Relative Wealth Concerns and Executive Compensation *

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Abstract

Empirical studies find that relative wealth concerns (RWCs) affect CEO compensation. In this paper, I incorporate RWCs into a standard principal-agent model and study the implications on CEO compensation. I first study the case in which a CEO's effort increases firm value without changing firm risk. In this case, RWCs will result in an increase in CEO incentives. This effect is larger if aggregate risk is higher, so RWCs can lead to a positive relation between CEO incentives and aggregate risk. CEOs with RWCs willingly risk exposure to aggregate shock to keep up with their peers. This help to reduce risk premium paid to the CEOs. As a result, RWCs can be beneficial to shareholders' payoffs. I also provide a simple explanation for the pay-for-luck puzzle. I next examine the case in which the CEO's effort affects both the mean and variance of firm value. I show that RWCs provide incentives for a CEO to take risk; thus if a firm's compensation policy has a large influence on other CEOs' risk-taking actions, then low (risk-taking) incentives can be optimal. Lastly, I show that RWCs render options preferable to stock where aggregate risk is much larger than idiosyncratic risk.

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1. Introduction

While standard economic models usually assume that an agent's utility is derived from the absolute level of her own consumption, economists have long believed that relative consumption effects are important (i.e., one's evaluation of her consumption depends on how much the other people are consuming, Veblen (1899)). A growing body of literature incorporates such effects into asset pricing models (Abel (1990), Constantinides (1990), Galí (1994), and Campbell and Cochrane (1999)). Such relative consumption effects, however, are not extensively studied in corporate finance. In this paper, I incorporate relative wealth concerns (RWCs) into a standard principal-agent model and study how CEO compensation is affected by such relative consumption effects.

Some empirical evidence already suggests that RWCs matter in CEO compensation. Bouwman (2011), for example, finds that CEO compensation depends on the compensation of geographicallyclose CEOs. An examination of the relation between geography and executive compensation shows that the results are most consistent with RWCs. Shue (2011) documents the phenomenon of "pay for friend's luck": pay responds to lucky industry-level shocks to the compensation of peers in distant industries. Therefore, peer effects in compensation are not driven by similarities in underlying managerial productivity; instead, RWCs might be the driving forces. Bereskin and Cicero (2011) study developments in Delaware caselaw in the mid-1990s and find that firms not directly impacted by legal changes increased their CEOs' compensation when the legal changes directly affected a substantial number of firms in their industry. RWCs could be a potential explanation.¹ If RWCs play a role in CEO compensation, then it is not surprising to see that the level of CEO pay is affected by peer compensation, e.g., Bouwman (2011), Bizjak, Lemmon, and Naveen (2008), Faulkender and Yang (2010) find that the level of CEO pay increases in the average level of CEO pay in a reference group.² Although the correlation among the level of CEO pay has been examined, the interaction among CEO incentives has not been studied empirically. In this paper, I will focus on the RWCs' effects on CEO incentives. I derive the closed-form solutions in the model. The model's tractability enables me to obtain a number of clear economic effects on CEO incentives and explain their

¹There are also some other explanations; in the paper, Bereskin and Cicero point to market competition as an explanation for their results.

²Note that the positive correlation in the level of CEO pay might be caused by many factors (e.g., competition, living costs, a common shock to managerial productivity, RWCs). Among these three papers, only Bouwman (2011) shows that her results are most consistent with RWCs.

intuitions transparently.

First, I study the case in which a CEO's effort increases firm value without changing firm risk. I set up the model with a continuum of firms and CEOs. I assume that a CEO's utility function exhibits a consumption externality. In particular, I define that a CEO's RWCs are given by the product of a parameter measuring her concerns and the sum of all the CEOs' compensation (see Definition 1 in the model). A CEO's utility is not determined by the absolute level of her compensation, but depends on the difference between her compensation and her RWCs. The participation constraint will require a premium for the CEO's RWCs, i.e., the CEO desires to have higher compensation when her peers' compensation is high. Therefore, a CEO's contract must achieve two goals: one is to induce the CEO to work; the other is to make the CEO satisfied with her contract when she compares herself to her peers. The contract has to pay a premium for the CEO's RWCs, which I refer to as a "RWCs' premium".

By granting CEOs equity to induce them to work, CEO pay is positively correlated with aggregate shock (i.e., the state of the whole economy); thus, their RWCs also depend on the aggregate shock. Because CEOs are risk-averse, it is less costly to pay the RWCs' premium by using a type of payment linked to the aggregate shock rather than cash; stock, then, is preferred to cash. So RWCs lead to an increase in CEO incentives. Since a CEO's RWCs also depend on how many incentives the other CEOs are granted, the model predicts that individual CEO incentives increase with an increase in aggregate CEO incentives. If aggregate risk is higher, then a CEO will face a higher risk to fall behind other CEOs; thus, stock compensation becomes more attractive than cash. Therefore, RWCs can result in a positive relation between CEO incentives and aggregate risk. On the other hand, according to the informativeness principle, firms should use fewer incentives when firm risk (which is the sum of aggregate risk and idiosyncratic risk) is higher, because the observable outcome (i.e., stock price) becomes less informative. Due to the two conflicting effects, the relation between CEO incentives and aggregate risk can be negative or positive, depending on the coefficient of a CEO's RWCs on aggregate shock. Empirical studies have found mixed evidence as to the relation between CEO incentives and firm risk. Many of them document no significant relation, or even a positive relation between incentives and firm risk (Prendergast (2002)), which is inconsistent with a standard principal-agent model. My model predicts that if the coefficient of a CEO's RWCs on aggregate shock is large, then there is a positive relation between incentives and

aggregate risk; otherwise, the relation remains negative.

The optimal CEO incentives depend on individual RWCs, which depend on other CEOs' incentives. In equilibrium, each firm will choose its contract optimally given other firms' contracts. I show that because each individual CEO incentives depend on the aggregate CEO incentives linearly, this linearity leads to a unique equilibrium. In addition, notice that in the presence of RWCs, a CEO is willing to risk exposure to aggregate shock to keep up with her peers to some extent: the CEO becomes less risk-averse due to her RWCs. Thus RWCs help to reduce the risk premium paid to the CEO. In other words, the presence of RWCs allows shareholders to grant a CEO more incentives to induce a higher effort, but without paying a higher risk premium. In particular, if a CEO's productivity is high and other CEOs' compensation is low, then choosing a CEO with RWCs is beneficial to shareholders' payoffs. In reality, however, shareholders may not be able to observe a CEO's RWCs, I also study the case where the CEO's RWCs are not observable. In this case with asymmetric information, a CEO can extract an information rent, i.e., the CEO can receive a contract which makes her utility above the reservation utility. I show that for each firm, a pooling on CEO types – giving the same contract to CEOs of different types – may exist, which could help to explain the practice of benchmark pay (Bizjak, Lemmon and Naveen (2008), Faulkender and Yang (2010)). Since CEOs' incentives depend on each other, and the CEO's type at the reservation utility also depends on other CEOs' compensation, I find that multiple equilibria can arise with asymmetric information.

Lastly, I show that if a CEO is also concerned about her friends or neighbors – entrepreneurs, investment bankers, and private equity investors for example – whose income is correlated with some lucky shock (market shock for example), then firms should compensate the CEO by a means that will correlate with that shock. Thus the model provides a simple explanation for the pay-for-luck puzzle documented by Bertrand and Mullainathan (2001).³

I then turn to study the case in which a CEO's effort affects not only the mean but also the variance of firm value. To improve firm value, a CEO may need to undertake risky projects; these decisions also increase the variance of firm value. By combining risk-taking efforts and RWCs, the model generates some new predictions on executive compensation. Because a CEO's effort also increases the variance of firm value and the CEO is risk-averse, higher incentives do not

³Gopalan, Milbourn and Song (2010), Noe and Rebello (2008), and Oyer (2004) provide alternative explanations.

always lead to a higher effort. As I demonstrate that an incentive-threshold exists such that the CEO's effort decreases when her incentives exceeds the threshold. If the CEO lacks RWCs, at the optimum, her effort is increasing in her incentives. However, if the CEO has RWCs, shareholders may have to grant the CEO incentives above the threshold to pay the RWCs' premium. Thus, in the presence of RWCs, at the optimum, her effort can be decreasing in her incentives. Because of this non-monotonicity between efforts and incentives, and because CEO incentives depend on peer incentives, I show that multiple equilibria could exist, such that in one equilibrium, CEOs' efforts increase in their incentives; in another equilibrium, CEOs' efforts decrease in their incentives.

So far in the analysis, I have assumed that there is a continuum of firms and CEOs. Thus, because each firm is infinitesimal, the change of one firm's compensation policy does not affect other firms and CEOs. In Section 3.3, I consider the case in which a firm's compensation policy has a large influence on other CEOs' effort decisions. A CEO's risk-taking effort is affected by her RWCs, which are, in turn, affected by other CEOs' compensation. If a firm's compensation policy affects other CEOs significantly, then high CEO incentives can increase the CEO's RWCs by increasing other CEOs' efforts, which, in turn, will increase the target CEO's compensation through participation constraint. Therefore, shareholders will prefer to limit CEO incentives at a lower level, if their executive compensation policy has a significant impact on other CEOs' effort decisions. This result helps to explain low CEO incentives documented by Jensen and Murphy (1990).

So far I have confined the discussions to linear contracts for tractability.⁴ In practice, however, options are widely used, I extended the model to account for non-linear contracts. In particular, I compare stock to options in the presence of RWCs in a simple example. I establish that RWCs have two effects when comparing stock to options. First, the effect of RWCs on incentivizing CEO effort is stronger for options than stock. This is because if the CEO chooses to shirk, stock still remains exposed to aggregate shock; in contrast, options are more likely to expire with value zero making them completely insensitive to the shock. But, as previously explained, in the presence of RWCs, CEOs willingly risk exposure to aggregate shock because it helps them to keep up with their peers. Thus the effect of RWCs on incentivizing a CEO's effort is larger for an option-based contract.

⁴Many papers specify the form of contracts for tractability (e.g., Holmstrom and Tirole (1993), Jin (2002), Oyer (2004), and Bolton, Scheinkman and Xiong (2006)).

Second, because RWCs make CEOs willing to risk exposure to aggregate shock, they can help to reduce the risk premium associated with aggregate risk. Notice that stock is fully exposed to risk, but options are only partially exposed to risk; thus, RWCs reduce the risk premium more for stock than options. I show that when aggregate risk is much larger than idiosyncratic risk, the first effect dominates the second one such that options become more preferable. Where idiosyncratic risk is not too small compared with aggregate risk, however, stock will be more efficient.

This paper is related to several strands of literature. First, it is related to the literature that examines RWCs' effects. This has been extensively studied in asset pricing models. For example, relative consumption effects are used to examine equity premium puzzle (Abel (1990), Constantinides (1990), Galí (1994), Campbell and Cochrane (1999)). García and Strobl (2011) study the complementarities in information acquisition using RWCs. DeMarzo, Kaniel and Kremer (2008) show how RWCs can result in financial bubbles. Roussanov (2010) explains the under-diversification puzzle by emphasizing the desire to "get ahead of the Joneses". In corporate finance, Goel and Thakor (2010) explore merger waves using envy-based preferences. The concerns for "equity" are applied to study the dynamics of workers' wages (Cabrales, Calvó-Armengol and Pavoni (2007)). This paper introduces RWCs into a standard principal-agent model and examines how CEO incentives will be affected. Miglietta (2010) finds that RWCs make agents prefer positively correlated payoffs. Using a case involving one principal and two agents, Miglietta (2008) explains the use of firm-wide incentive contracts to employees who do not seem to need any incentive. In this paper, I focus on the interactions between shareholders and a CEO within a firm, as well as interactions among firms. I derive the optimal contract for each firm and also show that RWCs may sometimes be beneficial to shareholders' payoffs. The case in which a CEO's effort affects both the mean and variance of firm value is also discussed.

Second, my paper is related to the literature on executive compensation. By incorporating RWCs into the model, I show that many features in CEO compensation can be rationalized. The standard principal-agent model predicts a negative relation between incentives and risk. However, many empirical studies find no significant relation, or even a positive relation between incentives and firm risk (Prendergast (2002)). RWCs can explain this contradiction: the relation between incentives incentives and aggregate risk can be either negative or positive, depending on the coefficient of a CEO's RWCs on aggregate shock. Bertrand and Mullainathan (2001) document that CEOs are

paid for market upswings that are beyond their control, a so-called "pay-for-luck" puzzle. Gopalan, Milbourn and Song (2010) argue that if a CEO can exert an effort to explore future market conditions and decide the firm's exposure to the market risk, then it is necessary to tie CEO pay to luck. In the model, I provide a simple explanation by showing that if a CEO compares her compensation with that of friends or neighbors, especially those whose income is correlated with lucky shock, then it is optimal to compensate the CEO by a means that is also correlated with the shock. Jensen and Murphy (1990) document that a CEO loses only \$3.25 for every \$1000 decline in firm value. I provide a rationale for this low level of CEO incentives by studying RWCs' effects on a CEO's risk-taking actions. I show that RWCs incentivize a CEO to exert more risk-taking efforts. Thus, if a firm's executive compensation policy has a significant impact on other CEOs' effort decisions, then high CEO incentives can increase the CEO's RWCs by increasing other CEOs' efforts. This, in turn, will increase the target CEO's compensation through participation constraint. So shareholders will be willing to keep CEO incentives at a low level to avoid a high premium for the CEO's RWCs. Dittmann and Maug (2007) calibrate a standard principal-agent model and find that options should not be used. In contrast, I provide a rationale for the use of options by demonstrating that the incentives provided by RWCs to induce efforts are stronger for options than stock, and this effect will be dominant if aggregate risk is much larger than idiosyncratic risk.

The remainder of the paper is organized as follows. Section 2 studies the case in which a CEO's effort increases firm value without changing firm risk. Section 3 studies the case in which a CEO's effort affects both the mean and variance of firm value. Section 4 extends to non-linear contracts and compares stock to options. Section 5 discusses empirical implications. Section 6 concludes. The Appendix contains proofs.

2. Efforts Only Affect the Mean of Firm Value

2.1 Model

Suppose there is a continuum of firms and CEOs: $i \in [0, 1]$. CEO *i* is assigned to firm i.⁵ For each firm *i*, consider the following principal-agent model: an all-equity firm is owned by risk-neutral

⁵Here, I do not study the problem of matching firms and CEOs. Gabaix and Landier (2008) and Edmans and Gabaix (2011) present competitive assignment models of the managerial labor market.

shareholders and managed by a risk-averse CEO. At t = 0, the CEO is offered a contract. At t = 1, she exerts an effort a_i , which is unobservable. The firm's terminal value at time t = 2 is given by

$$V_i = \pi_i a_i + \tilde{m} + \eta_i,\tag{1}$$

where \tilde{m} is the aggregate shock with mean 0 and variance σ_m^2 ; η_i is firm *i*'s idiosyncratic shock with mean 0 and variance $\sigma_{\eta i}^2$; \tilde{m} and $(\eta_i)_{i \in [0,1]}$ are independent of each other. I assume that $(\sigma_{\eta i}^2)_{i \in [0,1]}$ are uniformly bounded, i.e., there exists σ_{\max} such that $\sigma_{\eta i} \leq \sigma_{\max}$ for each $i \in [0,1]$. $\pi_i > 0$ is the measure of CEO *i*'s productivity.

For tractability, I confine the current discussion to linear contracts which consist of a base salary α_i^6 and $\beta_i \leq 1$ shares of stock. Then CEO *i*'s payoff at time t = 2 is

$$w_i = \alpha_i + \beta_i V_i = \alpha_i + \beta_i (\pi_i a_i + \tilde{m} + \eta_i).$$

At time t = 1, CEO *i* exerts an effort a_i to maximize her expected utility. I introduce RWCs into a CEO's utility function and assume that a CEO's utility is not determined by the absolute level of her wage, but depends on the difference between her wage and a portion of other CEOs' wage. Specifically, I assume that CEO *i*'s utility is given by

$$U_i = E[u_i(a_i), H_i] = E[w_i - H_i] - \frac{1}{2}\lambda_i Var[w_i - H_i] - \frac{1}{2}a_i^2,$$
⁷ (2)

where λ_i is the measure of CEO *i*'s risk-aversion; $\frac{1}{2}a_i^2$ is the cost of her effort. The novel part in the utility function is the incorporation of RWCs H_i . I define the CEO's RWCs as follows:

Definition 1. A CEO's RWCs are the product of a parameter measuring the CEO's concerns about other CEOs' pay and the sum of all the CEO pay. Specifically, for CEO i, her RWCs are denoted by

$$H_i = r_i \int_0^1 w_k dk$$

where $r_i \geq 0$ is a parameter measuring CEO i's concerns about other CEOs' pay. A larger r_i means

⁶I do not put restrictions on α_i . In Proposition 1, we will see that α_i depends on the CEO's reservation utility \underline{u}_i . So α_i can be negative if \underline{u}_i is low.

⁷In fact, this is the certainty-equivalent of exponential utility function $E\left[-e^{-\lambda_i(w_i-H_i-\frac{1}{2}a_i^2)}\right]$ if all the shocks are normally distributed.

that CEO i cares more about other CEOs' compensation. $\int_0^1 w_k dk$ is the sum of all the CEO pay.

The RWCs H_i can be simplified to

$$H_i = r_i (W + M\tilde{m}),^8 \tag{3}$$

where $W = \int_0^1 (\alpha_k + \beta_k \pi_k a_k) dk = r_i \int_0^1 E[w_k] dk$ is the sum of the expected CEO pay, and $M = \int_0^1 \beta_k dk = \int_0^1 \frac{\partial w_k}{\partial \tilde{m}} dk$ is the sum of the sensitivity of CEO pay to aggregate shock (i.e., aggregate CEO incentives in this case).

Shareholders will choose a contract to maximize the expected firm value net of CEO pay. For brevity, I drop the subscript i. For each firm, we have the following principal-agent problem:

Program 1.

$$\max_{\alpha,\beta} E[V-w] \ge 0^9$$

such that

IC:
$$a \in \arg \max_{\hat{a}} E[u(\hat{a}), H],$$

IR: $E[u(a), H] \ge \underline{u},$

where \underline{u} is the CEO's reservation utility.

Proposition 1.

1) Given a linear contract $w = \alpha + \beta V$, the optimal effort taken by the CEO is $a = \pi \beta$, thus $\frac{\partial a}{\partial \beta} > 0$, i.e., the CEO's effort is always increasing in incentives.

2) If $rM\sigma_m^2 \ge \sigma_m^2 + \sigma_\eta^2$, then $\beta^* = 1$. In this case, we have a corner solution, i.e., it is better for shareholders to sell the firm to the CEO.

If $rM\sigma_m^2 < \sigma_m^2 + \sigma_\eta^2$, then we have an interior solution. In this case, the optimal contract (α^*, β^*) is

$$\begin{cases} \alpha^* = \frac{(\pi^2 - \lambda r M \sigma_m^2)(\pi^2 + \lambda r M \sigma_m^2)}{2[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]} - \frac{\pi^2 (\pi^2 + \lambda r M \sigma_m^2)^2}{[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]^2} + rW + \frac{1}{2}\lambda r^2 M^2 \sigma_m^2 + \underline{u} \\ \beta^* = \frac{\pi^2 + \lambda r M \sigma_m^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}. \end{cases}$$

⁸See Appendix for the derivation.

⁹If the shareholders' expected payoff is negative, I assume that shareholders will not hire the CEO and all the agents get zero payoff in expectation. See Section 2.2 for discussions.

2i) β^* is increasing in π , r and M; decreasing in λ and σ_{η}^2 .

2*ii)* The relation between β^* and σ_m^2 is ambiguous. If $r \left[\frac{\partial M}{\partial \sigma_m^2} \sigma_m^2 + \left(1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)} \right) M \right] > \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}$, then β^* is increasing in σ_m^2 ; if $r \left[\frac{\partial M}{\partial \sigma_m^2} \sigma_m^2 + \left(1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)} \right) M \right] < \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}$, then β^* is decreasing in σ_m^2 .

2iii) The expected CEO pay is

$$E[w] = \frac{\pi^4}{2[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)]} + rW + \frac{1}{2}\lambda r^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}\right] + \underline{u},$$

which is increasing in r, W and M.

Now we discuss the intuitions behind Proposition 1. In part 1), since a CEO's effort only increases the mean of firm value, but has no effect on the variance, giving a CEO higher incentives will always induce her to exert a higher effort. Note that a CEO's effort decision is determined by her productivity of effort and her cost of effort; at the optimum, the marginal benefit of effort for the CEO equals to the marginal cost of effort. Also note that the other CEOs' contracts and efforts do not affect either the productivity or the cost of effort of the target CEO, so the target CEO's effort is not affected by other CEOs' contracts or efforts, even if she cares about other CEOs' compensation.¹⁰

In part 2i), the relations between β^* and π , λ , σ_{η}^2 are standard: a higher π (which is the measure of a CEO's productivity) means that the benefit of a CEO's effort is greater, thus shareholders would like to grant a CEO more incentives to induce a higher effort when π increases. Since the CEO is risk-averse, a rise in λ or σ_{η}^2 makes it more costly to induce a CEO's effort, so β^* is decreasing in λ and σ_{η}^2 . The novel results of part 2i) are the relations between β^* and r and M. In the presence of RWCs, a CEO's contract plays two roles: one is to induce the CEO to work; the other is to make the CEO satisfied with her contract when she compares it to those of her peers, i.e., the contract has to pay a "RWCs' premium" to the CEO. Since CEOs are risk-averse, note that the RWCs are positively correlated with aggregate shock, thus it is less costly for shareholders to make a payment that is related to the shock, rather than pay cash to the CEO. In other words, stock compensation is better. Thus as r or M goes up, CEO incentives will rise as well. Recall that M is the sum of the sensitivity of CEO pay to aggregate shock (aggregate CEO incentives, in this case),

 $^{^{10}}$ In Section 3.1, I will show that RWCs will affect a CEO's effort when her effort also affects the variance of firm value.

hence the model predicts that individual CEO incentives increase when aggregate CEO incentives increase. Similarly, since participation constraint requires a premium for RWCs, part 2iii) shows that the expected CEO pay is increasing in W (aggregate expected CEO pay) and M (aggregate CEO incentives). Bouwman (2011), Bizjak, Lemmon, and Naveen (2008), and Faulkender and Yang (2010) find that the level of CEO pay increases in the average level of CEO pay in a reference group,¹¹ which is consistent with the model. The model also predicts a positive relation between individual CEO incentives and aggregate CEO incentives, an interesting result to be tested in the future.

Part 2ii) shows that the relation between incentives and aggregate risk depends on a CEO's RWCs. In the benchmark case with no RWCs (r = 0), as aggregate risk goes up, it becomes more difficult to distinguish a CEO's effort from observable outcomes. According to the informativeness principle, a firm should use fewer incentives, thus the optimal incentives are decreasing in aggregate risk. This is consistent with the standard principal-agent model. When a CEO has RWCs (r > 0), however, there is another effect that affects the relation between incentives and aggregate risk. As explained above, it is less costly to pay the RWCs' premium by using stock compensation than using cash. Thus, when aggregate risk goes up, stock compensation becomes more attractive than cash compensation. Shareholders are more willing to use stock to pay the RWCs' premium as aggregate risk increases. Therefore, when r is large, RWCs' effects are dominant, which leads to a positive relation between incentives and aggregate risk. This result can be used to explain the mixed empirical evidence on the relation between CEO incentives and firm risk. The standard principalagent model predicts that there should be a negative relationship between CEO incentives and firm risk. Yet, some empirical studies show that there is no significant relationship, or even a positive relationship between incentives and firm risk (Prendergast (2002)). By incorporating a CEO's RWCs into the model, although the relation between CEO incentives and a firm's idiosyncratic risk is still negative, the relation between CEO incentives and aggregate risk can be positive.¹² Lastly, part 2iii) predicts that the expected CEO pay increases with aggregate expected CEO pay and aggregate CEO incentives. This result is intuitive: the participation constraint requires a premium

¹¹Bouwman (2011) use geographically-close firms as a reference group; Bizjak, Lemmon, and Naveen (2008), Faulkender and Yang (2010) use similar-sized firms in the same industry as a reference group.

 $^{^{12}}$ Jin (2002) obtains the same result under the assumption that a CEO is risk-averse and shareholders are riskneutral to a firm's idiosyncratic risk but risk-averse to aggregate risk.

for a CEO's RWCs, which depend on other CEOs' income. If aggregate expected CEO pay goes up, individual expected CEO pay goes up as well. The increase of aggregate CEO incentives makes each individual CEO face a higher risk to fall behind other CEOs' payoffs. This has to be compensated by increasing CEO pay.

2.2 Equilibrium

When a CEO has RWCs, the optimal CEO incentives will depend on other CEOs' incentives by Proposition 1. Thus, I define an equilibrium as follows:

Definition 2. A subgame perfect equilibrium is a set of functions $(\alpha_i, \beta_i, a_i)_{i \in [0,1]}$ that satisfy the following properties:

1) at time 0, for each firm *i*, given all the other firms' contracts $(\alpha_j, \beta_j)_{j \neq i}$, the contract (α_i, β_i) solves principal-agent problem optimally for firm *i*.

2) at time 1, for the CEO in firm i, given her contract and all the other firms' contracts, a_i maximizes CEO i's utility.

If a CEO's RWCs are too big, i.e., if r is large, then it becomes too costly to compensate the CEO. As a result, shareholders will decide not to hire the CEO, in which case all the agents get zero payoff. In the following lemma, I study the case in which each firm will hire its CEO. I provide a necessary and sufficient condition under which a unique (hiring) equilibrium exists.

Lemma 1. There is a unique equilibrium in which each firm will hire its CEO (and the optimal contract for each firm is given by Proposition 1), if and only if

$$\int_0^1 r_i di < 1, \quad and \tag{4}$$

$$r_i M \sigma_m^2 \le \sigma_m^2 + \sigma_{\eta i}^2 \text{ for each } i \in [0, 1], \text{ and}$$

$$\tag{5}$$

$$\frac{\pi_i^2(\pi_i^2 + \lambda_i r_i M \sigma_m^2)}{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta i}^2)} - E[w_i] \ge 0 \text{ for each } i \in [0, 1],$$

$$(6)$$

where
$$M = \frac{\int_0^1 \frac{\pi_i^2}{\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta_i}^2)} di}{1 - \int_0^1 \frac{\lambda_i r_i \sigma_m^2}{\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta_i}^2)} di} \text{ is aggregate CEO incentives;}$$
$$W = \frac{\int_0^1 \left[\frac{\pi_i^4}{2[\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta_i}^2)]} + \frac{1}{2}\lambda_i r_i^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda_i \sigma_m^2}{\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta_i}^2)}\right] + \underline{u}_i\right] di}{1 - \int_0^1 r_i di} \text{ is aggregate expected CEO pay;}$$

$$E[w_i] = \frac{\pi_i^4}{2[\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta i}^2)]} + r_i W + \frac{1}{2} \lambda_i r_i^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda_i \sigma_m^2}{\pi_i^2 + \lambda_i(\sigma_m^2 + \sigma_{\eta i}^2)} \right] + \underline{u}_i \text{ is the expected CEO pay for firm } i.$$

Note that (5) guarantees that the optimal incentives for each firm do not exceed 1, and (6) guarantees that the shareholders in each firm have nonnegative payoffs. From Proposition 1, it can be seen that a CEO's effort decision is determined only by her own contract, even if she cares about other CEOs' compensation. Also, a CEO's effort is always increasing in her own incentives linearly. As a result, although RWCs make a CEO's optimal incentives depend on the other CEOs' incentives, this dependence only results in a linear relation between individual CEO incentives and aggregate CEO incentives. By aggregating all the CEOs' incentives, we will eventually obtain unique aggregate CEO incentives, which in turn determine unique optimal CEO incentives for each firm. The equilibrium is therefore unique.

2.3 Are RWCs Good or Bad for Shareholders?

In this section, I explore the following question: if shareholders can observe the parameter r and choose a CEO from a pool of CEOs with a different r, then will they choose a CEO with r = 0 or a positive r? When a CEO has RWCs, it has two effects on shareholders' payoffs. On one hand, shareholders have to pay a premium for a CEO's RWCs due to the participation constraint; this will reduce shareholders' payoffs. On the other hand, shareholders' payoffs can also benefit from a CEO's RWCs for the following reason: in the presence of RWCs, a CEO willingly risks exposure to aggregate shock to keep up with her peers. Thus RWCs help to reduce the risk premium paid to the CEO. The risk premium paid to a CEO for the aggregate risk that she bears is not proportional to her owned shares, but is proportional to the difference between her owned shares and the coefficient of her RWCs on aggregate shock (which is $\beta - rM$). Hence, when r is positive, it allows shareholders to grant a CEO's productivity is high and other CEOs' compensation is low, then the second effect could dominate the first effect so that a positive r is optimal for shareholders.

Lemma 2. If

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$$\frac{\lambda M \sigma_m^2 \pi^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} - W > 0, \tag{7}$$

then the optimal r^* is positive, which is given by $r^* = \frac{\lambda M \sigma_m^2 \pi^2 - W[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)]}{\lambda M^2 \sigma_m^2(\pi^2 + \lambda \sigma_\eta^2)}$. The optimal r^* is increasing in π ; decreasing in W and σ_η^2 .

As we argued above, the presence of RWCs has two effects on shareholders' payoffs: first, it reduces the shareholders' payoffs, as the participation constraint requires a premium for a CEO's RWCs; second, it may also increase the shareholders' payoffs, since it allows shareholders to grant a CEO more incentives to induce a higher effort without paying a higher risk premium. If π (the productivity of a CEO's effort) is large, then it strengthens the second effect; therefore, the optimal r^* is increasing in π . If W (aggregate expected CEO pay) is large, then the first effect is stronger because it now requires a higher premium for CEOs' RWCs; thus, the optimal r^* is decreasing in W. The increase of a firm's idiosyncratic risk σ_{η}^2 reduces the optimal CEO incentives, which reduces the second effect; so the optimal r^* is decreasing in σ_{η}^2 .

2.4 Optimal CEO Incentives When CEOs' RWCs Are Unobservable

In reality, a CEO's RWCs may not be known by shareholders. If a CEO's RWCs are not observable, then how will they affect the optimal CEO incentives? Suppose that $r \in [0, r_{\text{max}}]$ is unobservable to shareholders. The shareholders' beliefs on r are represented by the probability density function f(r) with the associated cumulative distribution function F(r). I assume that the distribution function satisfies the relevant monotone hazard rate conditions: $\frac{F(r)}{f(r)}$ is increasing in r and $\frac{1-F(r)}{f(r)}$ is decreasing in r.¹³

The timeline of the model is: at t = 0, shareholders offer a menu of contracts to a CEO; at t = 1, the CEO chooses a contract and exerts an effort; at t = 2, all the agents' payoffs are realized. Note that when a CEO is choosing a contract and exerting an effort, both he and the shareholders do not know the other CEOs' types. As a result, the CEO and shareholders both face the uncertainty about the other CEOs' types when they are evaluating the contract and their utilities. In particular, the values of W (aggregate expected CEO pay) and M (aggregate CEO incentives) will affect a CEO's relative payoff. Given the distributions of r_i for $i \in [0, 1]$, each realization of $(r_i)_{i \in [0,1]}$ determines each CEO's choice of her contract, which determines the values of W and M. Thus the distributions of W and M depend on the distributions of $(r_i)_{i \in [0,1]}$.

¹³Many distribution functions satisfy this property, e.g., uniform, normal, logistic, exponential, and Laplace.

Assume that the distributions of $(r_i)_{i \in [0,1]}$ are independent of the distributions of aggregate shock \tilde{m} and idiosyncratic shocks $(\eta_i)_{i \in [0,1]}$, thus the distributions of W and M are independent of the distributions of aggregate shock \tilde{m} and idiosyncratic shocks $(\eta_i)_{i \in [0,1]}$. Let $F_W(x)$ and $F_M(x)$ denote the cumulative distribution functions for W and M, respectively.¹⁴

To distinguish CEOs of different types, shareholders offer a menu of contracts to a CEO. Each contract consists of a base salary and some shares of stock. Specifically, in response to a CEO's claimed RWCs \hat{r} , the contract specifies a base salary $\alpha(\hat{r})$ and $\beta(\hat{r})$ shares of stock. Let $U(\hat{r}|r)$ denote the CEO's expected utility, given that the CEO's type is r but he claims to be of type \hat{r} . Then the shareholders' objective is:

Program 2.

$$\max_{\alpha(r),\beta(r)} \int_0^{r_{\max}} E[V - w|r]f(r)dr$$

such that

IC:
$$U(r|r) \ge U(\hat{r}|r)$$
, for any \hat{r}, r
IR: $U(r|r) \ge \underline{u}$, for any $r, .$

Lemma 3. Given a menu of contracts $\{(\alpha(r), \beta(r))\}_{r \in [0, r_{\max}]}$, and suppose that $\beta(r)$ is differentiable, then the contracts satisfy the IC constraint, if and only if

1)

$$\frac{\partial \beta(r)}{\partial r} \ge 0;$$

2)

$$\frac{\partial \alpha(r)}{\partial r} = -\left\{ [\pi^2 - \lambda(\sigma_m^2 + \sigma_\eta^2)]\beta(r) + \lambda r E[M]\sigma_m^2 \right\} \frac{\partial \beta(r)}{\partial r}.$$

The coefficient of a CEO's RWCs on aggregate shock is rM. So to catch up with the other CEOs' exposure to aggregate shock, a CEO with a higher r is more willing to choose a contract with higher incentives. Thus to distinguish CEOs of different types, it is necessary to grant higher incentives to the CEO with a higher r.

Due to the asymmetric information between shareholders and a CEO, it is possible for a CEO

¹⁴Because there is a continuum of firms, it can be shown that by Kolmogorov's strong law of large numbers, M converges to E[M] and W converges to E[W] almost surely.

to receive a contract that gives her a utility above the reservation utility.¹⁵ I refer to $U(r|r) - \underline{u}$ as information rent. The optimal incentives will be a trade-off between the cost of information rent and the efficiency of incentives. A CEO with a higher r is more willing to risk exposure to aggregate shock because it helps her to keep up with her peers. Thus a CEO with a higher r will gain more from a high-incentives contract. In other words, $\Delta U = U(r + \delta r | r + \delta r) - U(r | r)$, the difference in utility between two near types, is increasing in incentives. Thus, if the CEO of type r_0 is at the reservation utility with her contract, then for $r < r_0$, information rent is decreasing in incentives $\beta(r)$; for $r > r_0$, information rent is increasing in incentives $\beta(r)$. Therefore, to lower the information rent, shareholders would prefer to grant high incentives to the CEOs of types below r_0 , and low incentives to the CEOs of types above r_0 . Further, because the IC constraint requires that incentives must be increasing in a CEO's type, there could be a pooling on CEOs' types around r_0 ; that is, the contracts are the same for the CEOs of types around r_0 . This result may help to explain why benchmark pay is so widely used in practice (Bizjak, Lemmon and Naveen (2008) and Faulkender and Yang (2010)). For firms with certain similarities, they may fall into the same region of pooling, so they will use other firms' contracts as a benchmark.

Proposition 2. Suppose $r_0 = \arg\min_{r \in [0, r_{\max}]} U(r|r)$ is the type such that the CEO of type r_0 is at the reservation utility with her contract, if $\max_{r \in [0, r_{\max}]} \sigma_m^2 E[M] \left[r + \frac{F(r)}{f(r)} \right] \leq \sigma_m^2 + \sigma_\eta^2$ (this is a sufficient condition to ensure an interior solution), then

1) if $r_0 = 0$, then the optimal incentives are $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r - \frac{1 - F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)};$ 2) if $r_0 = r_{\text{max}}$, then the optimal incentives are $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r + \frac{F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)};$ 3) if $0 < r_0 < r_{\text{max}}$, then there must exist $r_1 \le r_0 \le r_2$ such that $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r + \frac{F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$ for

 $0 \le r \le r_1, \ \beta^*(r) \ is \ constant \ for \ r_1 \le r \le r_2, \ and \ \beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r - \frac{1 - F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_n^2)} \ for \ r \ge r_2.$

Most of comparative statics are the same with Proposition 1: $\beta^*(r)$ is increasing in π , r; decreasing in λ and σ_{η}^2 ; the relation between $\beta^*(r)$ and σ_m^2 is ambiguous. But the relation between $\beta^*(r)$ and the aggregate CEO incentives E[M] becomes negative when $r > r_0$ and $r - \frac{1-F(r)}{f(r)} < 0$.

The reason why the incentives for the types above r_0 can be decreasing in aggregate CEO incentives is due to the cost of information rent. As argued above, when aggregate CEO incentives go up, it is less costly to use stock compensation than cash to pay a CEO's RWCs' premium. But

¹⁵Note that in the case where a CEO's RWCs are observable, the base salary is always set to make her utility at the reservation utility.

where information asymmetry is present, granting a CEO more shares of stock also increases the information rent extracted by the CEOs of types above r_0 . As a result, the optimal individual incentives may be decreasing in aggregate CEO incentives, if the cost of information rent is more important.

When r is unobservable, the optimal incentives for a CEO of type r depend on the relative position between r and r_0 . As argued above, when $r < r_0$, it is necessary to keep the CEO incentives as high as possible to reduce the information rent; when $r > r_0$, it is necessary to keep the CEO incentives as low as possible to reduce the information rent. Meanwhile, the location of r_0 depends on M (aggregate CEO incentives), i.e., r_0 depends on other CEOs' incentives. Hence, multiple equilibria can arise in the case with asymmetric information, such that r_0 is low in some equilibria and high in some other equilibria. In the following example, I show that there could exist multiple equilibria.

Example 1. Suppose all the firms are identical, i.e. $\lambda_i = \lambda$, $\pi_i = \pi$, $\sigma_{\eta i}^2 = \sigma_{\eta}^2$, r_i are independently and uniformly distributed over $[0, r_{\max}]$. Let $M_1 = \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}, W_1 = \frac{1}{2}[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)] \left[1 - \frac{1}{3} \left(\frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}\right)^2\right] M_1^2 + \underline{u}, M_2 = \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2) - \lambda \sigma_m^2 r_{\max}},$ $W_2 = \frac{1}{1 - r_{\max}} \left[\frac{1}{2}[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)] \left[1 - \frac{\lambda \sigma_m^2 r_{\max}(2 - r_{\max})}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)} - \frac{1}{3} \left(\frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}\right)^2\right] M_2^2 + \underline{u}\right].$ If

$$\lambda M_1^2 \sigma_m^2 \cdot \min\left(1 - \frac{1}{2} r_{\max}, 1 - \frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}\right) - W_1 \ge 0,$$

$$\lambda M_2^2 \sigma_m^2 \cdot \max\left(1 - \frac{1}{2} r_{\max}, 1 - r_{\max} + \frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}\right) - W_2 \le 0,$$

then there exist at least two equilibria such that

1) in one equilibrium, $r_0 = 0$. The optimal incentives for each firm are low and given by $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 M_1(2r - r_{\max})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$. The aggregate incentives are M_1 , and the aggregate expected CEO pay is W_1 .

2) in another equilibrium, $r_0 = r_{\text{max}}$. The optimal incentives for each firm are high and given by $\beta^*(r) = \frac{\pi^2 + 2r\lambda \sigma_m^2 M_2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$. The aggregate incentives are M_2 , and the aggregate expected CEO pay is W_2 .

2.5 A Simple Explanation of Pay-for-Luck

Empirical studies document that CEOs are often rewarded for factors that are beyond their control, i.e., paid for luck (Bertrand and Mullainathan (2001)). This phenomenon gives rise to criticism that executive compensation is inefficient and is in fact decided by executives themselves. In this section, I use RWCs to make a simple explanation of this puzzle. The intuition is as follows: suppose that firm value is affected by some lucky shock beyond a CEO's control. A CEO with RWCs will require a RWCs' premium due to the participation constraint. Because the CEO is risk-averse, if the CEO's RWCs depend on the lucky shock, then it is less costly to compensate the CEO by a means that correlates to the shock, rather than to pay cash to the CEO. In other words, shareholders should not subtract the lucky shock from the firm's terminal value when deciding CEO pay.

In (1), I refer to aggregate shock \tilde{m} as the lucky shock, and assume that it is observable and contractible. Hence for the CEO in firm *i*, her contract now has the following form:

$$w_i = \alpha_i + \beta_i (V_i - \tilde{m}) + \gamma_i \tilde{m} = \alpha_i + \beta_i (\pi_i a_i + \eta_i) + \gamma_i \tilde{m}$$

A CEO may not only care about other CEOs' pay, but also be concerned about her friends or neighbors – entrepreneurs, investment bankers, and private equity investors, etc – whose income is correlated with the lucky shock \tilde{m} . So I now assume that CEO *i*'s RWCs are given by

$$H_i = r_i \int_0^1 w_k dk + b_i \tilde{m},$$

where the first term still reflects CEO *i*'s concerns about other CEOs' compensation; the second term $b_i \ge 0$ reflects that the CEO will also compare her compensation to that of her friends, whose income is positively correlated with the lucky shock \tilde{m} .

Proposition 3. Suppose that $\int_0^1 r_k dk < 1$. Then the optimal contract $(\alpha_i^*, \beta_i^*, \gamma_i^*)$ for firm *i* is given by

$$\begin{cases} \alpha_i^* = \frac{(\lambda \sigma_{\eta i}^2 - \pi_i^2)h_i^4}{2(\pi_i^2 + \lambda_i \sigma_{\eta i}^2)^2} + rW + \underline{u}, \\ \beta_i^* = \frac{\pi_i^2}{\pi_i^2 + \lambda_i \sigma_{\eta i}^2}, \\ \gamma_i^* = \frac{r_i \int_0^1 b_k dk}{1 - \int_0^1 r_k dk} + b_i, \end{cases}$$

where W is the sum of the expected CEO pay. So as long as $\int_0^1 b_k dk > 0$, $\gamma_i^* > 0$ for each $i \in [0, 1]$,

3. Efforts Affect the Mean and Variance of Firm Value

3.1 Model

In the last section, a CEO can improve firm value without changing firm risk. In reality, however, a CEO's action may not only affect the mean of firm value, but also the variance of firm value. For example, if a CEO decides to invest in risky projects, investing in more projects will expose the firm to more risk, because the success of a project is subject to the state of the whole economy and the firm's own idiosyncratic risk. Previous literature has argued that one objective of CEO compensation is to induce a CEO's risk-taking actions that increase both the mean and variance of firm value. For example, risk-taking incentives are generally used to argue that options can be more efficient than stock (Dittmann and Yu (2008) and Feltham and Wu (2001)). In this section, I investigate the case in which CEO action increases both the mean and variance of firm value. I assume that at t = 1, if a CEO exerts effort a, then the firm's terminal value at time t = 2 is given by

$$V = \pi a + a(\tilde{m} + \eta), \tag{8}$$

Compared to (1), in (8) the CEO's effort also affects the volatility of firm value, which is proportional to the effort a. Except that the form of firm value is different from Section 2.1, all the other assumptions and specifications remain unchanged. In particular, I still confine the discussion to linear contracts for tractability.¹⁶

Now since CEOs' actions also affect their firms' exposure to risk and all CEOs' payoffs are subject to aggregate shock, for any target CEO, if the other CEOs' payoffs have a high exposure to aggregate shock, it will also incentivize the target CEO to exert a higher effort for the purpose of keeping up with the other CEOs' exposure to the aggregate shock. Therefore, other CEOs' incentives can also affect the target CEO's effort decision. This is different from the basic model

¹⁶Edmans and Gabaix (2011) and Ou-Yang (2003) explore situations where a CEO's effort affects the volatility of firm value in a general contracting problem (i.e., they do not restrict to linear contracts). But they have some other assumptions to get the closed form solution. Edmans and Gabaix (2011) assume that a CEO can observe the noise before taking the action to ensure the optimality of log-linear contracts; Ou-Yang (2003) uses a continuous time framework as in Holmstrom and Milgrom (1987).

in Section 2.1, in which a CEO's effort decision is only affected by her only contract – even if she has concerns about the other CEOs' payoffs.

Lemma 4. Given a linear contract $w = \alpha + \beta V$, the optimal effort taken by a CEO is

$$a = \frac{\beta \pi + \lambda \beta \sigma_m^2 r M}{\lambda (\sigma_m^2 + \sigma_\eta^2) \beta^2 + 1}.$$

1) The CEO's effort a is increasing in β if $\beta < \sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$ and decreasing in β if $\beta > \sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$. 2) a is increasing in π , r and M, decreasing in σ_η^2 ; but the relation between a and σ_m^2 is ambiguous. If $\frac{r}{\pi + \lambda \sigma_m^2 r M} \frac{\partial(\sigma_m^2 M)}{\sigma_m^2} > \frac{\beta^2}{\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 + 1}$, then a is increasing in σ_m^2 ; if $\frac{r}{\pi + \lambda \sigma_m^2 r M} \frac{\partial(\sigma_m^2 M)}{\sigma_m^2} < \frac{\beta^2}{\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 + 1}$, then a is decreasing in σ_m^2 .

Since a higher effort leads to a larger variance of firm value, a risk-averse CEO will be reluctant to exert a high effort when she is granted high incentives. Thus the CEO's effort is increasing in her incentives only when her incentives are not too high. This is different from the case in Section 2.1 where efforts are always increasing in incentives. Given the incentives β , a CEO's effort has two effects on her payoff: first, it increases the mean of firm value. The marginal benefit of this effect is $\pi\beta$, and the marginal cost of this effect is a. Second, the effort also increases the variance of firm value. Since the CEO is risk-averse, the marginal cost of increasing firm risk is $\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 a$. Due to the CEO's RWCs, the increase of firm risk also helps the CEO to catch up with the other CEOs' exposure to the aggregate shock. The marginal benefit of this effect is $\lambda \beta \sigma_m^2 r M$. So the total marginal benefit of the CEO's effort is $\beta \pi + \lambda \beta \sigma_m^2 r M$, and the total marginal cost of the CEO's effort is $a[\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 + 1]$. Equating the benefit to the cost yields the optimal effort in Lemma 4. Granting the CEO a few more shares will increase the marginal benefit of effort by $\pi + \lambda \sigma_m^2 r M$, and increase the marginal cost of effort by $2a\lambda(\sigma_m^2 + \sigma_\eta^2)\beta$. If the increase in the marginal benefit is larger (smaller) than the increase in the marginal cost, then granting the CEO more shares will increase (decrease) the CEO's effort. Comparing the change in the marginal benefit and marginal cost, it can be seen that there is a threshold $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, such that when CEO incentives exceed this threshold, the CEO's effort will become decreasing in the incentives.

I now turn to the results in part 2) of Lemma 4. Since the increase of π , r, and M will increase the marginal benefit of the CEO's effort, but has no effect on the marginal cost, the CEO's effort a is increasing in π , r and M. Similarly, the increase of σ_{η}^2 only increases the risk that the CEO has to bear, so it only increases the marginal cost of the CEO's effort. Thus the CEO's effort a is decreasing in σ_{η}^2 . Lastly, the increase of σ_m^2 increases both the marginal benefit and marginal cost. On one hand, the increase of σ_m^2 decreases the CEO's effort because the CEO is risk-averse and a higher CEO's effort exposes the CEO's payoff to more risk. On the other hand, because other CEOs' payoffs are correlated with aggregate shock, a higher effort can help the CEO to catch up with the other CEOs' exposure to aggregate shock. The benefit from this effect is more valuable when the variance of aggregate risk goes up; thus the increase of σ_m^2 also increases the CEO's effort due to her concerns about the other CEOs' payoffs. So the two conflicting effects make the effect of σ_m^2 on the CEO's effort ambiguous.

The shareholders will choose an optimal level of effort and a linear contract to implement the effort. Solving the model, we can obtain that

Proposition 4. If $rM\sigma_m^2 \ge \pi(\sigma_m^2 + \sigma_\eta^2)$, then $\beta^* = 1$, i.e., it is better to sell the firm to the CEO. If $rM\sigma_m^2 < \pi(\sigma_m^2 + \sigma_\eta^2)$, then the optimal linear contract $w = \alpha^* + \beta^* V$ is given by

$$\beta^* = \frac{2}{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}},$$

where $M = \int_0^1 \frac{\partial w_k}{\partial \tilde{m}} dk$ is the sum of the sensitivity of CEO pay to aggregate shock. The base salary α^* is set to make the participation constraint binding.

1) β^* is increasing in r and M; decreasing in π , λ , and σ_{η}^2 ;

2) The relation between β^* and σ_m^2 is ambiguous. When $rM + r\sigma_m^2 \frac{\partial M}{\partial \sigma_m^2} < \pi\beta^*$, β^* is decreasing in σ_m^2 ; when $rM + r\sigma_m^2 \frac{\partial M}{\partial \sigma_m^2} > \pi\beta^*$, β^* is increasing in σ_m^2 .

3) If $\lambda r M \sigma_m^2 < \pi$, then $\beta^* < \sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, which implies that $\frac{\partial a}{\partial \beta}|_{\beta^*} > 0$, i.e., granting the CEO more incentives will increase the CEO's effort. If $\lambda r M \sigma_m^2 > \pi$, then $\beta^* > \sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, which implies that $\frac{\partial a}{\partial \beta}|_{\beta^*} < 0$, i.e., granting the CEO more incentives will decrease the CEO's effort.

Compared to Proposition 1, there are two different results. In Proposition 4, the relation between β^* and π is reversed. Now β^* becomes decreasing in π . The reason is as follows: given CEO incentives, a CEO will exert an optimal effort *a* that is proportional to her productivity π . Thus the expected firm value will be increased by πa , which is proportional to π^2 (because *a* is proportional to π). But note that the CEO's effort also increases the variance of firm value, which is proportional to a^2 . Because a is proportional to π , the risk premium paid to the CEO is also proportional to π^2 . Hence from Proposition 4, we can see that when there are no RWCs (i.e. r = 0), the optimal incentives do not depend on π because the benefit and the cost are both proportional to π^2 . If the CEO has RWCs, they provide two benefits on shareholders' payoffs: Lemma 4 shows that the RWCs also provide incentives for the CEO to work. Such a benefit is proportional to the CEO's productivity π . Secondly, since the risk premium paid to the CEO for the aggregate risk that she bears is now equal to $\frac{1}{2}\lambda(\beta a - rM)^2\sigma_m^2$. If r is positive, it helps to reduce the risk premium. Savings on the risk premium are also proportional to π . Thus when π goes up, the benefits for the shareholders from the CEO's RWCs become relatively smaller when compared with the benefit from the CEO's effort and the cost of CEO compensation. Therefore, the optimal incentives will move toward to the ones in the case with no RWCs: the optimal incentives will decrease as π goes up.

Another different result is part 3) of Proposition 4. In Proposition 1, the CEO's effort only affects the mean, so her effort will always increase in the incentives. Here, the CEO's effort also affects the variance of firm value, so when the CEO's incentives are too high, it reduces the CEO's (risktaking) effort. In part 3), if the CEO has no RWCs, at the optimum, β^* is less than the threshold $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$: the CEO's effort is increasing in the incentives. Intuitively, it is never optimal for shareholders to grant a CEO more than $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$ shares of stock, since doing so will only reduce the CEO's effort and increase the risk premium paid to the CEO. However, if the CEO has RWCs (r > 0), it is less costly to pay the RWCs' premium by using stock than cash. When r is large, it could be optimal for shareholders to grant a CEO more than $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$ shares of stock to pay the RWCs' premium. Thus in equilibrium, granting the CEO more shares can decrease her (risk-taking) effort.¹⁷

As r increases, Lemma 4 shows that it directly increases CEO effort. On the other hand, the increase of r also increases the optimal CEO incentives by Proposition 4. If the CEO's RWCs are large, then at the optimum the CEO's risk-taking effort may be decreasing in her incentives. Thus the increase of CEO incentives may reduce a CEO's effort. Therefore the increase of r may have an

¹⁷Carpenter (2000) and Ross (2004) show that granting a CEO options may reduce the CEO's risk-taking incentives, but they do not endogenize the optimal CEO incentives in the model. Here I show that by endogenizing the optimal CEO incentives, granting her more incentives could reduce her risk-taking efforts only when the CEO has RWCs.

indirect effect through incentives to decrease a CEO's effort. I show in the following lemma that the total effects of a CEO's RWCs on her (risk-taking) effort are positive, i.e., the optimal CEO's effort is increasing in her RWCs. This suggests that when a CEO's RWCs go up, she will exert a higher effort. As a result, the covariance of firm value with aggregate shock goes up. Similarly, if the sensitivity of other CEOs' pay to aggregate shock goes up, the total effects on the target CEO's risk-taking action are also positive, which will push up the covariance of the target firm value with the aggregate shock. The results are summarized in the following lemma.

Lemma 5.

$$\frac{da}{dr}|_{\beta^*} = \frac{\partial a}{\partial \beta} \frac{\partial \beta}{\partial r} + \frac{\partial a}{\partial r} > 0.$$

Therefore, $\frac{d}{dr}Cov(V,\tilde{m}) > 0$. As a CEO's RWCs increase, firm value becomes more covariant with the aggregate shock.

Similarly, $\frac{da}{dM}|_{\beta^*} > 0$ implies that as the sum of the sensitivity of CEO pay to aggregate shock goes up, firm value becomes more covariant with the aggregate shock.

3.2 Equilibrium

When a CEO cares about other CEOs' compensation, Proposition 4 shows that the optimal incentives for one firm depend on other CEOs' incentives. Lemma 4 shows that each CEO's (risk-taking) effort also depends on the other CEOs' incentives and efforts. So an equilibrium requires that each firm chooses a contract optimally given the other firms' contracts, and each CEO exerts an optimal effort given the other CEOs' incentives and efforts.

Definition 3. A subgame perfect equilibrium is a set of functions $(\alpha_i, \beta_i, a_i)_{i \in [0,1]}$ that satisfy the following properties:

1) at time 0, for each firm *i*, given all the other firms' contracts $(\alpha_j, \beta_j)_{j \neq i}$, the contract (α_i, β_i) solves the principal-agent problem optimally for firm *i*.

2) at time 1, for the CEO in firm i, given the other firms' contracts and CEOs' efforts, a_i maximizes the CEO i's utility.

If r is too big, there could be two results. First, it can become too costly to compensate the CEO. As a result, shareholders will decide not to hire the CEO, which is not interesting. Second, as

CEOs' effort decisions are affected by each other, when some CEOs (a positive measure) increase their efforts, it will also incentivize the other CEOs to exert higher efforts. This, in turn, will increase the original CEO effort, and so on and so forth. Then RWCs eventually will have an amplifying effect on incentivizing a CEO's effort. Thus, if r is too big, the result could be that CEOs will exert an effort of infinite large, which is unrealistic. In the following lemma, I provide a sufficient condition under which equilibrium exists; in equilibrium, CEOs are hired and their efforts are bounded.

Lemma 6. Define that
$$\beta_i(M) = \frac{2}{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right) + \sqrt{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right)^2 + 4\lambda_i (\sigma_m^2 + \sigma_{\eta i}^2)}}$$
 for each $i \in [0, 1]$ and $\int_0^1 \left[\frac{1}{2} \frac{\beta_i(M)^2 (\pi_i - \lambda_i r_i M \sigma_m^2) (\pi_i + \lambda_i r_i M \sigma_m^2)}{2\pi_i (\pi_i - \lambda_i r_i M \sigma_m^2) + \frac{1}{2} \lambda_i r_i^2 M^2 \sigma_m^2}\right] di$

 $W(M) = \frac{\int_0^1 \left[\frac{1}{2} \frac{(K-Y) + (k-Y) + ($

$$\int_{0}^{1} r_{i} di < 1, \ and \tag{9}$$

$$r_i \sigma_m^2 \le \frac{2(\sigma_m^2 + \sigma_{\eta i}^2)}{1 + \sqrt{1 + 4\lambda_i(\sigma_m^2 + \sigma_{\eta i}^2)}} \text{ for each } i \in [0, 1], \text{ and}$$
(10)

$$r_i M \sigma_m^2 \le \pi_i (\sigma_m^2 + \sigma_{\eta i}^2) \text{ for each } i \in [0, 1], \text{ and}$$

$$(11)$$

$$\frac{\beta_i(M)\pi_i(\pi_i + \lambda_i r_i M \sigma_m^2)}{\lambda_i(\sigma_m^2 + \sigma_{\eta i}^2)\beta_i(M)^2 + 1} - E[w_i] \ge 0 \text{ for each } i \in [0, 1],$$

$$(12)$$

where
$$E[w_i] = \frac{1}{2} \frac{\beta_i(M)^2(\pi_i - \lambda_i r_i M \sigma_m^2)(\pi_i + \lambda_i r_i M \sigma_m^2)}{\lambda_i(\sigma_m^2 + \sigma_{\eta i}^2)\beta_i(M)^2 + 1} + \frac{1}{2} \lambda_i r_i^2 M^2 \sigma_m^2 + rW(M).$$

Note that (10) guarantees that each CEO's effort is bounded. (11) guarantees that the optimal CEO incentives do not exceed 1. (12) guarantees that the shareholders' payoffs are nonnegative for each firm. Since the CEO's effort not only affects the mean but also the variance of firm value, Lemma 4 sets forth that a CEO's effort is not monotonically increasing in the incentives. There is a threshold, $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, such that the CEO's effort is increasing in the incentives below the threshold and decreasing in the incentives above the threshold. By Proposition 4, each CEO's optimal incentives depend on other CEOs' incentives. If the other CEOs are granted low incentives, then the target CEO's incentives can be low, such that her effort is increasing in the incentives; but if the other CEOs are granted high incentives, then the target CEO's incentives has to be high

and may exceed the threshold, such that her effort becomes decreasing in the incentives. Therefore, there could exist multiple equilibria in this case. I illustrate this in the following example.

Example 2. Suppose there are two types of firms. For $i \in [0, k]$, $\lambda_i = \lambda$, $\pi_i = \pi$, $\sigma_{\eta i}^2 = \sigma_{\eta}^2$, $r_i = r$. For $i \in [1 - k, 1]$, $\lambda_i = \lambda$, $\pi_i = \pi$, but $\sigma_{\eta i}^2 = \bar{\sigma}_{\eta}^2$, $r_i = \bar{r}$. Assume that the parameters satisfy the following conditions: 1) $\lambda(\sigma_m^2 + \sigma_\eta^2) > 1$; 2) $\lambda r(\sigma_m^2 + \bar{\sigma}_\eta^2) = \lambda \bar{r}(\sigma_m^2 + \sigma_\eta^2)$; 3) $\frac{\bar{r}}{r} > 1$; 4) $\frac{r + \bar{r}}{2\bar{r}} \left(1 + \frac{\bar{r} - r}{\sqrt{(\bar{r} - r)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)\bar{r}r}} \right) < \frac{\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 r k}{\sigma_m^2 r (1 - k)} < 1$. ¹⁸ Then for firms in [0, k], there must exist at least two equilibria: in one equilibrium, the optimal CEO incentives are less than the threshold $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, thus the CEO's effort is increasing in the incentives; in another equilibrium, the optimal incentives are greater than the threshold $\sqrt{\frac{1}{\lambda(\sigma_m^2 + \sigma_\eta^2)}}$, thus the CEO's effort is decreasing in the incentives.

3.3 Two Firms with RWCs

In the previous sections, I have assumed that a continuum of firms and CEOs exists. Each firm represents an infinitesimal portion of the continuum, so the change of one CEO's incentives will not affect other CEOs' actions. This nice property leads to the result that CEO incentives are increasing in a CEO's RWCs r (Proposition 4), because the rise of a CEO's RWCs increases the CEO's effort and also helps to reduce the risk premium associated with the aggregate shock. However, if a firm's compensation policy affects some other CEOs' actions, then the result can be changed. In this section, I consider the case with two firms. If the two CEOs compare compensation with each other, then the increase of one CEO's incentives can increase the other CEO's effort. The rise of the other CEO's effort will increase her own income, which in turn reduces the target CEO's relative payoff because of the RWCs. As a result, the relation between CEO incentives and RWCs may change.

Suppose there are two firms with both CEOs having RWCs: firm 1 and firm 2. I use parameters for firm 2. For example, the measure of a CEO's risk aversion is λ for firm 1 and $\overline{\lambda}$ for firm 2. CEO 1's utility with RWCs is

$$U = E[u, H] = E[w - H] - \frac{1}{2}\lambda Var[w - H] - \frac{1}{2}a^{2},$$

¹⁸Note that since $\lambda(\sigma_m^2 + \sigma_\eta^2) > 1$ and $\bar{r} > r$, $\frac{r+\bar{r}}{2\bar{r}} \left(1 + \frac{\bar{r}-r}{\sqrt{(\bar{r}-r)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)\bar{r}r}} \right) < \frac{r+\bar{r}}{2\bar{r}} \left(1 + \frac{\bar{r}-r}{\sqrt{(\bar{r}-r)^2 + 4\bar{r}r}} \right) = 1$. So the condition 4) can be satisfied.

where $H = r\bar{w} = r\left(\bar{\alpha} + \bar{\beta}\left[\bar{\pi}\bar{a} + \bar{a}(\tilde{m} + \bar{\eta})\right]\right)$ denotes CEO 1's RWCs. A larger r means that the CEO 1 cares more about CEO 2's wage. Similarly, CEO 2's utility with RWCs is

$$\bar{U} = E[\bar{u}, \bar{H}] = E[\bar{w} - \bar{H}] - \frac{1}{2}\bar{\lambda}Var[\bar{w} - \bar{H}] - \frac{1}{2}\bar{a}^2,$$

where $\bar{H} = \bar{r}w = \bar{r}(\alpha + \beta [\pi a + a(\tilde{m} + \eta)])$ denotes CEO 2's RWCs. A larger \bar{r} means that the CEO 2 cares more about CEO 1's wage.

The equilibrium is a subgame perfect equilibrium. In the first stage (t = 0), each firm will choose an optimal contract given the other firm's contract. In the second stage (t = 1), after the contracts are signed, each CEO will exert an optimal effort given the other CEO's contract and effort. I will solve the problem using backward induction.

First, I consider the CEOs' effort decisions. Suppose that CEO 1 is granted β shares of stock, and CEO 2 is granted $\overline{\beta}$ shares of stock. Given CEO 2's effort \overline{a} , a similar derivation as in Lemma 4 shows that CEO 1's effort is given by

$$a = \frac{\beta(\pi + \lambda \sigma_m^2 r \bar{a} \bar{\beta})}{\lambda(\sigma_m^2 + \sigma_n^2)\beta^2 + 1}.$$
(13)

Similarly, given CEO 1's effort a, CEO 2's effort is given by

$$\bar{a} = \frac{\bar{\beta}(\bar{\pi} + \bar{\lambda}\sigma_m^2 \bar{r}a\beta)}{\bar{\lambda}(\sigma_m^2 + \sigma_{\bar{n}}^2)\bar{\beta}^2 + 1}.$$
(14)

I focus on solving the principal-agent problem for firm 1. The solution for firm 2, then, is straightforward by symmetry.

Lemma 7. Given the two contracts $w = \alpha + \beta V$ and $\bar{w} = \bar{\alpha} + \bar{\beta}\bar{V}$,

1) the optimal effort taken by CEO 1 is

$$a = \frac{\beta(\pi + \lambda \sigma_m^2 A)}{\lambda(\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 B)\beta^2 + 1},$$

where $A = \frac{r\bar{\beta}^2 \bar{\pi}}{\bar{\lambda}(\sigma_m^2 + \sigma_{\bar{\eta}}^2)\bar{\beta}^2 + 1}$ and $B = \frac{r\bar{\tau}\bar{\lambda}\bar{\beta}^2 \sigma_m^2}{\bar{\lambda}(\sigma_m^2 + \sigma_{\bar{\eta}}^2)\bar{\beta}^2 + 1}$. The optimal effort *a* is increasing in *r* and \bar{r} . 2) CEO 2's expected pay increase in CEO 1's incentives, i.e. $\frac{\partial E[\bar{w}]}{\partial \beta} > 0$. Thus, $\frac{\partial E[H]}{\partial \beta} > 0$.

The increase of r increases CEO 1's concerns about CEO 2's payoff; hence it increases CEO 1's

incentives to exert a higher (risk-taking) action to catch up with CEO 2's exposure to aggregate shock. Similarly, the increase of \bar{r} increases CEO 2's effort, and thus increases CEO 2's exposure to the aggregate shock. Since CEO 1 cares about CEO 2's payoff, then the increase of CEO 2's exposure to aggregate shock incentivizes CEO 1 to exert a higher effort. For the shareholders in firm 1, when they grant higher incentives to CEO 1, it will increase CEO 1's exposure to aggregate shock, thus it will increase CEO 2's incentives to exert effort. Higher incentives granted to CEO 1 will also induce CEO 2 to exert a higher effort (by (14), we see that \bar{a} is increasing in β), which in turn will affect CEO 1's utility through her RWCs. Lemma 7 shows that the rise of CEO 1's incentives increases CEO 2's expected payoff and hence reduces CEO 1's relative payoff. As a result, the shareholders in firm 1 may have to increase CEO pay due to the participation constraint. So the optimal CEO incentives may decrease in RWCs. In other words, RWCs can lead to low CEO incentives, which has been documented in empirical studies (Jensen and Murphy (1990)).

Proposition 5. The optimal linear contract $w = \alpha^* + \beta^* V$ for firm 1 is that β^* is a solution to the following equation,

$$\pi - \frac{2\lambda(\sigma^2 - \sigma_m^2 B)\pi\beta^2}{\lambda(\sigma^2 - \sigma_m^2 B)\beta^2 + 1} - \frac{\beta(\pi - \lambda\Gamma\sigma_m^2)}{\lambda\sigma^2\beta^2 + 1} + \left[\left(\frac{(\lambda\sigma_m^2)^2\beta^2}{\lambda\sigma^2\beta^2 + 1} - \lambda\bar{\sigma}^2 \right)\Gamma - \bar{\pi} \right] \frac{2B\beta}{\lambda(\sigma^2 - \sigma_m^2 B)\beta^2 + 1} = 0,$$

where $\Gamma = r\bar{a}\bar{\beta} = A + Ba\beta = A + \frac{\beta^2(\pi + \lambda \sigma_m^2 A)B}{\lambda(\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 B)\beta^2 + 1}$, A and B are defined in Lemma 7. α^* is set to make the participation constraint binding.

Moreover, if $(2\lambda\sigma_m^2+1)\pi < (2-\frac{\lambda}{\bar{r}\lambda})\bar{\pi}$, then $\frac{\partial\beta^*}{\partial r}|_{r=0} < 0$. This implies that if $\bar{\pi}$ and \bar{r} are big, then for small r, β^* is decreasing in r.

The intuition behind Proposition 5 is that when CEO 1's RWCs increase (i.e., r goes up), it increases CEO 1's effort and also helps to reduce the risk premium associated with aggregate shock. These two effects would push up the optimal incentives for CEO 1. However, if \bar{r} is big, then CEO 2 cares a lot about CEO 1's payoff, then increasing CEO 1's incentives will increase CEO 2's effort significantly. Since the productivity of CEO 2's effort is big (i.e., $\bar{\pi}$ is big), the increase in CEO 2's effort will increase CEO 2's pay greatly. Then because CEO 1 also cares about CEO 2's payoff, the shareholders may have to pay a high premium for CEO 1's RWCs. Thus increasing CEO 1's incentives may lead to a large increase in CEO 1's pay. If this cost is too big, then there will be a negative relation between CEO incentives and r.

4. Extension: An Example of Stock Versus Options

In Sections 2 and 3, I focus on linear contracts for the purpose of tractability. In practice, though, options are widely used. Options provide insurance from the downside of returns, giving a CEO convex payoffs and inducing her to take risk (Guay (1999), Coles, Daniel and Naveen (2006)). Dittmann and Maug (2007), however, calibrate a standard principal-agent model and find that options should not be used. In this section, I will compare stock to options in the presence of RWCs in a simple example.

Suppose there are a continuum of firms and CEOs: $i \in [0,1]$. CEO *i* is assigned to firm *i*. All firms are identical in the sense that the corresponding parameters are equal for each firm. For simplicity, I assume that for each CEO, there is a binary effort decision $\{0 \text{ (shirk)}, 1 \text{ (work)}\}$. If CEO *i* chooses to shirk, then there is no cost and firm value is $V_i = e_0(\tilde{m} + \eta_i)$. If she chooses to work, then there is a cost *c* for her and firm value is $V_i = \pi + e_1(\tilde{m} + \eta_i)$, where \tilde{m} refers to the aggregate shock that could be σ_m or $-\sigma_m$ with probability $\frac{1}{2}$ for each; η_i is firm *i*'s idiosyncratic shock that could be σ_η or $-\sigma_\eta$ with probability $\frac{1}{2}$ for each; \tilde{m} and $(\eta_i)_{i\in[0,1]}$ are independent of each other. e_0 is the firm's exposure to risk if the CEO shirks, and e_1 is the firm's exposure to risk if the CEO works. I assume that $e_1 \geq e_0$. If $e_0 = e_1$, then it corresponds to the situation in which the CEO's effort only increases the mean of firm value; if $e_0 < e_1$, then it corresponds to the situation in which the CEO's effort increases both the mean and variance. For simplicity, I make the following assumptions:

Assumption 1: It is always beneficial for shareholders to induce CEOs to work (i.e., 1 is the optimal effort);

Because of the convex payoff of options, I will relax the restriction on linear contracts and make the following assumption:

Assumption 2: For shareholders in each firm, they will use one of the two strategies to induce CEOs to work: either use a stock-based contract (i.e., salary + stock), or use an option-based contract of exercise price π (i.e., salary + options).

The CEO in firm i has RWCs with her expected utility defined in (2). The shareholders will choose either stock or options to induce the CEO to work. For simplicity, I assume that when there is no difference between stock and options, shareholders will choose to use stock to induce the CEO's effort. Then, since firms are identical, in equilibrium, either all the firms use options or all the firms use stock.

The presence of RWCs has two effects when comparing stock with options. First, if the CEO chooses to shirk, then a stock-based contract still has an exposure e_0 to aggregate shock; but an option-based contract becomes more insensitive to shock, because options are more likely to expire with value zero. Recall that in the presence of RWCs, CEOs are willing to risk exposure to aggregate shock because it helps them to keep up with their peers. Thus, the effect of RWCs on incentivizing a CEO's effort is larger for an option-based contract than a stock-based contract. This "incentive effect" makes options more preferable.

On the other hand, to some extent, RWCs make CEOs willing to expose to aggregate shock, which reduces the risk premium associated with aggregate shock. Because stock is completely exposed to the aggregate shock, while options are only partially exposed, the reduction in risk premium is larger for stock than options. For this symmetric case, the proof in the Appendix shows that the ratio of the risk premium of one unit of stock to the risk premium of one unit of options is

$$\kappa(r) = \frac{4[(1-r)^2\sigma_m^2 + \sigma_\eta^2]}{(1-r)^2\sigma_m^2 + \sigma_\eta^2 + \min(\sigma_m^2, \sigma_\eta^2)}$$

which is decreasing in r. Thus this "risk premium effect" makes stock preferable. Therefore, depending on which effect is more dominant, RWCs can render either stock or options more preferable.

Lemma 8. Suppose that r < 1 and $\pi \ge e_0(\sigma_m + \sigma_\eta)$. Then there exists a cutoff $\hat{\pi}$ such that shareholders in all the firms will use stock if and only if $\pi \ge \hat{\pi}$.

1) If
$$\frac{\sigma_{\eta}}{\sigma_{m}} \to 0$$
, then $\frac{\partial \kappa(r)}{\partial r}$ goes to 0, and $\frac{\partial \hat{\pi}}{\partial r} > 0$.
2) If $\sigma_{m}^{2} \ge \sigma_{\eta}^{2} > \frac{4\lambda c(1-r)^{2}}{(1-r)(1-8\lambda c)\sqrt{1-6\lambda c}-8\lambda c}\sigma_{m}^{2}$, then $\frac{\partial \hat{\pi}}{\partial r} < 0$.
3) If $\frac{\sigma_{m}}{\sigma_{\eta}} \to 0$, then $\frac{\partial \hat{\pi}}{\partial r} = 0$.

In part 1) of Lemma 8, when aggregate risk is much more important relative to idiosyncratic risk, we can see that the ratio of risk premiums converges to a constant, and the risk premium effect disappears. As a result, the incentive effect dominates and renders options more preferable. Part 2) indicates that if idiosyncratic risk is not too small compared with aggregate risk, then the risk premium effect can dominate the incentive effect such that stock becomes more preferable. Part 3) suggests that if aggregate risk becomes much smaller than idiosyncratic risk, then both effects tend to disappear. Therefore, RWCs do not affect the comparison between stock and options.

5. Empirical Implications

By introducing RWCs, the model generates a number of empirical predictions on executive compensation. I outline the predictions below.

1. Individual CEO compensation and aggregate CEO compensation

Due to RWCs, one expectes to see a correlation between CEOs' compensation. The model predicts a positive correlation in the level of CEO pay and CEO incentives. Proposition 1 shows that the expected individual CEO pay is increasing in the level of aggregate expected CEO pay and aggregate CEO incentives. Bouwman (2011), Bizjak, Lemmon, and Naveen (2008), as well as Faulkender and Yang (2010) find that the level of CEO pay increases in the average level of CEO pay in a reference group. Although market competition could also lead to a positive correlation among the level of CEO pay, it is not clear how market competition will affect the relation between CEO incentives. My RWCs-based model predicts a positive relation between CEO incentives. It will be interesting to test the positive relation between individual CEO incentives and aggregate CEO incentives. To test the relation, it is critical to identify the reference group. Bouwman (2011) uses geographically-close firms as a reference group, Bizjak, Lemmon, and Naveen (2008), Faulkender and Yang (2010) use similar-sized firms in the same industry as a reference group. So geography, industry and firm size might be the potential factors to identify a reference group in an empirical test. In addition, it is also a challenge to solve the causality problem, because the increase in individual incentives and aggregate incentives may be due to other factors – for example, a common shock to aggregate risk may increase both individual incentives and aggregate incentives simultaneously.

2. CEO incentives and firm risk

Prendergast (2002) provides a survey of the empirical evidence on the relation between CEO incentives and firm risk. The results are mixed: three papers find negative relation consistent with the informativeness principle, but eight other papers find no significant or even a positive relation. In Proposition 1, I show that while the relation between incentives and a firm's idiosyncratic risk is negative, the relation between incentives and aggregate risk could be negative or positive, depending

on the coefficient of a CEO's RWCs on aggregate shock (i.e., rM, where r measures the CEO's concerns about other CEOs' pay, and M is aggregate CEO incentives). In particular, the model predicts that if $r \left[\frac{\partial M}{\partial \sigma_m^2} \sigma_m^2 + \left(1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)} \right) M \right] > \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}$, then β^* is increasing in σ_m^2 . So if r is big, and (or) the aggregate incentives M is big, and (or) the idiosyncratic risk σ_η^2 is big, then there is a positive relation between incentives and aggregate risk. The prediction that aggregate CEO incentives also affect the relation between individual CEO incentives and aggregate risk is novel.

3. Pay for luck

Bertrand and Mullainathan (2001) document that CEOs are rewarded for factors that are beyond their control, i.e., paid for luck. Proposition 3 provides a simple explanation of pay-for-luck puzzle. Moreover, it predicts that a CEO is paid more for luck if her concerns about other CEOs' pay are more intensive (i.e., r is bigger), or if other CEOs in the reference group are paid more for luck.

4. RWCs and risk-taking incentives

Empirical studies have documented a negative relation between a CEO's sensitivity to stock price movement and the CEO's risk-taking behavior (Bizjak, Brickley and Coles (1993), Coles, Daniel and Naveen (2006), Aggarwal and Samwick (2006)). But because CEO compensation is endogenously determined, there is a causality problem when testing the relation between incentives and risk-taking behavior. Gormley, Matsa and Milbourn (2011) address this identification problem by studying unexpected changes in the firms' business environment that increase left-tail risk. They find that managers with incentives that provide a high sensitivity to stock price movements respond to the increased tail risk with greater risk-reducing activities. My model predicts that the relation between incentives and a CEO's risk-taking behavior depends on a CEO's RWCs. Proposition 4 shows that if rM exceeds some threshold, then granting a CEO more incentives can reduce her risk-taking effort. Some other papers (Carpenter (2000), Ross (2004)) also point out that granting a CEO stock options may reduce the CEO's risk-taking incentives, but those papers do not endogenize the optimal CEO incentives. My model shows that RWCs may cause the optimal CEO incentives to be at a high level, such that a negative relation between CEO incentives and risk-taking behavior arises at the optimum. Finally, Proposition 5 shows that if a firm's compensation policy has a large influence on other CEOs' risk-taking decisions, then the CEO's risk-taking incentives can be decreasing in r; thus the shareholders in the firm will be willing to keep CEO incentives at a lower level.

5. Stock versus options

Lemma 8 shows that if aggregate risk is much larger than idiosyncratic risk, then the incentive effect is dominant, so options are more preferable. But if idiosyncratic risk is not dominated by aggregate risk, then the risk premium effect is dominant such that stock becomes more efficient. To my knowledge, the comparison between stock and options has not been associated with the relative importance of aggregate risk and idiosyncratic risk in the previous literature.

6. Conclusion

Some recent empirical studies on executive compensation (Bouwman (2011), Shue (2011)) find that relative wealth concerns affect firm compensation policies. In this paper, I incorporate RWCs into a standard principal-agent model and study how CEO compensation will be affected. I focus on RWCs' effects on CEO incentives and find a number of interesting implications. I show that the relation between incentives and aggregate risk could be negative or positive, depending on the coefficient of the CEO's RWCs on aggregate shock. This can help to explain the mixed empirical evidence on the relation between incentives and risk. I also provide a simple explanation for payfor-luck puzzle. If a CEO's efforts also affect the variance of firm value, I show that if a firm's compensation policy has a large influence on other CEOs' risk-taking decisions, then low CEO incentives can be optimal. Finally, I compare stock to options in the presence of RWCs. I find that RWCs may render options or stock more preferable, depending on the relative importance of aggregate risk and idiosyncratic risk. This paper makes a number of predictions about RWCs' effects on CEO compensation, as well as some predictions concerning the relation between individual CEO compensation and aggregate CEO compensation (including the level of CEO pay and CEO incentives). The predictions, however, pose certain challenges for empirical tests: how to identify RWCs' effects; how to find an appropriate measure for a CEO's concerns about peer compensation; and how to solve the causality problem.

Appendix

Derivation of (3)

 $H_{i} = r_{i} \int_{0}^{1} w_{k} dk = r_{i} [\int_{0}^{1} (\alpha_{k} + \beta_{k} \pi_{k} a_{k}) dk + \int_{0}^{1} \beta_{k} \tilde{m} dk + \int_{0}^{1} \beta_{k} \eta_{k} dk] = r_{i} [W + M \tilde{m} + \int_{0}^{1} \beta_{k} \eta_{k} dk].$ Since $(\eta_{k})_{k \in [0,1]}$ are independent of each other, and the variance of $\beta_{k} \eta_{k}$ is bounded by $\sigma_{\eta k}^{2}$ which is bounded by σ_{\max}^{2} , by Kolmogorov's strong law of large numbers, $\int_{0}^{1} \beta_{k} \eta_{k} dk$ converges to 0 almost surely. So $H_{i} = r_{i} [W + M \tilde{m}].$

Proof of Proposition 1

Given a linear contract $w = \alpha + \beta V$, if a CEO exerts an effort a, then her expected utility is

$$U = \alpha + \beta \pi a - rW - \frac{1}{2}\lambda \left[(\beta - rM)^2 \sigma_m^2 + \beta^2 \sigma_\eta^2 \right] - \frac{1}{2}a^2.$$

Taking the first-order derivative w.r.t a yields that the optimal effort taken by the CEO is $a = \beta \pi$. Note that the base salary α does not affect the CEO's effort decision, it must be set to make the participation constraint bind, i.e.

$$\alpha + \beta \pi a - rW - \frac{1}{2}\lambda \left[(\beta - rM)^2 \sigma_m^2 + \beta^2 \sigma_\eta^2 \right] - \frac{1}{2}a^2 = \underline{u}.$$
 (15)

The shareholders' objective is to maximize the expected firm value net of CEO pay. Combining with the binding IR constraint (15), we can obtain that the shareholders' objective is

$$\max_{\beta}\beta\pi^2 - \left[rW + \frac{1}{2}\lambda\left[(\beta - rM)^2\sigma_m^2 + \beta^2\sigma_\eta^2\right] + \frac{1}{2}\beta^2\pi^2 + \underline{u}\right]$$

Taking the first-order derivative w.r.t β yields that the optimal CEO incentives are

$$\beta^* = \min\left(\frac{\pi^2 + \lambda r M \sigma_m^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}, 1\right).$$

So if $rM\sigma_m^2 \ge \sigma_m^2 + \sigma_\eta^2$, then $\beta^* = 1$.

If $rM\sigma_m^2 < \sigma_m^2 + \sigma_\eta^2$, $\beta^* = \frac{\pi^2 + \lambda rM\sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}$ is an optimal interior solution. From the solution of β^* , it is straight-forward to check that β^* is increasing in π , r, and M; decreasing in λ and σ_η^2 . For the

relation between β^* and σ_m^2 ,

$$\frac{\partial \beta^*}{\partial \sigma_m^2} = \frac{\lambda}{[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)]^2} \left[r \frac{\partial (M \sigma_m^2)}{\partial \sigma_m^2} [\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)] - (\pi^2 + \lambda r M \sigma_m^2) \right].$$

So if $r \frac{\partial (M\sigma_m^2)}{\partial \sigma_m^2} > \frac{\pi^2 + \lambda r M \sigma_m^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$, then β^* is increasing in σ_m^2 ; if $r \frac{\partial (M\sigma_m^2)}{\partial \sigma_m^2} < \frac{\pi^2 + \lambda r M \sigma_m^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$, then β^* is decreasing in σ_m^2 .

Plugging the solution of β^* into (15) yields that

$$\alpha^* = \frac{(\pi^2 - \lambda r M \sigma_m^2)(\pi^2 + \lambda r M \sigma_m^2)}{2[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]} - \frac{\pi^2 (\pi^2 + \lambda r M \sigma_m^2)^2}{[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]^2} + rW + \frac{1}{2}\lambda r^2 M^2 \sigma_m^2 + \underline{u} + \frac{1}{2}\lambda r^2$$

The expected CEO pay is

$$E[w] = \alpha^* + \beta^* \pi a = \frac{\pi^4}{2[\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)]} + rW + \frac{1}{2}\lambda r^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}\right] + \underline{u},$$

which is increasing in W and M.

Proof of Lemma 1

Proof of necessity

Suppose an (hiring) equilibrium exists and let M be the aggregate CEO incentives and W be the aggregate expected CEO pay in the equilibrium. Then by Proposition 1, for each firm i, the optimal CEO incentives are given by $\beta_i^* = \frac{\pi_i^2 + \lambda_i r_i M \sigma_m^2}{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}$. And the expected CEO pay is $E[w_i] = \frac{\pi_i^4}{2[\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)]} + r_i W + \frac{1}{2} \lambda_i r_i^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda_i \sigma_m^2}{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}\right] + \underline{u}_i$. Since $M = \int_0^1 \beta_i^* di$ and $W = \int_0^1 E[w_i] di$, we can solve that M has a unique solution $M = \frac{\int_0^1 \frac{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}{1 - \int_0^1 \frac{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}} di$; and W also has a unique solution $W = \frac{\int_0^1 \left[\frac{\pi_i^4}{2[\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)]} + \frac{1}{2} \lambda_i r_i^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda_i \sigma_m^2}{\pi_i^2 + \lambda_i (\sigma_m^2 + \sigma_{\eta_i}^2)}\right] + \underline{u}_i\right] di}{1 - \int_0^1 r_i di}$. Since W > 0 and M > 0, we must have $\int_0^1 r_i di < 1$. To ensure that each firm will hire its CEO, i.e. an interior solution exists, we must have $\beta_i^* \leq 1$ for each $i \in [0, 1]$, which is equivalent to $r_i M \sigma_m^2 \leq \sigma_m^2 + \sigma_{\eta_i}^2$ for each $i \in [0, 1]$. Also in each firm, shareholders' payoff must be nonnegative, i.e. (6) is satisfied.

Proof of sufficiency

Let

$$\begin{split} M &= \frac{\int_{0}^{1} \frac{\pi_{i}^{2}}{\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta i}^{2})} di}{1 - \int_{0}^{1} \frac{\lambda_{i} r_{i} \sigma_{m}^{2}}{\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta i}^{2})} di}, \\ W &= \frac{\int_{0}^{1} \left[\frac{\pi_{i}^{4}}{2[\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta i}^{2})]} + \frac{1}{2} \lambda_{i} r_{i}^{2} M^{2} \sigma_{m}^{2} \left[1 - \frac{\lambda_{i} \sigma_{m}^{2}}{\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta i}^{2})} \right] + \underline{u}_{i} \right] di}{1 - \int_{0}^{1} r_{i} di \end{split}$$

Since $\int_0^1 r_i di < 1$, W > 0 and M > 0. Then we can check that

$$\begin{cases} \alpha_{i}^{*} = \frac{(\pi_{i}^{2} - \lambda_{i}r_{i}M\sigma_{m}^{2})(\pi_{i}^{2} + \lambda_{i}r_{i}M\sigma_{m}^{2})}{2[\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta}^{2})]} - \frac{\pi_{i}^{2}(\pi_{i}^{2} + \lambda_{i}r_{i}M\sigma_{m}^{2})^{2}}{[\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta}^{2})]^{2}} + r_{i}W + \frac{1}{2}\lambda_{i}r_{i}^{2}M^{2}\sigma_{m}^{2} + \underline{u}_{i}, \\ \beta_{i}^{*} = \frac{\pi_{i}^{2} + \lambda_{i}r_{i}M\sigma_{m}^{2}}{\pi_{i}^{2} + \lambda_{i}(\sigma_{m}^{2} + \sigma_{\eta}^{2})} \leq 1, \\ a_{i}^{*} = \pi_{i}\beta_{i}^{*}. \end{cases}$$

is the unique equilibrium that satisfies the properties in Definition 2.

Proof of Lemma 2

The shareholders' payoffs are

$$\frac{\pi^2(\pi^2 + \lambda r M \sigma_m^2)}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]} - \left\{ \frac{\pi^4}{2[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]} + rW + \frac{1}{2}\lambda r^2 M^2 \sigma_m^2 \left[1 - \frac{\lambda \sigma_m^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} \right] + \underline{u} \right\},$$

which is quadratic in r. Therefore it is easy to see that if $\frac{\lambda M \sigma_m^2 \pi^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} - W \leq 0, r^* = 0$; if $\frac{\lambda M \sigma_m^2 \pi^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} - W > 0, r^* = \frac{\lambda M \sigma_m^2 \pi^2 - W[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)]}{\lambda M^2 \sigma_m^2 (\pi^2 + \lambda \sigma_\eta^2)} > 0.$

Proof of Lemma 3

Proof of necessity

Given any $\hat{r} < r$, if a CEO of type r chooses the contract $(\alpha(\hat{r}), \beta(\hat{r}))$ and exerts an effort a, then her relative payoff is

$$w - H = \alpha(\hat{r}) + \beta(\hat{r})\pi a + (\beta(\hat{r}) - rM)\tilde{m} + \beta(\hat{r})\eta - rW.$$

Then we can calculated that her expected utility is given by

$$\begin{aligned} U(\hat{r}|r) &= E[w-H] - \frac{1}{2}\lambda Var[w-H] - \frac{1}{2}a^2 \\ &= \alpha(\hat{r}) + \beta(\hat{r})\pi a - rE[W] \\ &- \frac{1}{2}\lambda \left[(\beta(\hat{r}) - rE[M])^2 \sigma_m^2 + \beta(\hat{r})^2 \sigma_\eta^2 + r^2 Var[M] \sigma_m^2 + r^2 Var[W] \right] - \frac{1}{2}a^2. \end{aligned}$$

Thus the optimal effort taken by the CEO is $a = \beta(\hat{r})\pi$ and her expected utility is

$$U(\hat{r}|r) = \alpha(\hat{r}) + \frac{1}{2}\beta(\hat{r})^{2}\pi^{2} - rE[W]$$

$$- \frac{1}{2}\lambda \left[(\beta(\hat{r}) - rE[M])^{2}\sigma_{m}^{2} + \beta(\hat{r})^{2}\sigma_{\eta}^{2} + r^{2}Var[M]\sigma_{m}^{2} + r^{2}Var[W] \right].$$
(16)

By the IC constraint, $U(\hat{r}|r) \leq U(r|r)$ implies that

$$\alpha(r) - \alpha(\hat{r}) \ge \frac{1}{2} [\pi^2 - \lambda(\sigma_m^2 + \sigma_\eta^2)] (\beta(\hat{r})^2 - \beta(r)^2) + \lambda r E[M] \sigma_m^2(\beta(\hat{r}) - \beta(r)).$$
(17)

Similarly, $U(\hat{r}|\hat{r}) \ge U(r|\hat{r})$ implies that

$$\alpha(r) - \alpha(\hat{r}) \le \frac{1}{2} [\pi^2 - \lambda(\sigma_m^2 + \sigma_\eta^2)] (\beta(\hat{r})^2 - \beta(r)^2) + \lambda \hat{r} E[M] \sigma_m^2(\beta(\hat{r}) - \beta(r)).$$
(18)

Thus, combining (17) and (18), and also note that $\hat{r} < r$, we must have $\beta(\hat{r}) \leq \beta(r)$ for any $\hat{r} < r$. So $\frac{\partial\beta(r)}{\partial r} \geq 0$. Dividing the both sides of (17) and (18) by $r - \hat{r}$ and letting $\hat{r} \to r$ yield

$$\lim_{\hat{r}\to r}\frac{\alpha(r)-\alpha(\hat{r})}{r-\hat{r}} = -\left\{\left[\pi^2 - \lambda(\sigma_m^2 + \sigma_\eta^2)\right]\beta(r) + \lambda r E[M]\sigma_m^2\right\}\frac{\partial\beta(r)}{\partial r},$$

i.e. $\frac{\partial \alpha(r)}{\partial r} = -\left\{ \left[\pi^2 - \lambda (\sigma_m^2 + \sigma_\eta^2) \right] \beta(r) + \lambda r E[M] \sigma_m^2 \right\} \frac{\partial \beta(r)}{\partial r}.$

Proof of Sufficiency

Given the two conditions in the lemma, we can calculate that

$$\frac{\partial U(\hat{r}|r)}{\partial \hat{r}} = \lambda E[M]\sigma_m^2(r-\hat{r})\frac{\partial \beta(r)}{\partial r}.$$

Then it is easy to see that $\frac{\partial U(\hat{r}|r)}{\partial \hat{r}}$ is positive when $\hat{r} < r$ and negative when $\hat{r} > r$. So $U(\hat{r}|r)$ is

maximized at $\hat{r} = r$. In other words, $U(\hat{r}|r) \leq U(r|r)$ for any \hat{r} and r, i.e. IC constraint holds.

Proof of Proposition 2

Suppose $U(r_0|r_0) = \min_{r \in [0, r_{\max}]} U(r|r)$, then we have $U(r_0|r_0) = \underline{u}$. Since from (16), we can obtain that

$$\frac{dU(r|r)}{dr} = \lambda E[M]\sigma_m^2\beta(r) - E[W] - \lambda r \left(E[M^2]\sigma_m^2 + Var[W]\right).$$
(19)

For brevity, let $Y(r) = E[W] + \lambda r \left(E[M^2] \sigma_m^2 + Var[W] \right)$. Then the CEO of type r has obtain a utility $U(r|r) = \underline{u} - \int_r^{r_0} \left[\lambda E[M] \sigma_m^2 \beta(r) - Y(r) \right] dr$. Then the expected CEO pay is

$$E[w(r)] = rE[W] + \frac{1}{2}\lambda \left[(\beta(r) - rE[M])^2 \sigma_m^2 + \beta(r)^2 \sigma_\eta^2 + r^2 Var[M] \sigma_m^2 + r^2 Var[W] \right] + \frac{1}{2}\beta(r)^2 \pi^2 + U(r|r) = rE[W] + \frac{1}{2}\lambda \left[(\beta(r) - rE[M])^2 \sigma_m^2 + \beta(r)^2 \sigma_\eta^2 + r^2 Var[M] \sigma_m^2 + r^2 Var[W] \right] (20) + \frac{1}{2}\beta(r)^2 \pi^2 + \underline{u} - \int_r^{r_0} \left[\lambda E[M] \sigma_m^2 \beta(r) - Y(r) \right] dr.$$

Then the shareholders' objective is to maximize the expected firm value net of CEO pay across CEOs of all possible types, i.e. $\max \int_0^{r_{\max}} f(r) \left[\beta(r)\pi^2 - E[w(r)]\right] dr$, which is equal to

$$\begin{split} &\int_{0}^{r_{0}} f(r) \Big\{ \beta(r)\pi^{2} - rE[W] - \frac{1}{2}\beta(r)^{2}\pi^{2} - \underline{u} + \frac{F(r)}{f(r)} \left[\lambda E[M]\sigma_{m}^{2}\beta(r) - Y(r) \right] \\ &- \frac{1}{2}\lambda \left[(\beta(r) - rE[M])^{2}\sigma_{m}^{2} + \beta(r)^{2}\sigma_{\eta}^{2} + r^{2}Var[M]\sigma_{m}^{2} + r^{2}Var[W] \right] \Big\} dr \\ &+ \int_{r_{0}}^{r_{\max}} f(r) \Big\{ \beta(r)\pi^{2} - rE[W] - \frac{1}{2}\beta(r)^{2}\pi^{2} - \underline{u} - \frac{1 - F(r)}{f(r)} \left[\lambda E[M]\sigma_{m}^{2}\beta(r) - Y(r) \right] \\ &- \frac{1}{2}\lambda \left[(\beta(r) - rE[M])^{2}\sigma_{m}^{2} + \beta(r)^{2}\sigma_{\eta}^{2} + r^{2}Var[M]\sigma_{m}^{2} + r^{2}Var[W] \right] \Big\} dr \end{split}$$

Proof of Part 1)

If $r_0 = 0$, then the pointwise optimization of the above objective function yields that the derivative w.r.t $\beta(r)$ is

$$\pi^2 + \lambda \sigma_m^2 E[M] \left[r - \frac{1 - F(r)}{f(r)} \right] - \left[\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2) \right] \beta(r).$$

Let $\beta_1(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r - \frac{1 - F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$. Then the objective function is increasing in $\beta(r)$ if $\beta(r) < 0$

 $\beta_1(r)$; and decreasing in $\beta(r)$ if $\beta(r) > \beta_1(r)$; So the optimal incentives in this case are $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r - \frac{1 - F(r)}{f(r)}\right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_n^2)}$.

 $\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)$

Proof of Part 2)

Similarly, if $r_0 = r_{\text{max}}$, let $\beta_2(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r + \frac{F(r)}{f(r)} \right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$, then the objective function is increasing in $\beta(r)$ if $\beta(r) < \beta_2(r)$; and decreasing in $\beta(r)$ if $\beta(r) > \beta_2(r)$; So the optimal incentives in this case are $\beta^*(r) = \frac{\pi^2 + \lambda \sigma_m^2 E[M] \left[r + \frac{F(r)}{f(r)} \right]}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$.

Proof of Part 3)

If $0 < r_0 < r_{\text{max}}$, then without IC constraint, for $r < r_0$, the optimal incentives are $\beta^*(r) = \beta_2(r)$; and $r > r_0$, the optimal incentives are $\beta^*(r) = \beta_1(r)$. However, the IC constraint requires that $\beta^*(r)$ is increasing in r, and also note that $\beta_1(r) < \beta_2(r)$, so in this case, we cannot get the first-best solution.

Suppose $\beta^*(r)|_{r\in[0,r_{\max}]}$ is the solution. I first show that for $r \ge r_0$, the optimal incentives must have the following form: there exists $r_2 \ge r_0$ such that $\beta^*(r)$ is constant for $r_0 \le r \le r_2$ and $\beta^*(r) = \beta_1(r)$ for $r \ge r_2$. If $\beta^*(r) < \beta_1(r)$ for for any $r \ge r_0$, then the shareholders' payoffs can be improved by increasing $\beta^*(r)$ to $\beta_1(r)$ for each $r \ge r_0$. Thus let r_2 be the minimum $r \in [r_0, r_{\max}]$ that satisfies $\beta^*(r_2) = \beta_1(r_2)$. Then similar argument implies that $\beta^*(r) = \beta_1(r)$ for $r \ge r_2$. For $r_0 \le r \le r_2$, by the definition of r_2 , we must have either 1) $\beta^*(r) < \beta_1(r)$ for any $r_0 \le r \le r_2$; or 2) $\beta^*(r) > \beta_1(r)$ for any $r_0 \le r \le r_2$. In the first case, the shareholders' payoffs can be improved by increasing $\beta^*(r)$ to $\beta_1(r)$. In the second case, if there exists $r_0 \le r_3 < r_2$ such that $\beta^*(r_3) < \beta^*(r_2)$, then the shareholders' payoffs can be improved by reducing $\beta^*(r)$ to $\max(\beta^*(r_3), \beta_1(r))$ for $r \in [r_3, r_2]$. So we must have $\beta^*(r)$ is constant for $r_0 \le r \le r_2$ and $\beta^*(r) = \beta_1(r)$ for $r \ge r_2$.

Similarly, for $r \leq r_0$, the optimal incentives must have the following form: there exists $r_1 \leq r_0$ such that $\beta^*(r) = \beta_2(r)$ for $0 \leq r \leq r_1$ and $\beta^*(r)$ is constant for $r_1 \leq r \leq r_0$. If $\lim_{r \to r_0-} \beta^*(r) < \lim_{r \to r_0+} \beta^*(r)$, then the shareholders' payoffs can be improved by reducing $\beta^*(r)$ around right side of r_0 a little bit. So there must exist $r_1 \leq r_0 \leq r_2$ such that $\beta^*(r) = \beta_2(r)$ for $0 \leq r \leq r_1$, $\beta^*(r)$ is constant for $r_1 \leq r \leq r_2$, and $\beta^*(r) = \beta_1(r)$ for $r \geq r_2$.

Proof of Example 1

Firstly, notice that by Kolmogorov's strong law of large numbers, M converges to E[M] and W

converges to E[W] almost surely. Thus I treat M and W as contants in the following proof and Var[M] = Var[W] = 0. Now I will characterize the two equilibria as follows:

Equilibrium 1

Suppose $r_0 = 0$ (i.e. the CEO of type r = 0 is at the reservation utility), then by Proposition 2, the optimal incentives for firm i are $\beta_{i1}(r) = \frac{\pi^2 + \lambda \sigma_m^2 M_1(2r - r_{\text{max}})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$. Aggregating all the CEO incentives yields that

$$M_1 = \int_0^{r_{\max}} \beta_{i1}(r) di = E_r \left[\frac{\pi^2 + \lambda \sigma_m^2 M_1 (2r - r_{\max})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} \right] = \frac{\pi^2}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}$$

From the proof of Proposition 2, the expected CEO pay in each firm is given by (20). Then aggregating the expected CEO pay from all the firms yields that

$$W_{1} = \frac{1}{2} \left[\pi^{2} + \lambda(\sigma_{m}^{2} + \sigma_{\eta}^{2})\right] \left[1 - \frac{1}{3} \left(\frac{\lambda \sigma_{m}^{2} r_{\max}}{\pi^{2} + \lambda(\sigma_{m}^{2} + \sigma_{\eta}^{2})}\right)^{2}\right] M_{1}^{2} + \underline{u}.$$

 $r_0 = 0$ means that $U(r|r) \ge U(0|0)$ for any $r \in [0, r_{\text{max}}]$. From (19), this is equivalent to

$$\int_0^r \left[\lambda M_1^2 \sigma_m^2 \left(1 + \frac{\lambda \sigma_m^2 (2r - r_{\max})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} \right) - W_1 - \lambda r M_1^2 \sigma_m^2 \right] dr \ge 0, \text{ for any } r \in [0, r_{\max}].$$

Finally, the above condition can be satisfied as long as

$$\lambda M_1^2 \sigma_m^2 \cdot \min\left(1 - \frac{1}{2} r_{\max}, 1 - \frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}\right) - W_1 \ge 0$$

Equilibrium 2

Suppose $r_0 = r_{\text{max}}$ (i.e. the CEO of type $r = r_{\text{max}}$ is at the reservation utility), then by Proposition 2, the optimal incentives for firm *i* are $\beta_{i2}(r) = \frac{\pi^2 + 2r\lambda\sigma_m^2 M_2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}$. Aggregating all the CEO incentives yields that

$$M_2 = \frac{\pi^2}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2) - \lambda \sigma_m^2 r_{\max}}$$

Then $\beta_{i2}(r) = \left(1 + \frac{\lambda \sigma_m^2 (2r - r_{\max})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)}\right) M_2$. Similarly, we can compute that the sum of the expected CEO

pay from all the firms is

$$W_{2} = \frac{1}{1 - r_{\max}} \left[\frac{1}{2} [\pi^{2} + \lambda(\sigma_{m}^{2} + \sigma_{\eta}^{2})] \left[1 - \frac{\lambda \sigma_{m}^{2} r_{\max}(2 - r_{\max})}{\pi^{2} + \lambda(\sigma_{m}^{2} + \sigma_{\eta}^{2})} - \frac{1}{3} \left(\frac{\lambda \sigma_{m}^{2} r_{\max}}{\pi^{2} + \lambda(\sigma_{m}^{2} + \sigma_{\eta}^{2})} \right)^{2} \right] M_{2}^{2} + \underline{u} \right]$$

 $r_0 = r_{\text{max}}$ means that $U(r|r) \ge U(r_{\text{max}}|r_{\text{max}})$ for any $r \in [0, r_{\text{max}}]$. From (19), this is equivalent to

$$\int_{r}^{r_{\max}} \left[\lambda M_2^2 \sigma_m^2 \left(1 + \frac{\lambda \sigma_m^2 (2r - r_{\max})}{\pi^2 + \lambda (\sigma_m^2 + \sigma_\eta^2)} \right) - W_1 - \lambda r M_2^2 \sigma_m^2 \right] dr \le 0, \text{ for any } r \in [0, r_{\max}].$$

The above condition can be satisfied as long as

$$\lambda M_2^2 \sigma_m^2 \cdot \max\left(1 - \frac{1}{2}r_{\max}, 1 - r_{\max} + \frac{\lambda \sigma_m^2 r_{\max}}{\pi^2 + \lambda(\sigma_m^2 + \sigma_\eta^2)}\right) - W_2 \le 0.$$

Proof of Proposition 3

Similar to the proof of Proposition 1.

Proof of Lemma 4

Given the contract $w = \alpha + \beta V$, the CEO's utility is given by

$$U = E[w - H] - \frac{1}{2}\lambda Var[w - H] - \frac{1}{2}a^{2}$$

= $\alpha + \beta \pi a - rW - \frac{1}{2}\lambda [(\beta a - rM)^{2}\sigma_{m}^{2} + (\beta a)^{2}\sigma_{\eta}^{2}] - \frac{1}{2}a^{2}.$

U is concave in a, so taking the derivative w.r.t a yields that the optimal effort is

$$a = \frac{\beta(\pi + \lambda \sigma_m^2 r M)}{\lambda(\sigma_m^2 + \sigma_n^2)\beta^2 + 1}.$$

Then the comparative statics are straight-forward.

Proof of Proposition 4

By Lemma 4, the CEO will exert an effort $a = \frac{\beta(\pi + \lambda r M \sigma_m^2)}{\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 + 1}$ if he is granted a contract $w = \alpha + \beta V$. Note that the base salary α does not affect the CEO's effort decision, it must be set to

make the participation constraint bind, that is

$$\alpha + \beta \pi a - rW - \frac{1}{2}\lambda[(\beta a - rM)^2 \sigma_m^2 + (\beta a)^2 \sigma_\eta^2] - \frac{1}{2}a^2 = \underline{u}.$$

So we have

$$E[\text{CEO pay}] = \alpha + \beta \pi a = rW + \frac{1}{2}\lambda[(\beta a - rM)^2 \sigma_m^2 + (\beta a)^2 \sigma_\eta^2] + \frac{1}{2}a^2 + \underline{u}.$$

Then the shareholders' payoffs are

$$\begin{split} E[V-w] &= \pi a - (\alpha + \beta \pi a) \\ &= \pi a - \left[rW + \frac{1}{2} \lambda [(\beta a - rM)^2 \sigma_m^2 + (\beta a)^2 \sigma_\eta^2] + \frac{1}{2} a^2 + \underline{u} \right] \\ &= \frac{\beta \pi (\pi + \lambda rM \sigma_m^2)}{\lambda (\sigma_m^2 + \sigma_\eta^2) \beta^2 + 1} - \frac{1}{2} \frac{\beta^2 (\pi - \lambda rM \sigma_m^2) (\pi + \lambda rM \sigma_m^2)}{\lambda (\sigma_m^2 + \sigma_\eta^2) \beta^2 + 1} - rW - \frac{1}{2} \lambda (rM)^2 \sigma_m^2 - \underline{u} d\tau \\ \end{split}$$

Taking the derivative w.r.t β yields that

$$\frac{\partial E[V-w]}{\partial \beta} = \frac{\pi(\pi + \lambda r M \sigma_m^2)}{[\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 + 1]^2} \left[-\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 - \left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)\beta + 1 \right].$$

 $\frac{\partial E[V-w]}{\partial \beta} \text{ is increasing in } \beta \text{ when } \beta < \frac{2}{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}} \text{ and decreasing in } \beta \text{ when } \beta > \frac{2}{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}}, \text{ therefore the optimal incentives are } \beta > \frac{2}{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}},$

$$\beta^* = \min\left(\frac{2}{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}}, 1\right).$$

If $rM\sigma_m^2 \ge h(\sigma_m^2 + \sigma_\eta^2)$, then $\beta^* = 1$.

If $rM\sigma_m^2 < h(\sigma_m^2 + \sigma_\eta^2)$, then $\beta^* = \frac{2}{\left(1 - \frac{\lambda rM\sigma_m^2}{\pi}\right) + \sqrt{\left(1 - \frac{\lambda rM\sigma_m^2}{\pi}\right)^2 + 4\lambda(\sigma_m^2 + \sigma_\eta^2)}}$. For the comparative

statics, let $F(\beta, q) = -\lambda(\sigma_m^2 + \sigma_\eta^2)\beta^2 - \left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)\beta + 1$, where q refers to basic parameters. Then $\frac{\partial F}{\partial \beta}|_{\beta^*} < 0$, so from $\frac{\partial F}{\partial \beta}\frac{\partial \beta}{\partial q} + \frac{\partial F}{\partial q} = 0$, $\frac{\partial \beta}{\partial q}|_{\beta^*}$ has the same sign with $\frac{\partial F}{\partial q}|_{\beta^*}$. Then it is straightforward to derive the results in part 1). For part 2), $\frac{\partial F}{\partial \sigma_m^2}|_{\beta^*} = -\lambda\beta^{*2} + \frac{\lambda r\beta^*}{\pi}\left(\frac{\partial M}{\partial \sigma_m^2}\sigma_m^2 + M\right)$. So when $rM + r\sigma_m^2 \frac{\partial M}{\partial \sigma_m^2} < \pi \beta^*$, β^* is decreasing in σ_m^2 ; when $rM + r\sigma_m^2 \frac{\partial M}{\partial \sigma_m^2} > \pi \beta^*$, β^* is increasing in σ_m^2 . Part 3) is obvious.

Proof of Lemma 5

Let $\sigma^2 = \sigma_m^2 + \sigma_\eta^2$. From lemma 4, we can calculate that

$$\frac{da}{dr} = \frac{\lambda\beta\sigma_m^2 M}{\lambda\sigma^2\beta^2 + 1} + (\pi + \lambda\sigma_m^2 r M) \frac{1 - \lambda\sigma^2\beta^2}{(\lambda\sigma^2\beta^2 + 1)^2} \frac{\partial\beta}{\partial r}.$$

Since $-\lambda\sigma^2\beta^2 - \left(1 - \frac{\lambda r M \sigma_m^2}{\pi}\right)\beta + 1 = 0,$

$$\frac{\partial\beta}{\partial r} = \frac{\frac{\lambda M \sigma_m^2 \beta}{\pi}}{2\lambda \sigma^2 \beta + \left(1 - \frac{\lambda r M \sigma_m^2}{h}\right)} = \frac{\frac{\lambda M \sigma_m^2 \beta^2}{\pi}}{\lambda \sigma^2 \beta^2 + 1}.$$

 So

$$\frac{da}{dr} = \frac{\lambda M \beta \sigma_m^2}{\lambda \sigma^2 \beta^2 + 1} \left[1 + (\pi + \lambda \sigma_m^2 r M) \frac{\beta}{\pi} \frac{1 - \lambda \sigma^2 \beta^2}{(\lambda \sigma^2 \beta^2 + 1)^2} \right].$$

Note that $(\pi + \lambda \sigma_m^2 r M) \frac{\beta}{\pi} = \lambda \sigma^2 \beta^2 + 2\beta - 1$,

$$\frac{da}{dr} = \frac{\lambda M \beta \sigma_m^2}{(\lambda \sigma^2 \beta^2 + 1)^3} \left[(\lambda \sigma^2 \beta^2 + 1)^2 - (\lambda \sigma^2 \beta^2 + 2\beta - 1)(1 - \lambda \sigma^2 \beta^2) \right] > 0.$$

So $\frac{da}{dr} > 0$. By the similar argument, $\frac{da}{dM} > 0$.

Proof of Lemma 6

The procedure of solving an equilibrium is as follows:

1) Guess a value for $M = \int_0^1 a_i \beta_i di$, the sum of the sensitivity of CEO pay to the aggregate shock.

2) Then for each firm *i*, we can solve the optimal incentives and CEO's effort from Proposition 4 and Lemma 4. Specifically, given *M*, we have $\beta_i(M) = \frac{2}{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right) + \sqrt{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right)^2 + 4\lambda_i (\sigma_m^2 + \sigma_{\eta i}^2)}}$ and $a_i = \frac{\beta_i(M)(\pi_i + \lambda_i \sigma_m^2 r_i M)}{\lambda_i (\sigma_m^2 + \sigma_{\eta i}^2)\beta_i(M)^2 + 1}$.

3) Plugging the expressions for $\beta_i(M)$ and a_i into $M = \int_0^1 a_i \beta_i di$, we can solve for M.

4) Plugging the solution of M into $\beta_i(M) = \frac{2}{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right) + \sqrt{\left(1 - \frac{\lambda_i \sigma_m^2 r_i M}{\pi_i}\right)^2 + 4\lambda_i (\sigma_m^2 + \sigma_{\eta i}^2)}}$ to check that the incentives are not greater than 1 (ensured by (11)).

5) Bounded efforts: let $a_k = \max_{i \in [0,1]} a_i$. Then by Lemma 4, $a_k = \frac{\beta_k(\pi_k + \lambda_k r_k M \sigma_m^2)}{\lambda_k(\sigma_m^2 + \sigma_{\eta k}^2)\beta_k^2 + 1}$. Since $M = \int_0^1 a_i \beta_i di \leq a_k \int_0^1 \beta_i di \leq a_k$, $a_k \leq \frac{\beta_k(\pi_k + \lambda_k r_k a_k \sigma_m^2)}{\lambda_k(\sigma_m^2 + \sigma_{\eta k}^2)\beta_k^2 + 1}$. Note that $\beta_k \geq \frac{2}{1 + \sqrt{1 + 4\lambda_k(\sigma_m^2 + \sigma_{\eta k}^2)}}$, so by (10), $a_k \leq \frac{\beta_k \pi_k}{\lambda_k \beta_k [\beta_k(\sigma_m^2 + \sigma_{\eta k}^2) - r_k \sigma_m^2] + 1} \leq \pi_k$.

The procedure above is summarized in Lemma 6.

Proof of Example 2

Let β and a denote the optimal incentives and CEO's effort for firms in [0, k]. Let $\bar{\beta}$ and \bar{a} denote the optimal incentives and CEO's effort for firms in [1-k, 1]. Then $M = \int_0^1 a_i \beta_i di = ka\beta + (1-k)\bar{a}\bar{\beta}$. For brevity, let $\sigma^2 = \sigma_m^2 + \sigma_\eta^2$ and $\bar{\sigma}^2 = \sigma_m^2 + \bar{\sigma}_\eta^2$. By Lemma 4, $a = \frac{\beta(\pi + \lambda \sigma_m^2 rM)}{\lambda \sigma^2 \beta^2 + 1}$, and $\bar{a} = \frac{\bar{\beta}(\pi + \lambda \sigma_m^2 \bar{r}M)}{\lambda \bar{\sigma}^2 \bar{\beta}^2 + 1}$. Then

$$M = ka\beta + (1-k)\bar{a}\bar{\beta} = \frac{k\beta^2(\pi + \lambda\sigma_m^2 rM)}{\lambda\sigma^2\beta^2 + 1} + \frac{(1-k)\bar{\beta}^2(\pi + \lambda\sigma_m^2\bar{r}M)}{\lambda\bar{\sigma}^2\bar{\beta}^2 + 1}$$

which implies that

$$M = \frac{\frac{k\beta^2\pi}{\lambda\sigma^2\beta^2+1} + \frac{(1-k)\beta^2\pi}{\lambda\bar{\sigma}^2\bar{\beta}^2+1}}{1 - \lambda\sigma_m^2 \left[\frac{rk\beta^2}{\lambda\sigma^2\beta^2+1} + \frac{\bar{r}(1-k)\bar{\beta}^2}{\lambda\bar{\sigma}^2\bar{\beta}^2+1}\right]}$$

By the proof of Proposition 4, β satisfies that

$$-\lambda\sigma^2\beta^2 - \left(1 - \frac{\lambda\sigma_m^2 rM}{\pi}\right)\beta + 1 = 0.$$
(21)

 $\bar{\beta}$ satisfies that

$$-\lambda\bar{\sigma}^2\bar{\beta}^2 - \left(1 - \frac{\lambda\sigma_m^2\bar{r}M}{\pi}\right)\bar{\beta} + 1 = 0.$$
(22)

From (21) and (22), we obtain that

$$\frac{\bar{r}}{r} = \frac{\lambda \bar{\sigma}^2 \bar{\beta} + 1 - 1/\bar{\beta}}{\lambda \sigma^2 \beta + 1 - 1/\beta}.$$
(23)

Thus we can solve $\bar{\beta}$ in terms of β , i.e. $\bar{\beta} = \bar{\beta}(\beta)$. Then β is a solution to

$$-\lambda\sigma^2\beta^2 - \left(1 - \frac{\lambda\sigma_m^2 r M(\beta)}{\pi}\right)\beta + 1 = 0.$$

Let $F(\beta)$ denote the left side of the above equation. It is obvious that F(0) > 0. Note that $\bar{\beta}(1) = 1$ by (23) and the condition 2) in the lemma (i.e. $\lambda r \bar{\sigma}^2 = \lambda \bar{r} \sigma^2$), then it is easy to check that F(1) > 0 if and only if $\sigma_m^2(1-k) > \frac{\sigma^2}{r} - \sigma_m^2 k$. Similarly, $F(\sqrt{\frac{1}{\lambda\sigma^2}}) < 0$ if and only if

$$\sigma_m^2(1-k)\left(\lambda\sigma^2 + \lambda\sigma^2\frac{\bar{r}}{r}\right)\frac{1}{\lambda\sigma^2\frac{\bar{r}}{r} + \frac{1}{\bar{\beta}(\sqrt{\frac{1}{\lambda\sigma^2}})^2}} < \frac{\sigma^2}{r} - \sigma_m^2k.$$
(24)

From (23), we can solve that $\bar{\beta}(\sqrt{\frac{1}{\lambda\sigma^2}}) = \frac{(\bar{r}-r)+\sqrt{(\bar{r}-r)^2+4\lambda\sigma^2\bar{r}r}}{2\lambda\sigma^2\bar{r}}$. Plugging it into (24) yields that $F(\sqrt{\frac{1}{\lambda\sigma^2}}) < 0$ if and only if $\frac{r+\bar{r}}{2\bar{r}} \left(1 + \frac{\bar{r}-r}{\sqrt{(\bar{r}-r)^2+4\lambda\sigma^2\bar{r}r}}\right) < \frac{\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 rk}{\sigma_m^2 r(1-k)}$. Hence given the four conditions in the lemma, we have F(0) > 0, $F(\sqrt{\frac{1}{\lambda\sigma^2}}) < 0$, and F(1) > 0. Thus for firms in [0, k], there is an equilibrium in which β is less than $\sqrt{\frac{1}{\lambda\sigma^2}}$; and there is another equilibrium in which β is greater than $\sqrt{\frac{1}{\lambda\sigma^2}}$.

Proof of Proposition 5

Given a linear contract $w = \alpha + \beta V$ for CEO 1 and $\bar{w} = \bar{\alpha} + \bar{\beta} \bar{V}$ for CEO 2, CEO 1's effort is $a = \frac{\beta(\pi + \lambda \sigma_m^2 A)}{\lambda(\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 B)\beta^2 + 1}$ by Lemma 7. Also note that CEO 1's RWCs are

$$H = r\bar{w} = r\bar{\alpha} + \Gamma(\bar{\pi} + \tilde{m} + \bar{\eta}),$$

where $\Gamma = r\bar{a}\bar{\beta} = A + Ba\beta = A + \frac{\beta^2(\pi + \lambda\sigma_m^2 A)B}{\lambda(\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 B)\beta^2 + 1}$ and recall that $A = \frac{r\bar{\beta}^2\bar{\pi}}{\bar{\lambda}(\sigma_m^2 + \sigma_\eta^2)\bar{\beta}^2 + 1}$ and $B = \frac{r\bar{\lambda}\bar{\lambda}\bar{\beta}^2\sigma_m^2}{\bar{\lambda}(\sigma_m^2 + \sigma_\eta^2)\bar{\beta}^2 + 1}$. Combining with the binding participation constraint, the shareholders' objective in firm 1 is to maximize

$$\frac{\beta\pi(\pi+\lambda\sigma_m^2A)}{\lambda(\sigma_m^2+\sigma_\eta^2-\sigma_m^2B)\beta^2+1} - \frac{1}{2}\frac{\beta^2(\pi-\lambda\Gamma\sigma_m^2)(\pi+\lambda\Gamma\sigma_m^2)}{\lambda(\sigma_m^2+\sigma_\eta^2)\beta^2+1} - \frac{1}{2}\lambda(\sigma_m^2+\sigma_{\bar{\eta}}^2)\Gamma^2 - \bar{\pi}\Gamma - r\bar{\alpha}.$$

For brevity, let $\sigma^2 = (\sigma_m^2 + \sigma_\eta^2)$ and $\bar{\sigma}^2 = (\sigma_m^2 + \sigma_{\bar{\eta}}^2)$. Taking the first-order condition w.r.t β yields that

$$\begin{aligned} &\frac{\pi(\pi+\lambda\sigma_m^2A)}{\lambda(\sigma^2-\sigma_m^2B)\beta^2+1} - \frac{2\lambda(\sigma^2-\sigma_m^2B)\pi(\pi+\lambda\sigma_m^2A)\beta^2}{[\lambda(\sigma^2-\sigma_m^2B)\beta^2+1]^2} - \frac{\beta(\pi-\lambda\Gamma\sigma_m^2)(\pi+\lambda\Gamma\sigma_m^2)}{(\lambda\sigma^2\beta^2+1)^2} \\ &+ \left[\left(\frac{(\lambda\sigma_m^2)^2\beta^2}{\lambda\sigma^2\beta^2+1} - \lambda\bar{\sigma}^2\right)\Gamma - \bar{\pi} \right] \frac{2(\pi+\lambda\sigma_m^2A)B\beta}{[\lambda(\sigma^2-\sigma_m^2B)\beta^2+1]^2} = 0. \end{aligned}$$

Note that $a = \frac{\beta(\pi + \lambda \sigma_m^2 A)}{\lambda(\sigma_m^2 + \sigma_\eta^2 - \sigma_m^2 B)\beta^2 + 1} = \frac{\beta(\pi + \lambda \sigma_m^2 \Gamma)}{\lambda \sigma^2 \beta^2 + 1}$, the above equation can be simplified to

$$\pi - \frac{2\lambda(\sigma^2 - \sigma_m^2 B)\pi\beta^2}{\lambda(\sigma^2 - \sigma_m^2 B)\beta^2 + 1} - \frac{\beta(\pi - \lambda\Gamma\sigma_m^2)}{\lambda\sigma^2\beta^2 + 1} + \left[\left(\frac{(\lambda\sigma_m^2)^2\beta^2}{\lambda\sigma^2\beta^2 + 1} - \lambda\bar{\sigma}^2 \right)\Gamma - \bar{\pi} \right] \frac{2B\beta}{\lambda(\sigma^2 - \sigma_m^2 B)\beta^2 + 1} = 0.$$

Let $F(\beta, r)$ denote the left side of the above equation. Then since $\frac{\partial F}{\partial \beta} < 0$, $\frac{\partial \beta}{\partial r}$ has the same sign with $\frac{\partial F}{\partial r}$. We can calculate that

$$\begin{split} \frac{\partial F}{\partial r}|_{r=0} &= \frac{2\lambda\pi\beta^2}{(\lambda\sigma^2\beta^2+1)^2} \frac{\bar{r}\bar{\lambda}\bar{\beta}^2(\sigma_m^2)^2}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} + \frac{\lambda\sigma_m^2\beta}{\lambda\sigma^2\beta^2+1} \left[\frac{\bar{\beta}^2\bar{\pi}}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} + \frac{\beta^2\pi}{\lambda\sigma^2\beta^2+1} \frac{\bar{r}\bar{\lambda}\bar{\beta}^2\sigma_m^2}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} \right] \\ &+ \left(\frac{(\lambda\sigma_m^2)^2\beta^2}{\lambda\sigma^2\beta^2+1} - \lambda\bar{\sigma}^2 \right) \frac{\partial}{\partial r} \left[\frac{2\Gamma B\beta}{\lambda(\sigma^2-\sigma_m^2B)\beta^2+1} \right] - \frac{2\bar{\pi}\beta}{\lambda\sigma^2\beta^2+1} \frac{\bar{r}\bar{\lambda}\bar{\beta}^2\sigma_m^2}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} \\ &< \frac{\bar{r}\bar{\lambda}\bar{\beta}^2\sigma_m^2}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} \frac{\beta}{\lambda\sigma^2\beta^2+1} \left[\frac{2\lambda\sigma_m^2\pi\beta}{\lambda\sigma^2\beta^2+1} + \frac{\bar{\pi}\lambda}{\bar{r}\bar{\lambda}} + \frac{\lambda\sigma_m^2\pi\beta^2}{\lambda\sigma^2\beta^2+1} - 2\bar{\pi} \right] \\ &< \frac{\bar{r}\bar{\lambda}\bar{\beta}^2\sigma_m^2}{\bar{\lambda}\bar{\sigma}^2\bar{\beta}^2+1} \frac{\beta}{\lambda\sigma^2\beta^2+1} \left[2\lambda\sigma_m^2\pi + \frac{\bar{\pi}\lambda}{\bar{r}\bar{\lambda}} + \pi - 2\bar{\pi} \right]. \end{split}$$

So if $(2\lambda\sigma_m^2+1)\pi < (2-\frac{\lambda}{\bar{r}\lambda})\bar{\pi}$, then $\frac{\partial F}{\partial r}|_{r=0} < 0$. Thus $\frac{\partial \beta}{\partial r}|_{r=0} < 0$.

Proof of Lemma 8

Since firms are identical, we can assume that in equilibrium, for firms in [0, k], the shareholders are using an option-based contract $(\alpha_O + \beta_O \max(e_1(\tilde{m} + \eta), 0))$ to induce their CEOs' efforts; for firms in [k, 1], the shareholders are using a stock-based contract $\alpha_S + \beta_S(\pi + e_1(\tilde{m} + \eta))$ to induce their CEOs' efforts. Then the sum of all CEOs' wages is $C = \int_0^1 w_i di = k\alpha_O + (1 - k)\alpha_S + k\beta_O \int_0^1 \max(e_1(\tilde{m} + \eta_i), 0)di + (1 - k)\beta_S(\pi + e_1\tilde{m})$ (because $\int_0^1 \eta_i di = 0$).

Then for any representative firm (i.e. I drop subscript *i* for brevity), if shareholders use a stock contract, then if the CEO exerts an effort at a cost *c*, her relative payoff is $w_S - rC = \alpha_S + \beta_S(\pi + e_1(\tilde{m} + \eta)) - rC$. Since $\tilde{m} = \sigma_m$, or $-\sigma_m$ with probability $\frac{1}{2}$ for each, $C = C^+ = k\alpha_O + (1 - k)\alpha_S + k\beta_O E_{\eta}[\max(e_1(\sigma_m + \eta), 0)] + (1 - k)\beta_S(\pi + e_1\sigma_m)]$ with probability $\frac{1}{2}$ and $C = C^- = k\alpha_O + (1 - k)\alpha_S + k\beta_O E_{\eta}[\max(e_1(-\sigma_m + \eta), 0)] + (1 - k)\beta_S(\pi - e_1\sigma_m)]$ with probability $\frac{1}{2}$. Then we can calculate that the mean of $w_S - rC$ is $\alpha_S + \beta_S \pi - r\frac{C^+ + C^-}{2}$, and the variance of $w_S - rC$ is $\left(\beta_S e_1 \sigma_m - r\frac{C^+ - C^-}{2}\right)^2 + \beta_S^2 e_1^2 \sigma_\eta^2$. Therefore the CEO's expected utility under working is $\alpha_S + \beta_S \pi - r\frac{C^+ + C^-}{2} - \frac{1}{2}\lambda \left[\left(\beta_S e_1 \sigma_m - r\frac{C^+ - C^-}{2}\right)^2 + \beta_S^2 e_1^2 \sigma_\eta^2 \right] - c$. Similarly, if the CEO does not work and given that all the other CEOs are working, then her relative pay-

off is $\alpha_S + \beta_S e_0(\tilde{m} + \eta) - rC$. It can be calculated that her expected utility under shirking is $\alpha_S - r\frac{C^+ + C^-}{2} - \frac{1}{2}\lambda \left[\left(\beta_S e_0 \sigma_m - r\frac{C^+ - C^-}{2} \right)^2 + \beta_S^2 e_1^2 \sigma_\eta^2 \right]$. So to guarantee that the CEO is working, we must have

$$\begin{aligned} \alpha_{S} + \beta_{S}\pi - r\frac{C^{+} + C^{-}}{2} - \frac{1}{2}\lambda \left[\left(\beta_{S}e_{1}\sigma_{m} - r\frac{C^{+} - C^{-}}{2} \right)^{2} + \beta_{S}^{2}e_{1}^{2}\sigma_{\eta}^{2} \right] - c \\ \geq \alpha_{S} - r\frac{C^{+} + C^{-}}{2} - \frac{1}{2}\lambda \left[\left(\beta_{S}e_{0}\sigma_{m} - r\frac{C^{+} - C^{-}}{2} \right)^{2} + \beta_{S}^{2}e_{1}^{2}\sigma_{\eta}^{2} \right], \end{aligned}$$

which can be simplified to $\beta_S \pi - \frac{1}{2} \lambda \beta_S^2 \sigma_\eta^2 (e_1^2 - e_0^2) - \frac{1}{2} \lambda \beta_S \sigma_m (e_1 - e_0) [\beta_S (e_1 + e_0) \sigma_m - r(C^+ - C^-)] - c \ge 0$. The base salary α_S must be set to make the participation constraint binding, thus the expected CEO pay is equal to $E[w_S] = r \frac{C^+ + C^-}{2} + \frac{1}{2} \lambda \left[\left(\beta_S e_1 \sigma_m - r \frac{C^+ - C^-}{2} \right)^2 + \beta_S^2 e_1^2 \sigma_\eta^2 \right] + c + \underline{u}$. Therefore, if the shareholders use stock, then their objective is

$$\min_{\beta_S} r \frac{C^+ + C^-}{2} + \frac{1}{2} \lambda \left[\left(\beta_S e_1 \sigma_m - r \frac{C^+ - C^-}{2} \right)^2 + \beta_S^2 e_1^2 \sigma_\eta^2 \right] + c + \underline{u},$$

subject to $\beta_S \pi - \frac{1}{2} \lambda \beta_S^2 \sigma_\eta^2 (e_1^2 - e_0^2) - \frac{1}{2} \lambda \beta_S \sigma_m (e_1 - e_0) [\beta_S (e_1 + e_0) \sigma_m - r(C^+ - C^-)] - c \ge 0.$

Due to the assumption that shareholders will use stock compensation if there is no difference between stock and options, and since each firm is identical, we must have that k = 0 or k = 1. If all the firms use stock (i.e. k = 0), then we have $C^+ - C^- = 2\beta_S e_1 \sigma_m$. Then it is easy to see that the optimal $\beta_S^* \geq 0$ must be the minimum solution to the constraint $\beta_S \pi - \frac{1}{2}\lambda\beta_S^2\sigma_\eta^2(e_1^2 - e_0^2) - \frac{1}{2}\lambda\beta_S\sigma_m(e_1 - e_0)[\beta_S(e_1 + e_0)\sigma_m - r(C^+ - C^-)] - c \geq 0$, which yields that $\beta_S^* = \frac{2c}{\pi + \sqrt{\pi^2 - 2c\lambda[(e_1^2 - e_0^2 - 2re_1(e_1 - e_0))\sigma_m^2 + (e_1^2 - e_0^2)\sigma_\eta^2]}}$. Also note that $\frac{C^+ + C^-}{2} = \alpha_S + \beta_S \pi = E[w_S]$, so we get that the cost of stock is $E[w_S] = \frac{1}{1-r} \left[\frac{1}{2}\lambda[(\beta_S^* e_1\sigma_m)^2(1 - r)^2 + (\beta_S^* e_1\sigma_\eta)^2] + c + \underline{u}\right]$.

If shareholders use options, then we will calculate the cost of options in two cases: $\sigma_{\eta} \leq \sigma_{m}$ or $\sigma_{\eta} > \sigma_{m}$.

Case 1: $\sigma_{\eta} \leq \sigma_{m}$.

In this case, if the CEO works, then we can compute that the mean of $w_O - rC$ is $\alpha_O + \frac{1}{2}\beta_O e_1\sigma_m - r\frac{C^+ + C^-}{2}$, and the variance of $w_O - rC$ is $\left(\frac{1}{2}\beta_O e_1\sigma_m - r\frac{C^+ - C^-}{2}\right)^2 + \frac{1}{2}(\beta_O e_1\sigma_\eta)^2$. Similarly, we can

obtain that shareholders' objective is

$$\min_{\beta_O} r \frac{C^+ + C^-}{2} + \frac{1}{2} \lambda \left[\left(\frac{1}{2} \beta_O e_1 \sigma_m - r \frac{C^+ - C^-}{2} \right)^2 + \frac{1}{2} (\beta_O e_1 \sigma_\eta)^2 \right] + c + \underline{u},$$

subject to $\frac{1}{2}\beta_{O}e_{1}\sigma_{m} - \frac{1}{2}\lambda_{\frac{1}{2}}^{1}(\beta_{O}e_{1}\sigma_{\eta})^{2} - \frac{1}{2}\lambda_{\frac{1}{2}}^{1}\beta_{O}e_{1}\sigma_{m}\left(\frac{1}{2}\beta_{O}e_{1}\sigma_{m} - r(C^{+} - C^{-})\right) - c \ge 0.$

If all the firms use options (i.e. k = 1), then $C^+ - C^- = \beta_O e_1 \sigma_m$ and $\frac{C^+ + C^-}{2} = E[w_O]$. So similarly we can derive the optimal number of options is $\frac{1}{2}\beta_O^* = \frac{2c}{e_1\sigma_m + \sqrt{(e_1\sigma_m)^2 - 2c\lambda[e_1^2(1-2r)\sigma_m^2 + 2e_1^2\sigma_\eta^2]}}$, and the cost of options is $E[w_O] = \frac{1}{1-r} \left[\frac{1}{2}\lambda \left[\left(\frac{1}{2}\beta_O^* e_1\sigma_m \right)^2 (1-r)^2 + 2 \left(\frac{1}{2}\beta_O^* e_1\sigma_\eta \right)^2 \right] + c + \underline{u} \right]$. **Case 2**: $\sigma_\eta > \sigma_m$.

Similarly, we have that the optimal number of options is $\frac{1}{2}\beta_O^* = \frac{2c}{e_1\sigma_\eta + \sqrt{(e_1\sigma_\eta)^2 - 2c\lambda[e_1^2(2-2r)\sigma_m^2 + e_1^2\sigma_\eta^2]}}$, and the cost of options is $E[w_O] = \frac{1}{1-r} \left[\frac{1}{2}\lambda \left[\left(\frac{1}{2}\beta_O^* e_1\sigma_m \right)^2 \left(1 + (1-r)^2 \right) + \left(\frac{1}{2}\beta_O^* e_1\sigma_\eta \right)^2 \right] + c + \underline{u} \right].$

Combining the two cases, we can conclude that the cost of options is

$$E[w_O] = \frac{1}{1-r} \left[\frac{1}{2} \lambda \left[\left(\frac{1}{2} \beta_O^* e_1 \sigma_m \right)^2 (1-r)^2 + \left(\frac{1}{2} \beta_O^* e_1 \sigma_\eta \right)^2 + \left(\frac{1}{2} \beta_O^* e_1 \sigma_{\min} \right)^2 \right] + c + \underline{u} \right],$$

where $\frac{1}{2}\beta_O^* = \frac{2c}{e_1\sigma_{\max} + \sqrt{(e_1\sigma_{\max})^2 - 2c\lambda[e_1^2(1-2r)\sigma_m^2 + e_1^2\sigma_\eta^2 + e_1^2\sigma_{\min}^2]}}, \sigma_{\min} = \min(\sigma_m, \sigma_\eta), \sigma_{\max} = \max(\sigma_m, \sigma_\eta).$

Then it is easy to see that the ratio of the risk premium of one unit of stock to the risk premium of one unit of options is $\kappa(r) = \frac{4[(1-r)^2\sigma_m^2 + \sigma_\eta^2]}{(1-r)^2\sigma_m^2 + \sigma_\eta^2 + \sigma_{\min}^2}$.

Comparing the cost of stock with the cost of options, there exists $\hat{\pi}$ such that all the firms will use stock if and only if $\pi \geq \hat{\pi}$.

$$\hat{\pi} = \frac{1}{4} e_1 \sigma_{\max} \sqrt{\kappa(r)} \left[\left(1 + \frac{4Q}{\kappa(r)P} \right) + \left(1 - \frac{4Q}{\kappa(r)P} \right) \sqrt{1 - \frac{4Pc}{e_1^2 \sigma_{\max}^2}} \right],\tag{25}$$

where $P = \frac{1}{2}\lambda[e_1^2\sigma_m^2(1-2r) + e_1^2\sigma_\eta^2 + e_1^2\sigma_{\min}^2], Q = \frac{1}{2}\lambda[(e_1^2 - e_0^2 - 2re_1(e_1 - e_0))\sigma_m^2 + (e_1^2 - e_0^2)\sigma_\eta^2].$

Proof of Part 1)

As $\frac{\sigma_{\eta}}{\sigma_m}$ goes to 0, $\frac{\partial \kappa(r)}{\partial r}$ goes to 0. Thus it can be calculated that

$$\frac{\partial \hat{\pi}}{\partial r} = \frac{e_0 \sigma_m}{2(1 + \sqrt{1 - 2c\lambda(1 - 2r)})^2 \sqrt{1 - 2c\lambda(1 - 2r)}} \left(\sqrt{1 - 2c\lambda(1 - 2r)} + 1 - 2\lambda c + 2\lambda cr + \frac{e_0 \lambda c}{e_1}\right) > 0$$

Proof of Part 2)

Note that $\left(1 + \frac{4Q}{\kappa(r)P}\right) + \left(1 - \frac{4Q}{\kappa(r)P}\right)\sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}$ can be rewritten as

$$D(r) = 1 + \sqrt{1 - \frac{4Pc}{e_1^2 \sigma_{\max}^2}} + \frac{16Qc}{\kappa(r)e_1^2 \sigma_{\max}^2} \frac{1}{1 + \sqrt{1 - \frac{4Pc}{e_1^2 \sigma_{\max}^2}}}$$

So the sign of $\frac{\partial \hat{\pi}}{\partial r}$ is the same as the sign of $\frac{\partial \sqrt{\kappa(r)}D(r)}{\partial r} = \frac{1}{2\sqrt{\kappa(r)}} \frac{\partial \kappa(r)}{\partial r} D(r) + \sqrt{\kappa(r)} \frac{\partial D(r)}{\partial r}$. Note that $\frac{\partial \kappa(r)}{\partial r} < 0$ and $\frac{r}{1+\sqrt{1-\frac{4Pc}{e_1^2\sigma_{\max}^2}}}$ is increasing in r. We can obtain that

$$\begin{aligned} \frac{\partial\sqrt{\kappa(r)}D(r)}{\partial r} &< \frac{1}{2\sqrt{\kappa(r)}}\frac{\partial\kappa(r)}{\partial r} \left[1 + \frac{16Qc}{\kappa(r)e_1^2\sigma_{\max}^2} \frac{1}{1 + \sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}} \right] \\ &+ \sqrt{\kappa(r)} \left[\frac{2\lambda c\sigma_m^2}{\sigma_{\max}^2} \frac{1}{\sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}} - \frac{16Qc}{\kappa(r)^2 e_1^2 \sigma_{\max}^2} \frac{1}{1 + \sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}} \frac{\partial\kappa(r)}{\partial r} \right] \\ &= \frac{1}{2\sqrt{\kappa(r)}}\frac{\partial\kappa(r)}{\partial r} \left[1 - \frac{16Qc}{\kappa(r)e_1^2 \sigma_{\max}^2} \frac{1}{1 + \sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}} \right] + \sqrt{\kappa(r)}\frac{2\lambda c\sigma_m^2}{\sigma_{\max}^2} \frac{1}{\sqrt{1 - \frac{4Pc}{e_1^2\sigma_{\max}^2}}} \right] \end{aligned}$$

Since $2 \leq \kappa(r) \leq 4$, and $Q \leq \frac{1}{2}\lambda(e_1^2 - e_0^2)(\sigma_m^2 + \sigma_\eta^2)$, $\frac{\partial\sqrt{\kappa(r)}D(r)}{\partial r} < 0$ if the following condition is satisfied.

$$\left(\frac{1}{2}-4\lambda c\right)\frac{(1-r)\sigma_m^2\sigma_{\min}^2}{\left[(1-r)^2\sigma_m^2+\sigma_\eta^2+\sigma_{\min}^2\right]\left[(1-r)^2\sigma_m^2+\sigma_\eta^2\right]} > \frac{\lambda c}{\sqrt{1-6\lambda c}}.$$

Suppose that $\sigma_m^2 \ge \sigma_\eta^2$, then the above condition is satisfied if $\frac{(1-r)\sigma_\eta^2}{2[(1-r)^2\sigma_m^2+\sigma_\eta^2]} > X$, where $X = \frac{2\lambda c}{(1-8\lambda c)\sqrt{1-6\lambda c}}$. This can be simplified to $\sigma_\eta^2 > \frac{2X(1-r)^2\sigma_m^2}{(1-r)-4X}$.

Proof of Part 3)

As $\frac{\sigma_m}{\sigma_\eta}$ goes to 0, $\kappa(r) \to 4$, $\frac{P}{e_1^2 \sigma_{\max}^2} \to \frac{1}{2}\lambda$, and $\frac{Q}{P} \to \frac{e_1^2 - e_0^2}{e_1^2}$. So from (25), we can see that $\frac{\partial \hat{\pi}}{\partial r}$ goes to 0 as well.

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