Differences of Opinion and International Equity Markets[1]

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Abstract

We develop an international financial market model in which domestic and foreign residents differ in their beliefs about the information in economic signals. Similar to models of asymmetric information, we consider how informational advantages by domestic investors about local output impacts equity markets. In contrast to these models, however, all information is publicly available, but domestic investors are better equipped to understand the information in local news. We show that our model can help explain four standard international pricing anomalies: (i) home equity preference; (ii) the co-movement of returns and international capital flows; (iii) the dependence of firm returns on local and foreign factors; and (iv) abnormal returns around foreign firm cross-listing in the local market.

1 Introduction

Informational differences across countries have long been used to explain international equity portfolios and their flows. For example, Gehrig (1993) posits that domestic residents have more certainty about domestic fundamentals information than do foreigners, thereby reducing the optimal holdings of foreign assets.[1] Brennan and Cao (1997) show that foreign purchases of domestic equities are positively correlated with domestic stock returns. They argue that these capital inflows occur because domestic residents see more precise signals about local fundamentals, while less informed foreigners react to price changes, buying more domestic equity.[2] Home-equity preference provides a second example. These international portfolio arguments are typically framed in models in which domestic residents are simply endowed with better private signals about the local market.

Informational advantages between local and foreign investors are also used anecdotally to explain the covariance structure of firm-level equity pricing behavior in at least two cases. First, domestic firm returns depend upon both local and foreign market factors. This return behavior is often loosely considered evidence of capital market segmentation, with differing information across home and foreign investors as a potential source.[3] As a second case, the stock prices on foreign firms that cross-list in domestic markets tend to increase, generating abnormal returns around this event. Some have argued that returns are higher for cross-listing foreign firms because the firm commits to abide by the requirements of the home country and thereby reduces uncertainty to investors.[4] These informational differences are generally simply posited as potential explanations of the international pricing behavior.

Although international informational differences is a commonly-cited explanation of international equity market regularities, there has so far been no unified framework to jointly address the wide-range of portfolio flow and pricing relationships in the examples above. The lack of a common framework stems from two main features of the standard "noisy rational expectations" (NRE)

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[1] This argument extends the Merton (1987) model of investor familiarity to international equity.

[2] A number of papers such as Grinblatt and Keloharju (2000), and Brennan *et al.* (2005) have found the pattern of co-movement in foreign capital flows and both actual returns and beliefs about these returns.

[3] For studies finding local and foreign factors in international returns and the associated debate on the number of factors, see Heston and Rouwenhorst (1994), Brooks and Del Negro (2004, 2006), and Bekaert *et al.* (2009), among others.

[4] This argument has been termed the "bonding hypothesis." See Coffee (1999, 2002) and the discussion in Karolyi (2006).

model typically used to consider capital flows. First, the equilibrium dynamics of the wealth and consumption of the investors are not tightly modeled, as resources come from and go to exogenous noise traders and also come and go from an exogenous riskless security available in infinitely elastic supply. Investors simply receive payoffs and consume in a final period.[5] Second, NRE models generally assume constant absolute risk-aversion investors. However, the assumption seems to be fairly restrictive since changes in risk-aversion and wealth effects are an important determinant in firm-level returns as well as general asset pricing. Therefore, it is difficult to use these models to directly address firm-level pricing observations.

By contrast, in this paper, we propose a model based upon international differences in perceptions of economic information that both incorporates the equilibrium consumption of investors and allows for constant relative risk-aversion. As such, the model can be used to address all four of the international equity pricing regularities mentioned above. In doing so, we build upon two recent papers that examine the equilibrium behavior of asset prices when investors have differences of opinion about the informativeness in commonly observed signals. Scheinkman and Xiong (2003) develop a model in which investors must infer some features of the current output process by observing current outputs and public signals, but these investors have different beliefs about the correlations between the signals and the unobserved features. In their model, investors are risk-neutral and face short-sale constraints. Dumas, *et al.* (2009) consider a similar setting but allow for risk averse investors and remove the short-sale restrictions.

There is no denying that the approach we take, often labelled "the difference-of-opinion" approach, entails some irrational learning on the part of investors. But there is ample precedent for it in the Economics literature. Previous contributions include: Harris and Raviv (1993), and Cecchetti, Lam, and Mark (2000). See Morris (1995) for a discussion of this approach and Kandel and Pearson (1995) for empirical evidence. Kandel and Pearson cite Ariel Rubinstein (1993):

> "In almost all models of economic theory, behavioral differences among consumers
> are attributed to differences in preferences or in the information they possess. In real
> life, differences in consumer behavior are often attributed to varying intelligence and

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[5] See Lowenstein and Willard (2006).

ability to process information. Agents reading the same morning newspapers with the same stock price lists will interpret the information differently." [our emphasis]

The international focus of our analysis requires a fuller model than Scheinkman and Xiong (2003) and Dumas *et al.* (2009). In our model, representative investors live in two countries, each endowed with an output-paying process. These investors do not observe the time-varying expected value of the growth rate of either output, but they observe a signal in each country that is conditionally correlated with the change in the expected growth rate of the corresponding country's output. Each investor correctly perceives that the information in his own country's signal is valuable, but incorrectly believes that the information in the other country's signal is just noise. This perception captures the idea that, while foreigners may see domestic information as well as home residents, they do not know how to interpret it. Thus, while the earlier models about capital flows and home equity preference typically assume that the informational advantage to domestic residents arises from more precise, privately observed signals, we assume that the informational advantage arises from the country's ability to interpret public signals. Domestic investors have some form of informational advantage, in the same spirit as the asymmetric information literature. As such, the assumption is motivated by the same evidence used to suggest asymmetric information models.[6] However, our assumption has further intuitive appeal because we do not require the informational advantage to be hidden. Even Brennan and Cao (1997) acknowledge that "communication across countries is now close to instantaneous."

Our investors will differ in their ability to interpret signals about expected foreign growth rates of output. For that, we require an information structure that provides two important features. First, we assume that domestic residents have an informational advantage about their own output relative to foreigners. Second, in order to analyze equilibrium with complete markets, domestic and foreign investors must be able to see the same variables. We incorporate both of these features into the following simple information assumption: while all investors see the same information,

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[6] Hau (2001) finds that the profits of professional traders in German stocks are higher for those located in Germany or in German-speaking countries. Brennan *et al.* (2005) analyze surveys of expectations and the purchases of domestic assets by foreign institutional investors and find that these investors are at an informational disadvantage. For an attempt at discriminating empirically between the two meanings of "information advantage", see Banerjee (2010). In a panel-data analysis of Norwegian household portfolios, Doskeland and Hvide (2010) find employer-equity preference with no return-based evidence of informational advantage.

domestic investors are better equipped to understand the information about conditional growth rates conveyed in the domestic signal. Since home investors interpret their own signal differently from foreign investors, observations of output and signals generate differences of opinion about the future growth rate of output. As a result, investors in each country accurately perceive that the other country's investors interpret the signal differently than they do. Moreover, the difference in their beliefs creates an additional source of risk which we call "foreign-sentiment risk", building on Dumas *et al.* (2009).

This assumption may be motivated from at least three different perspectives. First, it could be that domestic investors have had a longer time to study the relationship between the signal and the growth rate. From this perspective, it may be taking foreigners a longer time to learn how to interpret the signal and we are looking at a long transitional time period. Second, foreigners may simply have chosen not to become informed about the signal because they have viewed domestic investment as too risky in the past and consider becoming informed too costly. For example, Van Nieuwerburgh and Veldkamp (2009) show in a NRE model that when investors are endowed with a small home information advantage, they choose not to learn what foreigners know. Third, foreign investors may not be able to learn the same information as domestic residents. Acemoglu, Chernozhukov, and Yildiz (2009) show that when individuals are uncertain about the conditional distribution of signals and outputs, even a small amount of individual uncertainty can lead to significant differences in asymptotic beliefs. We do not take a stand on which of these three motivations is the correct one, but maintain the assumption that home investors are better at understanding home signals throughout. In particular, investors in each country have correct beliefs about the information in the signal that is correlated with the conditional expected growth rate of output generated in their own country, but misinterpret the information coming from the foreign signal. We stress, however, that in our setup both countries are equally rational or irrational, since both of them (symmetrically) misinterpret the information about the other country.

To show how foreign-sentiment risk can produce the international asset pricing regularities described above, we consider two versions of our basic model with as much parsimony as possible to generate the basic stylized facts. First, to examine home-equity preference within this framework, we focus on the stock markets of the two countries. In this version of the model, investors in

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each country have an identical set of five assets that complete the market of four independent subjective Brownian motions: a risk-free asset, two equities with payoffs in the output process in each country, and two futures contracts with payoffs marked in part to signal innovations (but rendered independent of stock returns). When there are no current differences of opinion, domestic investors shy away from holding foreign equity because they perceive that the variance of the unobserved future growth rate is higher on foreign output. During periods when domestic investors are relatively optimistic about foreign output, they may actually prefer foreign equity. However, during equally likely periods when domestic investors are relatively pessimistic about foreign output, they shun foreign equity by more. In the borderline case of no current disagreement between all investors, domestic investors tend to hold more domestic equity at any given point in time because they shy away from the risk created by the behavior of foreigners.

We then consider the effects of difference of opinion on the capital flows. We show that, when domestic investors have an informational advantage in interpreting local signals, foreign capital inflows will be positively related to domestic stock returns. The intuition in our model is similar to that in Brennan and Cao (1997), though the actual economic mechanism is different since there is no information asymmetry in our model. When domestic output increases, the domestic stock price increases. Foreigners over-react to this increase in output because they believe it means that the future growth rate will increase as well. Since domestic investors have an advantage in interpreting signals related to domestic output, they do not increase their view of the future domestic growth rate by as much and therefore sell their equity to the foreigners. As such, foreign capital inflows and domestic stock returns positively co-move.[7]

Next, to consider the effects of difference of opinion on firm-level pricing behavior, we require a slightly richer version of our basic model. We introduce an additional firm that operates in a domestic market. Like the stock market, this firm has an output process with an unobserved mean growth rate and a signal about this growth rate. Since the firm operates in the domestic market, only local investors understand that the signal conveys information about the future growth rate. To make markets complete in this version of the model, we allow investors to hold the equity on

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[7]A similar question is raised in Xiouros (2010). Evidence on information-based international trades is provided in Dong (2009).

this firm as well as futures contracts on the firm's signal.

We then use this expanded three-equities model version to consider the two firm-level pricing observations. First, we examine the local and foreign factor asset pricing relationships. We start by showing how foreign-sentiment risk can generate a local and world factor model in consumption. We then show that, with foreign sentiment, the betas from regressing our firm-level excess returns on the home and foreign country stock excess returns would imply a higher beta on the local market than the foreign market, consistent with typical empirical findings. For the second pricing observation, we examine the impact of cross-listing. For this purpose, we assume that our domestic firm now cross-lists in the foreign market. We conjecture that this cross-listing enables the foreign country's investor to correctly interpret information about the expected growth rate of the domestic firm's cash flow. Because cross-listing aligns perceptions about the information in public signals, the resulting decline in disagreement risk decreases the required return and increases the price.

The structure of the paper is as follows. The next section describes the information set up of the model and the equilibrium, including consumption and the state-price density. Section 3 describes the securities-market implementation of the equilibrium. Section 4 describes how the model can generate home equity preference and can produce the positive covariation between capital flows and stock returns. Section 5 describes the factor structure of returns within our model and demonstrates how the final empirical regularity – the apparently abnormal returns of firm equity around the time of cross-listing – can be explained. Section 6 demonstrates that the explanations we have provided are enduring ones. Concluding remarks follow in Section 7.

2 The Foreign-Sentiment Risk Model

How much current economic variables and public signals help predict future variables is at the core of standard asset pricing theories. Moreover, differences in perceptions about this informativeness across countries have long been proposed as an explanation for various international asset pricing anomalies, as described above. In this section, we provide a simple framework that captures both these features. First, the informativeness of current economic variables and public signals affects forecasts of future variables and hence current prices of financial securities. And, second, investors

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differ across countries in their beliefs about the informativeness of these currently observed public signals.

The basic features of these differing beliefs and their impact on future expectations can be shown most parsimoniously using a model with two *ex ante* identical countries. In each of the two countries, labeled A and B, live representative investors. The countries are completely integrated in that they are open to international trade in securities and in a single, perishable good. Investors in each country are initially endowed with one share of their own output process, itself initialized at the value 1. The financial market is complete.

2.1   Exogenous outputs and public signals

The output delivered by economy i, $i = A, B$, at time t is denoted $\delta_{i,t}dt$. The stochastic process for $\delta_{i,t}$ is:

$$\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t}dt + \sigma_\delta dz_{i,t}^\delta, \ \ i = A, B \tag{1}$$

where z_i^δ are Brownian motions under the objective probability measure, which governs empirical realizations of the process. The conditional growth rates $f_{i,t}$ of outputs are also stochastic:

$$df_{i,t} = -\zeta \times \left(f_{i,t} - \overline{f}\right) dt + \sigma_f dz_{i,t}^f, \ i = A, B$$

where $\zeta > 0$ and z_i^f are also Brownian motions under the objective probability measure.

The conditional growth rates of outputs f_i are not observed by any investor. All investors must estimate, or filter out, the current value of f_i in order to value how future conditional mean growth rates affect forward-looking asset prices. They estimate this value by observing current outputs and two public signals (s_A, s_B). The signals follow the processes:

$$ds_{i,t} = \phi dz_{i,t}^f + \sqrt{1 - \phi^2}dz_{i,t}^s, \ \ i = A, B$$

where $|\phi| \in [0, 1]$ and where z_i^s is a third pair of Brownian motions, under the objective probability measure as well. The term $\phi dz_{i,t}^f$ in the stochastic differential equation for the signals means that the signals are truly informative about output growth shocks $dz_{i,t}^f$. For parsimony, we assume that the

six Brownian motions $\left(z_{A,t}^{\delta}, z_{A,t}^{f}, z_{A,t}^{s}, z_{B,t}^{\delta}, z_{B,t}^{f}, z_{B,t}^{s} \right)$ are independent of each other. As we show below, this independence, together with symmetry, significantly simplifies our analysis. Clearly, however, some realism is lost by making an assumption of uncorrelated outputs across countries.

Note that, in these output and signal processes, the parameters are identical across countries. Thus, the variances of the outputs and conditional growth rates, σ_{δ} and σ_f, the long-run means of the conditional growth rates and their mean reversion parameter, $\overline{f}$ and ζ, and the information in the signal, ϕ, do not depend upon the country. Again, we maintain this assumption so that the model is symmetric across the two countries.

2.2   The econometrician's viewpoint

In the information model we develop below, no investor knows the true state of the economy. Hence, the objective (or empirical) measure is not defined on either investor's σ-algebra and we can ignore it for the purpose of calculating the equilibrium.

In order to relate our equilibrium to empirical findings, however, we use a representation of the way in which "the econometrician" would collect and process data on returns. Our abstract description of the econometrician is based on the idea that he observes the same information as do both sets of investors. Like the investors in each country, the econometrician does not observe the true conditional growth rate of outputs and must filter this process. As in Xiong and Yan (2010), however, we assume that the econometrician knows the structure of the economy, i.e., that he works under that null hypothesis. Accordingly, the econometrician in his estimation of the model filters the signal process under the hypothesis that:

$$ds_{i,t} = \phi dz_{i,t}^{f} + \sqrt{1-\phi^2} dz_{i,t}^{s} \quad i = A, B$$

While no investor's learning is as correct as that of the econometrician, we use the econometrician's probability measure as our benchmark in all calculations of empirical regularities. It will prove convenient to calculate the probability measures of the two groups as deviations from the econometrician's probability measure.

Thus, the econometrician conducts his analysis under the null hypothesis that the signals convey information about the conditional growth rates in accordance with the correlation ϕ. Unlike investors, he is not deluded. To calculate his probability measure, we rewrite the stochastic differential equations in terms of processes that are Brownian motions under the econometrician's probability measure. For this purpose, we define the four-dimensional process $w_t^E = \left(w_{\delta_A,t}^E, w_{\delta_B,t}^E, w_{s_A,t}^E, w_{s_B,t}^E\right)^{\mathsf{T}}$, where each of the components of w_t^E corresponds to a Brownian motion of each of the four observed variables under the probability measure of the econometrician. Defining $\widehat{f}_{i,t}^E$ as the conditional mean of the growth rate of output in Country i as estimated by the econometrician, we use filtering theory (see Lipster and Shiryaev (2000, Theorem 12.7, page 36)) to compute these conditional expected values. For $i = A, B$, these expectations are given by:

$$d\widehat{f}_{i,t}^E = -\zeta \times \left(\widehat{f}_{i,t}^E - \overline{f}\right) dt + \frac{\gamma^E}{\sigma_\delta} dw_{\delta_i,t}^E + \phi\sigma_f dw_{s_i,t}^E \tag{2}$$

where the number γ^E is the steady-state variance of $\widehat{f}_A^E$ and $\widehat{f}_B^E$, these variances being equal to each other by virtue of symmetry:

$$\gamma^E \triangleq \sigma_\delta^2 \left(\sqrt{\zeta^2 + \left(1 - \phi^2\right) \frac{\sigma_f^2}{\sigma_\delta^2}} - \zeta\right). \tag{3}$$

This variance would normally be a deterministic function of time. But for simplicity we assume, as did Scheinkman and Xiong (2003) and Dumas *et al.* (2009), that there has been a sufficiently long period of learning for people of both countries to converge to their long-run level of variance, independent of their prior. Equation (2) shows how the econometrician filters the conditional growth rate with observations of outputs and signals. When he sees an increase in the output of Country i, he updates his estimate of the conditional mean growth rate by the ratio of its steady state variance γ^E and the variance of the output σ_δ. When he sees an increase in the signal of Country i, he increases his view of $f_{i,t}$ according to ϕ, which measures the information precision in the signal about this growth rate.

By definition of the growth rates $\widehat{f}^E_{A,t}$ and $\widehat{f}^E_{B,t}$, we can then write:

$$\frac{d\delta_{A,t}}{\delta_{A,t}} = \widehat{f}^E_{A,t}dt + \sigma_\delta dw^E_{\delta_A,t} \tag{4}$$

$$\frac{d\delta_{B,t}}{\delta_{B,t}} = \widehat{f}^E_{B,t}dt + \sigma_\delta dw^E_{\delta_B,t} \tag{5}$$

Comparing these equations to (1) implies that the relationship between the output Brownian motions under the objective measure and the output Brownian motions under the econometrician's (E's) measure is given by:

$$dw^E_{\delta_i,t} = \frac{f_{i,t} - \widehat{f}^E_{i,t}}{\sigma_\delta}dt + dz^\delta_{i,t} \ , \ i = A, B \tag{6}$$

The signals having zero drift both objectively and in the econometrician's eyes, the processes for the signals can be written under E's measure as:[8]

$$ds_{i,t} = dw^E_{s_i,t}, \ i = A, B \tag{7}$$

2.3 The investors' viewpoint

The difference in information processing by the investors of the two countries is implemented as follows. Investors in Country A perform their filtering under the belief that the signal s_A has the correct conditional correlation with f_A but they believe incorrectly that the signal s_B has zero correlation with f_B. The "model" they have in mind is:

$$ds_{A,t} = \phi dz^f_{A,t} + \sqrt{1-\phi^2}dz^s_{A,t} \tag{8}$$

$$ds_{B,t} = dz^s_{B,t} \tag{9}$$

Notice that investors of Country A have the same model of signal s_A as the econometrician (incorporating the true correlation ϕ) but a different one (incorporating a correlation equal to zero) for

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[8]While the signal process under the objective measure depends upon two different processes z^f and z^s, it can be shown that this signal can be written as a single Brownian motion.

the signal s_B. Symmetrically, the "model" that investors of Country B have in mind is:

$$ds_{A,t} = dz^s_{A,t} \tag{10}$$

$$ds_{B,t} = \phi dz^f_{B,t} + \sqrt{1-\phi^2}dz^s_{B,t} \tag{11}$$

Notice that investors of Country B have the same model of signal s_B as the econometrician but a different one for the signal s_A.

Defining $\widehat{f}^i_j$ as the conditional mean of the output in Country j as estimated by investors of Country i, we implement filtering theory one more time, to write:

$$d\widehat{f}^i_{i,t} = -\zeta \times \left(\widehat{f}^i_{i,t} - \overline{f}\right)dt + \frac{\gamma^E}{\sigma^2_\delta}\left(\frac{d\delta_{i,t}}{\delta_{i,t}} - \widehat{f}^i_{i,t}dt\right) + \phi\sigma_f ds_{i,t} \tag{12}$$

$$d\widehat{f}^i_{j,t} = -\zeta \times \left(\widehat{f}^i_{j,t} - \overline{f}\right)dt + \frac{\gamma^X}{\sigma^2_\delta}\left(\frac{d\delta_{j,t}}{\delta_{j,t}} - \widehat{f}^i_{j,t}dt\right), \ i \neq j \tag{13}$$

where the number γ^X is the steady-state variance of the "transnational" estimates $\widehat{f}^B_A$ and $\widehat{f}^A_B$, their variances being equal by virtue of symmetry:

$$\gamma^X = \gamma^E\big|_{\phi=0} = \sigma^2_\delta\left(\sqrt{\zeta^2 + \sigma^2_f\frac{1}{\sigma^2_\delta}} - \zeta\right).$$

Note from (3) that γ^E decreases as the information in the signal measured by ϕ^2 rises. Intuitively, the signal s_i allows the econometrician and investors in country i to get a more precise estimate of f_i, thereby reducing the steady-state variance for investors in country i's estimate. By contrast, investors in country $j \neq i$ ignore the information in the signal s_i and thereby attribute more of the variability to f_i. As a result, the steady-state variance for the conditional growth rate forecast of home output will be lower for home investors than the corresponding variance of the foreign output forecast; i.e., $\gamma^E < \gamma^X = \gamma^E\big|_{\phi=0}$, a relationship we will use below.

Since the econometrician's hypothesis about signals is, in fact, not in line with the assumptions made by investors in any of the two countries, differences in beliefs are generated. We define the

"disagreements" between the econometrician and the investors as:

$$\widehat{g}_i^j \equiv \widehat{f}_{i,t}^E - \widehat{f}_{i,t}^j; \quad i, j = A, B$$

In principle, $\widehat{g}_i^j$ represents two pairs of disagreements for each country's investor. However, the econometricians and the investors agree about the estimate of the conditional growth rate of their own output since they are filtering in the same way. Therefore, $\widehat{g}_A^A \equiv \widehat{f}_{A,t}^E - \widehat{f}_{A,t}^A = 0$ and $\widehat{g}_B^B = \widehat{f}_{B,t}^E - \widehat{f}_{B,t}^B = 0$ so that only $\widehat{g}_A^B \equiv \widehat{f}_{A,t}^E - \widehat{f}_{A,t}^B$ and $\widehat{g}_B^A = \widehat{f}_{B,t}^E - \widehat{f}_{B,t}^A$, the disagreements between the econometrician and the foreign country's **forecast of the domestic output growth rate**, move over time. Using the last four equations, we get the dynamics for the disagreements:

$$d\widehat{g}_{i,t}^j = -\left(\zeta + \frac{\gamma^X}{\sigma_\delta^2}\right)\widehat{g}_{i,t}^j dt + \left(\frac{\gamma^E - \gamma^X}{\sigma_\delta}\right) dw_{\delta i,t}^E + \phi\sigma_f dw_{s_i,t}^E; \quad i \neq j; \ i, j = A, B \qquad (14)$$

Given the econometrician's filter in equation (2), it is clear what drives the disagreements in (14). On the one hand, an increase in the output of, say, Country A, $dw_{\delta_A,t}^E$, causes the econometrician to update his estimate of $\widehat{f}_A^E$ according to $\frac{\gamma^E}{\sigma_\delta}$. However, this same output change will induce investors in country B to increase their estimate by $\frac{\gamma^X}{\sigma_\delta}$. Since these investors ignore the signal information, they update their estimate by more than informed investors in Country A (and more than the econometrician). Thus, since $\gamma^E < \gamma^X$, Country B investors over-react to the output and the disagreement between these investors and the econometrician declines, $d\widehat{g}_{A,t}^B < 0$. In this case, Country B investors become relatively optimistic about Country A output. On the other hand, an increase in the signal $dw_{s_A,t}^E$ will induce the econometrician to increase his estimate of the conditional mean $\widehat{f}_A^E$. Since Country B investors do not use the signal information, the signal increases the disagreement about Country A output, $d\widehat{g}_{A,t}^B > 0$. In this case, Country B investors become relatively pessimistic about Country A. Foreign investors underreact to a domestic signal and overreact to domestic output. Domestic investors react properly to both domestic output and domestic signal.

To examine further how the information looks to each country's investor, we rewrite the dividend and signal processes from the viewpoint of each investor. For this purpose, we also consider a set of four-dimensional processes for each country that is Brownian under the probability measure of

investors in Country j; $w_t^j = \left(w_{\delta_A,t}^j, w_{\delta_B,t}^j, w_{s_A,t}^j, w_{s_B,t}^j \right), j = A, B$. Since we use the viewpoint of the econometrician as a reference, the probabilities of events will look different from the point of view of the investors in the two countries. How the probabilities evolve according to the econometrician and the two investors is captured by the change in measure between the set of Brownians perceived by the econometrician, w_t^E, and by the investor in country j, w_t^j. These processes differ according to:

$$dw_{\delta_i,t}^E = dw_{\delta_i,t}^j - \frac{\widehat{g}_{i,t}^j}{\sigma_\delta} dt \tag{15}$$

$$dw_{s_i,t}^E = dw_{s_i,t}^i \tag{16}$$

for, $i, j = A, B$. It can be shown that $w_{s_i,t}^B$ and $w_{s_i,t}^A$ can be treated as independent Brownian motions. From Equations (15) and (16) and Girsanov theorem, we can obtain the changes from the probability measure of the econometrician to those of investors in Country A and B, capturing the differences of probability beliefs between these countries, as being given by:

$$d\eta_{A,t} = -\frac{\widehat{g}_{B,t}^A}{\sigma_\delta} \eta_{A,t} dw_{\delta_B,t}^E \tag{17}$$

$$d\eta_{B,t} = -\frac{\widehat{g}_{A,t}^B}{\sigma_\delta} \eta_{B,t} dw_{\delta_A,t}^E \tag{18}$$

We give the difference in beliefs η_i the picturesque name of "Country i sentiment" (relative to the rational econometrician's beliefs). Disagreements $\widehat{g}_{B,t}^A$ and $\widehat{g}_{A,t}^B$ are the drivers of the instantaneous volatilities of the sentiment variables. The change of measure between an investor of a country and the econometrician is perfectly (positively or negatively) correlated with the output in the other country. For example, $\eta_{A,t}$ depends upon realizations of the output in Country B, according to $dw_{\delta_B,t}^E$. The size of this effect depends upon the current disagreement between the econometrician and investors in Country A, $\widehat{g}_{B,t}^A$. If investors in Country A are currently optimistic about Country B, then $\widehat{g}_{B,t}^A < 0$. Since Country A investors over-react to the output process in Country B, this will further increase the difference in probabilities and $\eta_{A,t}$ increases. The evolution of $\eta_{A,t}$ does not depend upon Country A outputs since the econometrician and the Country A investors agree about the filter of that process.

We have now defined the evolution of the state vector. The Markovian system comprised of Equations (4), (5), (2), (14), (17) and (18) completely characterize the dynamics of eight exogenous state variables that drive the economy, defined by the vector:

$Y_t \equiv \left(\delta_{A,t}, \widehat{f}^E_{A,t}, \widehat{g}^B_{A,t}, \eta_{B,t}, \delta_{B,t}, \widehat{f}^E_{B,t}, \widehat{g}^A_{B,t}, \eta_{A,t} \right)^\mathsf{T}$. However, the first four components of the vector are only driven by the Brownians on the output and signal of Country A, while the last four components of the vector are driven by the corresponding Brownians for Country B. Therefore, the state vector can be written as two independent processes: $Y_t = \{Y_{A,t}, Y_{B,t}\}$ where:

$$Y_{i,t} = \left\{ \delta_{i,t}, \widehat{f}^E_{i,t}, \widehat{g}^j_{i,t}, \eta_{j,t} \right\}^\mathsf{T}$$

for $i, j = A, B$; $i \neq j$. Each of these two processes is driven by separate Brownians but they have equal diffusion matrices. In particular,

$$dY_{i,t} = \mu_{i,t}dt + \Omega_{i,t}dw^E_{i,t}$$

where

$$dw^E_{i,t} = \left\{ dw^E_{\delta_i,t}, dw^E_{s_i,t} \right\}$$

and

$$\Omega_{i,t} = \begin{bmatrix} \sigma_\delta \delta_{i,t} & 0 \\ \left(\frac{\gamma^E}{\sigma_\delta} \right) & \phi\sigma_f \\ \left(\frac{\gamma^E - \gamma^X}{\sigma_\delta} \right) & \phi\sigma_f \\ -\eta_{j,t} \left(\frac{\widehat{g}^j_{i,t}}{\sigma_\delta} \right) & 0 \end{bmatrix} ; j \neq i$$

Thus, the state vector can be evaluated as two independent processes, each governing the evolution of views about each country's output. For instance, the full vector of eight state variables Y_t can be written as:

$$dY_t = \mu_t dt + \Omega_t d\overrightarrow{w}^E_{i,t}$$

where

$$d\overrightarrow{w}^E_{i,t} = \left\{ dw^E_{\delta_A,t}, dw^E_{\delta_B,t}, dw^E_{s_A,t}, dw^E_{s_B,t} \right\}$$

and

$$\Omega_t = \begin{bmatrix} \Omega_{A,t} & 0 \\ 0 & \Omega_{B,t} \end{bmatrix}$$

The block diagonal structure of Ω_t will be exploited in our solution of the equilibrium below.

2.4   Individual optimization problem

We now use the information structure to derive equilibrium pricing relationships. For this purpose, we assume that the investors in the two countries have identical time-separable utility functions in a common perishable consumption good. For Country B investors, the problem can be written as:

$$\sup_{c_B} \mathbb{E}_0^E \int_0^\infty e^{-\rho t} \frac{1}{\alpha} c_{B,t}^\alpha \eta_{B,t} dt; \ \alpha < 1 \tag{19}$$

subject to the lifetime budget constraint:

$$\mathbb{E}_0^E \int_0^\infty \xi_t^E \left(c_{B,t} - \delta_{B,t} \right) dt \leq 0 \tag{20}$$

where ξ_t^E is the state price density under the econometrician's measure. Note that we write the optimization in equation (19) using the expectation of the econometrician at initial time 0. We indicate this expectation with the superscript E in the expectation operator, $\mathbb{E}_0^E$. We multiply the period utility of B at time t by the change of measure variable, $\eta_{B,t}$, to get back to the expectation under the measure of B. The first-order conditions of this optimization imply that consumption of Country B residents is:

$$c_{B,t} = \left(\frac{\lambda_B}{\eta_{B,t}} e^{\rho t} \xi_t^E \right)^{-\frac{1}{1-\alpha}} \tag{21}$$

where λ_B is the Lagrange multiplier of the budget constraint (20). Country A residents face a symmetric optimization problem, that can be written identically to (19), (20), and (21) above, with B replaced by A.

2.5   Equilibrium pricing measure

We now use the optimal consumption plan for each country to solve for the stochastic price density. For this purpose, we set the sum of the optimal consumption for B in (21) and the counterpart for Country A to the sum of the two output processes.[9] Thus, we have:

$$\left(\frac{\lambda_A}{\eta_{A,t}}e^{\rho t}\xi_t^E\right)^{-\frac{1}{1-\alpha}} + \left(\frac{\lambda_B}{\eta_{B,t}}e^{\rho t}\xi_t^E\right)^{-\frac{1}{1-\alpha}} = \delta_{A,t} + \delta_{B,t} \tag{22}$$

Solving this equation for the state price density ξ_t^E implies:

$$\xi_t^E(\delta_{A,t}, \delta_{B,t}, \eta_{A,t}, \eta_{B,t}) = e^{-\rho t}\left[\left(\frac{\eta_{A,t}}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_{B,t}}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}(\delta_{A,t} + \delta_{B,t})^{\alpha-1} \tag{23}$$

The state price density relative to the econometrician's measure, ξ_t^E, depends upon the changes in probability measure of both countries. In fact, it is homogeneous of degree 1 in these two variables.

Proposition 1: *The state-price density (23) contains two priced factors: world output, $\delta_{A,t} + \delta_{B,t}$, and world average[10] sentiment $\left[\left(\frac{\eta_{A,t}}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_{B,t}}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}$.*

The two priced factors, however, are conditionally correlated: as we have seen (Equation (17)), sentiment η_A is perfectly correlated with output innovations B, $dw_{\delta_B,t}^E$, but the sign of that correlation depends on the sign of the current disagreement $\widehat{g}_{B,t}^A$, and vice versa. If and when there is full agreement today ($\widehat{g}_{B,t}^A = \widehat{g}_{A,t}^B = 0$), these correlations become equal to zero. In all cases, the pricing measure (23) means that, in the presence of sentiment, equilibrium prices now contain an additional risk premium, over and above the classic premium based on total consumption.

In equilibrium, each country's share of world consumption is given by a monotonic transformation of the ratio of the two changes of probability measure. Defining Country A's share as ω:

$$\omega \triangleq \frac{c_{A,t}}{\delta_{A,t} + \delta_{B,t}}; \quad 1 - \omega = \frac{c_{B,t}}{\delta_{A,t} + \delta_{B,t}} \tag{24}$$

---

[9]The econometrician is not a participant in the economy.

[10]This is a harmonic average.

its equilibrium value is given by:

$$\omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right) = \frac{\left(\frac{\lambda_B}{\lambda A}\frac{\eta_{A,t}}{\eta_{B,t}}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{\lambda_B}{\lambda A}\frac{\eta_{A,t}}{\eta_{B,t}}\right)^{\frac{1}{1-\alpha}}} \tag{25}$$

or:

$$c_{A,t} = \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)(\delta_{A,t}+\delta_{B,t})$$

As in Dumas et al. (2009), the consumption-sharing rule is linear in aggregate output, $\delta_A + \delta_B$, and its slope, ω, is driven by the ratio of Country A beliefs to Country B beliefs, $\left(\eta_{A,t}/\eta_{B,t}\right)$. This relationship can be understood intuitively as follows. When the investors of Country A have deemed an event more likely to occur than did investors of Country B, they have bet on that event through $\left(\eta_{A,t}/\eta_{B,t}\right)$ and, when it occurs, they get to consume more. Isomorphically, one could interpret the allocation coming out of the complete markets problem: when $\left(\eta_{i,t}/\eta_{j,t}\right)$ is high, the ratio of marginal utility of Country i divided by marginal utility of Country j is high. Investors will then trade contingent assets across countries that will pay off in these states. In this way, changes of probability measure act as taste shocks in each country.

In the standard case without foreign sentiment $\frac{dc_{W,t}}{c_{W,t}}$, $\frac{dc_{A,t}}{c_{A,t}}$ and $\frac{dc_{B,t}}{c_{B,t}}$ are perfectly correlated. By contrast, in our case the cross-country consumption correlation is below one, except at exceptional times of full agreement. Hence, the sentiment adjustment is particular to each consumption process considered.

It will be useful to note, as did Yan (2008) and Dumas *et al.* (2009), that, if we assume that risk aversion $1 - \alpha$ is a positive integer (which can be true only when investors have risk aversion greater than or equal to 1), one can use the binomial theorem to expand the state-price density.

The square bracket expression in equation (23) is then written as a sum of powers:[11]

$$
\left[\left(\frac{\eta_{A,t}}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_{B,t}}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} = \frac{1}{\lambda^B} \sum_{j=0}^{1-\alpha} \binom{1-\alpha}{j} \left(\frac{\eta_{A,t}}{\eta_{B,t}} \frac{\lambda^B}{\lambda^A} \right)^{\frac{j}{1-\alpha}}
$$

$$
= \frac{1}{\lambda^B} \sum_{j=0}^{1-\alpha} \binom{1-\alpha}{j} \left[\frac{\omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)}{1 - \omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)} \right]^{j} \tag{26}
$$

We next specify a menu of assets to examine the implications of this model for the first two regularities discussed in the introduction.

3 Financial securities prices and international portfolio choice

3.1 Securities market implementation

To consider the aggregate equity market implications described above, we require a set of securities that both complete the market and make our difference-of-opinion effects most transparent. To complete the market, we need five securities with non linearly dependent payoffs since we have four linearly independent Brownians that are observable by investors. Given the aggregate equity market focus of the two regularities in this section, stocks that pay off on claims to each country's output are a natural choice.

Therefore, the first two securities in the menu of assets are equities with equilibrium stock prices denoted $S_{A,t}$ and $S_{B,t}$. Equities are infinitely long-lived and pay amounts equal to outputs perpetually at every instant, so that the equilibrium prices are:

$$
S_{i,t} = \int_t^\infty \mathbb{E}_t^E \left[\frac{\xi_u^E}{\xi_t^E} \delta_{i,u} \right] du; \ i = A, B \tag{27}
$$

---

[11]The state price is the sum of $1 - \alpha$ terms. The term $j = 0$ can be interpreted as the influence of investors in Country B on prices: if one increases their weight in the market, the sum approaches the value of this term (calculated at $\omega = 0$). In the same interpretation, the term $j = 1 - \alpha$ represents the influence of investors in Country A.

Using our equilibrium state price density, stock prices can be written as:

$$
S_{A,t}\left(\frac{\eta_{A,t}}{\eta_{B,t}}, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^E, \widehat{f}_{B,t}^E, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A\right) = \int_t^\infty \mathbb{E}_t^E\left[\frac{\xi_u^E}{\xi_t^E}\delta_{A,u}\right] du
$$

$$
= \left[1 - \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\right]^{1-\alpha} \delta_{A,t} \int_t^\infty e^{-\rho(u-t)} \sum_{j=0}^{1-\alpha} \left[\binom{1-\alpha}{j}\left(\frac{\omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)}{1 - \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)}\right)^j \right. \tag{28}
$$

$$
\times \mathbb{E}_t^E\left[\left(\frac{\eta_{A,u}}{\eta_{A,t}}\right)^{\frac{j}{1-\alpha}}\left(\frac{\eta_{B,u}}{\eta_{B,t}}\right)^{1-\frac{j}{1-\alpha}}\left(\frac{\delta_{A,u}}{\delta_{A,t}}\right)^\alpha\left(\frac{1+\frac{\delta_{B,u}}{\delta_{A,u}}}{1+\frac{\delta_{B,t}}{\delta_{A,t}}}\right)^{\alpha-1}\right]\right] du
$$

$$
S_{B,t}\left(\frac{\eta_{A,t}}{\eta_{B,t}}, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^E, \widehat{f}_{B,t}^E, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A\right) = \int_t^\infty \mathbb{E}_t^E\left[\frac{\xi_u^E}{\xi_t^E}\delta_{B,u}\right] du
$$

$$
= \left[1 - \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\right]^{1-\alpha} \delta_{B,t} \int_t^\infty e^{-\rho(u-t)} \sum_{j=0}^{1-\alpha} \left[\binom{1-\alpha}{j}\left(\frac{\omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)}{1 - \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)}\right)^j \right.
$$

$$
\times \mathbb{E}_t^E\left[\left(\frac{\eta_{A,u}}{\eta_{A,t}}\right)^{\frac{j}{1-\alpha}}\left(\frac{\eta_{B,u}}{\eta_{B,t}}\right)^{1-\frac{j}{1-\alpha}}\left(\frac{\delta_{B,u}}{\delta_{B,t}}\right)^\alpha\left(\frac{1+\frac{\delta_{A,u}}{\delta_{B,u}}}{1+\frac{\delta_{A,t}}{\delta_{B,t}}}\right)^{\alpha-1}\right]\right] du
$$

Subsection 3.4 shows how to obtain the expectation terms as explicit functions of $\left\{\widehat{f}_{A,t}^E, \widehat{f}_{B,t}^E, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A, \frac{\delta_{A,t}}{\delta_{B,t}}, u-t\right\}$.

As a way of defining expected returns and volatilities of equities, we write the dynamics of returns on the stocks $S_{A,t}$ and $S_{B,t}$ as:

$$
\frac{dS_{i,t} + \delta_{i,t}dt}{S_{i,t}} = \widehat{\mu}_{S_i}^E dt + \sigma_{S_i}^{\delta_A} dw_{\delta_A,t}^E + \sigma_{S_i}^{\delta_B} dw_{\delta_B,t}^E + \sigma_{S_i}^{s_A} dw_{s_A,t}^E + \sigma_{S_i}^{s_B} dw_{s_B,t}^E, \quad i = A, B \tag{29}
$$

where $\widehat{\mu}_{S_i}^E$ are the conditional mean returns under the reference measure, and $\sigma_{S_i} = \begin{bmatrix} \sigma_{S_i}^{\delta_A} & \sigma_{S_i}^{\delta_B} & \sigma_{S_i}^{s_A} & \sigma_{S_i}^{s_B} \end{bmatrix}$ are the endogenous diffusions of stock returns. All of these quantities are obtained in equilibrium as explicit functions of the state variables, following the same methodology as in Dumas et al. (2009).

We need three more securities to complete the market. We want to choose a menu of securities that allows investors to allocate their risk exposures to output shocks exclusively through country

equities.[12] In this way, we can ensure that the equity preference is truly the result of higher exposures to own country output shocks and is not polluted by indirect allocations via non equity securities. To that aim, we add to the menu three securities that are neutral vis-à-vis output risk. The first one is the riskless instantaneous bank deposit with interest rate r_t.

The other two are zero-net supply futures contracts in part marked to the fluctuations of the public signal from each country, but designed to be uncorrelated with the stock prices. The prices of the two additional "signal futures contracts" are defined as $F_{A,t}$ and $F_{B,t}$, and follow the processes:

$$dF_{i,t} = \widehat{\mu}_{F_i}^E dt + \sigma_{F_i}^{\delta_A} dw_{\delta_A,t}^E + \sigma_{F_i}^{\delta_B} dw_{\delta_B,t}^E + \sigma_{F_i}^{s_A} dw_{s_A,t}^E + \sigma_{F_i}^{s_B} dw_{s_B,t}^E, \quad i = A, B$$

where $\widehat{\mu}_{F_i}^E$ are continuously determined so that the aggregate demand for each of these securities is always zero, and $\sigma_{F_i} = \begin{bmatrix} \sigma_{F_i}^{\delta_A} & \sigma_{F_i}^{\delta_B} & \sigma_{F_i}^{s_A} & \sigma_{F_i}^{s_B} \end{bmatrix}$, for $i = A, B$, are *exogenously chosen* diffusions of futures price changes. We choose this exogenous diffusion in such a way that futures prices are uncorrelated with the stock prices. The diffusion of risky financial securities prices $\{S_{A,t}, S_{B,t}, F_{A,t}, F_{B,t}\}$ is then given by

$$\Sigma_t = \begin{bmatrix} S_{A,t}\sigma_{S_A}^{\delta_A} & S_{A,t}\sigma_{S_A}^{\delta_B} & S_{A,t}\sigma_{S_A}^{s_A} & S_{A,t}\sigma_{S_A}^{s_B} \\ S_{B,t}\sigma_{S_B}^{\delta_A} & S_{B,t}\sigma_{S_B}^{\delta_B} & S_{B,t}\sigma_{S_B}^{s_A} & S_{B,t}\sigma_{S_B}^{s_B} \\ \sigma_{F_A}^{\delta_A} & \sigma_{F_A}^{\delta_B} & \sigma_{F_A}^{s_A} & \sigma_{F_A}^{s_B} \\ \sigma_{F_B}^{\delta_A} & \sigma_{F_B}^{\delta_B} & \sigma_{F_B}^{s_A} & \sigma_{F_B}^{s_B} \end{bmatrix} \tag{30}$$

The first two rows of this matrix are obtained following the same steps as in Dumas *et al.* (2009), i.e. by premultiplying the diffusion matrix of state variables Ω_t by the gradients of the two equity prices with respect to the state variables. The last two rows are set in such a way that the variance-covariance matrix of financial securities prices $\Sigma\Sigma^{\mathsf{T}}$ is block diagonal.

3.2   Two-factor consumption CAPM

Proposition 1 and the state-price density in Equation (23) reveal that only *two* factors are priced: world output and world average sentiment. The next proposition says that our model is consistent

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with a two-factor consumption CAPM.

Proposition 2: *The following two-factor CAPMs hold equally well:*

$$\widehat{\mu}_{S_i}^E - r_t = (1-\alpha)\, Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{dc_{W,t}}{c_{W,t}}\right) - Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{d\eta_{W,t}}{\eta_{W,t}}\right); i = A, B$$

$$\widehat{\mu}_{S_i}^E - r_t = (1-\alpha)\, Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{dc_{A,t}}{c_{A,t}}\right) - Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{d\eta_{A,t}}{\eta_{A,t}}\right); i = A, B$$

$$\widehat{\mu}_{S_i}^E - r_t = (1-\alpha)\, Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{dc_{B,t}}{c_{B,t}}\right) - Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{d\eta_{B,t}}{\eta_{B,t}}\right); i = A, B$$

where $c_{W,t} = c_{A,t} + c_{B,t}$ is world consumption and $\eta_{W,t}$ is a measure of world average sentiment risk with $\frac{d\eta_{W,t}}{\eta_{W,t}} = \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\frac{d\eta_{A,t}}{\eta_{A,t}} + \left[1 - \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\right]\frac{d\eta_{B,t}}{\eta_{B,t}}$.

Proof: The market price of risk is obtained by applying Itô Lemma to the state price density, and identifying its diffusion vector. The CAPM risk premiums are derived from the market price of risk.

The conditionally expected excess returns on the left-hand side of the CAPM correspond to the way in which *the econometrician* would collect and process data on returns. The factors in the CAPM represent consumption risk and sentiment risk. As in the standard consumption-based CAPM, a security risk premium is positively correlated with the covariance of its return with the country (world) consumption growth. The risk premium is decreasing in the covariance of the security's return with the country (world) sentiment.

The three CAPMs set in the proposition are equivalent to each other, and they imply that the relevant sentiment risk measure depends on the consumption risk considered. For instance, world consumption risk must be accompanied in the CAPM by world average sentiment risk, and country consumption risk must be accompanied by the sentiment risk associated with the vagaries from investors in that particular country. The first CAPM equation considered above shows that a security risk premium is explained by a world consumption risk factor and a world average sentiment risk factor. In this case, if a national risk factor appears to be priced empirically, our interpretation is that it is priced only as a proxy for world sentiment risk; its apparent pricing

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derives from its correlation with world average sentiment risk. The latter is the true unobserved risk factor. Similarly, the next two CAPM equations show that a security risk premium is explained by a country consumption risk factor and the corresponding country sentiment risk factor. In this case, our interpretation is that the world risk factor is priced only as a proxy for national sentiment risk. By the same reasoning as in the first CAPM equation, the world risk factor appears to be priced due to its correlation with world sentiment risk. The latter is the true unobserved risk factor.

3.3 International portfolio choice

In order to analyze the exposures and portfolios of the representative international investors given the menu of securities described above, we first need to derive their total wealth processes. In doing so, we view total wealth as the price of a security with payoffs equal to optimal consumption.[13] The wealths of the representative country investors are given by:[14]

$$
W_t^A \left(\frac{\eta_{A,t}}{\eta_{B,t}}, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^E, \widehat{f}_{B,t}^E, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A \right) = \int_t^\infty \mathbb{E}_t^E \left[\frac{\xi_u^E}{\xi_t^E} c_{A,u} \right] du
$$

$$
= \left[\omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right) \right]^{1-\alpha} \delta_{A,t} \left(1 + \frac{\delta_{B,t}}{\delta_{A,t}} \right) \int_t^\infty e^{-\rho(u-t)} \sum_{j=0}^{-\alpha} \left[\binom{-\alpha}{j} \left(\frac{\omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)}{1 - \omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)} \right)^{-j} \right.
$$

$$
\left. \times \mathbb{E}_t^E \left[\left(\frac{\eta_{A,u}}{\eta_{A,t}} \right)^{1-\frac{j}{1-\alpha}} \left(\frac{\eta_{B,u}}{\eta_{B,t}} \right)^{\frac{j}{1-\alpha}} \left(\frac{\delta_{A,u}}{\delta_{A,t}} \right)^\alpha \left(\frac{1 + \frac{\delta_{B,u}}{\delta_{A,u}}}{1 + \frac{\delta_{B,t}}{\delta_{A,t}}} \right)^\alpha \right] \right] du
$$

$$
W_t^B \left(\frac{\eta_{A,t}}{\eta_{B,t}}, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^E, \widehat{f}_{B,t}^E, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A \right) = \int_t^\infty \mathbb{E}_t^E \left[\frac{\xi_u^E}{\xi_t^E} c_{B,u} \right] du
$$

$$
= \left[1 - \omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right) \right]^{1-\alpha} \delta_{A,t} \left(1 + \frac{\delta_{B,t}}{\delta_{A,t}} \right) \int_t^\infty e^{-\rho(u-t)} \sum_{j=0}^{-\alpha} \left[\binom{-\alpha}{j} \left(\frac{\omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)}{1 - \omega \left(\frac{\eta_{A,t}}{\eta_{B,t}} \right)} \right)^{j} \right.
$$

$$
\left. \times \mathbb{E}_t^E \left[\left(\frac{\eta_{A,u}}{\eta_{A,t}} \right)^{\frac{j}{1-\alpha}} \left(\frac{\eta_{B,u}}{\eta_{B,t}} \right)^{1-\frac{j}{1-\alpha}} \left(\frac{\delta_{A,u}}{\delta_{A,t}} \right)^\alpha \left(\frac{1 + \frac{\delta_{B,u}}{\delta_{A,u}}}{1 + \frac{\delta_{B,t}}{\delta_{A,t}}} \right)^\alpha \right] \right] du
$$

---

[13] We could have focused, instead on financial wealth defined as the present value of future consumption minus country outputs. But we wish to obtain the total country exposure of an investor, whether arising from his portfolio or from his endowments. In a complete market, the two approaches are obviously equivalent.

[14] As was the case for equities prices, the next subsection shows how to obtain the expectation terms as a function of $\left\{ \widehat{f}_{A,t}^B, \widehat{f}_{B,t}^B, \widehat{g}_{A,t}^B, \widehat{g}_{B,t}^A, \frac{\delta_{B,t}}{\delta_{A,t}}, u-t \right\}$.

We define the 1×4 vector of desired, or "target", exposures for the investor in Country i as $x_{i,t}$:

$$x_{i,t} = \left[\begin{array}{cccc} x_{i,t}^{\delta_A} & x_{i,t}^{\delta_B} & x_{i,t}^{s_A} & x_{i,t}^{s_B} \end{array} \right], \quad i = A, B \tag{31}$$

as the diffusion of these wealth processes. These are computed as the gradient of wealth with respect to the seven state variables post-multiplied by the diffusion matrix Ω_t of the seven state variables. If the investors had available to them elementary claims on output and signal shocks, the exposures would indicate the desired amount of holdings of these claims.

With the menu of the two country stocks and the two futures contracts written on the signals that we have described, investors must replicate or engineer the desired exposures. The 1×4 vector θ_i represents the numbers of units held by investors in Country i of each available financial security:

$$\theta_{i,t} = \left[\begin{array}{cccc} \theta_{S_A,t}^i & \theta_{S_B,t}^i & \theta_{F_A,t}^i & \theta_{F_B,t}^i \end{array} \right], \quad i = A, B$$

This vector can be computed directly from a system of linear equations:

$$x_{i,t} = \theta_{i,t} \times \Sigma_t \tag{32}$$

where Σ is the diffusion of securities defined in Equation (30).

3.4 Transform Analysis

In order to obtain the prices of financial securities as well as the country wealth processes (needed for constructing the portfolios), we need to compute the expected values of the product of the change of measure with the payoffs. From the equations for the equilibrium state price density, and the expressions obtained for the stock prices and wealths, it is clear that we need the joint conditional distribution of $\left(\eta_{A,u}, \eta_{B,u}, \delta_{A,u}, \delta_{B,u} \right)^{\mathsf{T}}$ at some future date u given the current state $\left(\eta_A, \eta_B, \delta_A, \delta_B, \widehat{f}_A^E, \widehat{f}_B^E, \widehat{g}_A^B, \widehat{g}_B^A \right)^{\mathsf{T}}$ at current time t. While we cannot determine the full distribution, we can derive a moment function or Fourier transform which allows us to obtain the required

expressions, $\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{1+\frac{\delta_{j,u}}{\delta_{i,u}}}{1+\frac{\delta_j}{\delta_i}}\right)^\psi\right]$ for $i=A,B$ and $i\neq j$.

In a "one-tree" version of our economy, Dumas et al. (2009) show that, by assuming that risk aversion $1-\alpha$ is a positive integer (which can be true only when investors have risk aversion greater than or equal to 1) and using the binomial theorem as in Equation (26), the moment function of outputs and sentiment is enough for obtaining prices and portfolios. While this property is useful in our setup, our problem is further complicated by the fact that the state price density (see Equation (23)), in our model, contains a power of the sum of two outputs: $(\delta_{A,t}+\delta_{B,t})^{\alpha-1}$. To see why this complicates our problem, note that, since investors are risk averse, $\alpha-1<0$, the binomial theorem cannot be used to expand that term. Clearly, obtaining exact solutions for the stock prices and portfolio choice is more challenging in our "two-trees" setup.

In order to overcome this problem, we first note that the marginal utility of aggregate outputs can be expressed as:

$$(\delta_{A,t}+\delta_{B,t})^{\alpha-1}=\delta_{A,t}^{\alpha-1}\left(1+\frac{\delta_{B,t}}{\delta_{A,t}}\right)^{\alpha-1}=\delta_{B,t}^{\alpha-1}\left(1+\frac{\delta_{A,t}}{\delta_{B,t}}\right)^{\alpha-1}$$

The moment generating function in Proposition 3 contains precisely these types of elements, and can be used to obtain stock prices and wealths.

Proposition 3. *The moment generating function needed for solving stock prices and wealths is given by*

$$\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{1+\frac{\delta_{j,u}}{\delta_{i,u}}}{1+\frac{\delta_j}{\delta_i}}\right)^\psi\right]=H\left(\widehat{g}_i^B,t,u,\chi_i\right)\times H\left(\widehat{g}_j^A,t,u,\chi_j\right)$$

$$\times J\left(\widehat{f}_i^E,\widetilde{g}_i^j,t,u,\varepsilon_i,\chi_j\right)\times G\left(\frac{\delta_j}{\delta_i},\widehat{f}_i^E,\widehat{f}_j^E,\widetilde{g}_i^j,\widetilde{g}_j^i,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j,\psi\right)$$

with

$$H\left(\widehat{g}_t,t,u,\chi\right)=\exp\left\{B_1\left(t,u,\chi\right)+\widehat{g}_t^2B_4\left(t,u,\chi\right)\right\},$$

$$J\left(\widehat{f}_i^E,\widetilde{g}_i^j,t,u,\varepsilon_i,\chi_j\right)=\exp\left\{\varepsilon_i\left[K_2\left(t,u\right)+\widehat{f}_i^EK_3\left(t,u\right)+\widetilde{g}_i^jB_3\left(t,u,\chi_j\right)\right]+\varepsilon_i^2\left[K_1\left(t,u\right)+B_2\left(t,u,\chi_j\right)\right]\right\}$$

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and

$$G\left(\frac{\delta_j}{\delta_i}, \widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j, \psi\right) = \int\limits_{-\infty}^{+\infty} \left(1 + \frac{\delta_j}{\delta_i} \exp\left(\frac{y - \mu_y\left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_j^i, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right)}{\sigma_y\left(t, u, \chi_i, \chi_j\right)}\right)\right)^{\psi} n\left(y\right) dy$$

where

$$\mu_y\left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right) = \left(\widehat{f}_j^E - \widehat{f}_i^E\right) K_3\left(t, u\right) + \widehat{g}_j^i B_3\left(t, u, \chi_i\right) - \widehat{g}_i^j B_3\left(t, u, \chi_j\right)$$

$$-2\varepsilon_i\left[K_1\left(t, u\right) + B_2\left(t, u, \chi_j\right)\right]$$

$$\sigma_y\left(t, u, \chi_i, \chi_j\right) = \sqrt{2K_1\left(t, u\right) + B_2\left(t, u, \chi_i\right) + B_2\left(t, u, \chi_j\right)}$$

and $n\left(\cdot\right)$ is a univariate standard normal density function. The functions K_1, K_2, K_3, B_1, B_2, B_3, and B_4 are given explicitly in the proof in the appendix.

4 Country-Level Equity and Empirical Regularities

We now consider the two country-level empirical regularities described in the introduction. First, domestic investors tend to have a bias toward holding domestic equity, despite the potential portfolio gains from holding more foreign equity, a phenomenon called "home-equity preference". Second, foreign purchases of domestic equity increase when domestic equity returns rise. We call this phenomenon the "co-movement of foreign capital and domestic returns". Below, we discuss each of these regularities in turn using our model.

4.1 Benchmark numerical values

To calculate our numerical examples, we base our parameter values and benchmark values for the state variables on those in Dumas et al. (2009). In turn, they take their parameters from Brennan and Xia (2001) who use these parameters to match features of US returns. The standard deviation of the conditional expected growth rate of output, σ_f, as well as its speed of mean reversion, ζ, are chosen to be higher than these studies in order to match the regularities as we describe below. We derive all the results for Country B alone since our model is fully symmetric. The parameter values

we use are given in Table 1.[15,16]

Table 1: Parameter values and benchmark values for the state variables used in all the numerical examples

Parameter	Value	State variable	Benchmark value
ρ	0.2	δ_A	1
$\overline{f}$	0.02	δ_B	1
σ_δ	0.125	δ_C	1
ϕ	0.95	$\widehat{f}_A^E$	$\overline{f}$
$1-\alpha$	3	$\widehat{f}_B^E$	$\overline{f}$
σ_f	0.12	$\widehat{f}_C^E$	$\overline{f}$
ζ	0.27	$\widehat{g}_A^B$	0
		$\widehat{g}_B^A$	0
		$\widehat{g}_C^B$	0
		$\omega(\frac{\eta_{A,t}}{\eta_{B,t}})$	0.5

Note that the benchmark values of state variables reflect a current situation of full agreement $(\widehat{g}_A^B = \widehat{g}_B^A = 0)$ both in the presence and absence of foreign-sentiment risk. Of course, even when there is agreement today, there will be disagreement tomorrow because of different interpretations of signals.

As a first indication of the influence of foreign-sentiment risk, we use our model to calculate the price of each stock and the wealth of investors in each country. Due to symmetry in the benchmark values of the state variables, stock prices and wealth will be the same across countries in this case. Moreover, given our definition of wealth, and stock prices will also be the same. These numbers are reported in the first four rows of Table 2.

As the table shows, the presence of sentiment reduces the stock price. Moreover, sentiment risk reduces the volatility of stocks, reported in the fifth and sixth rows. In particular, the equity volatilities are equal to 0.53 in the absence of sentiment. With sentiment, these drop to 0.27. The intuition is clear. Since half of the investors ignore foreign signals, volatility is lower with sentiment than under rational learning.

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[15] The growth condition, which ensures existence of stock prices, is satisfied by these parameter values. That is, equity prices have a finite value.

[16] Firm C will be introduced in Subsection 5.1.

Table 2: Effect of sentiment on prices and on second moments of equity returns

	No sentiment	Sentiment
Price of Stock A	19.22	7.42
Price of Stock B	19.22	7.42
Wealth of A	19.22	7.42
Wealth of B	19.22	7.42
Volatility of Stock A return	0.53	0.27
Volatility of Stock B return	0.53	0.27
Correlation of Stock A and Stock B returns	0.97	0.84

The foreign-sentiment risk model also gives results that are similar to a segmentation model. In many models, stock return correlations tend to be lower in a segmented market than in an integrated one.[17] As the last rows shows, the equity correlation is equal to 0.97 without sentiment, but this correlation drops to 0.84 with sentiment. The high correlation of stock returns is particularly noteworthy since we have assumed the output correlation to be equal to zero.

We now assess the ability of the model under the benchmark values to reproduce the "home-equity preference" and the "co-movement of foreign capital and domestic returns". To that aim, we evaluate these empirical regularities using both our model with sentiment and our model without sentiment in which all investors interpret correctly the information in both countries. We then compare the results.

4.2 Home equity preference

Home-equity preference, or "home bias" as it is often called, is the observation that home residents tilt their portfolios towards home equity. Evidence has been provided many times, including recently in Ahearne, *et al.* (2004). Table 3 reports the proportion of foreign securities held in aggregate US porftolios from the IMF coordinated survey.

A number of explanations have been proposed for "home bias". Our sentiment-risk model could be viewed as a complement rather than a substitute for these explanations. Nevertheless, it is useful to note how our analysis compares to these standard groups. One set of explanations assumes that

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[17]See Dumas, Harvey, Ruiz (2003).

Table 3: Source: IMF coordinated survey

Year	Foreign securities's share in U.S. portfolios
2001	20%
2002	21%
2003	23%
2004	26%
2005	29%
2006	29%

investors have utility functions that are non-separable between tradeable goods and items that are not traded, such as leisure or non-tradeable goods.[18] The plausibility of these arguments depends upon whether the effects of non-tradeables risk is sufficiently large and of the right sign to explain home bias.[19] By contrast, our explanation does not depend upon non-separabilities in utility since investors in both countries consume the same, single good. A second group of explanations depends upon incomplete or segmented markets. In these models, some investors are restricted from holding some securities or do not otherwise have access to assets capable of spanning the state space.[20] In our model, asset markets are complete and all investors have access to the same set of securities. The third group of explanations assumes that investors receive asymmetric information as in Noisy-Rational Expectations equilibria.[21] Our explanation most closely resembles this group of explanations. However, these explanations typically depend upon a static portfolio problem and the presence of noise traders, who are not utility maximizers and who have outside resources. By contrast, we endogenously determine equilibrium prices of financial securities and the optimal portfolio of financial securities held by investors of both countries, in a fully closed dynamic general equilibrium under symmetric information.

Using our solutions for the portfolio and wealth of country investors, we obtain an exact expression for the share of foreign (Country A) stocks held by local investors (in Country B), given by:

$$\frac{S_{A,t}\theta^B_{S_A,t}}{S_{A,t}\theta^B_{S_A,t} + S_{B,t}\theta^B_{S_B,t}}$$

---

[18] See for example Stockman and Tesar (1995) and Pesenti and van Wincoop (2002).

[19] See Lewis (1996) and Baxter and Jermann (1997).

[20] See Baxter and Crucini (1991), among others.

[21] This argument was posited by Gehrig (1993) and made dynamic by Brennan and Cao (1997).

To evaluate the portfolio decisions and home equity preference using our model, we calculate the target exposures in equation (31), the portfolio share of Country B investors in the home and foreign stock, and the consumption correlation across countries.

Table 4 reports these numbers for the benchmark case. The impact of foreign sentiment on the equilibrium variables in the row is shown for each sentiment case in the column. Without foreign-sentiment risk, given in the column labeled "No Sentiment", the share invested abroad is, of course, equal to 50%. With foreign-sentiment risk and with the benchmark numbers given above, the share drops to 38% when there is current agreement about the growth prospects of the two-countries, $\widehat{g}_A^B = \widehat{g}_B^A = 0$.

Table 4: The impact of foreign sentiment on the equilibrium numbers. Negative numbers for disagreements mean that investors are currently optimistic about foreign investment compared to the econometrician's beliefs

	No sentiment	Sentiment with $\widehat{g}_A^B = \widehat{g}_B^A = 0$	Sentiment with $\widehat{g}_A^B = \widehat{g}_B^A = -0.001$	Sentiment with $\widehat{g}_A^B = \widehat{g}_B^A = -0.01$
Expos. of B to A output shock	0.17	-1.19		
Expos. of B to B output shock	0.17	0.02		
Expos. of B to A signal shock	-7.09	-0.27		
Expos. of B to B signal shock	-7.09	-2.19		
B's portfolio share invested in S_A	50.00%	38.01%	40.39%	61.47%
B's portfolio share invested in S_B	50.00%	61.99%	69.61%	38.53%
Consumption correlation	1.00	1.00	0.9964	0.6920

The exposure numbers are useful as a check that the home preference does not arise from the menu of assets. The numbers verify that sentiment generates a target exposure toward domestic output. Without sentiment, the desired exposures to output shocks and signals shocks are symmetric at 0.17 and -7.09, respectively. However, in the presence of sentiment, the B investor prefers negative exposure to the foreign output shock at -1.19 and similarly reduces desired exposure to the foreign signal.

Increasing the level of disagreement (last two columns of the table) produces a very interesting result. A large increase/decrease represents a high degree of optimism/pessimism of one group of

investors regarding the foreign country so that portfolio investment is proportionately affected. The last column in Table 4 confirms that investors invest in a country when they are bullish about it. Small increases in disagreement are more interesting. Recall that disagreement causes sentiment to become more volatile (Equations (17, 18)). As we just saw, foreign-sentiment risk is a deterrent to investing abroad. When disagreement of any sign is small, it is not large enough to create a potent enough optimism or pessimism but it does increase the volatility of sentiment risk. Even with small disagreements *of either sign*, there is some home bias. This reasoning is reflected in the numbers of the last-but-one column of Table 4, labeled $\widehat{g}_A^B = \widehat{g}_B^A = -0.001$.

The table also reports the effects of sentiment risk on consumption correlations. Lewis (1999, 2000) has noted the connection between home-biased portfolio composition and less than perfect correlation of consumption across countries. Intuitively, when investors of different countries hold portfolios with different compositions, the correlation in the consumption resulting from these portfolio choices should be reduced. Indeed, in our model the random components of the consumption of investors from different countries differ by the "sentiment wedge" η already described. Consumptions are less than perfectly correlated to the degree that η is more volatile. When it so happens that $\widehat{g}_A = \widehat{g}_B = 0$, consumptions are for a brief moment perfectly correlated across countries. But, as the disagreements, $\left(\widehat{g}_A^B, \widehat{g}_B^A\right)$, fluctuate around zero, the consumption correlation changes accordingly. For instance, as shown in the table above, consumption correlation drops to 0.69 with a disagreement of only $\widehat{g}_A = \widehat{g}_B = -0.01$. Consumption correlation, in some states of nature, can be very low and even negative, for some extreme values of disagreement. Contrary to the majority of empirical evidence, however, the correlation of consumptions in most cases remains above the correlation of outputs, which in our model is by assumption equal to zero.

4.3   Foreign capital inflow and domestic return co-movement

We now turn to our next international empirical regularity: the co-movement between returns and capital flows. Grinblatt and Keloharju (2000) have shown that capital flows into countries at the same time the stock market of that country experiences above average returns. According to the asymmetric-information model of Brennan and Cao (1997), the interpretation would be that foreigners are less informed about a country's firms than are the country's residents. As they

see domestic stock prices going up, less informed foreigners speculate that residents have received good news. They then buy the domestic country's stocks. In this section, we show that our general-equilibrium model can deliver this same relationship for a different but somewhat analogous reason.

Our framework mimics the domestic investor's informational advantage as found in the asymmetric information literature. Recall that Brennan and Cao (1997) show that less informed investors should buy shares at the same time the prices of these shares rises. By way of empirical verification, they regress quarterly gross capital flow from home to foreign country – deflated by the average gross capital flow of the last four quarters – on quarterly foreign-market return. They generally find positive slope coefficients. A representative sample across countries is shown in Table 5.

Table 5: Source: Brennan and Cao (1997). They regress quarterly gross capital flow from home to foreign country – deflated by the average gross capital flow of the last four quarters – on quarterly foreign-market return. They obtain the positive slope coefficients shown here.

Country	Regression coefficient
Canada	7.19
Germany	6.83
Japan	4.79
United Kingdom	1.53
Argentina	5.44
Mexico	6.05
Indonesia	35.11
Turkey	22.8

In our setup, gross international capital flows are interpreted as the dollar demand of home-country (say, Country B) investors for foreign stock $S_{A,t} d\theta^B_{S_A,t}$ (the "target" country is then Country A). However, barring the use of a simulation, it is not possible for us to replicate exactly the deflation by the last four quarters of capital flows that was done by Brennan and Cao. Presumably they did that because they wanted a measure of flows relative to some outstanding amounts and they did not have available the outstanding amounts. We can produce the outstanding amounts $\theta^B_{S_A,t}$ directly from our model, however. Therefore, as the closest approximation available, we calculate the theoretical coefficient of a regression of $\frac{d\theta^B_{S_A,t}}{\theta^B_{S_A,t}}$ on $\frac{dS_{A,t}}{S_{A,t}}$. In addition, we report the correlation of capital inflows and stock returns as well as the volatility of capital of outflows. Table 6 gives these results.

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Table 6: Theoretical coefficient of a regression of $\frac{d\theta^B_{S_A,t}}{\theta^B_{S_A,t}}$ on $\frac{dS_{A,t}}{S_{A,t}}$

	No sentiment	Sentiment
Correlation of capital flows and stock return	0.06	0.49
Volatility of capital outflows	0.02	2.19
Brennan&Cao regression beta	0.005	6.37

Sentiment increases the correlation of capital flows and stock returns to about 0.5. Since foreigners weight domestic output in their forecast of future dividends, they over-react to dividends, thus driving up the correlation between capital inflows and domestic stock returns. Similarly, sentiment implies an increase in volatility in these capital flows. When we calculate the implication for the regression beta, the value of that regression coefficient is essentially zero at 0.005 without sentiment, but increases to 6.37 with sentiment.

In our model, there is no information asymmetry. But investors have a different interpretation of the publicly available information. Thus, public information produces a joint reaction of portfolios and prices that mimics their behavior under information asymmetry, as illustrated, for instance, by the model of Brennan and Cao (1997).

5 Firm-Level Equity and Empirical Regularities

We now consider the firm-level empirical regularities described in the introduction. We evaluate the ability of the model to reproduce the "pricing puzzle" and the "abnormal cross-listing return". For that purpose, we evaluate these empirical regularities using our model, and for the standard case when all investors interpret correctly the information in both countries. We then compare the results.

5.1 Extension of the model

Until now, we have considered one firm in each country for simplicity. In this section, we consider two firms in Country A. In particular, let there be one firm listed in Country B, which we call Firm B, and let there be *two* firms listed in Country A, which we call Firm A and Firm C. We index these firms by $i = \{A, B, C\}$.

Under the objective measure, the outputs of Firms A and B are as they were before but we now introduce the output of the new Firm C as:

$$\frac{d\delta_{C,t}}{\delta_{C,t}} = f_{C,t}dt + \sigma_\delta dz_{C,t}^\delta$$

where $z_{C,t}^\delta$ is an independent Brownian motion under the objective probability measure, which governs empirical realizations of the process. The conditional expected growth rate $f_{C,t}$ of output is also stochastic:

$$df_{C,t} = -\zeta \times \left(f_{C,t} - \overline{f}\right) dt + \sigma_f dz_{C,t}^f$$

As before, all investors must estimate, or filter out, the current value of $f_{i,t}$ and its future behavior. They do so by observing the current cash flows and the three public signals (s_A, s_B, s_C) The signal correlated with $dz_{C,t}^f$ evolves according to the following processes:

$$ds_{C,t} = \phi dz_{C,t}^f + \sqrt{1 - \phi^2} dz_{C,t}^s$$

where z_C^s is a Brownian motion, under the objective probability measure as well. All the Brownian motions are independent from each other.

Investors in Country A (where firms A and C are listed) perform their filtering under the belief that the signals s_A and s_C have the correct correlation with f_A and f_C; but they believe incorrectly that the signal s_B has zero correlation with f_B, which means that they ignore the information about the firms listed in the other country. The "model" they have in mind, in addition to Equations (8) and (9), posits that:

$$ds_{C,t} = \phi dz_{C,t}^f + \sqrt{1 - \phi^2} dz_{C,t}^s$$

Investors in Country B (where Firm B is listed) perform their filtering under the belief that the signal s_B has the correct correlation with f_B; but they believe incorrectly that the signals s_A and s_C have zero correlation with f_A and f_C, which means that they ignore the information about the firms listed in the other country. The "model" they have in mind, in addition to Equations (10) and (11), posits that:

$$ds_{C,t} = dz_{C,t}^s$$

By following exactly the same steps as in Section 2, we can show that the vector of exogenous states variables under the reference measure of the econometrician is given by the Markovian system comprised of Equations (4), (5), (2), (14), (17) and (18), to which we now add the following three analogous equations for Firm C's dividends, expected conditional growth rate, and disagreement, respectively:

$$
\frac{d\delta_{C,t}}{\delta_{C,t}} = \widehat{f}^E_{C,t}dt + \sigma_\delta dw^E_{\delta_C,t}
$$

$$
d\widehat{f}^E_{C,t} = -\zeta \times \left(\widehat{f}^E_{C,t} - \overline{f} \right) dt + \frac{\gamma^E}{\sigma_\delta}dw^E_{\delta_C,t} + \phi\sigma_f dw^E_{s_C,t}
$$

$$
d\widehat{g}^B_{C,t} = -\left(\zeta + \frac{\gamma^X}{\sigma_\delta^2} \right) \widehat{g}^B_{C,t}dt + \frac{\gamma^X - \gamma^E}{\sigma_\delta}dw^E_{\delta_C,t} + \phi\sigma_f dw^E_{s_C,t}
$$

and the new change $\widetilde{\eta}_{B,t}$ from the measure of investor B to that of the econometrician is:

$$
\frac{d\widetilde{\eta}_{B,t}}{\widetilde{\eta}_{B,t}} = -\frac{1}{\sigma_\delta} \left(\widehat{g}^B_{A,t}dw^E_{\delta_A,t} + \widehat{g}^B_{C,t}dw^E_{\delta_C,t} \right)
$$

Therefore, we get an extended vector of ten exogenous state variables which drive the economy:
$Y_t \equiv \left(\delta_{A,t}, \widehat{f}^E_{A,t}, \widehat{g}^B_{A,t}, \eta_{B,t}, \delta_{B,t}, \widehat{f}^E_{B,t}, \widehat{g}^A_{B,t}, \eta_{A,t}, \delta_{c,t}, \widehat{f}^E_{c,t}, \widehat{g}^B_{C,t}, \eta_{C,t} \right)^{\mathsf{T}}$ where η_C is defined as:

$$
\frac{d\eta_{C,t}}{\eta_{C,t}} = -\frac{1}{\sigma_\delta}\widehat{g}^B_{C,t}dw^E_{\delta_C,t}
$$

such that $\widetilde{\eta}_{B,t} = \eta_{B,t} \times \eta_{C,t}$. We get a structure that is very similar to that of our benchmark model of Section 2. But there is a now third system $Y_{C,t}$ where

$$
Y_{C,t} = \left\{ \delta_{C,t}, \widehat{f}^E_{C,t}, \widehat{g}^B_{C,t}, \eta_{C,t} \right\}^T
$$

Therefore this system can be written:

$$
dY_{C,t} = \mu_{C,t}dt + \Omega_{C,t}dw^E_{C,t}
$$

where

$$
dw^E_{C,t} = \left\{ dw^E_{\delta_C,t}, dw^E_{s_C,t} \right\}
$$

and

$$\Omega_{C,t} = \begin{bmatrix} \sigma_\delta \delta_{i,t} & 0 \\ \left(\frac{\gamma^E}{\sigma_\delta}\right) & \phi\sigma_f \\ \left(\frac{\gamma^E - \gamma^X}{\sigma_\delta}\right) & \phi\sigma_f \\ -\eta_{C,t}\left(\frac{\hat{g}^B_{C,t}}{\sigma_\delta}\right) & 0 \end{bmatrix}$$

Thus, the state vector can be evaluated as three independent sets of processes. For instance, the full vector of twelve state variables $\widetilde{Y}_t$ can be written as:

$$d\widetilde{Y}_t = \widetilde{\mu}_t dt + \ \widetilde{\Omega}_t d\overrightarrow{w}^E_{i,t}$$

where

$$d\overrightarrow{w}^E_{i,t} = \left\{dw^E_{\delta_A,t}, dw^E_{\delta_B,t}, dw^E_{\delta_C,t}, dw^E_{s_A,t}, dw^E_{s_B,t}, dw^E_{s_C,t}\right\}$$

and

$$\widetilde{\Omega}_t = \begin{bmatrix} \Omega_{A,t} & 0 & 0 \\ 0 & \Omega_{B,t} & 0 \\ 0 & 0 & \Omega_{C,t} \end{bmatrix}$$

where Ω_t is still block diagonal, which is again exploited in our solution of the equilibrium in this extended model.

In this setting we now have six different Brownian motions, hence we need seven linearly independent securities in order to complete the market. Since we intend to replicate a regression of a firm excess stock return on a local country excess stock return and a foreign country excess stock return, we choose our menu of securities accordingly. In particular we consider three stocks: (i) a firm stock S_C which is a claim on the output of Firm C (δ_C); (ii) a local stock $S_{A,C}$ which is a claim on the aggregate output of Country A ($\delta_A + \delta_C$); and (iii) a foreign stock S_B which is a claim on the foreign output δ_B. As in Subsection 3.1, we complete the market with a riskless, instantaneously maturing bank deposit and three zero-net supply futures contracts whose prices are in part marked to the three signals, but are uncorrelated with all the stock prices.

In general, the return on the stock market of Country A would be given by

$$\frac{dS_{A,C}}{S_{A,C}} = \left(1 - \frac{S_C}{S_A + S_C}\right)\frac{dS_A}{S_A} + \frac{S_C}{S_A + S_C}\frac{dS_C}{S_C}$$

where S_A is the price of a claim on the output of Firm A (δ_A). This corresponds to a capitalization weighted index of the two stocks traded in Country A. Since we intend to run a regression of $\frac{dS_C}{S_C} - r$ on $\frac{dS_{A,C}}{S_{A,C}} - r$ and $\frac{dS_B}{S_B} - r$, we make sure not to include Firm C in the index for the stock market of Country A, as this would bias the beta against that index. In terms of the equation above, this is equivalent to let $\frac{S_C}{S_A + S_C} \to 0$, and, consequently, $\frac{dS_{A,C}}{S_{A,C}} \to \frac{dS_A}{S_A}$.

5.2 Pricing puzzle: factor models

In integrated markets, risk factors are common to all securities and all securities are priced with these same factors. In our model, with no market segmentation and with all tradeable goods, equilibrium prices, such as (28), are functions of seven variables, each of which is driven by four Brownian motions. Therefore, it is possible to recognize seven factors in securities returns. However, these factors are not independent of each other since their covariation structure has rank four. With or without foreign-sentiment risk, each of these four factors can be seen as having a home or a foreign dimension. For instance, of the four Brownians, two are home and foreign output shocks and two are signals shocks that are correlated with home or foreign expected growth rates. Ultimately, the only two priced factors are aggregate output $\delta_{A,t} + \delta_{B,t} + \delta_{C,t}$ and the world average sentiment $\left[\left(\frac{\eta_{A,t}}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\tilde{\eta}_{B,t}}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}$.

The CAPM Subsection 3.2 may help explain why the risk premium of financial securities' contain both national and world *consumption* risk factors. The factors in these CAPMs are national consumption risk and country sentiment risk.[22] By construction (in a manner similar to Equations (17, 18)), we know that sentiment and the world output are correlated. As has been indicated in Subsection 3.2, our model provides one interpretation of the empirical findings of two priced factors, one being national and the other a world factor, a result commonly ascribed to some form of segmentation. Our interpretation is that the world consumption risk is priced only as a proxy for sentiment risk. The fact that it appears to be priced derives from its correlation with sentiment

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[22] Empirical studies on two-factor international CAPMs are usually conducted on some form of market-return CAPM, as opposed to a consumption CAPM.

risk, the true unobserved risk factor.

To consider this interpretation, we calculate the correlation between country sentiment $\frac{d\eta_{i,t}}{\eta_{i,t}}$, and the world consumption growth, $\frac{dc_{W,t}}{c_{W,t}}$, where $c_{W,t} = c_{A,t} + c_{B,t} = \delta_{A,t} + \delta_{B,t}$, due to market clearing. In the fully symmetric case when $\delta_{A,t} = \delta_{B,t} = 1$ we obtain a correlation of sentiment in country A, $\frac{d\eta_{A,t}}{\eta_{A,t}}$, and world consumption, $\frac{dc_{W,t}}{c_{W,t}}$, equal to $0.71 \times sign\left(\widehat{g}_{B,t}^A\right)$. The sign of the correlation depends on the disagreements between investors in Country A and the econometrician on the prospects about the growth rate of output in Country B. Symmetrically, the correlation of sentiment in country B, $\frac{d\eta_{B,t}}{\eta_{B,t}}$, and world consumption, $\frac{dc_{W,t}}{c_{W,t}}$, is equal to $0.71 \times sign\left(\widehat{g}_{A,t}^B\right)$.

However, much of the empirical literature on international stock returns focuses not upon a CAPM, but rather on the factor structure of the returns. Empiricists typically find that international firm returns depend upon local home factors as well as foreign or world factors. Dependence of firm returns on local and foreign factors is often loosely interpreted as evidence for market segmentation or non-tradeable risks. Ever since Agmon (1973) and Lessard (1976), many authors have dissected international stock returns to find out how much of their variance was due to country factors and how much was due to industry factors, which are worldwide factors (see, for instance, Heston and Rouwenhorst (1994), Cavaglia *et al.* (2000), Cavaglia and Moroz (2002), Brooks and Del Negro (2004), and Brooks and Del Negro (2006)). The ballpark figure is that country factors represent at least 30% of variance.[23] Moreover, for large multinational firms, the beta on the local market is typically higher than the foreign market. For example, using a sample of multinational firms that list in foreign markets, Sarkissian and Schill (2009) find a beta on the local factor of 0.8 and a beta for the foreign factor of 0.16. These firms also provide a useful benchmark for the section on cross-listed firms below.

To consider factor variances and betas for firms in the presence of foreign sentiment, we now evaluate our model's ability to replicate these results. First, recall that the cross-country correlation of returns is lower with foreign-sentiment risk than without it. Second, we replicate linear regressions of financial securities return in both national and world stock market risk factors. We then evaluate the foreign-sentiment risk impact on the regression coefficients.Consider regressing the firm stock

---

[23] Bekaert, Hodrick and Zhang (2009) conclude that this number is an overstatement.

excess return $\frac{dS_C}{S_C} - r_t$ on the country stock excess return $\frac{dS_A}{S_A} - r_t$ and the foreign stock excess return $\frac{dS_B}{S_B} - r_t$. We obtain the estimated regression coefficients $\left(\widehat{\beta}_A, \ \widehat{\beta}_B \right)$ from the associated normal equations:

$$\mathbb{E}\left[\left(\left(\frac{dS_C}{S_C} - r_t \right) - \widehat{\beta}_A \left(\frac{dS_A}{S_A} - r_t \right) - \widehat{\beta}_B \left(\frac{dS_B}{S_B} - r_t \right) \right) \left(\frac{dS_A}{S_A} - r_t \right) \right] = 0$$

$$\mathbb{E}\left[\left(\left(\frac{dS_C}{S_C} - r_t \right) - \widehat{\beta}_A \left(\frac{dS_A}{S_A} - r_t \right) - \widehat{\beta}_B \left(\frac{dS_B}{S_B} - r_t \right) \right) \left(\frac{dS_B}{S_B} - r_t \right) \right] = 0$$

The regression coefficients depend on the elements of the variance covariance matrix of the vector $\left(\frac{dS_C}{S_C} - r_t, \ \frac{dS_A}{S_A} - r_t, \ \frac{dS_B}{S_B} - r_t \right)^{\mathsf{T}}$.

We compute the stock prices using the same procedure as in the case with two stocks. The interest rate follows from applying Itô lemma to the state price density expression. The diffusion of excess returns is obtained from the corresponding gradients and the diffusion of the state variables, as in the case with only two stocks. We determine the variance-covariance matrix by multiplying the diffusion of excess returns by its transpose. This variance-covariance matrix includes all the necessary second moments so that we obtain the regression coefficients in closed-form.

Table 7: Betas and shares of variance explained in our model by the various factors.

	No sentiment	Sentiment
Two factor CAPM regression coefficient for the local factor	0.4614	0.3868
Two factor CAPM regression coefficient for the foreign factor	0.4614	0.3786
% of variance contributed by local factor	26.93%	31.89%
% of variance contributed by foreign factor	26.93%	29.90%
% of variance contributed by covariance between them	46.14%	38.21%

The verdict on the ability of the model to explain the empirical share of variance due to country factors is contained in Table 7. In the benchmark case without sentiment the beta of the local factor is identical to that of the foreign factor, and the share of variance explained by the local factor is around 27%. When we add foreign-sentiment the beta of the local factor gets slightly higher than the beta of the foreign factor, and the share of variance explained by the local factor increases to around 32%. Although we do not match the empirical betas precisely, we obtain qualitative results that are closer to the data by adding foreign-sentiment risk.

5.3 "Abnormal" Cross-Listing Returns

Cross-listing events present a final feature of international security-return behavior, which has often been associated with differing economic perceptions across countries. When foreign firms list in domestic markets, the returns on equity shares of the cross-listing firm become abnormally high relative to the market. In particular, the cumulative abnormal returns (CAR) generally increase and peak near the cross-listing event.[24] Karolyi (2006) surveys the literatures that finds an abnormal return in the week of cross-listing (the "impact" effect) which is between 1.5% and 7%. After the cross listing (the risk-premium "after-effect"), Hail and Leuz (2005) find a change in the cost of capital which is between -0.7% and -1.2%[25] An often-cited explanation for this behavior is that the event provides new information about the future behavior of the firm. Here again Ahearne, et al. (2004) is an inspiration. They show that U.S. investors exhibit a greater willingness to invest in firms from countries that list on U.S. exchanges. Their interpretation is that cross-listing provides U.S investors with information that is easier for them to interpret.

We now consider cross-listing as a mental experiment conducted on the three-firms extended model. As before, let there be one firm in Country B, which we call Firm B, and let there be two firms in Country A, which we call Firm A and Firm C. We conduct a comparative-dynamics exercise comparing the equilibrium in which Firm C is listed in Country A to the equilibrium in which it is listed both in Country A and in Country B. We postulate that, when the firm is cross-listed, investors in Country B know how to correctly interpret the public information about Firm C. Therefore, under cross-listing, the "model" that investors in Country B have in mind, in addition to Equations (10) and (11), posits that:

$$ds_{C,t} = \phi dz_{C,t}^f + \sqrt{1 - \phi^2} dz_{C,t}^s$$

---

[24] Moreover, the correlation of these foreign firms relative to the domestic market increases.

[25] See also Sarkissian and Schill (2009) and King and Segal (2003).

Following the same steps as in Section 2, the Firm C diffusion block under cross-listing becomes:

$$
\Omega_{C,t} =
\begin{bmatrix}
\sigma_\delta \delta_{i,t} & 0 \\
\left(\frac{\gamma^E}{\sigma_\delta} \right) & \phi \sigma_f \\
0 & 0 \\
-\eta_{C,t} \left(\frac{\widehat{g}^B_{C,t}}{\sigma_\delta} \right) & 0
\end{bmatrix}
$$

where $\widehat{g}_C$ is now deterministic. This result is intuitive: due to the cross listing the perception of information in the signal of Firm C is aligned across investors from different countries. Therefore, disagreement about the cross-listed firm stops being a relevant risk factor. The new moment functions for Firm C with and without cross listing are very similar to those computed in Proposition 3.
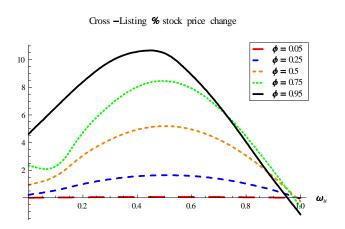
Using our model and the benchmark numbers, we calculate and display in Table 8 the effect of Firm C cross listing from Country A to Country B, interpreted in the manner described above. Regarding first the impact effect, it is obvious that, without sentiment, the effect is equal to zero. With sentiment, the change in price on impact is: 10.21%, when there happened to be full agreement at the time of the event, while the after-effect on the cost of capital is equal to -0.08%.

Table 8: The effect of Firm C cross listing from Country A to Country B

	No sentiment	Sentiment
Cross-listing price change (%)	0.00%	10.21%
Cross-listing risk premium change (%)	0.00%	-0.08%

Figure 1 illustrates the extent to which the relative sizes of the two countries in the cross-listing modifies the effect of cross-listing, for different levels of ϕ. Due to cross listing, disagreement about the output growth rate of firm C is reduced. We can call that the "disagreement-risk" effect. However, that cuts two ways. Without cross-listing, investors in Country B ignore completely the signals that are correlated with f_C. With cross listing, they take them into account correctly. Their increased sensitivity to the signals (the "signal-risk" effect) means that the return of the stock from Firm C becomes more volatile because there is new uncertainty about the learning process of

Figure 1: The effect of the relative sizes of the two countries in the cross-listing effect for different levels of ϕ.

investors from Country B. This is true even in the eyes of the investors from Country B themselves. As has been pointed out by Brennan (1998), an investor sees his own future learning as a source of risk that must be hedged. Clearly, the disagreement-risk effect operates more strongly when $\omega \to 0.5$, as this is the case in which the impact of the heterogeneity of beliefs is highest. In the middle range, the interaction between investors from the two countries (i.e., the degree to which they are scared of *each other*) dominates and adds up to a reduction in the amount of foreign-sentiment risk: investors from Country A realize that those from Country B will now behave as correctly as themselves so that, in their eyes, future price movements, that arose previously from their disagreements, will be less erratic than before. As a result, the required return decreases. Because this is anticipated, there must be an immediate increase in the stock price in order to allow for future lower returns.

Towards the edges, when heterogeneity is minimal, the interaction of these risks differs. When $\omega \to 0$, B investors, who learn to interpret the signals of Firm C, are predominant. So the reduction in disagreement risk generates a price increase. When $\omega \to 1$, A investors are predominant, so the

reduction in disagreement risk is less important, and higher signal risk is the strongest effect. As a result, there may be price decreases in that edge. In section 6 below, we show that our model is stationary and there is a tendency for the consumption share to revert to the value of 0.5. Therefore, ω tends to be in the middle range where the disagreement-risk effect is more important than signal risk, and we expect in general an increase in prices from cross-listing, as observed empirically.[26]

6 Survival

So far, we have found that a number of international financial-market empirical regularities can be explained by our model. But, to generate sentiment risk, our model requires the active presence of both home and foreign investors. Given the symmetric irrationality, it is not obvious how long these investors may survive in the market. In this last section, we verify that the effects of sentiment defined in this paper persist in the long-run.
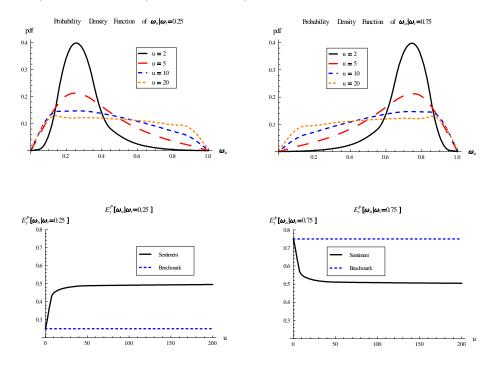
Existing studies on the survival of irrational traders ask whether excessively pessimistic or optimistic agents survive in the long run in an economy in which one population of agents knows the true probability distribution. These studies include Kogan *et al.* (2006), Yan (2008) and Dumas *et al.* (2009). These studies conclude that, although "irrational" traders do not survive in the long-run, they disappear very slowly, in terms of consumption shares. In our model, both types of investors are symmetrically underconfident about the foreign signals. As a result, their individual and symmetric irrationalities offset each other and we obtain a stationary equilibrium, in which both countries' investors continue to participate in the economy in the long run. Foreign sentiment, therefore, provides a rationale for the empirical regularities described above that is valid both in the short run and in the long run.

To show this, we obtain, as in Dumas *et al.* (2009), the expected value and distribution under the objective probability measure of the future consumption share of investors from Country A (ω_u) by Fourier Inversion of the characteristic function of *relative* sentiment $\frac{\eta_A}{\eta_B}$. We study the case where Country A is currently small ($\omega_t = 0.25$) and the case when it is currently large ($\omega_t = 0.75$).

---

[26] Note that even in the extreme where $\omega \to 1$ and Country B has a negligible size we obtain an increase in signal risk and an associated decrease in price. This is because at both edges the fact that ω reverts to 0.5 in the future enters in play, as both investors are non-myopic.

The left upper panel in Figure 2 plots the probability density function of ω_u given a current value of $\omega_t = 0.25$. We see from this figure that, as time passes, the density moves to the right and therefore when the consumption share of investors in Country A is currently small, it tends to increase. The left lower panel plots the expected value of this share against time measured in years. It shows that, compared to the benchmark case without sentiment, the consumption share of investors in Country A reverts to the steady state value of 0.5.

Figure 2: Top panel: the probability density function of ω_u given a current value of $\omega_t = 0.25$ (left-hand side) and $\omega_t = 0.75$ (right-hand side). Bottom panel: expected values of the same.

The symmetry imposed in the model implies that the two exponential martingales which arise from the foreign sentiment of each country investor, η_A and η_B, offset each other in the sense that even though each of them tends to zero over time independently, the ratio of the two converges to one, which brings ω_u to 0.5 in the long run. The plot shows that, starting from a small consumption share of 0.25, it takes about 100 years to investors in Country A to reach the steady state value of 0.5.

The right-hand plots of Figure 2 show that the exact same logic applies when Country A is

currently large ($\omega_t = 0.75$). In that case, as time passes, the density moves to the left, i.e., it decreases. We conclude that our model is stationary and that all sentiment-based explanations for the empirical regularities presented above hold both in the short and in the long run.

7   Conclusions

By allowing international investors to differ in their interpretation of domestic and foreign public information (a feature which we label foreign-sentiment risk), we show that four of the most striking regularities in international finance can be at least partially resolved, including: (i) home-equity preference; (ii) the co-movement of returns and international capital flows; (iii) the dependence of firm returns on local and foreign factors; and (iv) abnormal returns around foreign firm cross-listing in the domestic market.

Home equity preference is a natural consequence of domestic investors perceiving the foreign public information as less valuable than the one generated at home, even when, in fact, both domestic and foreign public information are equally valuable. The foreign-sentiment risk, or the risk of the behavior of others that is created by this difference in perception deters investors from investing abroad and is priced in the market.

We generate a co-movement of returns and international capital flows due to the perceived advantage of domestic investors in processing public information about their own fundamentals. When a positive output shock hits the domestic economy, foreign investors misinterpret public information about domestic fundamentals, inducing a higher demand for domestic stocks. These shocks then generate a simultaneous increase in returns in the domestic stocks, and capital flows from the foreign to the domestic country.

Our model also implies a two-factor consumption CAPM, involving consumption-risk and foreign-sentiment risk factors that are priced. In the model, the cross-country correlation of returns is lower with foreign-sentiment risk than without it, a fact which would commonly be ascribed to segmentation instead of foreign-sentiment risk. The two-factor conditional structure of returns that we calculate from the model shows that foreign-sentiment has some potential to explain the relative importance of local factors in explaining the expected returns of multinational companies.

Finally, our model produces an increase in stock prices for firms that cross list. Here, cross-listing is a thought experiment, or comparative-dynamic exercise, in which the investors of the country where the new listing occurs know how to interpret the public information about the cross-listed firm. Due to cross-listing, disagreement is reduced, hence the amount of foreign-sentiment risk and the required return decrease. Because this is anticipated, there is an immediate increase in the stock price in order to allow for future lower returns.

Overall, we have shown that a single type of informational friction can theoretically explain a range of international pricing anomalies.

Proof of Proposition 3. We want to compute the expectation given in the proposition:

$$\mathbb{E}_t^E \left[\left(\frac{\eta_{i,u}}{\eta_i} \right)^{\chi_i} \left(\frac{\eta_{j,u}}{\eta_j} \right)^{\chi_j} \left(\frac{\delta_{i,u}}{\delta_i} \right)^{\varepsilon_i} \left(\frac{1 + \frac{\delta_{j,u}}{\delta_{i,u}}}{1 + \frac{\delta_j}{\delta_i}} \right)^{\psi} \right] .$$

Consider first obtaining a similar moment function: $\mathbb{E}_t^E \left[\left(\frac{\eta_{i,u}}{\eta_i} \right)^{\chi_i} \left(\frac{\eta_{j,u}}{\eta_j} \right)^{\chi_j} \left(\frac{\delta_{i,u}}{\delta_i} \right)^{\varepsilon_i} \left(\frac{\delta_{j,u}}{\delta_j} \right)^{\varepsilon_j} \right]$.

We have:

$$
\begin{aligned}
\mathbb{E}_t^E \left[\left(\frac{\eta_{i,u}}{\eta_i} \right)^{\chi_i} \left(\frac{\eta_{j,u}}{\eta_j} \right)^{\chi_j} \left(\frac{\delta_{i,u}}{\delta_i} \right)^{\varepsilon_i} \left(\frac{\delta_{j,u}}{\delta_j} \right)^{\varepsilon_j} \right] &= \mathbb{E}_t^E \left[\left(\frac{\eta_{i,u}}{\eta_i} \right)^{\chi_i} \left(\frac{\delta_{j,u}}{\delta_j} \right)^{\varepsilon_j} \right] \mathbb{E}_t^E \left[\left(\frac{\eta_{j,u}}{\eta_j} \right)^{\chi_j} \left(\frac{\delta_{i,u}}{\delta_i} \right)^{\varepsilon_i} \right] \\
&= Q \left(\widehat{f}_j^E, \widehat{g}_j^i, t, u, \varepsilon_j, \chi_i \right) Q \left(\widehat{f}_j^E, \widehat{g}_j^i, t, u, \varepsilon_j, \chi_i \right)
\end{aligned}
$$

Therefore, we can split the required moment function into the product of two separate ones, one for each subset of independent state variables. Since the dynamics of the state variables in each of these groups are almost identical to those in Dumas *et al.* (2009), and since the object is also the same as in that paper, we obtain a very similar result:

$$Q \left(\widehat{f}_j^E, \widehat{g}_j^i, t, u, \varepsilon_j, \chi_i \right) = H_{\widehat{f}_j^E} \left(\widehat{f}_j^E, t, u, \varepsilon_j \right) H_{\widehat{g}_j^i} \left(\widehat{g}_j^i, t, u, \varepsilon_j, \chi_i \right)$$

with

$$
\begin{aligned}
H_{\widehat{f}_j^E} \left(\widehat{f}_j^E, t, u, \varepsilon_j \right) &= \exp \left\{ \varepsilon_j^2 K_1 (t, u) + \varepsilon_j \left[K_2 (t, u) + \widehat{f}_j^E K_3 (t, u) \right] \right\} \\
H_{\widehat{g}_j^i} \left(\widehat{g}_j^i, t, u, \varepsilon_j, \chi_i \right) &= \exp \left\{ B_1 (t, u, \chi_i) + \varepsilon_j^2 B_2 (t, u, \chi_i) + \varepsilon_j \widehat{g}_j^i B_3 (t, u, \chi_i) + \left(\widehat{g}_j^i \right)^2 B_4 (t, u, \chi_i) \right\}
\end{aligned}
$$

and:

$$
\begin{aligned}
K_1 (t, u) &= \left(\frac{\gamma^E}{\zeta} + \frac{1}{2} \sigma_\delta^2 + \frac{1}{2 \zeta^2} \left[\left(\frac{\gamma^E}{\sigma_\delta} \right)^2 + (\phi \sigma_f)^2 \right] \right) (u - t) + \frac{1 - e^{-2 \zeta (u - t)}}{4 \zeta^3} \left[\left(\frac{\gamma^E}{\sigma_\delta} \right)^2 + (\phi \sigma_f)^2 \right] \\
&\quad - \frac{1 - e^{-\zeta (u - t)}}{\zeta} \left(\frac{\gamma^E}{\zeta} + \frac{1}{\zeta^2} \left[\left(\frac{\gamma^E}{\sigma_\delta} \right)^2 + (\phi \sigma_f)^2 \right] \right) \\
K_2 (t, u) &= \left(\overline{f} - \frac{1}{2} \sigma_\delta^2 \right) (u - t) - \frac{1}{\zeta} \overline{f} \left[1 - e^{-\zeta (u - t)} \right] \\
K_3 (t, u) &= \frac{1}{\zeta} \left[1 - e^{-\zeta (u - t)} \right]
\end{aligned}
$$

$$B_1\left(t, u, \chi_i\right) = \frac{a}{2} \int_t^u B_4\left(t, \tau, \chi_i\right) d\tau$$

$$B_2\left(t, u, \chi_i\right) = \int_t^u B_3\left(t, u, \chi_i\right) \left[m + n e^{-\zeta(\tau-t)} + \frac{a}{4} B_3\left(t, u, \chi_i\right)\right] d\tau$$

$$B_3\left(t, u, \chi_i\right) = \frac{\sum_{i=1}^5 \vartheta_i e^{-\upsilon_i(u-t)}}{q + b + (q-b)\, e^{-2q(u-t)}}$$

$$B_4\left(t, u, \chi_i\right) = \frac{c\left(1 - e^{-2q(u-t)}\right)}{q + b + (q-b)\, e^{-2q(u-t)}}$$

where:

$$a = 2\left[\left(\frac{\gamma^E - \gamma^X}{\sigma_\delta}\right)^2 + (\phi\sigma_f)^2\right] \qquad l = \chi_i \frac{\gamma^E}{\zeta} \frac{1}{\sigma_\delta^2}$$

$$b = \zeta + \frac{\gamma^X}{\sigma_\delta^2} + \chi_i\left(\frac{\gamma^E - \gamma^X}{\sigma_\delta^2}\right) \qquad m = \gamma^E - \gamma^X + \frac{\gamma^E}{\zeta}\left(\frac{\gamma^E - \gamma^X}{\sigma_\delta^2}\right) + \frac{1}{\zeta}(\phi\sigma_f)^2$$

$$c = \frac{1}{2}\chi_i(\chi_i - 1)\frac{1}{\sigma_\delta^2} \qquad n = \frac{\gamma^E}{\zeta}\left(\frac{\gamma^E - \gamma^X}{\sigma_\delta^2}\right) - \frac{1}{\zeta}(\phi\sigma_f)^2$$

$$k = -\chi_i\left[1 + \frac{\gamma^E}{\zeta}\frac{1}{\sigma_\delta^2}\right] \qquad q = \sqrt{b^2 - ac}$$

and:

$$\upsilon_1 = 0 \qquad \vartheta_1 = \frac{2cm + k(b+q)}{q}$$

$$\upsilon_2 = 2q \qquad \vartheta_2 = \frac{2cm + k(b-q)}{q}$$

$$\upsilon_3 = \zeta \qquad \vartheta_3 = \frac{2cn + l(b+q)}{q-\zeta}$$

$$\upsilon_4 = 2q + \zeta \qquad \vartheta_4 = \frac{2cn + l(b-q)}{q+\zeta}$$

$$\upsilon_5 = q \qquad \vartheta_{5=} - (\vartheta_1 + \vartheta_2 + \vartheta_3 + \vartheta_4)$$

Using this solution we can write:

$$
\mathbb{E}_t^E \left[\left(\frac{\eta_{i,u}}{\eta_i} \right)^{\chi_i} \left(\frac{\eta_{j,u}}{\eta_j} \right)^{\chi_j} \left(\frac{\delta_{i,u}}{\delta_i} \right)^{\varepsilon_i} \left(\frac{\delta_{j,u}}{\delta_j} \right)^{\varepsilon_j} \right] = \mathbb{E}_t^E \left[\exp \left\{ \chi_j \ln \frac{\eta_{j,u}}{\eta_j} + \chi_i \ln \frac{\eta_{i,u}}{\eta_i} + \varepsilon_i \ln \frac{\delta_{i,u}}{\delta_i} + \varepsilon_j \ln \frac{\delta_{j,u}}{\delta_{j,t}} \right\} \right]
$$

$$
= \exp \left\{ B_1 \left(t, u, \chi_i \right) + \left(\widehat{g}_j^i \right)^2 B_4 \left(t, u, \chi_i \right) \right\}
$$

$$
\exp \left\{ B_1 \left(t, u, \chi_j \right) + \left(\widehat{g}_i^j \right)^2 B_4 \left(t, u, \chi_i \right) \right\}
$$

$$
\exp \left\{ \varepsilon_i \left[K_2 \left(t, u \right) + \widehat{f}_i^E K_3 \left(t, u \right) + \widehat{g}_i^j B_3 \left(t, u, \chi_j \right) \right] \quad \text{(A1)} \right.
$$

$$
\left. + \varepsilon_i^2 \left[K_1 \left(t, u \right) + B_2 \left(t, u, \chi_j \right) \right] \right\}
$$

$$
\exp \left\{ \varepsilon_j \left[K_2 \left(t, u \right) + \widehat{f}_j^E K_3 \left(t, u \right) + \widehat{g}_j^i B_3 \left(t, u, \chi_i \right) \right] \right.
$$

$$
\left. + \varepsilon_j^2 \left[K_1 \left(t, u \right) + B_2 \left(t, u, \chi_i \right) \right] \right\}
$$

The product of the last two exponentials in Equation (4) corresponds to the moment function of a bivariate normal distribution:

$$
\begin{pmatrix} \ln \frac{\delta_{i,u}}{\delta_i} \\ \ln \frac{\delta_{j,u}}{\delta_j} \end{pmatrix} \sim N \left(\mu_\delta \left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j \right), \Sigma_\delta \left(t, u, \chi_i, \chi_j \right) \right)
$$

where

$$
\mu_\delta \left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j \right) = \begin{bmatrix} K_2 \left(t, u \right) + \widehat{f}_i^E K_3 \left(t, u \right) + \widehat{g}_i^j B_3 \left(t, u, \chi_j \right) \\ K_2 \left(t, u \right) + \widehat{f}_j^E K_3 \left(t, u \right) + \widehat{g}_j^i B_3 \left(t, u, \chi_i \right) \end{bmatrix}
$$

$$
\Sigma_\delta \left(t, u, \chi_i, \chi_j \right) = \begin{bmatrix} 2 \left[K_1 \left(t, u \right) + B_2 \left(t, u, \chi_j \right) \right] & 0 \\ 0 & 2 \left[K_1 \left(t, u \right) + B_2 \left(t, u, \chi_i \right) \right] \end{bmatrix}
$$

From properties of normal distributions, it follows that the product of the last two exponentials in Equation (4) also corresponds to this alternative bivariate normal distribution:

$$
\begin{pmatrix} \ln \frac{\delta_{i,u}}{\delta_i} \\ \ln \frac{\delta_{j,u}}{\delta} - \ln \frac{\delta_{i,u}}{\delta_i} \end{pmatrix} \sim N \left(\widetilde{\mu}_\delta \left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j \right), \widetilde{\Sigma}_\delta \left(t, u, \chi_i, \chi_j \right) \right)
$$

where

$$\widetilde{\mu}_{\delta}\left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right) = \begin{bmatrix} K_2\left(t,u\right) + \widehat{f}_i^E K_3\left(t,u\right) + \widehat{g}_i^j B_3\left(t,u,\chi_j\right) \\ \left(\widehat{f}_j^E - \widehat{f}_i^E\right) K_3\left(t,u\right) + \widehat{g}_j^i B_3\left(t,u,\chi_i\right) - \widehat{g}_i^j B_3\left(t,u,\chi_j\right) \end{bmatrix}$$

$$\widetilde{\Sigma}_{\delta}\left(t,u,\chi_i,\chi_j\right) = \begin{bmatrix} 2\left[K_1\left(t,u\right) + B_2\left(t,u,\chi_j\right)\right] & -2\left[K_1\left(t,u\right) + B_2\left(t,u,\chi_j\right)\right] \\ -2\left[K_1\left(t,u\right) + B_2\left(t,u,\chi_j\right)\right] & 4K_1\left(t,u\right) + 2\left[B_2\left(t,u,\chi_i\right) + B_2\left(t,u,\chi_j\right)\right] \end{bmatrix}$$

Therefore we obtain that the moment function given in Equation (4) is equivalent to:

$$\begin{aligned}
& \mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_{i,u}}\bigg/\frac{\delta_j}{\delta_i}\right)^{v}\right] \\
=\ & \mathbb{E}_t^E\left[\exp\left\{\chi_j\ln\frac{\eta_{j,u}}{\eta_j} + \chi_i\ln\frac{\eta_{i,u}}{\eta_i} + \varepsilon_i\ln\frac{\delta_{i,u}}{\delta_i} + v\ln\left(\frac{\delta_{j,u}}{\delta_j} - \ln\frac{\delta_{i,u}}{\delta_i}\right)\right\}\right] \\
=\ & H\left(\widehat{g}_i^B, t, u, \chi_i\right) \times H\left(\widehat{g}_j^A, t, u, \chi_j\right) \times J\left(\widehat{f}_i^E, \widehat{g}_i^j, t, u, \varepsilon_i, \chi_j\right) \\
& \exp\left\{v\left[\mu_y\left(\widehat{f}_i^E, \widehat{f}_j^E, \widehat{g}_i^j, \widehat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right)\right] + v^2\sigma_y^2\left(t, u, \chi_i, \chi_j\right)\right\}
\end{aligned}$$

where the functions H, J, μ_y and σ_y^2 are given in the proposition.

In order to use this moment function to compute the required expectation, we integrate over the log output ratio conditional normal distribution, which is implicit in this moment function. By doing so, we obtain the expression in the Proposition.

REFERENCES

Acemoglu D., V. Chernozhukov, and M. Yildi, 2009, "Fragility of Asymptotic Agreement under Bayesian Learning", MIT working paper

Agmon T., 1973, "Country Risk: The Significance of the Country Factor for the Share Price Movements in the United Kingdom, Germany and Japan", *Journal of Business*, Vol. 46, No. 1, pp. 24-32

Ahearne A., W. Griever, and F. Warnock, 2004, "Information Costs and Home Bias: An Analysis of US Holdings of Foreign Equities", *Journal of International Economics*, Vol. 62, No. 2, pp. 313-336

Banerjee, S., 2010, "Learning from Prices and the Dispersion in Beliefs," working paper, Kellogg School of Management.

Baxter M. and M. Crucini, 1995, "Business Cycles and the Asset Structure of Foreign Trade", *International Economic Review*, Vol. 36, No. 4, pp. 821-54

Baxter M. and U. Jermann, 1997, "The International Diversification Puzzle Is Worse Than You Think", *American Economic Review*, Vol. 87, No. 1, pp. 170-180

Bekaert, G., R. J. Hodrick and X. Zhang, 2009, "International Stock Return Comovements", *Journal of Finance*, Vol. 64, No. 6, pp. 2591–2626

Brennan M., 1998, "The Role of Learning in Dynamic Portfolio Decisions", *European Finance Review*, Vol. 1, No. 3, pp. 295-306

Brennan M. and H. Cao, 1997, "International Portfolio Investment Flows", *Journal of Finance*, Vol. 52, No. 5, pp. 1851-80

Brennan M., H. Cao, N. Strong, and X. Xu, 2005, "The dynamics of international equity market expectations", *Journal of Financial Economics*, Vol. 77, No. 2, pp. 257-288

Brennan, M., and Y. Xia, 2001, "Stock Price Volatility and Equity Premium," *Journal of Monetary Economics, 47,249-83.*

Brooks R. and M. Del Negro, 2004, "A Latent Factor Model with Global, Country, and Industry Shocks for International Stock Returns", Federal Reserve Bank of Atlanta Working Paper 2002-23b.

Brooks R. and M. Del Negro, 2006, "Firm-Level Evidence on International Stock Market Co-movement", *Review of Finance*, Vol. 10, No. 1, pp. 69-98

Cavaglia S., C. Brightman and M. Aked, 2000, "The Increasing Importance of Industry Factors", *Financial Analysts Journal*, Vol. 56, No. 5, pp. 41-54

Cavaglia S. and V. Moroz, 2002, "Cross-Industry, Cross-Country Allocation", *Financial Analysts Journal*, Vol. 58, No. 6, pp. 78-98

Cecchetti S., P. Lam and N. Mark, 2000, "Asset pricing with distorted beliefs: Are equity returns too good to be true?", *American Economic Review*, Vol. 90, No. 4, pp. 787–805

Coffee J., 1999, "The future as history: the prospects for global convergence in corporate governance and its implications", *Northwestern University Law Review*, Vol. 93, No.3, pp.641-708

Coffee J., 2002, "Racing towards the top? The impact of cross-listings and stock market competition on international corporate governance", *Columbia Law Review* Vol. 102, No. 7, pp.1757–1831

Dong, X., 2009, "Informational vs. Risk-sharing Trade in Foreign and Domestic Stocks: A New Perspective of Home Bias," working paper, Caroll School of Management, Boston College and INSEAD.

Doskeland, T. M. and H. K. Hvide, 2009, "Do Individual Investors Have Asymmetric Information Based on Work Experience?" working paper, Norwegian School of Economics and Business Administration, forthcoming *Journal of Finance.*

Dumas B., C. Harvey and P. Ruiz, 2003, "Are Correlations in International Stock Returns Justified by Subsequent Changes in National Outputs?" *Journal of International Money and Finance*, Vol. 22, No. 6, pp. 777-811

Dumas B., A. Kurshev and R. Uppal, 2009, "Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility", *Journal of Finance*, Vol. 64, No. 2, pp. 579 - 629

Gehrig, T., 1993, "An Information Based Explanation of the Domestic Bias in International Equity Investment" *Scandinavian Journal of Economics*, Vol. 95, No. 1, pp. 97-109

Grinblatt M. and M. Keloharju, 2000, "The investment behavior and performance of various investor types: A study of Finland's unique data set", *Journal of Financial Economics*, Vol. 55, No. 1, pp. 43-67

Hail L. and C. Leuz, 2009, "Cost of Capital Effects and Changes in Growth Expectations around U.S. Cross-Listings", *Journal of Financial Economics*, Vol. 93, No. 3, pp. 428-454

Harris M., and A. Raviv, 1993, "Differences of opinion make a horse race", *Review of Financial Studies*, Vol. 6, No. 3, pp. 473-506

Hau H., 2001, "Geographic Patterns of Trading Profitability in Xetra" *European Economic Review*, Vol. 45, No. 4-6, pp. 757-769

Heston S. and G. Rouwenhorst, 1994, "Does Industrial Structure Explain the Benefits of International Diversification?" *Journal of Financial Economics*, Vol. 36, No. 1, pp. 3-27

Kandel E., and N. D. Pearson, 1995, "Differential interpretation of public signals and trade in speculative markets", *Journal of Political Economy*, Vol. 103, No. 4, pp. 831-872

King, M. R. and D. Segal, 2003, "Corporate Governance, International Cross Listing and Home Bias," working paper, Rotman School of Management, University of Toronto.

Karolyi G, 2006, "The World of Cross-Listings and Cross-Listings of the World: Challenging Conventional Wisdom" *Review of Finance*, Vol. 10, No. 1, pp. 99-152

Lessard D., 1976, "World, Country, and Industry Relationships in Equity Returns" *Financial Analysts Journal*, Vol. 32, No. 1, pp. 32-38

Lewis K., 1996, "What Can Explain the Apparent Lack of International Consumption Risk-Sharing?", *Journal of Political Economy*, Vol. 104, No. 2, pp. 267-297

Lewis K., 1999, "Trying to Explain Home Bias in Equities and Consumption" *Journal of Economic Literature*, Vol 37, No. 2, pp. 571-608

Lewis K., 2000, "Why Do Stocks and Consumption Imply Such Different Gains from International Risk Sharing?" *Journal of International Economics*, Vol. 52, No. 1, pp. 1-35.

Lipster R. and A. Shiryaev. (2000). Statistics and Random Processes II: Applications. Berlin: Springer-Verlag.

Loewenstein, M., and G. Willard, 2006, "The limits of investor behavior", *Journal of Finance*, Vol. 61, No. 1, pp. 231-258

Merton R., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information", *Journal of Finance*, Vol. 42, No. 3, pp. 483-510

Morris S., 1995, "The common prior assumption in economic theory", *Economics and Philosophy*, Vol. 11, pp. 227-253

Pesenti P. and E. Van Wincoop, 2002, "Can Nontradables Generate Substantial Home Bias?" *Journal of Money, Credit, and Banking*, Vol. 34, No.1, pp. 25-50

Rubinstein A., 1993, "On Price Recognition and Computational Complexity in a Monopolistic Model", *Journal of Political Economy*, Vol. 101, No. 3, pp. 473-484

Sarkissian S. and M. Schill, 2009, "Are There Permanent Valuation Gains to Overseas Listing?", *Review of Financial Studies*, Vol. 22, No. 1, pp. 371-412

Scheinkman J. and W. Xiong, 2003, "Overconfidence and speculative bubbles", *Journal of Political Economy*, Vol. 111, No. 6, pp. 1183-1219

Stockman A. and L. Tesar, 1995, "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements", *American Economic Review*, Vol. 85, No.1, pp. 168-85

Van Nieuwerburgh S. and L. Veldkamp, 2009, "Information acquisition and under-diversification", *Review of Economic Studies*, Vol. 77, No. 2, pp. 779 - 805.

Xiong W. and H. Yan, 2010, "Heterogeneous Expectations and Bond Markets", *Review of Financial Studies*, Vol. 23, No. 4, pp. 1433-1466

Xiouros, C., 2010, "Differences of Opinion and the Price Volume Relation," working paper, BI Norwegian School of Management.

Yan, H. "Natural Selection in Financial Markets: Does It Work?" *Management Science*, 2008, Vol. 54, No 11, pp. 1935-1950.