Online Appendix for "Governing Through Communication and Intervention"

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The Online Appendix has two sections. The first section contains supplemental results for Section 3. The second section contains two extensions to the baseline model: continuum of actions and verifiable information.

A Supplemental results for Section 3

Proposition 7 *A non-influential equilibrium always exists. In any non-influential equilibrium there are b_{NI}^{**} and $\mu_{NI}^{**} \in (0,1)$ such that following hold: the agent chooses action L if and only if $\beta \leq b_{NI}^{**}$. If the agent chooses action L then the principal intervenes if and only if $\theta > c_P$, and upon failed intervention the agent revises his initial decision with probability one. If the agent chooses action R then the principal intervenes if and only if $\theta < -\frac{c_P}{1-(1-\lambda)\mu_{NI}^{**}}$, and upon failed intervention the agent revises his initial decision with probability $1-\mu_{NI}^{**}$, where μ_{NI}^{**} solves $\mu_{NI}^{**} = \phi\left(\mu_{NI}^{**}, b_{NI}^{**}\right)$ and ϕ is given by (19).*

Proof. Let μ_{a_A} be the probability that $a_F = R$ if $a_A \in \{L, R\}$ and intervention fails. Suppose $a_A = L$. The principal gets $\lambda\theta + (1-\lambda)\mu_L\theta - c_P$ if she intervenes, and zero otherwise. Therefore, $e = 1 \Leftrightarrow \theta > \frac{c_P}{\lambda+(1-\lambda)\mu_L}$. Since $\mathbb{E}\left[\theta + \beta | \theta > \frac{c_P}{\lambda+(1-\lambda)\mu_L}\right] > 0$ for all $\beta > 0$ and $\mu_L \in [0,1]$, then in any equilibrium it must be $\mu_L = 1$. The agent's expected utility from $a_A = L$ is

$$\Pr\left[\theta > c_P\right]\left(\mathbb{E}\left[\theta + \beta | \theta > c_P\right] - \lambda c_A\right). \tag{39}$$

Suppose $a_A = R$. The principal gets $(1-\lambda)\mu_R\theta - c_P$ if she intervenes, and θ otherwise. Therefore, $e = 1 \Leftrightarrow \theta < -\frac{c_P}{1-(1-\lambda)\mu_R}$. If $e = 1$ and $\chi = 0$, then $a_F = R \Leftrightarrow \mathbb{E}\left[\theta + \beta | \theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right] \geq$

0. The agent's expected utility from $a_A = R$ is

$$\Pr\left[\theta \geq -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right]\mathbb{E}\left[\theta+\beta|\theta \geq -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right] \tag{40}$$
$$+\Pr\left[\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right]\left(-\lambda c_A + (1-\lambda)\max\left\{0, \mathbb{E}\left[\theta+\beta|\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right]\right\}\right).$$

Comparing (40) and (39), the agent chooses $a_A = L$ if and only if

$$\beta \leq \min\left\{-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right], \eta'(\mu_R)\right\} \tag{41}$$

or

$$-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right] < \beta \leq \delta'(\mu_R), \tag{42}$$

where

$$\eta'(\mu_R) = \lambda c_A \frac{\Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right] - \Pr\left[\theta > c_P\right]}{\Pr\left[-\frac{c_P}{1-(1-\lambda)\mu_R} < \theta < c_P\right]} - \mathbb{E}\left[\theta| -\frac{c_P}{1-(1-\lambda)\,\mu_R} < \theta < c_P\right]$$

$$\delta'(\mu_R) = \lambda c_A \frac{\Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right] - \Pr\left[\theta > c_P\right]}{\Pr\left[\theta < c_P\right] - \lambda \Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right]}$$
$$+\frac{\lambda \Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right]\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right] - \Pr\left[\theta < c_P\right]\mathbb{E}\left[\theta|\theta < c_P\right]}{\Pr\left[\theta < c_P\right] - \lambda \Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right]}.$$

Consider the following condition,

$$\lambda c_A \frac{\Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right] - \Pr\left[\theta > c_P\right]}{\Pr\left[\theta < c_P\right]} > \mathbb{E}\left[\theta|\theta < c_P\right] - \mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right]. \tag{43}$$

It can be verified that if (43) holds then

$$-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R}\right] < \eta'(\mu_R) < \delta'(\mu_R), \tag{44}$$

and otherwise,

$$\delta'(\mu_R) < \eta'(\mu_R) < -\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right]. \tag{45}$$

Define

$$\begin{aligned} l'(x) &\equiv \Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)x}\right] - \Pr[\theta > c_P] \\ h'(x) &\equiv \frac{\Pr[\theta < c_P]}{\lambda} \frac{\mathbb{E}[\theta|\theta < c_P] - \mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)x}\right]}{l'(x)}. \end{aligned}$$

Condition (43) holds if and only if

$$l'(\mu_R) > 0 \text{ and } c_A > h'(\mu_R).$$

Note that $l'(x)$ decreases in x and $h'(x)$ increases in x, and hence, (43) holds if and only if

$$\mu_R < \overline{\mu} \equiv \min\left\{l'^{-1}(0), h'^{-1}(c_A)\right\}.$$

Note that $\overline{\mu}$ can be greater than one or smaller than zero. Based on (41) and (42):

- If $\mu_R \leq \overline{\mu}$, then (43) holds and the agent chooses $a_A = L$ if and only if $\beta \leq \delta'(\mu_R)$. Note that $-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right] \leq \delta'(\mu_R)$ implies $\mu_R = 1$. Therefore, it has to be $\overline{\mu} \geq 1$.
- If $\mu_R > \overline{\mu}$, then (43) is violated and the agent chooses $a_A = L$ if and only if $\beta \leq \eta'(\mu_R)$. Since $\eta'(\mu_R) < -\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right]$, μ_R must also solve $\mu_R = \phi(\mu_R, \eta(\mu_R))$. Therefore, it has to be $\overline{\mu} < 1$. Suppose $\overline{\mu} < 1$. Then, $\eta'(1) < -\mathbb{E}\left[\theta|\theta < -\frac{c_P}{\lambda}\right]$, and hence, $\phi(1, \eta'(1)) < 1$. If $\mu_R = \overline{\mu} \geq 0$ then $\eta(\overline{\mu}) = -\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\overline{\mu}}\right]$, and hence, $\phi(\overline{\mu}, \eta'(\overline{\mu})) = 1$. If $\mu_R = 0 > \overline{\mu}$ then $\eta'(0) < -\mathbb{E}[\theta|\theta < -c_P]$, and hence, $\phi(0, \eta'(0)) > 0$. Either way, if $\overline{\mu} < 1$ then $\mu_R = \phi(\mu_R, \eta'(\mu_R))$ has a solution in $[\max\{0, \overline{\mu}\}, 1]$.

We conclude, there are two cases:

1. If $\overline{\mu} \geq 1$, then the agent chooses $a_A = L$ if and only if $\beta \leq \delta'(1)$, where $\mu_{NI}^{**} = 1$. Note that $b_{NI}^{**} \equiv \delta'(1) > 0$.

2. If $\overline{\mu} < 1$, then the agent chooses $a_A = L$ if and only if $\beta \leq \eta'(\mu^{**}_{NI})$, where $1 > \mu^{**}_{NI} > \max\{0, \overline{\mu}\}$ and μ^{**}_{NI} solves $\mu^{**}_{NI} = \phi(\mu^{**}_{NI}, \eta'(\mu^{**}_{NI}))$. Note that $b^{**}_{NI} = \eta'(\mu^{**}_{NI})$.

Since f is symmetric and $\mathbb{E}[\theta] \geq 0$, $\Pr\left[\theta \leq -\frac{c_P}{1-(1-\lambda)\mu_R}\right] - \Pr[\theta > c_P] \leq 0$ for all $\mu_R \in [0,1]$. Therefore, $\overline{\mu} = -\infty$, (43) is violated and only the second case applies, that is, $b^{**}_{NI} = \eta'(\mu^{**}_{NI})$. Note that $\eta'(\mu^{**}_{NI})$ can be positive or negative. Moreover, since $\overline{\mu} = -\infty$, $\mu^{**}_{NI} < 1$. ■

Lemma 4 *If the equilibrium is influential according to Definition 3, then it is also influential according to Definition 1.*

Proof. Suppose on the contrary there is an equilibrium that is influential according to Definition 3 but it is not influential according to Definition 1. Therefore, for any $m_1, m_2 \in M$ and $\beta \in [0, \overline{\beta}]$, $a_A(m_1, \beta) = a_A(m_2, \beta)$. Moreover, there are $m_1, m_2 \in M$ and β_0 such that $a_F(m_1, \beta_0) \neq a_F(m_2, \beta_0)$. Similar to the arguments in the proof of Proposition 7, $\mu_L = 1$. Since $a_A(m, \beta)$ is invariant to m but $a_F(m_1, \beta_0) \neq a_F(m_2, \beta_0)$, it has to be $a_A(\beta, m) = R$ for all $\beta \in [0, \overline{\beta}]$ and $m \in M$. Suppose that, without the loss of generality

$$\Pr[a_F = R | a_A = R, e = 1, \chi = 0, m_1] < \Pr[a_F = R | a_A = R, e = 1, \chi = 0, m_2].$$

Note that if $a_A = R$ and $e = 1$ then $\theta < 0$. Therefore, the principal strictly prefers m_1 over m_2, a contradiction to $m_2 \in M$. ■

Lemma 5 *Suppose $\lambda = 0$ and $c_P < -\underline{\theta}\frac{G(-\underline{\theta}) - G(-\mathbb{E}[\theta|\theta<0])}{1-G(-\mathbb{E}[\theta|\theta<0])}$. An influential equilibrium in which intervention is off the equilibrium path does not survive the Grossman and Perry (1986) criterion. Moreover, an influential equilibrium that survives the Grossman and Perry (1986) criterion always exists, and it satisfies Proposition 5 part (ii.b).*

Proof. Consider an equilibrium in which intervention is off the equilibrium path. In this equilibrium, the agent always follows the recommendation to choose action R. The agent follows the recommendation to choose action L if and only if $\beta \leq -\mathbb{E}[\theta|\theta < 0]$. Suppose $\theta < 0$ and the principal recommends the agent to choose action L but the agent decides on R. Let

$$\hat{\mu}_R = \frac{1 - G\left(-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-\hat{\mu}_R}\right]\right)}{1 - G(-\mathbb{E}[\theta|\theta < 0])}, \tag{46}$$

and note that since $c_P < -\underline{\theta}\frac{G(-\underline{\theta})-G(-\mathbb{E}[\theta|\theta<0])}{1-G(-\mathbb{E}[\theta|\theta<0])}$ then a solution such that $\underline{\theta} < -\frac{c_P}{1-\hat{\mu}_R}$ always exists. Consider the following deviation: the principal intervenes if and only if $\theta < -\frac{c_P}{1-\hat{\mu}_R}$. If the agent expects that upon deviation the principal intervenes if and only if $\theta < -\frac{c_P}{1-\hat{\mu}_R}$, the agent has incentives to revise the decision from R to L if and only if $\beta \leq -\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-\hat{\mu}_R}\right]$. Given this behavior, the principal has incentives to deviate and intervene if and only if $\theta < -\frac{c_P}{1-\hat{\mu}_R}$. Indeed, since $\hat{\mu}_R$ solves (46), if the principal deviates and intervenes, she expects the agent to revise his decision with probability $1-\hat{\mu}_R$. Therefore, the benefit from intervention is $\hat{\mu}_R\theta - c_A$. If the principal does not intervene, then her payoff is θ. Therefore, the principal intervenes if and only if $\theta < -\frac{c_P}{1-\hat{\mu}_R}$. The existence of this deviation violates the Grossman and Perry (1986) criterion.

Next, note that any equilibrium that is described by Proposition 5, both $e = 1$ and $e = 0$ are on the equilibrium path, and hence, the Grossman and Perry (1986) criterion is trivially satisfied. Based on the proof of Proposition 5, if $\lambda = 0$ then $c_A < h(\mu_R)$ for any $\mu_R \in [0, 1)$. Clearly, if $\mu_R^{**} = 1$ then the $\lambda = 0$ implies that the principal never intervenes. Therefore, if there is an influential equilibrium in which $e = 1$ is on the equilibrium path, then b^{**} and $\mu_R^{**} < 1$ must satisfy part (ii.b). Since $c_P < -\underline{\theta}\frac{G(-\underline{\theta})-G(-\mathbb{E}[\theta|\theta<0])}{1-G(-\mathbb{E}[\theta|\theta<0])}$ then

$$\mu_R = \frac{1 - G\left(-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-\mu_R}\right]\right)}{1 - G\left(-\mathbb{E}\left[\theta| -\frac{c_P}{1-\mu_R} < \theta < 0\right]\right)} \tag{47}$$

has a solution where $\underline{\theta} < -\frac{c_P}{1-\mu_R}$, μ_R^{**} is given by this solution and b^{**} is given by (20). By construction, an influential equilibrium as in Proposition 5 part (ii.b) indeed exists. ■

Proposition 8 *All the equilibria in Proposition 5 continue to exist when the agent is allowed to revise his initial decision when $e = 0$.*

Proof. Consider an equilibrium as described by Proposition 5. Suppose $m \in M_R$. In any equilibrium described by Proposition 5, $a_A = R$ for sure. Since $m \in M_R \Rightarrow \theta > 0$, based on (3), the agent has incentives to maintain his original decision even if the principal does not intervene, and as in Proposition 5, $e = 0$ for sure. Suppose $m \in M_L$. Based on Proposition 5, $a_A = R \Leftrightarrow \beta > b^{**}$, and $e = 1$ if and only if $a_A = R$ and $\theta < -\frac{c_P}{1-(1-\lambda)\mu_R^{**}}$. If $a_A = L$ then

$e = 0$ for sure. The agent does not infer new information from $e = 0$ and his expected payoff is therefore zero. If $a_A = R$ and $e = 0$ then the agent infers $-\frac{c_P}{1-(1-\lambda)\mu_R^{**}} < \theta < 0$, and he revises his decision to L if and only if

$$\beta \leq -\mathbb{E}\left[\theta | -\frac{c_P}{1-(1-\lambda)\,\mu_R^{**}} < \theta < 0\right]. \tag{48}$$

Suppose $\beta > b^{**}$. From the proof of Proposition 5, it can be verified that the right hand side of (48) is strictly smaller than b^{**}. Therefore, $\beta > b^{**}$ implies that (48) never holds. That is, the agent has no incentives to revise his decision to L if $e = 0$. The agents's expected payoff from choosing R is given by (32). By construction, $\beta > b^{**}$ implies that (32) is non-negative and the agent is better off choosing action R as prescribed by Proposition 5.

Suppose $\beta \leq b^{**}$. If (48) is violated then the agent has no incentives to revise his decision to L if $e = 0$, and his expected payoff from choosing R is given by (32). By construction, $\beta < b^{**}$ implies that (32) is negative. Therefore, the agent is better off choosing action L as prescribed by Proposition 5. If (48) holds then the agent revises his decision to L if $e = 0$. Note that (48) implies $\mathbb{E}\left[\theta + \beta | \theta < -\frac{c_P}{1-(1-\lambda)\mu_R^{**}}\right] \leq 0$. Therefore, if $e = 1$ and $\chi = 0$ then the agent always revises his decision to L. The agent's expected payoff from choosing R is $-\lambda c_A \Pr\left[\theta < -\frac{c_P}{1-(1-\lambda)\mu_R^{**}} | \theta < 0\right] < 0$. Therefore, the agent is better off choosing action L as prescribed by Proposition 5.

To conclude, the agent's initial decision as prescribed by Proposition 5 does not change if he has option to revise it when $e = 0$. Therefore, the principal's communication and intervention strategies, as prescribed Proposition 5, are also incentive compatible, and all the equilibria in Proposition 5 continue to exist as required. ■

Proposition 9 *With voluntary revision, intervention harms communication if and only if*

$$c_A < \frac{1}{\lambda}\left(\mathbb{E}\left[\theta | \theta < 0\right] - \mathbb{E}\left[\theta | \theta < -\frac{c_P}{1-(1-\lambda)\,\mu_R^{**}\left(c_A, c_P\right)}\right]\right), \tag{49}$$

where $\mu_R^{**}\left(c_A, c_P\right)$ is given by Proposition 5 part (ii.b).

Proof. Based on the proof of Proposition 5, if $c_A \geq h(1)$, then $b^{**} = \delta(1, c_A) > 0$. Note that $\delta(1, h(1)) = -\mathbb{E}\left[\theta | \theta < -\frac{c_P}{\lambda}\right]$. Since $c_A > h(1) \Rightarrow \delta(1, c_A) > \delta(1, h(1))$, and since $-\mathbb{E}\left[\theta|\theta < -\frac{c_P}{\lambda}\right] > -\mathbb{E}[\theta|\theta < 0]$, $c_A \geq h(1)$ implies $\delta(1, c_A) > -\mathbb{E}[\theta|\theta < 0]$, and intervention enhances communication. If $c_A < h(1)$, then $b^{**} = \eta(\mu_R^{**}(c_A), c_A)$ and

$$\eta(\mu_R^{**}(c_A), c_A) > -\mathbb{E}[\theta|\theta < 0] \Leftrightarrow c_A > \frac{\mathbb{E}[\theta|\theta < 0] - \mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right]}{\lambda},$$

and note that

$$\frac{\mathbb{E}[\theta|\theta < 0] - \mathbb{E}\left[\theta|\theta < -\frac{c_P}{1-(1-\lambda)\mu_R}\right]}{\lambda} < \frac{\Pr[\theta < 0]}{\Pr\left[\theta < -\frac{c_P}{\lambda}\right]} \frac{\mathbb{E}[\theta|\theta < 0] - \mathbb{E}\left[\theta|\theta < -\frac{c_P}{\lambda}\right]}{\lambda} = h(1).$$

Combined, intervention enhances communication if and only if (49) holds. ■

B Extensions

B.1 Continuum of actions

Consider a variant of the baseline model in which the action space is a continuum. Specifically, suppose $a \in \mathbb{R}$ and let

$$v(\theta, a) = -(\theta - a)^2. \tag{50}$$

For simplicity, I assume that the agent's bias β is a common knowledge and strictly positive. I make the following assumptions about intervention. First, intervention is always successful, that is, $\lambda = 1$. Second, if the agent chooses a_A and the principal intervenes and chooses a_P, the principal incurs an additional cost of $c_P(a_P - a_A)^2$ and the agent incurs an additional cost of $c_A(a_P - a_A)^2$, where $c_P \geq 0$ and $c_A \geq 0$. These functional forms capture the idea that as $|a_P - a_A|$ increases, both the principal and the agent incur larger costs due to intervention. As in the baseline model, I denote by $a_A(m)$ the agent's action strategy and by $\rho(\theta)$ the principal's messaging strategy. I also denote by $\Delta(a_A, \theta)$ the difference between a_P and a_A when principal intervenes, as a function of a_A and θ.

Proposition 10 *Let $\Lambda(\beta, c_P, c_A)$ be the set of equilibria of the game. In any equilibrium, $\Delta^*(a_A, \theta) = \frac{\theta - a_A}{1+c_P}$. Moreover,*

$$\left(a_A^*(m), \rho^*(\theta), \Delta^*(a_A, \theta)\right) \in \Lambda(\beta, c_P, c_A)$$

if and only if

$$\left(a_A^*(m), \rho^*(\theta), \Delta^*(a_A, \theta)\right) \in \Lambda\left(\beta \times \frac{c_P + c_P^2}{c_A + c_P^2}, \infty, c_A\right).$$

Proof. Given the agent's decision a_A and the observation of θ, regardless of the message that the principal sent the agent, the principal solves

$$\begin{aligned} \Delta(a_A, \theta) &\in \arg\max_{\Delta} \left\{-(\theta - (a_A + \Delta))^2 - c_P \Delta^2\right\} \\ &\Rightarrow \Delta(a_A, \theta) = \frac{\theta - a_A}{1 + c_P}. \end{aligned}$$

Thus, if the agent chooses action a_A, the principal's utility conditional on θ is

$$\begin{aligned} u_P &= -(\theta - (a_A + \Delta(a_A, \theta)))^2 - c_P \Delta(a_A, \theta)^2 \\ &= -\frac{c_P}{1 + c_P}(\theta - a_A)^2. \end{aligned}$$

The agent expects the principal to follow intervention policy $\Delta(a_A, \theta)$, and therefore, given message m, he solves

$$\begin{aligned} a_A^* &\in \arg\max_{a_A} \mathbb{E}\left[-(\theta + \beta - (a_A + \Delta(a_A, \theta)))^2 - c_A \Delta(a_A, \theta)^2 \,|m\right] \\ &\Rightarrow a_A^* = \mathbb{E}[\theta|m] + \beta \frac{c_P + c_P^2}{c_A + c_P^2}. \end{aligned}$$

It follows, at the communication stage, the principal behaves as if her preferences are represented by the utility function $-(\theta - a_A)^2$, and the agent behaves as if $c_P = \infty$ and his preferences are represented by the utility function $-\left(\theta + \beta \frac{c_P + c_P^2}{c_A + c_P^2} - a_A\right)^2$. ■

Proposition 10 implies that the quality of communication between the principal and the agent in equilibrium is equivalent to the quality of communication when intervention is not possible and the agent's bias is $\beta\frac{c_P+c_P^2}{c_A+c_P^2}$ instead of β. Note that Crawford and Sobel's (1982) setup with a quadratic loss function is a special case of this model when $c_P=\infty$. Therefore, intervention harms communication if and only if

$$\frac{c_P+c_P^2}{c_A+c_P^2}>1\Leftrightarrow c_P>c_A.$$

B.2 Verifiable information

Consider a variant of the baseline model in which θ is verifiable. I argue that intervention can harm the principal's ability to affect the agent's decision through disclosure in this setup as well. When information is verifiable, $\rho(\theta)\in\{\theta,\phi\}$, where $\rho=\phi$ is interpreted as the principal's decision not to disclose information, and $\rho(\theta)=\theta$ is the principal's decision to disclose the exact value of θ. To keep the analysis simple, I assume $\lambda=1$ and $c_A>0$.

Suppose the principal discloses θ. If $\theta\leq -c_P$, then the principal intervenes if and only if the agent chooses R. Since $c_A>0$, the agent will avoid intervention and choose L. If $\theta>-c_P$, the principal intervenes if and only if the agent chooses L and $\theta>c_A$. However, according to (3), the agent will choose L if and only if $\theta<-\beta$. To conclude,

$$\Pr\left[a_A=R|\rho(\theta)=\theta\right]=\begin{cases}0 & \textit{if } \theta\leq -c_P\\ \Pr\left[\theta\geq-\beta\right] & \textit{if } \theta>-c_P,\end{cases}\tag{51}$$

and note that if $\rho(\theta)=\theta$, then the principal never intervenes. The next result characterizes the equilibria of the game with verifiable information.

Proposition 11 *Let $\Upsilon^*\equiv\{\theta:\rho(\theta)=\phi\}$ and $\varphi^*=\Pr\left[a_A=R|\rho=\phi\right]$. In any equilibrium the principal intervenes with zero probability. Moreover:*

(i) For any $\Theta\subseteq\left[0,\overline{\theta}\right]$ there is an equilibrium in which $\Upsilon^*=\Theta$, $\varphi^*=1$ and $a_A=L$ if and only if $\theta<\max\{-c_P,-\beta\}$.

(ii) An equilibrium with $\varphi^* = 0$ exists if and only if $\overline{\beta} \leq \bar{b}_{\text{ver}}$, where

$$\bar{b}_{\text{ver}} \equiv \max_{\Upsilon:\left[\max\left\{-c_P,-\overline{\beta}\right\},0\right]\subseteq\Upsilon\subseteq\left[\underline{\theta},0\right]} \left\{ c_A \frac{\Pr\left[\theta < -c_P | \theta \in \Upsilon\right]}{1 - \Pr\left[\theta < -c_P | \theta \in \Upsilon\right]} - \mathbb{E}\left[\theta | \theta \geq -c_P, \theta \in \Upsilon\right] \right\}. \tag{52}$$

In this equilibrium, $\left[\max\left\{-c_P, -\overline{\beta}\right\}, 0\right] \subseteq \Upsilon^* \subseteq \left[\underline{\theta}, 0\right]$ and $a_A = L$ if and only if $\theta < 0$.

(iii) No other equilibrium exists.

Proof. First, consider an equilibrium with $\varphi^* = 1$. If $\theta < 0$, the principal strictly prefers disclosing θ and thereby reducing the probability that R is chosen from one to $\Pr\left[a_A = R | \rho\left(\theta\right) = \theta\right] < 1$ as given by (51). If $\theta \geq 0$ the principal is indifferent with respect to her disclosure policy, since in both cases the agent chooses R with probability one. Moreover, the principal has no incentives to intervene if $\theta > 0$ and $a_A = R$. Therefore, for any $\Upsilon \subseteq \left[0, \overline{\theta}\right]$, if $\rho = \phi$, then the agent infers that $\theta > 0$ for sure, and according to (3), he strictly prefers choosing R. Overall, in this equilibrium, $a_A = L$ if and only if $\theta < \max\left\{-c_P, -\beta\right\}$, and the principal never intervenes.

Second, consider an equilibrium with $\varphi^* \in (0, 1)$. If $\theta < \max\left\{-c_P, -\overline{\beta}\right\}$ and $\rho = \theta$, then the principal expects the agent to choose L with probability one. Since $\varphi^* > 0$, the principal strictly prefers disclosing θ, thereby saving the cost of intervention when the agent chooses R. If $\theta > 0$ and $\rho = \theta$ then the principal expects the agent to choose R with probability one. Since $\varphi^* < 1$, the principal strictly prefers disclosing θ in this range. Combined, it is necessary that $\Upsilon^* \subseteq \left[\max\left\{-c_P, -\overline{\beta}\right\}, 0\right]$. Since $\left[\max\left\{-c_P, -\overline{\beta}\right\}, 0\right] \subseteq \left[-c_P, 0\right]$ the agent knows that upon non-disclosure $\theta \in \left[-c_P, 0\right]$, and hence, that the principal will not intervene. It follows, the agent will choose R upon non-disclosure if and only if $-\beta \leq \mathbb{E}\left[\theta | \theta \in \Upsilon^*\right]$. Therefore, it must be $\varphi^* = \Pr\left[-\beta \leq \mathbb{E}\left[\theta | \theta \in \Upsilon^*\right]\right]$. Let $\hat{\theta} \in \Upsilon^*$ be such that $\hat{\theta} < \mathbb{E}\left[\theta | \theta \in \Upsilon^*\right]$. If $\rho = \hat{\theta}$ then the agent will choose R if and only if $-\beta \leq \hat{\theta}$. Therefore, by disclosing $\hat{\theta}$, the principal strictly increases the probability that the agent chooses L from $1 - \varphi^*$ to $\Pr[-\beta > \hat{\theta}]$. Since $\hat{\theta} \in \Upsilon^* \Rightarrow \hat{\theta} < 0$, the principal has strict incentives to deviate and disclose $\hat{\theta}$. By this logic, if $\varphi^* \in (0, 1)$, then $\Upsilon^* \in \{\varnothing, \{0\}\}$. In both cases, $a_A = L$ if and only if $\theta < \max\left\{-c_P, -\beta\right\}$, which is a special case of part (i).[19]

[19] If $\Upsilon^* \in \{\varnothing, \{0\}\}$, then $\Pr\left[\theta \in \Upsilon^*\right] = 0$, and hence, φ^* can take any value without changing the outcome of the equilibrium.

Last, consider an equilibrium with $\varphi^* = 0$. Since $\varphi^* = 0$, the principal has strict incentives to disclose θ when $\theta > 0$. Moreover, if $\theta \in \left[\max\left\{-c_P, -\overline{\beta}\right\}, 0\right]$, the principal has strict incentives to conceal θ, since if she discloses θ, there is a strictly positive probability that the agent chooses $a = R$. If $\theta \in \left[\underline{\theta}, \max\left\{-c_P, -\overline{\beta}\right\}\right]$, the principal is indifferent between disclosing and concealing θ, as in both cases the agent chooses L for sure. Therefore, it is necessary that $\left[\max\left\{-c_P, -\overline{\beta}\right\}, 0\right] \subseteq \Upsilon^* \subseteq [\underline{\theta}, 0]$. If $\rho = \phi$, the agent infers $\theta \in \Upsilon^*$. Since $\Upsilon^* \subseteq [\underline{\theta}, 0]$, the agent expects that if he chooses L the principal never intervenes and his payoff will be zero. Instead, if the agent chooses R, his expected utility is

$$\Pr\left[\theta \geq -c_P | \theta \in \Upsilon^*\right] \mathbb{E}\left[\theta + \beta | \theta \geq -c_P, \theta \in \Upsilon^*\right] - c_A \Pr\left[\theta < -c_P | \theta \in \Upsilon^*\right].$$

Therefore, if $\rho = \phi$, the agent chooses L if and only if

$$\beta \leq c_A \frac{\Pr\left[\theta < -c_P | \theta \in \Upsilon^*\right]}{1 - \Pr\left[\theta < -c_P | \theta \in \Upsilon^*\right]} - \mathbb{E}\left[\theta | \theta \geq -c_P, \theta \in \Upsilon^*\right].$$

Note that $\varphi^* = 0$ requires $\overline{\beta}$ being smaller than the RHS of the above condition. Therefore, an equilibrium with $\varphi^* = 0$ exists if and only if $\overline{\beta} \leq \overline{b}_{\text{ver}}$. If $\overline{\beta} \leq \overline{b}_{\text{ver}}$, then the agent effectively chooses $a_A = R$ if and only if $\theta > 0$, and the principal never intervenes. This argument proves part (ii). Part (iii) and the claim that in any equilibrium the principal intervenes with a zero probability, follow by noting that all cases where $\varphi^* \in [0, 1]$ have been covered by the proof. ■

When $\varphi^* = 0$, the principal can conceal enough information to convince the agent to choose action L whenever $\theta < 0$. In this respect, the agent is following the principal's demand, and the principal's first best is obtained in equilibrium. By contrast, when $\varphi^* = 1$ (and $c_P > 0$), the principal's expected payoff is strictly less than her first best. In this respect, equilibria with $\varphi^* = 0$ ($\varphi^* = 1$) are the analog of influential (non-influential) equilibria in the baseline model. Proposition 11 shows that the existence of an equilibrium with $\varphi^* = 0$ depends on how $\overline{\beta}$ compares with $\overline{b}_{\text{ver}}$. The next result gives an example where an equilibrium with $\varphi^* = 0$ exists without intervention, but it does not exist with intervention. In this respect, intervention harms the principal's ability to influence the agent through communication, even with verifiable information. The intuition behind the result is similar to the one in the baseline model, and this can be seen by the similarity between expression (52) and expression (8).

Proposition 12 *Suppose $c_P \in (0, -\underline{\theta})$ and $-\mathbb{E}[\theta| - c_P \leq \theta < 0] < \overline{\beta} \leq -\mathbb{E}[\theta|\theta < 0]$. There is $\bar{c}_A > 0$ such that if $c_A \in (0, \bar{c}_A)$, then the principal's first best is obtained in equilibrium without intervention, but is not obtained in equilibrium with intervention.*

Proof. I start by arguing that $\overline{\beta} \leq \bar{b}_{\text{ver}}(c_A = 0, c_P)$ if and only if $\overline{\beta} \leq -\mathbb{E}[\theta| - c_P \leq \theta < 0]$. Consider three cases. First, suppose $\overline{\beta} \leq -\mathbb{E}[\theta| - c_P \leq \theta < 0]$. Let $\Upsilon = [-c_P, 0]$ and note that $\overline{\beta} \leq -\mathbb{E}[\theta| - c_P \leq \theta < 0] = -\mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon]$. Since $-\mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon] \leq \bar{b}_{\text{ver}}$, we have $\overline{\beta} \leq \bar{b}_{\text{ver}}(c_A = 0, c_P)$ as required. Second, suppose $\overline{\beta} > -\mathbb{E}[\theta| - c_P \leq \theta < 0]$ and $\overline{\beta} \geq c_P$. Then $[\max\{-c_P, -\overline{\beta}\}, 0] \subseteq \Upsilon$ implies $[-c_P, 0] \subseteq \Upsilon$. Therefore, $\mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon]$ is invariant to Υ and is equal to $\mathbb{E}[\theta| - c_P \leq \theta < 0]$. Therefore, $\bar{b}_{\text{ver}} = -\mathbb{E}[\theta| - c_P \leq \theta < 0]$. Since $\overline{\beta} > -\mathbb{E}[\theta| - c_P \leq \theta < 0]$, $\bar{b}_{\text{ver}}(c_A = 0, c_P) < \overline{\beta}$, as required. Third, suppose $c_P > \overline{\beta} > -\mathbb{E}[\theta| - c_P \leq \theta < 0]$. Relative to $\Upsilon' = [-c_P, 0]$, any $[-\overline{\beta}, 0] \subseteq \Upsilon \subseteq [\underline{\theta}, 0]$ such that $[-c_P, 0] \backslash \Upsilon \neq \varnothing$ is missing from its pool $\theta \in [-c_P, -\overline{\beta}]$. Since $-\overline{\beta} < \mathbb{E}[\theta| - c_P \leq \theta < 0]$, $\mathbb{E}[\theta| - c_P \leq \theta < 0] \leq \mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon]$. This implies $-\overline{\beta} < \mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon]$ for all $[\max\{-c_P, -\overline{\beta}\}, 0] \subseteq \Upsilon \subseteq [\underline{\theta}, 0]$. Therefore, $\bar{b}_{\text{ver}}(c_A = 0, c_P) < \overline{\beta}$ as required.

A special case of Proposition 11 is $c_P \geq -\underline{\theta}$, that is, intervention is not allowed. Based on Proposition 11, without intervention, an equilibrium with $\varphi^* = 0$ exists if and only if $\overline{\beta} \leq \bar{b}_{\text{ver}}(c_A, -\underline{\theta})$. Note that $\bar{b}_{\text{ver}}(c_A, -\underline{\theta}) = \bar{b}_{\text{ver}}(0, -\underline{\theta})$ for any c_A. According to the argument above, without intervention, an equilibrium with $\varphi^* = 0$ exists if and only if $\overline{\beta} \leq -\mathbb{E}[\theta|\theta < 0]$. Next, suppose $c_P \in (0, -\underline{\theta})$ and $-\mathbb{E}[\theta| - c_P \leq \theta < 0] < \overline{\beta} \leq -\mathbb{E}[\theta|\theta < 0]$. Note that for all c_A and c_P we have $\bar{b}_{\text{ver}}(c_A, c_P) \leq H(c_A, c_P)$ where

$$\begin{aligned} H(c_A, c_P) \quad \equiv \quad & c_A \frac{\Pr\left[\theta < -c_P | \theta \in [\underline{\theta}, -c_P] \cup [\max\{-c_P, -\overline{\beta}\}, 0]\right]}{1 - \Pr\left[\theta < -c_P | \theta \in [\underline{\theta}, -c_P] \cup [\max\{-c_P, -\overline{\beta}\}, 0]\right]} \\ & + \max_{\Upsilon : [\max\{-c_P, -\overline{\beta}\}, 0] \subseteq \Upsilon \subseteq [\underline{\theta}, 0]} \{-\mathbb{E}[\theta|\theta \geq -c_P, \theta \in \Upsilon]\} \end{aligned}$$

and $H(c_A, c_P)$ is continuous and increasing in c_A. Moreover, $\lim_{c_A \to 0} H(c_A, c_P) = \bar{b}_{\text{ver}}(0, c_P)$. Therefore, for any $\varepsilon \in \left(0, \overline{\beta} + \mathbb{E}[\theta| - c_P \leq \theta < 0]\right)$ there is $\bar{c}_A > 0$ such that if $c_A \in (0, \bar{c}_A)$ then

$$\bar{b}_{\text{ver}}(c_A, c_P) \leq H(c_A, c_P) < -\mathbb{E}[\theta| - c_P \leq \theta < 0] + \varepsilon$$

Therefore, if $c_A \in (0, \bar{c}_A)$, then $\overline{b}_{\text{ver}}(c_A, c_P) < \overline{\beta}$, and according to Proposition 11, an equilibrium with $\varphi^* = 0$ exists without intervention, but not with intervention. ■