current and future output from their own country in exchange for claims to current and future world output. Residents in country j sell their equity at price $\underline{Q}_t^j$ and for each unit of

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[17]See, for example, Ingersoll (1987).

[18]Note that, as with equity, this specification implies an approximation that both the sum of the output processes and the output processes themselves are log-normally distributed. See Lewis (1996) for Monte Carlo simulations that suggest this approximation is relatively innocuous.

good, they will purchase the quantity $1/\underline{Q}_t$ of shares in the mutual fund. Therefore, in equilibrium, $\underline{\kappa}_t^j = \underline{Q}_t^j/\underline{Q}_t$.

4.3. Calculating welfare gains

Using the autarky equilibrium in Section (3.1) and the open economy equilibrium in Section (3.2), welfare gains can be calculated using Eq. (5). Note that, under autarky, the initial level of consumption, $C_0 = Y_0^j$, while the optimal initial level of consumption depends upon the value of the economy's share in world output, $\underline{C}_0 = \underline{\kappa}_0^j \underline{Y}_0 = (\underline{Q}_0^j/\underline{Q}_0)\underline{Y}_0$. Therefore, the gain function can be rewritten as in Eq. (5) where $X_0/\underline{X}_0 = (Y_0^j\underline{Q}_0)/(\underline{Y}_0\underline{Q}_0^j)$. Since these prices depend only upon the parameters of the distribution and the utility function, the gains can be calculated for each country given the consumption means and variances as detailed in Eq. (7) $\delta = \delta(M_c; \Omega^{\text{plaus}}; C_0/\underline{C}_0)$.

Table 2 reports the results of calculating the welfare gains for each of the G-7 countries against the world. Following Obstfeld (1994b), I use an updated version through 1992 of the data in the Penn World Tables (Summers and Heston, 1991). For the purpose of comparison with the stock market data, the sample begins in 1969. Panel (A) reports summary statistics for the G-7 countries as well as a world index that corresponds with the population-weighted world stock market index discussed in Section 2.

Many consumption-based measures of welfare gains from risk-sharing treat consumption growth rates as equal across countries. For this reason, Panel (B) reports the welfare gains from risk-sharing when all countries are assumed to share the same growth rate as the world. To conserve space, the gains are reported for the two extreme cases examined in Table 1, i.e. for γ and θ equal to 2 and 5. The lowest welfare gains are represented by the case where risk aversion is lowest at $\gamma = 2$ and where the inverse of intertemporal substitutability is highest at $\theta = 5$. The gains are slightly larger for the time-additive cases, $\gamma = \theta = 2$ and 5, and are largest when risk aversion and intertemporal substitution is high with $\gamma = 5$ and $\theta = 2$. However, the gains are significantly lower than those in Table 1 based upon similar utility parameters.

In most cases, the gains are quite low. For the US, the maximum gain is 0.25% of current consumption. Certain countries measure higher gains from risk-sharing relative to other countries. The higher gains typically arise from two sources. First, for countries such as Canada or the UK, their autarky consumption paths are more variable than the rest of the world. Second, countries such as Italy have a lower correlation with the rest of the world, thereby increasing the value of their equity on world markets.

Panel (C) examines the effects of allowing the mean growth rates to differ. The overall magnitude of the gains tend to increase with differing means. For the lowest gain parameters, the US gains increase to 0.06% from 0.04%. Strikingly,

Table 2
Consumption-based model gains from diversification using consumption data $\delta(M_c; \Omega^{\text{plaus}}; C_0/\underline{C}_0)$

	Mean	S.D.	Correlation matrix							
			Canada	US	Japan	France	Germany	Italy	UK	World
(A) Summary statistics										
Canada	2.61	2.70	1.00	0.62	0.20	0.24	0.42	0.32	0.34	0.62
US	1.86	1.91	–	1.00	0.57	0.50	0.54	0.02	0.63	0.95
Japan	3.18	1.91	–	–	1.00	0.73	0.45	0.11	0.68	0.74
France	2.07	1.06	–	–	–	1.00	0.58	0.17	0.46	0.65
Germany	2.26	1.67	–	–	–	–	1.00	0.26	0.39	0.65
Italy	2.98	1.76	–	–	–	–	–	1.00	0.18	0.21
UK	2.58	2.93	–	–	–	–	–	–	1.00	0.77
World	2.34	1.53	–	–	–	–	–	–	–	1.00
				$\theta = 5$		$\theta = 5$		$\theta = 2$		$\theta = 2$
				$\gamma = 2$		$\gamma = 5$		$\gamma = 2$		$\gamma = 5$
(B) Same means										
US				0.04		0.08		0.10		0.25
Canada				0.39		0.97		1.01		2.47
Japan				0.15		0.36		0.38		0.95
France				0.13		0.34		0.32		0.83
Germany				0.17		0.42		0.42		1.05
Italy				0.40		1.00		1.01		2.53
UK				0.35		0.86		0.92		2.30
(C) Differing means										
US				0.06		0.16		0.85		1.11
Canada				0.79		1.30		1.68		3.09
Japan				2.16		2.36		4.91		5.47
France				0.00		0.19		0.40		0.83
Germany				0.10		0.35		0.37		1.00
Italy				1.76		2.38		3.91		5.60
UK				0.68		1.11		1.44		2.66

the maximum welfare gains for Japan increase to 5.5% relative to only 0.95% in Panel (B). Thus, much of the gains to Japan derive from the strong equity value of its high growth rate. I illustrate this effect graphically below.

Although these values are larger than when the means are assumed to be the same, they remain significantly smaller than the stock return gains in Table 1.

4.4. Graphical representation of the gains

Fig. 3 shows the source of these gains with differing means for the low gain case where $\gamma = 2$ and $\theta = 5$. The other parameter values imply similar trade-off. For each country, the figure shows the certainty equivalent consumption growth paths for autarky as the dashed line (labeled as $\mu - \frac{1}{2}\gamma\sigma^2$) and for risk-sharing as the
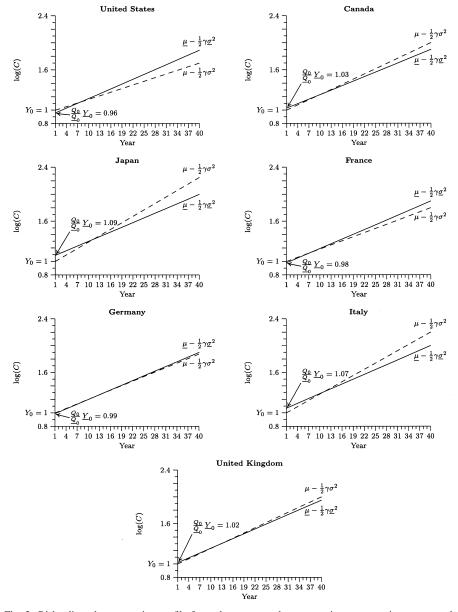
Fig. 3. Risk-adjusted consumption profile for endogenous stock returns using consumption means and variances ($\gamma = 2$, $\theta = 5$).

solid line (labeled as $\mu - \frac{1}{2}\gamma\underline{\sigma}^2$). The intercepts show the level of consumption associated with the path at time 0.

This figure makes clear why Japan has relatively large gains while the US has low gains. The Japanese sell off their claims to their high growth economy by receiving a relatively high price in the beginning of time. The Japanese participate because they substitute current consumption for future consumption. The rest of the world wants the higher growth and provides this intertemporal substitution. Therefore, Japan enjoys large gains.

On the other hand, the US has low gains because Americans trade off current consumption for future consumption. Furthermore, since the US output stream is highly correlated with the world output stream, the price of US equity is relatively low. Similar trade-offs are shown for the other countries as well.

4.5. Conclusions from consumption gains

Overall, the evidence in Tables 1 and 2 shows that the equity-based approach generates significantly higher gains than the consumption-based approach. While both measures are based upon a similar gain function, the analysis so far points to two main potential reasons.

First, the consumption approach treats stock prices as endogenous and thereby allows for an intertemporal substitution across countries as well as the opportunity to reduce risk. This intertemporal substitution is missing in the exogenous stock-based approach. Clearly, this intertemporal substitution can potentially exacerbate the gains for some countries such as Japan with high growth rates. Therefore, this explanation alone does not appear to explain the puzzle.

Second, the measures of risk-adjusted growth paths are different. The consumption approach calculates this path from the means and variances of consumption growth. The equity approach generates this path using the means and variances of equity returns. This approach treats the underlying utility as generated from the stock return process. This distinction suggests a simple, counterfactual experiment for studying the implications of this assumption, as I describe next.

5. Consumption-based gains using equity returns

To investigate the importance of treating the underlying determinant as utility of stock returns as opposed to consumption, I recalculate the consumption approach gains using stock return data instead of consumption data. Although calculating the gains in this way is clearly counterfactual, it addresses the question: are the higher gains measured by equity generated by the higher variability and/or differences in mean returns in stocks as opposed to consumption growth?

5.1. Measuring the gains

I calculate the general equilibrium gain using Eq. (5) where as for the general equilibrium results in Table 2, $X_0/\underline{X}_0 = (Y_0^j \underline{Q}_0)/(\underline{Y}_0 \underline{Q}_0^j) \neq 1$. In this case, however, the stock prices depend upon the means and variances of stock returns as opposed to consumption and the initial consumption level is treated as the wealth level, $X_0/\underline{X}_0 = (Y_0^j \underline{Q}_0)/(\underline{Y}_0 \underline{Q}_0^j) = W_0/\underline{W}_0$. Furthermore, the utility level itself depends upon the means and variances, so that the gain function can be written in the general form given in Eq. (8): $\delta = \delta(M_s; \Omega^{\mathrm{plaus}}; W_0/\underline{W}_0)$.

Panel (A) of Table 3 reports the summary statistics used in the calculation. As with consumption, the correlation of US equity returns with the world portfolio is higher than that of the other countries.

Panels (B) and (C) give the gains for the extreme case of low and high gain parameter combinations: $\theta = 5$, $\gamma = 2$ and $\theta = 2$, $\gamma = 5$. Panel (B) reports the gains for the case where the world mean return is assumed the same for all countries, the

Table 3
Consumption-based model gains from diversification using equity data $\delta(M_s; \Omega^{\mathrm{plaus}}; W_0/\underline{W}_0)$

	US	Canada	Japan	France	Germany	Italy	UK	World	Interest rate
(A) Summary statistics									
Mean	4.64	2.64	10.72	6.48	6.34	−0.17	−7.02	5.86	2.98
S.D.	16.51	16.94	30.46	27.45	23.88	32.96	29.36	17.61	2.62
Corr(X, World)	0.873	0.735	0.791	0.689	0.510	0.615	0.672	1.000	–
			$\theta = 5$		$\theta = 5$		$\theta = 2$		$\theta = 2$
			$\gamma = 2$		$\gamma = 5$		$\gamma = 2$		$\gamma = 5$
(B) Same means									
US			4.51		NA[a]		12.79		NA[a]
Canada			9.15		NA[a]		26.25		NA[a]
Japan			25.03		NA[b]		54.78		NA[b]
France			21.98		NA[a]		52.96		NA[b]
Germany			23.80		NA[a]		61.31		NA[a]
Italy			43.62		NA[a]		79.26		NA[b]
UK			27.44		NA[a]		61.46		NA[b]
(C) Differing means									
US			3.85		NA[a]		11.99		88.78
Canada			14.80		NA[a]		35.38		NA[b]
Japan			26.49		NA[a]		87.08		NA[a]
France			21.10		NA[a]		53.50		NA[a]
Germany			24.04		NA[a]		64.33		NA[a]
Italy			NA		NA[b]		NA		NA[b]
UK			25.18		NA[a]		63.12		NA[a]

[a] Not available, because domestic stock price at world prices is not defined: $\beta^{-1} \leq \underline{M}^{-\theta} H_j$ (see Eq. (B.15) in Appendix B).

[b] Not available, because autarky utility is not defined: $\beta^{-1} \leq \underline{M}_j^{1-\theta}$ (see Eq. (13) in the text).

counterpart of Panel (B) in Table 2. Panel (C) reports the gains when the means are allowed to differ by country, the standard assumption in the equity approach literature.

Clearly these gains are substantially larger than those in Table 2. As with the consumption-based gains, the gains are lower for the US than the other countries since its correlation with the world is higher. For the other countries, the gains are quite large. Even for the low gain parameter case, the benefits to these countries from risk-sharing range from about 15% to 26% of permanent consumption.

5.2. Graphical representation of the gains

Fig. 4 shows why these gains are so much larger when stock return means and variances are used. As before, the figure shows the autarky certainty equivalent growth path of consumption as the dashed line, while the risk-sharing path is depicted by the solid line. As in Fig. 3, the parameter values are the low gains case: $\gamma = 2$ and $\theta = 5$.

For countries such as Canada, the differences in variances based upon risk-sharing relative to autarky imply significant increases in the tilt to risk-adjusted consumption growth. Since Italy's autarky risk-adjusted growth rate is negative, the gains from moving to risk-sharing are unbounded and undefined. For France, Germany and the UK, risk-sharing not only puts the economies on a higher risk-adjusted consumption growth path, it also raises initial consumption as well. The reason is that these countries obtain a high price for their equity on world markets because of their relatively high mean and low correlation with the world return. Even though their autarky deterministic mean returns are higher than the world, these countries achieve a higher risk-adjusted growth rate of consumption because of the lower variance resulting from diversification. This improvement in variance measured with consumption data is significantly lower since consumption variances are so much smaller.

As in Fig. 3, Japan gains by intertemporally substituting higher current consumption levels for future consumption growth. Using stock return means and variances exacerbates this effect, however. The significantly higher mean of Japanese stocks together with its diversification benefits imply that Japan can command a significantly high price of stocks on world markets. With a growth rate of nearly 11% in the stock index, the Japanese stock price is so high that the risk-sharing consumption levels are above the autarky levels for over 25 years. As a result, the gains to Japan are substantial.

As this figure shows, an important contributing factor to why stock returns suggest such different gains of international risk-sharing is the dramatically higher variance of stocks. This high variance implies both that investors are willing to pay significantly for claims on equities that reduce overall consumption risk and that risk-adjusted consumption growth is dramatically increased by reductions in variance.
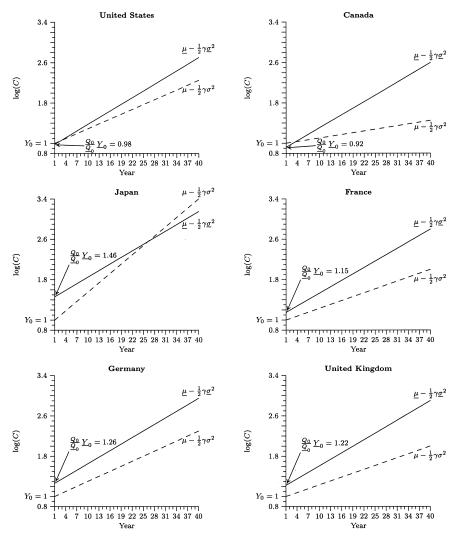
Fig. 4. Risk-adjusted consumption profile for endogenous stock returns using stock returns means and variances ($\gamma = 2$, $\theta = 5$).

These calculations are counterfactual, however. They treat stock returns as though they characterize the consumption process. It is well-known that the general equilibrium framework cannot explain the means or variances of stock returns with plausible utility parameters. Therefore, an alternative explanation of the puzzle may be that the preference parameter values for the consumption-based

model are inconsistent with the behavior of stock returns. I consider this possibility next.

6. Consumption-based gains using preference parameters that match equity returns

The inconsistency between stock return behavior and its predictions from consumption-based models using conventional utility parameter values has been established repeatedly in the literature. For example, Mehra and Prescott (1985) show that, with time-additive utility, the mean excess return on equity over the risk-free rate in the US requires very high levels of risk-aversion. This relationship is verified in Weil (1989) who shows that high levels of risk-aversion also generate implausibly high levels of the risk-free rate. Kandel and Stambaugh (1991) argue that, while the equity premium and the risk-free rate can be explained by high levels of risk-aversion, other features of stock returns cannot.

In this section, I ask whether parameter values that match features of return behavior can help reconcile the difference in implied gains between equity and consumption risk-sharing.

6.1. Matching the mean equity returns and risk-free rate

I begin by solving for the risk-aversion parameter γ and the intertemporal elasticity parameter θ that match the mean of the equity returns and the risk-free rate implied by the model to their counterparts in the data.[19] That is, the consumption-based model in Section 3 implies stock returns given the moments of consumption, M_c, and parameters, Ω. The form of these returns is derived in Appendix D. Table 2, Panel (A) reports the means and variances of these returns. Therefore, solving the two equations for the mean of equity and the mean of the risk-free rate for the two unknown parameters, γ and θ, provides the values for the matching parameters, Ω^{match}.

Table 4 reports these combinations of γ and θ matching three different equity returns: the autarky US stock price in Panel (A), the US stock price in integrated world markets in Panel (B), and the world mutual fund price in Panel (C). For each combination of preference parameters, the table provides summary statistics for each country's implied returns from the model.

In all three panels of Table 4, θ is quite low while γ is quite high. In fact, to explain both the high world mutual fund price of 5.86% and the 3% risk-free rate

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[19]For the purposes of calculating the risk-free rate, I use the dollar London-Interbank Offer Rate (LIBOR) deflated by US inflation. See Appendix A.

Table 4
Preference parameters that match mean returns and their implied international returns

	US	Canada	Japan	France	Germany	Italy	UK	World
(A) Matching US equity for autarky and risk-free rate								
Implied parameters: $\theta = 1.69$, $\gamma = 52.72$								
Implied returns:								
Mean	4.64	5.31	7.00	5.47	5.51	6.75	5.02	5.01
S.D.	2.00	2.82	2.04	1.12	1.77	1.87	3.08	1.60
(B) Matching US equity for integrated markets and risk-free rate								
Implied parameters: $\theta = 0.91$, $\gamma = 69.8$								
Implied returns:								
Mean	4.64	4.50	4.20	3.40	3.84	3.04	5.13	4.32
S.D.	2.00	2.82	1.99	1.10	1.74	1.81	3.08	1.59
(C) Matching world equity and risk-free rate								
Implied parameters: $\theta = 2.6$, $\gamma = 132.5$								
Implied returns:								
Mean	6.47	6.20	5.63	4.09	4.93	3.40	7.42	5.86
S.D.	2.04	2.87	2.02	1.10	1.76	1.82	3.15	1.61
Corr(X, World)	0.623	0.947	0.742	0.651	0.652	0.206	0.767	1.000

in Panel (C), the risk aversion parameter is 132.5 while the inverse of the intertemporal elasticity of substitution is only 2.6.[20]

Notably, even when the means of returns are matched as in Table 4, the variances are not. In all cases, the implied standard deviations are significantly lower than the actual standard deviations in the data. I will return to this problem below.

6.2. Implied welfare gains from international diversification

I now ask whether the consumption-based gains implied by preference parameters that match the equity returns are consistent with the equity-based gains. Table 5, Panel (A) therefore reports the gains as given in Eq. (9) $\delta = \delta(M_c; \Omega^{\text{match}}; C_0/\underline{C}_0)$. These gains correspond to the same gains calculated as in Table 2, but using values of θ and γ that match returns.

The first column of Table 5 Panel (A) gives welfare gain estimates when the utility parameters are set to match the US returns under autarky and in Panel (A) of

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[20]The differences between the closed US economy case in Panel (A) and other estimates in the literature largely derive from differences in the risk-free rate and the equity premium over this period. Using century-long data, Mehra and Prescott (1985) report an estimate of 0.9 for the risk-free rate and about 6% for the excess of equity over the risk-free rate.

Table 5
Consumption-based model gains from diversification using matching parameters $\delta(M_c; \Omega^{\text{match}}; C_0/\underline{C}_0)$

	US equity (autarky) $\gamma=52.72$ $\theta=1.69$	US equity (integrated) $\gamma=69.8$ $\theta=0.91$	World equity $\gamma=132.5$ $\theta=2.62$
(A) Matching means			
US	7.3	12.4	31.4
Canada	33.2	55.2	NA[b]
Japan	18.3	40.3	32.8
France	11.3	27.5	34.3
Germany	14.2	30.1	33.4
Italy	42.4	NA[a]	89.5
UK	33.5	51.7	NA[b]
	US equity (autarkey) $\gamma=5.94$ $\theta=123.96$	US equity (integrated) $\gamma=8.39$ $\theta=94.61$	World equity $\gamma=11.98$ $\theta=109.55$
(B) Matching variances			
US	0.00	0.00	0.00
Canada	0.29	0.30	0.30
Japan	0.90	0.90	0.87
France	0.00	0.00	0.00
Germany	0.00	0.00	0.00
Italy	0.77	0.84	0.90
UK	0.22	0.20	0.17

[a] Not available, because domestic stock price at world prices is not defined: $\beta^{-1} \leq \underline{M}^{-\theta}H_j$.
[b] Not available, because autarky utility is not defined: $\beta^{-1} \leq M_j^{1-\theta}$ (see Eq. (13) in the text).

Table 4. While the gains for some countries such as Italy, the UK and Canada are quite large in excess of 33%, the gain for the US is lower at about 7%.

The second column uses the parameters matched to the US equity return under open markets. Again, the costs for the US are larger, but remain less than those implied by stock returns at about 12%.

Finally, the third column matches the equity premium to the return on the world equity mutual fund. In this case, with a risk aversion parameter of 132.5, the welfare gains are in the range suggested by the stock returns. Furthermore, the risk-adjusted growth rates for the high variances countries of the UK and Canada now become negative and the effective discount rate exceeds unity, implying that utility is not defined.

Overall, the evidence suggests that when utility parameters match the world equity premium, even consumption data imply large gains to risk-sharing.

6.3. Graphical representation of the gains

Fig. 5 shows why the gains from international diversification are so large with high risk aversion. For the purposes of illustration, I use the parameters $\gamma=132.5$
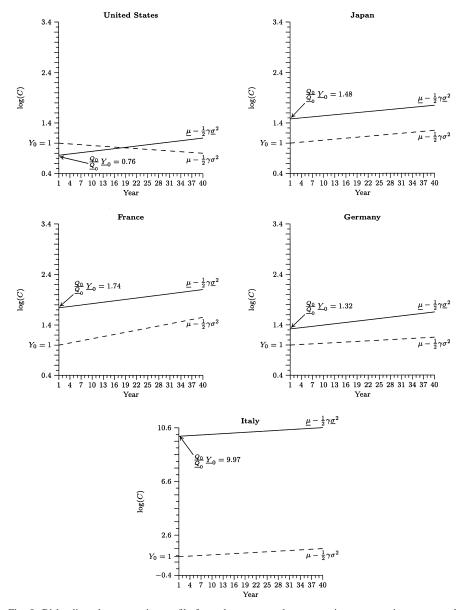
Fig. 5. Risk-adjusted consumption profile for endogenous stock returns using consumption means and variances ($\gamma = 132.5$, $\theta = 2.6$).

and $\theta = 2.6$ which match the world equity premium and the risk-free rate [Table 4, Panel (C)]. With high risk aversion, the autarky consumption profiles are significantly flatter than their low risk aversion counterparts depicted in Fig. 3. For the US, in fact, the high risk aversion implies that the autarky risk-adjusted consumption profile is negative. Furthermore, the risk-adjusted consumption paths for the UK and Canada are sufficiently negative to violate the condition in Eq. (13) that the discount rate is less than 1. As a result, utility is undefined for these two countries at these parameter values.

The risk-sharing equilibrium leads to significant welfare gains. With high risk-aversion, investors in each country value greatly even small reductions in consumption variances. As a result, the stocks from countries such as Italy having low covariances with the rest of the world become much more valuable. This phenomenon also shows why the US has lower gains than the other countries. Since the US has the highest covariances with the rest of the world, its stock is less valuable on the world market. This is depicted in Fig. 5 by a drop in the intercept, representing a decline in initial consumption. However, the gains from buying an increasing risk-adjusted consumption path significantly outweigh the losses from the low equity value.

Comparing Figs. 4 and 5 shows that the profiles measured using stock return data and plausible preference parameters are quite different from those measured using consumption data and preference parameters that make consumption data match stock returns in the model. The stock return calculations in Fig. 4 using low risk aversion imply steeply rising consumption paths. The gains derive from a higher growth rate, an intertemporal substitution toward higher current consumption, or both. On the other hand, Fig. 5 show that high risk aversion coupled with consumption data lower the risk-adjusted consumption growth rate both in autarky and under risk-sharing. Thus, for countries other than the US, the gains derive largely from a higher value of domestic equity on world markets, depicted by the shift in intercept.

6.4. Matching variances

Above, I described the gains from diversification using preference parameters that match the means of stock returns. However, Table 4 demonstrates that the models imply variances that are too low to be consistent with actual stock return variances. I therefore consider a different set of parameters that match the variance, instead of the mean, of stock returns. These calculations are detailed in Appendix D.

Panel (B) of Table 5 reports the results for the three stock returns described previously: the autarky domestic stock return, the domestic stock return at world prices, and the world equity return. As the table shows, the intertemporal substitutability must be very low to match the stock return variances. θ, the inverse of this substitution parameter, ranges from 94.61 in the case of domestic equity at

world prices to 123.96 for domestic equity at autarky. Intuitively, for investors to be willing to accept such high variability, they must be relatively indifferent to changes in marginal utility over time. For this indifference to be true, the elasticity of substitution must be extremely low (θ high). At the same time, risk aversion is moderate with γ ranging from about 6 to 12.

As described above, low intertemporal substitution mitigates the gains from risk-sharing. In this case, investors do not value strongly the future gains of lower variability in the consumption profile. Thus, some countries such as the US that must substitute current for future consumption in the risk-sharing equilibrium gain less than 0.01% of permanent consumption. These gains are clearly lower than the more standard consumption-based gains measured in Table 2.

7. Conclusion

In this paper, I have examined the sources of differences between equity-based and consumption-based calculations of the welfare gains from international risk-sharing. The differences largely come from differences in the utility gain of realized variability. This utility gain can arise in two ways. First, the variability of the determinant of utility can itself be high. Thus, when the determinant of utility is assumed to be equity, the high return variance implies significantly higher variability of utility over time compared to the case when this determinant is assumed to be consumption. Second, the value of reducing the variability may be high. For example, when risk aversion is sufficiently high to explain the equity premium on an international diversified world mutual fund, the implied gains from risk-sharing measured from consumption is comparable to the gains based upon equity directly.

The paper shows that either of these factors can explain the differences between the high gains from equity-based risk-sharing compared to consumption-based risk-sharing. However, the analysis also demonstrates that these two effects operate through somewhat different channels. Using stock return data in a consumption-based model, the gains largely arise from increases in the growth rate of the certainty equivalent path of consumption. However, using high risk aversion implies flatter certainty equivalent consumption paths but greater gains from intertemporal substitution.

The paper also rules out the importance of some other plausible explanations for differences in measured gains. Conventional wisdom suggests that calculations based upon exogenous stock returns do not allow the investor to internalize the effects of his actions on the stock return and thereby exacerbate the gains from international diversification. However, I show that internalizing these effects can actually increase the gains from diversification for high growth countries.

This paper pinpoints the reasons for significant differences in apparent gains to risk sharing between equity-based and consumption-based measures. Therefore,

these results should be useful for future research on measuring international risk-sharing gains.

Acknowledgements

Appendix A. Data sources and construction

The data for the stock market series are the country stock market indexes from Morgan Stanley (MSCI) with gross dividends reinvested. The series are measured in dollars and converted into December year-over-year growth rates in real US terms by deflating the index using the 'consumer price index for all goods' reported in the *Economic Report of the President*. The foreign mutual fund used in Fig. 1 and Table 1 is a 1969 population-weighted average of the non-US G-7 countries excluding Italy. The stock on Italy was excluded since its negative average return and higher variance made it clearly dominated by the other stocks. The risk-free rate is the six month London Interbank Offer Rate (LIBOR) from the *London Financial Times*. Following standard practice in the literature, the consumption data were taken from an updated version of the Penn World Tables described in Summers and Heston (1991).

Appendix B. Deriving the theoretical welfare gains

B.1. Deriving expected utility under the assumption of log-normality

Assume that the distributions for, alternatively, wealth in the equity case and consumption in the consumption case are log-normally distributed:

$$x_t = x_{t-1} + \mu - \frac{1}{2}\sigma^2 + \varepsilon_t \quad \text{where } \xi_t \sim N(0, \sigma^2) \tag{B.1}$$

Solving for the utility at time t requires forming a guess about the solution and

verifying. By straightforward iteration of Eq. (1) in the text and using properties of the log-normal distribution, the guess for the case when $\theta \neq 1$ is taken to be:

$$U_t = X_t \left\{ 1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2}\gamma\sigma \right) \right] \right\}^{-(1/(1-\theta))} \tag{B.2}$$

Leading Eq. (B.2) one period and substituting the result into (1) implies:

$$U_t = \left[X_t^{(1-\theta)} + \left[\beta \Big/ \left\{ 1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2}\gamma\sigma \right) \right] \right\} \right] \right.$$
$$\left. \times (E_t[X_{t+1}^{(1-\gamma)}])^{(1-\theta)/(1-\gamma)} \right]^{(1/1-\theta)}$$
$$= X_t \left[1 + \left[\beta / \left\{ 1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2}\gamma\sigma \right) \right] \right\} \right] \right.$$
$$\left. \times (E_t[(X_{t+1}/X_t)^{(1-\gamma)}])^{(1-\theta)/(1-\gamma)} \right]^{(1/1-\theta)} \tag{B.3}$$

But by the property of log-normal distributions:

$$E((X_{t+1}/X_t)^{(1-\gamma)}) = \exp\left[(1 - \gamma)\left(\mu - \frac{1}{2}\sigma^2 \right) + \frac{1}{2}(1 - \gamma)^2\sigma^2 \right]$$
$$= \exp\left[\mu - \frac{1}{2}\gamma\sigma^2 \right]^{(1-\gamma)} \tag{B.4}$$

Substituting Eq. (B.4) into Eq. (B.3) and rearranging yields Eq. (B.2) so that the guess is verified.

B.2. Deriving the welfare gains

The gain function δ is defined by Eq. (2) in the text: $U[(1 + \delta)X_t, \mu, \sigma^2] = U[\underline{X}_t, \underline{\mu}, \underline{\sigma}^2]$ where $X_t(\underline{X}_t)$ is the initial autarky (optimal) time t wealth in the stock case and consumption in the consumption case. Also, μ and σ are the means and variances along the autarky path and $\underline{\mu}$ and $\underline{\sigma}$ are the means and variances along the optimal path. Substituting the utility function in Eq. (B.2) into Eq. (2) implies:

$$(1 + \delta)X_t \left\{ 1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2}\gamma\sigma^2 \right) \right] \right\}^{-(1/(1-\theta))}$$
$$= \underline{X}_t \left\{ 1 - \beta \exp\left[(1 - \theta)\left(\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2 \right) \right] \right\}^{-(1/(1-\theta))} \tag{B.5}$$

Solving for δ for $t = 0$ implies Eq. (5) in the text.

B.2.1. Welfare gains for the equity based approach (Table 1)

In this case, initial wealth is unaffected by risk-sharing. Therefore, $W_0 = X_0 = \underline{X}_0$ so that the gain function Eq. (5) becomes:

$$\delta = \left\{ \left(1 - \beta \exp\left[(1 - \theta)\left(\mu - \frac{1}{2} \gamma\sigma^2 \right) \right] \right) \right.$$
$$\left. \div \left(1 - \beta \exp\left[(1 - \theta)\left(\underline{\mu} - \frac{1}{2} \gamma\underline{\sigma}^2 \right) \right] \right) \right\}^{(1/(1-\theta))} - 1 \qquad \text{(B.6)}$$

Note that this gain function is identical to the gain function obtained by Obstfeld (1994a) [p. 1478, Eq. (11)] since Obstfeld considers a representative agent in a closed economy who cannot change his initial endowment level by risk-sharing.

B.2.2. Welfare gains for the general equilibrium case (Tables 2, 3 and 5)

In this case, initial wealth is affected by risk-sharing since high growth countries can intertemporally substitute future consumption into present consumption and vice versa. In this case, the initial autarky endowment level for country j is: $X_0 = Y_0^j$. The initial risk-sharing endowment level is: $\underline{X}_0 = (\underline{Q}_0^j / \underline{Q}_0) \underline{Y}_0$ where $\underline{Q}_t^j$ is the world price of equity which pays out country j's endowment every period, $\underline{Q}_t$ is the price of a world mutual fund which pays out the world endowment every period, and $\underline{Y}_t$ is the world endowment at time t. In this case, introducing the subscript j to indicate the country, the gain function becomes:

$$\delta = (Y_0^j \underline{Q}_0)/(\underline{Y}_0 \underline{Q}_0^j)\left\{ \left(1 - \beta \exp\left[(1 - \theta)\left(\mu_j - \frac{1}{2} \gamma\sigma_j^2 \right) \right] \right) \right.$$
$$\left. \div \left(1 - \beta \exp\left[(1 - \theta)\left(\underline{\mu} - \frac{1}{2} \gamma\underline{\sigma}^2 \right) \right] \right) \right\}^{(1/(1-\theta))} - 1 \qquad \text{(B.7)}$$

This gain function differs from that in Obstfeld (1994a) according to the multiplying term, $(Y_0^j \underline{Q}_0)/(\underline{Y}_0 \underline{Q}_0^j)$, which reflects the initial substitution of country j's autarky endowment for country j's share of the world endowment.

B.3. Deriving the general equilibrium stock prices

The following describes the derivation of three prices: (a) the autarky price of domestic equity, Q^j; (b) the world mutual fund, $\underline{Q}$; and (c) the world price of domestic equity, $\underline{Q}^j$.

B.3.1. Autarky price of domestic equity

Eq. (16) in the text gives the first-order condition as:

$$1 + R_{t+1}^j = (Y_{t+1}^j + Q_{t+1}^j)/Q_t^j \qquad \text{(B.8)}$$

The stock price can be solved by guessing and verifying the solution. In this case, the guess is:

$$Q^j_t = Y^j_t \beta \exp\left[(1-\theta)\left(\mu_j - \frac{1}{2}\gamma\sigma^2_j \right) \right]$$
$$\div \left(1 - \beta \exp\left[(1-\theta)\left(\mu_j - \frac{1}{2}\gamma\sigma^2_j \right) \right] \right) \tag{B.9}$$

Defining the certainty-equivalent consumption path as: $M_j \equiv \exp(\mu_j - \frac{1}{2}\gamma\sigma^2_j)$, the stock price guess in Eq. (B.9) can be rewritten as:

$$Q^j_t = Y^j_t \beta M^{(1-\theta)}_j / (1 - \beta M^{(1-\theta)}_j) \tag{B.9'}$$

Substituting Eq. (B.9′) into Eq. (B.8) and the result into Eq. (16) in the text verifies the conjecture.

B.3.2. Price of the world mutual fund

Deriving the price of the world mutual fund follows exactly the same steps as the domestic price in autarky. The first-order condition becomes:

$$\beta^{(\gamma-1)/(\theta-1)} E_t\{(c^j_{t+1}/c^j_t)^{-\theta(\gamma-1)/(\theta-1)}(1+\underline{R}_{t+1})^{(\gamma-1)/(\theta-1)}\} = 1 \tag{B.10}$$

where

$$1 + \underline{R}_{t+1} = (\underline{Y}_{t+1} + \underline{Q}_{t+1})/\underline{Q}_t \tag{B.11}$$

The stock price guess now becomes:

$$\underline{Q}_t = \underline{Y}_t \beta \underline{M}^{(1-\theta)} / (1 - \beta \underline{M}^{(1-\theta)}) \tag{B.12}$$

where the certainty-equivalent mutual fund consumption path is: $\underline{M} \equiv \exp(\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2)$. Following the same steps as above verifies the guess.

B.3.3. World price of domestic equity

Eq. (20) in the text gives the first-order condition where:

$$1 + \underline{R}^j_{t+1} = (Y^j_{t+1} + \underline{Q}^j_{t+1})/\underline{Q}^j_t \tag{B.13}$$

The stock price guess is:

$$\underline{Q}^j_t = Y^j_t \beta \exp\left[-\theta\underline{\mu} + \mu_j + \frac{1}{2}\gamma\underline{\sigma}^2(1+\theta) - \gamma\underline{\sigma}_j \right]$$
$$\div \left(1 - \beta \exp\left[-\theta\underline{\mu} + \mu_j + \frac{1}{2}\gamma\underline{\sigma}^2(1+\theta) - \gamma\underline{\sigma}_j \right] \right) \tag{B.14}$$

where $\underline{\sigma}_j$ is $\mathrm{Cov}(\underline{R}, R^j)$, the covariance between the return on the world mutual fund and the return on the domestic equity in world markets. Using the definition for $\underline{M}$ along with the definition, $H_j \equiv \exp[\mu_j + \frac{1}{2}\gamma\underline{\sigma}^2 - \gamma\underline{\sigma}_j]$, the stock price can be rewritten as,

$$\underline{Q}^j_t = Y^j_t \beta \underline{M}^{-\theta} H_j / (1 - \beta \underline{M}^{-\theta} H_j) \tag{B.15}$$

Substituting Eq. (B.15) into Eq. (B.13) and using this solution for $\underline{R}^j$ (along with the solution for $\underline{R}$) in the first-order condition Eq. (20) in the text verifies the conjecture.

Appendix C. Calculating the welfare gain estimates

C.1. Equity-based model results in Table 1

As derived in Appendix B, Eq. (B.6) gives the equity-based gain function where μ and σ^2 are, respectively, the mean and variance for the return on the domestic country stock return, while $\underline{\mu}$ and $\underline{\sigma}^2$ are the mean and variance for the return on the optimal portfolio given the utility parameters.

C.1.1. Calculating the optimal portfolio

To obtain closed form solutions, I have assumed that the individual country returns are log-normally distributed. As an approximation, I have also assumed that the optimal portfolio returns are log-normally distributed, although this is strictly incorrect. Lewis (1996) reports Monte Carlo experiments that suggest this approximation is innocuous.

To obtain the optimal portfolio, I take the risk-return trade-off between the US market and the foreign mutual fund as given. This trade-off is depicted in Fig. 1 determined by the means and variances of the portfolio process (Eq. (11) in the text.) For each set of parameter values, I then conduct a grid search over increments of $1/1000$ of a portfolio share in the foreign stock, derive $\underline{\mu}$ and $\underline{\sigma}$, and use these estimates to calculate the utility using Eq. (B.2). The share that maximizes the utility for these parameters is chosen as the optimal portfolio with corresponding mean $\underline{\mu}$ and variance $\underline{\sigma}^2$. This is equivalent to maximizing Eq. (4′) subject to Eq. (10′) and Eq. (11) in the text.

C.1.2. Calculating the welfare gains

Using the optimal $\underline{\mu}$ and $\underline{\sigma}$ determined by γ and the US stock return means and standard deviations μ_{us} and σ_{us}, the next step is to calculate the welfare gains. Plugging these means and variances into Eq. (B.6) above gives the welfare gains in Table 1.

C.2. Consumption-based model results in Tables 2, 3 and 5

Substituting into Eq. (B.7) the solutions for the stock prices Q_t [Eq. (B.9′)] and $\underline{Q}_t^j$ [Eq. (B.15)] and using the definitions of M_j, $\underline{M}$ and H_j, the gain function can be rewritten:

$$\delta = [\{[\underline{M}^{-\theta}H_j/(1 - \beta\underline{M}^{-\theta}H_j)] \div [\underline{M}^{(1-\theta)}/(1 - \beta\underline{M}^{(1-\theta)})]\}\{[1 - \beta M_j^{(1-\theta)}]$$
$$\div [1 - \beta\underline{M}^{(1-\theta)}]\}] - 1 \tag{C.1}$$

C.2.1. Table 2, Panel (C) gains

For each country, I take the consumption means, variances and covariances with the world given in Table 2, Panel (A) and use these estimates as μ_j, σ_j^2 and $\underline{\sigma}_j$, respectively. I also take the means and variances of the world to be estimates of $\underline{\mu}$ and $\underline{\sigma}^2$. Plugging these moments into Eq. (C.1), I vary θ and γ. For each pair of $\overline{\theta}$ and γ, I report the gains.

C.2.2. Table 2, Panel (B) gains

These gains are calculated the same as Table 2, Panel (C) gains except that the assumption is imposed that the individual country means are equal to the world mean; i.e. $\mu_j = \underline{\mu}\ \forall j$.

C.2.3. Table 3, Panel (B) gains

These gains are calculated the same as Table 2, Panel (C) gains except that for each country's equity, I take the means, variances and covariances with the world equity fund given in Table 3, Panel (A) and use these estimates as μ_j, σ_j^2 and $\underline{\sigma}_j$, respectively. I also take the means and variances of the world equity fund to be estimates of $\underline{\mu}$ and $\underline{\sigma}^2$.

C.2.4. Table 3, Panel (B) gains

These gains are calculated the same as Table 3, Panel (C) gains except that the assumption is imposed that the individual country means are equal to the world mean; i.e. $\mu_j = \underline{\mu}\ \forall j$.

C.2.5. Table 5 gains

These gains are calculated as for the Table 2, Panel (C) gains described in Section C.2.1. However, instead of varying the pair of parameters γ and θ exogenously, these parameters are determined by matching moments of the risk-free rate and a particular stock price. For example, Panel (A) reports the gains when matching the means of the risk-free rate and, alternatively, the US stock return under autarky (Column 1), the US stock return under world prices (Column 2), and the world stock return (Column 3). (The implied returns for these cases are reported in Table 4.) Table 5, Panel (B) reports the gains when γ and σ are chosen to match the variances of these same pairs of returns.

Appendix D. Calculation of the return series

D.1. Deriving means and variances of equity returns

The equity returns all have the general form: $(1 + R_t) = (y_t/y_{t-1})(1/A)$ where $A = \beta M_j^{(1-\theta)}$ when $R = R^j$, the autarky domestic return; $A = \beta \underline{M}^{(1-\theta)}$ when $R = \underline{R}$, the world mutual fund return; and $A = \beta \underline{M}^{-\theta} H_j$ when $R = \underline{R}^j$, the domestic equity at world prices. Therefore, the mean returns have the form: $E(1 + R) =$

$\exp(\mu)A^{-1}$. Similarly, the variances of the returns have the form: $\mathrm{Var}(1 + R) = \exp(2\mu)(\exp(\sigma^2) - 1)A^{-2}$.

D.2. Deriving the risk-free rates

Setting R_t in the Euler equation Eq. (20) equal to the risk-free rate, R_t^{rf}, implies:

$$1 + \underline{R}_t^{rf} = \beta^{-1} \exp\left(\theta\underline{\mu} - \frac{1}{2}\gamma\underline{\sigma}^2(1 + \theta) \right) \quad \text{Risk-free rate at world prices}$$

(D.1)

The counterpart for the autarky domestic economy is:

$$1 + R_t^{rf} = \beta^{-1} \exp\left(\theta\mu_j - \frac{1}{2}\gamma\sigma_j^2(1 + \theta) \right) \quad \text{Risk-free rate at autarky prices}$$

(D.2)

D.3. Matching parameters

The parameters in Table 4 are determined through the following steps. First, the consumption means and variances from Table 2, Panel (A) are used to obtain values for μ_j, σ_j, $\underline{\mu}$, $\underline{\sigma}$ and $\underline{\sigma}_j$. Second, the means of stock returns and the risk-free rates given in Table 3, Panel (A) are used to obtain estimates for the risk-free rate and the means of US and world stock returns. Third, the means of stock returns are set equal to its theoretical value and the mean of the risk-free rate is set equal to its theoretical value implying two equations in the two unknown parameters, θ and γ. The solution implies parameter pairs which match the mean returns. Thus, Table 4 Panel (A) is determined by setting the theoretical autarky equity return equal to the mean of US stock returns and the theoretical risk-free rate equal to the mean of the risk-free rate implying $\theta = 1.69$ and $\gamma = 52.72$. In addition to this restriction from the risk-free rate, Panel (B) is derived by setting the theoretical domestic equity return on world markets equal to the mean of US returns. Finally, Panel (C) sets the theoretical world equity return equal to the mean of the world stock return and treats the risk-free rate as in Panels (A) and (B).

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