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CAN LEARNING AFFECT EXCHANGE-RATE BEHAVIOR?
The Case of the Dollar in the Early 1980's

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This paper demonstrates that a change in parameters of the money market that is not immediately understood will affect the behavior of exchange-rate forecast errors while the market is learning. The analysis is applied to the behavior of U.S. dollar–German mark forecast errors during the early 1980's when the market appeared to systematically underpredict the strength of the dollar. Consistent with this observation, empirical estimates of implied forecast errors based upon Bayesian learning suggest that forecast errors would have been on-average negative during the period as the market came to realize the increase in U.S. money demand.

1. Introduction

During the early 1980's, foreign exchange-market participants appear to have been systematically surprised at the strength of the dollar, a phenomenon that some have claimed represents market irrationality.[1] As one measure of these expectations, table 1 presents sample averages and mean-squared errors of monthly forward prediction errors for the German mark (DM), the Japanese yen, and the British pound from the period of October 1979 through 1984. For all three currencies, these forward-rate errors are on-average negative and relatively large in absolute value, ranging from -1.1% for the DM to -0.4% for the yen.[2] Comparing the mean-squared errors and the exchange-rate

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[1]Evidence of systematic expectation errors come from survey data [Frankel and Froot (1987)] and forward prediction errors [Levich (1985)]. The poor forecasting performance of the forward rate more generally is well-established; see Hansen and Hodrick (1980) and Cumby and Obstfeld (1981), for example.

[2]The exchange rates are from the IMF's 'International Statistics Monthly', while the forward rates are constructed from interest-rate data from Morgan Guaranty's 'World Financial Markets' assuming covered interest parity.

Table 1

Forward prediction-error means, mean-squared errors, and exchange-rate variances.[a]

	German mark	Japanese yen	British pound
	(A) Error means		
October 1979–June 1984	−1.06	−0.43	−0.74
	(B) Mean-squared errors		
July 1973–September 1979	10.11	8.82	7.11
October 1979–June 1984	11.44	14.10	10.72
	(C) Exchange-rate variances		
July 1973–September 1979	11.38	10.90	10.22
October 1979–June 1984	12.40	13.13	13.72

[a]All prediction errors and exchange-rate changes are in monthly rates of change relative to the dollar. Data sources are *International Financial Statistics* for the exchange rates and *Morgan Guaranty* for interest rates. Forward prediction errors are calculated as the excess returns on the excess returns from selling dollars forward one month and then buying dollars at the spot market prevailing in one month. Implied forward rates are calculated from the spot and interest-rate series assuming covered interest parity.

variances for the floating-rate period before and after this change also indicates that the variances increased for all three currencies.

Systematic forecast errors appear to violate the premise of rational expectations that the market uses efficiently all available information, and, therefore, would seem to contradict the paradigm of a rational market. On the other hand, this paper investigates whether the dollar forecast-error behavior described above was consistent with the hypothesis that the market was learning *rationally* about a shift in the 'fundamentals' process. As a key switch in the fundamentals process, the analysis focuses upon the increase in the growth rate of U.S. money demand during the early 1980's. To characterize the learning process, market participants are assumed to use Bayesian forecasts of the parameters in money demand and discover the new parameters only over time.[3] The empirical evidence relates this analysis to the German mark–U.S. dollar rate since, of the major currencies, this rate exhibits the largest systematic misprediction. Using two extreme priors, the dollar forecast errors that would be implied by the money-market equilibrium condition are calculated and compared to the behavior of actual forward prediction errors. Interestingly, when using a prior that weights past observations of the fundamentals,

[3]In a related issue, Flood and Garber (1980) and Baxter (1985) study agents' beliefs about the credibility of government reforms using Bayesian methods.

the forecast errors implied by the empirical learning model are significantly correlated with forward prediction errors and appear to account for roughly one-half of the underprediction of the dollar.

The paper is organized as follows. Section 2 explains the basic learning process using a simple example and shows how it can contribute to systematic misprediction by the market. Section 3 relates the analysis empirically to the behavior of the U.S. dollar–German mark during the early 1980's. Concluding remarks follow.

2. Learning and forecast-error means

2.1. A simple theoretical example: Forecast-error means

The following simple example illustrates how forecast errors may be systematically wrong while the market is learning rationally.[4] In general, the results in this section apply to the behavior of variables that depend upon the expectation of their own future values, for example, stock prices [Schiller (1981)] and hyperinflation [Sargent and Wallace (1973)]. However, this representative variable will be called the 'exchange rate' since the results will be related empirically to the U.S. dollar–German mark exchange rate in the next section.

Assume that s_t, the logarithm of the exchange rate, is determined by the following simple equation:

$$s_t = n_t - z_t + \alpha\, \mathrm{E}_t(s_{t+1} - s_t), \tag{1}$$

where $\mathrm{E}_t(.)$ is the conditional expectations operator and where z_t and n_t are 'fundamentals' variables that determine the exchange rate with coefficients that have arbitrarily been set equal to 1 and -1, respectively. While the distribution of n_t is assumed stationary and ergodic throughout, the analysis below will allow the process for z_t to switch from one stationary process to another. To focus upon the effect of learning about the switch, z_t and n_t are assumed uncorrelated.[5] Solving eq. (1) gives the solution of the exchange rate

[4]The behavior to be described is similar to the effects upon employment described by Taylor (1975) as an economy converges to equilibrium.

[5]To avoid imbedding learning behavior in a fully determined model of exchange-rate determination, the analysis relates learning about particular fundamentals to the exchange-rate behavior. The learning model will have the property that forecasts are rational only with respect to convergence to long-run equilibrium as noted by Bray and Savin (1986) and Marcet and Sargent (1986).

in terms of future expected 'fundamentals',

$$
\begin{aligned}
s_t &= (1/(1+\alpha)) \sum_{j=0}^{\infty} (\alpha/(1+\alpha))^j \mathrm{E}_t(n_{t+j} - z_{t+j}) \\
&\equiv {}_tN_t - (1/(1+\alpha)) \sum_{j=0}^{\infty} (\alpha/(1+\alpha))^j \mathrm{E}_t(z_{t+j}),
\end{aligned}
\tag{2}
$$

where

$$
{}_iN_k \equiv (1/(1+\alpha)) \sum_{j=0}^{\infty} (\alpha/(1+\alpha))^j \mathrm{E}_i(n_{k+j}).
$$

Since the n_t are stationary, they have a time-series representation with white-noise, i.i.d. innovations.

Before describing the effects of learning, consider first the exchange-rate forecast errors under the standard assumption that the market knows all of the parameters of the probability distribution of fundamentals. Furthermore, assume that the z_t process is stationary after first-differencing and given by the general representation

$$
\Delta z_t = \delta_0 x_t + v_t = \delta_0 + v_t, \tag{3}
$$

where Δ is the difference operator, x_t are predetermined variables, δ are parameters, and v_t is a white-noise, normally distributed disturbance term. Although, for simplicity, x_t is assumed to contain only a constant, the standard x_t notation is retained below for clarity. In this case, eq. (2) can be written as

$$
s_t = -(1+\alpha)\delta_0 - v_t - z_{t-1} + {}_tN_t. \tag{4}
$$

Taking the conditional expectation operator across eq. (4) gives the white-noise, mean-zero forecast errors:

$$
s_t - \mathrm{E}_{t-1}s_t = -v_t + ({}_tN_t - {}_{t-1}N_t). \tag{5}
$$

Now suppose that market participants were uncertain about the underlying parameters of the z_t process. In this case, they would learn about δ only over time. To characterize this learning process, market participants are assumed to use Bayesian forecasts. At every point in time, these agents have a prior distribution of the parameter that includes a prior mean, $\hat{\delta}_t$, and a prior precision estimate given by H_t. Their parameter estimates evolve with each additional observation of z_t according to the Bayesian updating rule:[6]

$$\hat{\delta}_t = [H_{t-1} + x_t'x_t]^{-1}[H_{t-1}'\hat{\delta}_{t-1} + x_t'\Delta z_t]. \tag{6}$$

Substituting this evolution of $\hat{\delta}$ during learning into the exchange-rate eq. (4) implies an exchange-rate solution that depends upon the market's beliefs about the process governing z_t:

$$s_t = -(1+\alpha)\hat{\delta}_t - \hat{v}_t - z_{t-1} + {}_tN_t \quad \text{for} \quad \hat{v}_t = \Delta z_t - \hat{\delta}_t \Delta x_t. \tag{7}$$

The market has only an estimate of the current disturbance term, $\hat{v}_t$, since it does not know the parameter δ with certainty. The market's conditional forecast of the exchange rate at time t based upon observing fundamentals at time $t-1$ depends upon an initial prior estimate of the parameter at some point in time and the sequence of the 'fundamental' z_t's observed since then, information embodied in the market's estimate of $\hat{\delta}_{t-1}$ and H_{t-1}. Subtracting the expected future exchange rate conditional upon learning up until time $t-1$ gives the following forecast errors:

$$(s_t - \mathrm{E}_{t-1}s_t|\hat{\delta}_{t-1}) = -(1+\alpha)(\hat{\delta}_t - \hat{\delta}_{t-1}) - \hat{v}_t + ({}_tN_t - {}_{t-1}N_t). \tag{8}$$

These learning forecast errors evolve according to market observations of z_t which affect not only the estimates of δ themselves, but also the estimates of the residual, v_t.

Now suppose that the z_t process changes at a point in time, τ. In particular, for $\delta_1 > \delta_0$, the z_t process in eq. (3) switches to

$$\Delta z_t = \delta_1 x_t + v_t = \delta_1 + v_t, \qquad t \geq \tau. \tag{3'}$$

For now, the increase in δ represents an arbitrary switch in the fundamentals process that strengthens the exchange rate, but will represent the increase in U.S. money demand during the early 1980's in the following section.

Clearly, the market's beliefs about the z_t process evolve according to the estimates of δ. These parameter estimates move over time in response to realizations of the random variable z and converge asymptotically to the true parameter; i.e., plim $\hat{\delta}_t \to \delta$. Therefore, one may consider the stochastic behavior of eq. (8) based upon the true process for z_t. From repeated drawings of the sequence $z_\tau, \ldots, z_{\tau+T}$ from the true distribution in eq. (3'), the expected

[6]See Zellner (1971) or Box and Tiao (1973). Since v_t is normally distributed, the prior distribution is assumed to be the natural conjugate prior.

value of 'learning forecast error' sample means may be written as

$$\mathrm{E}\left\{\left[\frac{\sum_{t=\tau}^{T+\tau}\left(s_t - \mathrm{E}_{t-1}s_t|\hat{\delta}_{t-1}\right)}{T}\right]\middle|\,\delta\right\}$$

$$= \mathrm{E}\left\{\left[\frac{\sum_{t=\tau}^{T+\tau}\left(-(1+\alpha)\left(\hat{\delta}_t - \hat{\delta}_{t-1}\right) - \hat{v}_t\right)}{T}\right]\middle|\,\delta\right\} < 0, \tag{9}$$

where the expectation, $\mathrm{E}\{\cdot|\delta\}$, is based upon the true process and the mean-zero terms, $({}_tN_t - {}_{t-1}N_t)$, have been suppressed (the inequality is explained next).

Since δ rose at time τ, it seems reasonable to suppose that the initial estimate, $\hat{\delta}_\tau$, was less than the true δ_1 generating the process. In this case, the expected small sample mean of $(\hat{\delta}_t - \hat{\delta}_{t-1})$ is positive since the probability limit of $\hat{\delta}_t$ is greater than the initial $\hat{\delta}_\tau$. Similarly, when $\hat{\delta}_t < \delta_1$, the market's error estimate, $\hat{v}_t$, is larger than the true error, v_t. For both reasons, the inequality in eq. (9) holds.[7] As $\hat{\delta}_t$ converges to δ_1, the expected value of forecast errors goes to zero.

2.2. A simple theoretical example: Prior conditional variances

During the learning period, market participant's beliefs about the variance of their forecast errors will include uncertainty about their parameter estimates, as the following discussion illustrates (where the constant variance of ${}_tN_t - {}_{t-1}N_t$ is suppressed throughout).[8] Taking conditional variances of eq. (5) gives the conditional exchange-rate variance when the market knows the parameters, σ_v^2. On the other hand, finding the market's beliefs about this variance when the parameters are unknown, requires taking the variance of eq. (8) conditional upon the prior distribution of δ,

$$\mathrm{var}_{t-1}\left[s_t|\hat{\delta}_{t-1}\right] = (1+\alpha)^2\mathrm{var}_{t-1}(\hat{\delta}) + {}_{t-1}\hat{\sigma}_v^2. \tag{10}$$

[7] Of course, since $\hat{\delta}$ is a random variable, a sequence of particularly small z's could occur causing the mean to be positive.

[8] In addition, the process of learning will increase the unconditional variance of exchange rates as demonstrated by Tabellini (1986) and Lewis (1988).

This variance is based upon the market's subjective distribution of $\hat{\delta}$ at $t-1$, which implies in turn the market's time $t-1$ estimate of the variance of v_t.[9]

Subtracting σ_v^2 from eq. (10) provides an interesting comparison between the market's beliefs about the variance in its forecast errors and the standard conditional variance when parameters are known,

$$\mathrm{var}_{t-1}\left[s_t|\hat{\delta}_{t-1}\right] - \mathrm{var}_{t-1}\left[s_t|\delta\right] = a^2\,\mathrm{var}_{t-1}(\hat{\delta}) + \left({}_{t-1}\hat{\sigma}_v^2 - \sigma_v^2\right), \qquad (11)$$

where $a = (1+\alpha)$. The first component, $a^2\,\mathrm{var}_{t-1}(\hat{\delta})$, arises only during learning and captures the degree of confidence market participants place upon their parameter estimates. When the market has little confidence in its parameter estimates, this term can be quite large, as, for example, when the prior is diffuse. The second component in (11) is the difference between the market's estimate and the true variance of v_t. When the prior variance is large as in the diffuse case, the prior variance exceeds the true variance. But when the market's prior estimate of ${}_{t-1}\hat{\sigma}_v^2$ is less than σ_v^2, the prior conditional variance could be smaller than the known-parameter variance. Over time, of course, the market's estimate of the variance converges to σ_v^2.

2.3. *Discussion of the application*

This section has demonstrated that a change in the probability distribution of an underlying fundamental variable can induce exchange-rate forecast errors that appear systematically wrong *ex post* during a learning period. The next section applies this analysis to the behavior of the U.S. dollar–German mark rate during the period of unstable money demand in the early 1980's, and argues that this instability contributed to systematic market forecasts of a weaker dollar than occurred *ex post*. The analysis below focuses solely upon the money market, although some have asserted that other fundamentals such as fiscal policy also affected the exchange rate during this period. To the extent these other fundamentals affected the exchange rate, they are assumed uncorrelated with the money-market fundamentals and are therefore subsumed into the n_t process described above. This assumption, though strong, allows analyzing the effects of different fundamentals components separately as a first pass at investigating exchange-rate learning behavior.

[9] The market's variance estimate depends upon the inverse of the precision estimate,

$$\mathrm{var}_{t-1}\left(s_t|\hat{\delta}_{t-1}\right) = (1+\alpha)^2(H_\tau + X_{t-1})^{-1} + \mathrm{E}_{t-1}\hat{\sigma}_v^2,$$

$$\mathrm{E}_{t-1}\hat{\sigma}_v = r_{t-1}\left[\sqrt{(\nu/2)}\,\Gamma((\nu-1)/2)\right]/\Gamma(\nu/2),$$

where X and Z are the vectors of x and Δz from time τ to t and $r_t^2 \equiv (Z_t - X'_t\hat{\delta}_t)'(Z_t - X'_t\hat{\delta}_t)/\nu$ for ν, the degrees of freedom, and Γ, the gamma function. See Box and Chiao (1973, pp. 86–89) and Zellner (1971, pp. 70–72 and, on asymptotic convergence, pp. 371–373).

The following analysis focuses upon the money market for two reasons. First, the money-market equilibrium condition is an ingredient in most models of exchange-rate determination. By contrast, the channels through which other fundamentals affect the exchange rate are more likely to be model-specific. Second, the behavior of U.S. money demand changed sharply during the early 1980's. In particular, the apparent increase in money demand was preceded by a period when estimates of the money-demand equation became highly unstable following the 1979 change in Federal Reserve operating procedures, making plausible the conjecture that the market did not immediately understand this change.

3. Learning forecast errors and the monetary model

During the first several years following the 1979 change in Federal Reserve operating procedures, academics and Fed staff economists alike noted increased instability in the previously stable U.S. money-demand relationships.[10] In hindsight, the money-demand equation appears to have shifted positively during the early 1980's, especially evident in a dramatic decline in the growth rate of velocity. This section investigates the implied effects of this instability and the apparent increase in money demand upon the behavior of exchange-rate forecast errors.

The monetary model of the exchange rate focuses upon the relationship between domestic relative to foreign money demand and the behavior of the exchange rate [e.g., Mussa (1982)]. For this reason, the model is used below to calculate the component of forecast errors due to learning about the money-demand equation. A strict interpretation of the specified model as *the* correct model of exchange-rate determination would imply that these estimates should explain the overall behavior of exchange-rate forecast errors. An alternative interpretation is that the true model includes other fundamentals that affect the exchange rate. In this richer model, the errors from the monetary model represent errors from the money-market equilibrium condition that comprise only part of the total. Both views will be discussed below.

3.1. A standard monetary model

Using an empirical version of the monetary model, the effects of learning about changes in money demand were calculated and are reported below. In order to distinguish the effects of learning from differences arising from the empirical specification of the model, only empirical versions of the model found in the literature were investigated. In particular, the empirical results to

[10]On the variability of the money-demand estimates following 1979 that the Fed used to implement policy, see Bryant (1983) and Brunner and Meltzer (1983). On the shift in U.S. money demand, especially around the fall of 1981, see Rose (1986), Baba, Hendry, and Starr (1987), and the references therein.

be reported below used the Meese (1986) specification in which the disturbances to fundamentals are assumed to contain unit roots.[11]

Real money demand in each country depends upon a growth parameter, δ, the nominal interest rate, i, and income, y:[12]

$$\begin{aligned} m_t^u - p_t^u &= \delta^u - \alpha i_t^u + h y_t^u + e_t^u, \qquad e_t^u = e_{t-1}^u + v_t^u, \\ m_t^g - p_t^g &= \delta^g - \alpha i_t^g + h y_t^g + e_t^g, \qquad e_t^g = e_{t-1}^g + v_t^g, \end{aligned} \tag{12}$$

where v_t^u and v_t^g are white noise, and u and g refer to the U.S. and Germany, respectively, all lower-case variables are in natural logarithm form except for the interest rates, m indicates the money supply, and p the price level. Domestic and foreign interest rates are linked through open interest parity:

$$i_t^u = i_t^g + \mathrm{E}_t(s_{t+1} - s_t), \tag{13}$$

where s is the logarithm of the price of DM in terms of dollars. The remaining link required to close the model is a goods-market equilibrium condition. Deviations from purchasing power parity are assumed to follow a random walk:

$$\Delta p_t^u = \Delta p_t^g + \Delta s_t + w_t, \tag{14}$$

where w_t is the deviation from PPP, assumed to be uncorrelated with v_t.

Substituting eqs. (13) and (14) into (12) and solving the difference equation forward gives the exchange rate as a function of expected future variables:

$$\begin{aligned} s_t = &-(1+\alpha)(\delta^u - \delta^g) - (v_t^u - v_t^g) \\ &+ (1/(1+\alpha)) \sum_{j=0}^{\infty} (\alpha/(1+\alpha))^j \\ &\times \mathrm{E}_t\{(m_{t+j}^u - m_{t+j}^g) - h(y_{t+j}^u - y_{t+j}^g)\} \\ \equiv &-(1+\alpha)\delta - v_t + (1/(1+\alpha)) \\ &\times \sum_{j=0}^{\infty} (\alpha/(1+\alpha))^j \mathrm{E}_t\{m_{t+j} - h y_{t+j}\}, \end{aligned} \tag{15}$$

[11]Forecast errors were also calculated using the lagged-adjustment specification in Woo (1985). However, Woo includes a deterministic time trend to adjust for the growth rate in the fundamentals. In the present context, this trend implies both implausibly strong learning effects and sensitivity to the starting date of the trend. Empirical results using the Woo model are available upon request from the author. Another empirical monetary model in the literature, Huang (1981), does not allow for money-demand errors and therefore was not investigated. West (1987) compares these three models using variance-bounds tests.

[12]Gandolfi and Lothian (1983) and Hafer and Hein (1980) report evidence suggesting that the disturbance to money demand contains a unit root.

where, for notational simplicity, the variables without superscripts are defined as the U.S. minus the German variable; e.g., $\delta \equiv \delta^{u} - \delta^{g}$, $v_t \equiv v_t^{u} - v_t^{g}$, etc.

To solve for the exchange rate, the expected future forcing variables must be specified. As in Meese (1986), the time-series processes of money and income follow a first-order autoregressive process after first-differencing:

$$\begin{aligned} \Delta m_t &= c_m + \rho_m \Delta m_{t-1} + u_{m,t}, \\ \Delta y_t &= c_y + \rho_y \Delta y_{t-1} + u_{y,t}, \end{aligned} \tag{16}$$

where c are constants, ρ are the autocorrelation coefficients, and u are white-noise disturbance terms. For empirical tractability in the analysis below, these disturbance terms are assumed to be uncorrelated with the innovation in relative money demand. In actuality, these errors are likely to be correlated since the money market is determined simultaneously by both money demand and supply.[13]

Solving for the expected future paths of money and income using (16) and substituting the result into the exchange-rate eq. (15) gives

$$\begin{aligned} s_t = &-(1+\alpha)\delta - v_t + (m_{t-1} - h y_{t-1}) \\ &+ \left[\frac{1+\alpha}{1+\alpha(1-\rho_m)}\right](\Delta m_t + \alpha c_m) - \left[\frac{h(1+\alpha)}{1+\alpha(1-\rho_y)}\right](\Delta y_t + \alpha c_y). \end{aligned} \tag{17}$$

When the parameters of the model are known, the conditional forecast errors of eq. (15) are simply

$$\begin{aligned} s_t - \mathrm{E}_{t-1} s_t = &-v_t + \left[\frac{1+\alpha}{1+\alpha(1-\rho_m)}\right] u_{m,t} \\ &- \left[\frac{h(1+\alpha)}{1+\alpha(1-\rho_y)}\right] u_{y,t} - w_t. \end{aligned} \tag{18}$$

[13] In future research, covariance between money-demand and fundamentals processes could be allowed by incorporating the Wishart distribution of the covariance matrix in the prior distribution as described in Zellner (1971, pp. 270–276). In support of the exogeneity assumption, Meese (1986) reports Granger-causality tests of the fundamentals on the exchange rate in which he cannot reject exogeneity.

Since v_t, $u_{m,t}$, $u_{y,t}$, and w_t have mean zero, the sample average of the forecast errors has expectation zero as illustrated in eq. (5).

3.2. The fundamentals and parameter stability

The apparent shift in U.S. money demand discussed in the beginning of this section would suggest an increase in δ. To verify the rise in money demand found in other studies, stability tests for the parameters in eqs. (12) were conducted using end-of-month money supply and monthly industrial production for the U.S. and Germany from the Meese (1986) data set. Consistent with other studies, the growth rate of U.S. money demand increases during the period, particularly evident beginning in the fall of 1981. Specifically, in the U.S. money-demand equation, a Chow test that δ^{u} was constant over the period was rejected at a marginal significance level of 2%. Also, a Chow test for the hypothesis that δ, the growth rate in the U.S. relative to Germany money-demand equation, was equal before and after October 1979 produced an F-statistic of 3.2, exceeding the 95% critical value of 2.7. Stability tests were also conducted on the constancy of the growth rates, c_m and c_y, and autocorrelation parameters of the processes, ρ_m and ρ_y, specified in eqs. (16). However, these tests could not reject the hypothesis of constant coefficients.[14]

Three interpretations of these results will be considered in the empirical results below. First, in light of no *ex post* evidence of a change in the ARI(1, 1) parameters $(c_m, \rho_m, c_y, \rho_y)$, the markets knew these parameters with certainty. Hence, the errors arising from Δm_t and Δy_t were white noise. Second, the increase in money demand was partially accommodated by money supply, and this money-supply reaction was not captured by the exogenous process in (16). In this case, observed shifts in the money-market equilibrium condition arose largely from shifts in money demand.[15] Under either of these interpretations, eq. (8) gives the form of the exchange-rate forecast errors, where ${}_tN_t - {}_{t-1}N_t$ include either the errors arising from the $u_{m,t}$, $u_{y,t}$, and w_t components in eq. (18) under the first interpretation, or else include the errors to other uncorrelated processes under the second interpretation. In both cases, eq. (9) describes the behavior of forecast-error means as the market learns about the strength of U.S. money demand. A third interpretation, described further in section 3.4, is that the market was also uncertain about the parameters in the m_t and y_t fundamentals processes.

[14]A number of tests were conducted using different forms of money demand, using instrumental variables, and different forms of seasonal adjustment. The results concerning parameter instability were essentially the same.

[15]Brunner and Meltzer (1983) and Bryant (1983) discuss the Fed's use of money-demand estimates to decide monetary policy under the period of nonborrowed reserves targeting, 1979–1982.

Table 2

Estimates of fundamentals across time periods.[a]

(A) U.S.–German money demand

$\Delta m_t - \Delta p_t = \delta - \alpha\Delta i_t + h\Delta y_t$

	δ	α	h	D.W.	S.E.R.
January 1973–September 1979	−0.47 (0.29)	−0.88[c] (0.35)	0.07[b] (0.04)	2.5	2.62
October 1979–June 1984	0.02 (0.40)	0.31[b] (0.22)	0.13 (0.05)	2.3	2.95

(B) U.S.–German money supply

$\Delta m_t = c_m + \rho_m \Delta m_{t-1}$

	c_m	ρ_m	D.W.	S.E.R.
January 1973–September 1979	−0.27 (0.29)	−0.31[c] (0.11)	2.2	2.58
October 1979–June 1984	0.37 (0.40)	−0.28[c] (0.13)	2.2	3.02

(C) U.S.–German income

$\Delta y_t = c_y + \rho_y \Delta y_{t-1}$

	c_y	ρ_y	D.W.	S.E.R.
January 1973–September 1979	0.20 (0.85)	−0.04 (0.12)	1.9	7.57
October 1979–June 1984	0.34 (1.03)	−0.18 (0.13)	2.0	7.78

(D) U.S. money supply

$\Delta m_t^u = c_m^u + \rho_m^u \Delta m_{t-1}^u$

	c_m^u	ρ_m^u	D.W.	S.E.R.
January 1973–September 1979	0.65 (0.24)	−0.22 (0.11)	2.1	2.06
October 1979–June 1984	0.74[c] (0.30)	−0.16 (0.13)	2.1	2.17

[a]The estimates without superscripts are for U.S. minus German variables. The variables are defined as the natural logarithms of the following variables: m = M1 money supply, p = CPI price levels, i = short-term interest rates, y = industrial production. Δ is the backward difference operator. Data are nonseasonally adjusted. Standard errors are in parentheses. D.W. refers to the Durbin–Watson statistic. S.E.R. is the standard error of the regression.

[b]Significantly different from zero at the 90% confidence level.

[c]Significantly different from zero at the 95% confidence level.

Table 2 reports equation estimates for relative money demand in eq. (12) and the autoregressive processes in eq. (16).[16] The equations reported are estimated in terms of U.S. relative to German variables, although the essential results remained when the U.S. and German equations were estimated separately. The money-demand equation from 1973 to 1979 gives parameter estimates that conform to the predicted sign. The negative growth rate, δ, roughly corresponds with the growth rate of velocity observed in the U.S. from 1947 through the early 1980's. However, this trend has reversed with the decline in velocity, captured by the change in sign on δ for the 1979–1984 period.[17] Consistent with the hypothesis that this upward shift in money demand was not fully accomodated, the interest elasticity becomes insignificant with a positive sign during this second period. The table also reports estimates for the U.S. relative to German money and income processes. As indicated by the Chow tests, the parameter estimates vary insignificantly across time periods.

In panel D, table 2 reports estimates of the process of U.S. money supply alone. Despite the common perception that monetary policy was 'tight' during this period, the growth rate of the nominal money stock actually *increased*, demonstrated by the increase in the monthly growth rate from 0.65% to 0.74%.[18] This evidence also accords with an increase in money demand that was partially, but not completely, accomodated.

3.3. The implied empirical forecast errors: Money-demand component

As described in section 3.2 above, two of the three interpretations of how learning may have affected the dollar–DM exchange rate imply exchange-rate forecast errors as in eq. (8). Forecast errors take this form when either (a) the model is given as specified and the autoregressive process parameters are known, or (b) the shift in money demand is accomodated and the other unspecified exchange-rate fundamentals are uncorrelated with the money demand error. The third interpretation will be considered in section 3.4.

To investigate how instability in U.S. money demand during the early 1980's would affect the market's ability to make exchange-rate forecasts, the errors described in eq. (8) (ignoring the ${}_tN_t - {}_{t-1}N_t$ terms) were generated using two extreme market priors about the money-demand equations in eq. (12). The

[16] In forming the implied errors, seasonality posed a problem. Meese and Rogoff (1983b) and Meese (1986) regress the variables on seasonal dummies. But including these dummies during learning would require the market to update 12 additional coefficients, implying almost a year between each update. Therefore, in the interest of providing enough degrees of freedom, nonseasonally adjusted data were used.

[17] See, for example, the Council of Economic Advisers (1987, pp. 51–53).

[18] The increase remains when the second subsample is restricted to October 1979 through August 1982 as well, although the largest rate of growth occurs after 1981.

first prior was diffuse in October 1979. In view of the new money-demand instability, this prior represents a market that had no confidence in its estimates of the money-demand parameters. Below, this prior will be called CU for 'Change to Unknown' parameters The second prior, on the other extreme, was an informative prior based upon the past history of money demand during the flexible-rate period since July 1973. This model represents a market that believed money demand in the 1980's was unchanged from the past and is defined as NC for 'No Change'. Of course, if demand in fact shifted during this period, a market using diffuse priors would on-average learn more quickly and make less systematic prediction errors than a market using a prior based upon the past.

Calculating the forecast errors in eq. (8) requires a value for the semi-elasticity of money demand, α, that is assumed known in order to insure convergence.[19] The results reported below use 0.5 as the semi-elasticity of money demand, a value roughly in the middle of the range of estimates reported by Laidler (1977). Calculations based upon values for α in the range of 0.2 to 1 did not substantially alter the results. The interest-elasticity estimates in money demand are left unconstrained so that this restriction does not contaminate estimates of δ.

To gauge the sensitivity of these implied forecast errors to the income variable, two forms of money demand were estimated. In one, the income elasticity was set equal to zero, and in the other, the income elasticity was unconstrained.

To investigate the correlation between actual exchange-rate forecast errors and those implied by the model, table 3a reports the results of regressing forward prediction errors on the implied errors for the period October 1979 to June 1984.[20] Since these regressions implicitly treat the implied forecast errors as the true forecast errors, the standard errors in the regressions may understate the true standard errors, as in standard two-step procedures.[21] If the forward prediction errors only contain forecast errors and if the components of the exchange-rate forecast errors are uncorrelated (i.e., v_t is uncorrelated with ${}_tN_t - {}_{t-1}N_t$), then the regression coefficients should be positive and equal to one.

[19]Through eq. (2), if α and, therefore, the 'discount factor', $\alpha/(1+\alpha)$, is stochastic, the exchange-rate solution need not exist.

[20]For the same regressions beginning the period in October 1981, the parameter estimates for both priors were largely unaffected.

[21]However, the current regression is fundamentally different from the standard two-step procedure. Conditional on the econometrician having the market's correct prior at τ, the implied errors are the *true* errors since the econometrician updates the parameter estimates as would agents facing the same observations. The variance of the parameter estimates only affect agents' uncertainty about their forecasts as discussed in section 2.2.

Table 3a

Means and regressions of forward prediction errors on implied forecast errors.[a]

Regression: $(s_t - {}_{t-1}f_t) = -b[(1+\alpha)(\hat{\delta}_t + \hat{\delta}_{t-1}) + \hat{v}_t] + e_t$

Prior	Income elasticity	Coefficient	S.E.R.	Regressor mean
NC	Unconstrained	0.20[b] (0.14)	3.36	−0.46
	$h=0$	0.25[b] (0.14)	3.24	−0.50
CU	Unconstrained	−0.16 (0.15)	3.39	−0.16
	$h=0$	−0.13 (0.13)	3.39	−0.35

[a]All data are monthly for the period October 1979 to June 1984. Left-hand-side variable is the forward prediction errors given by the difference between (the logarithm of) the current exchange rate and the previous month's forward rate for exchange in the current period. The right-hand side is the exchange-rate forecast error implied by the learning model conditional upon each prior and the assumption about the income elasticity. The NC prior is based upon an estimate using data from January 1973 to September 1979. The CU prior is a diffuse prior beginning in October 1979. The column labeled 'Coefficient' is the estimate of the parameter b. Standard errors conditional upon the initial prior are in parentheses. The column labeled 'Regressor mean' is the mean of the forward prediction errors over the period.

[b]Significantly different from zero at the 90% confidence level.

The prior used in the first panel, 'No Change', was formed by estimating money demand from 1973 to 1979. Both forms of money demand give coefficient estimates near 0.2 that are significantly positive but also significantly different from one.[22] Table 3a also reports regression results using the 'Change to Unknown' prior. Since the initial prior variance of parameter estimates is much larger in this case, the relationship between the implied forecast errors and the forward prediction errors becomes relatively imprecise. In both regressions, the coefficient estimates are insignificantly different than zero. Overall, the 'No Change' prior yields a stronger relationship between forward and implied prediction errors, suggesting that the actual prior used by the market may have weighted the past.

Table 3a also reports the mean of the components of the implied forecast errors for each prior. In support of the discussion in section 2, the components arising from money demand, $-(1+\alpha)(\hat{\delta}_t + \hat{\delta}_{t-1}) - \hat{v}_t$, are on average negative in all four cases. Using the 'NC' prior, the money-demand component

[22]Also, although regressions of the prediction errors on the changes in the money-demand estimates, $\hat{\delta}_t - \hat{\delta}_{t-1}$, should theoretically be zero under no learning, these coefficient estimates based upon separate regressions were significantly positive.

Table 3b

Means and regressions of forward prediction errors on implied forecast errors.[a]

Regression: $(s_t - {}_{t-1}f_t) = -b_1[(1+\alpha)(\hat{\delta}_t + \hat{\delta}_{t-1}) + \hat{v}_t] + b_2U_t + e_t$						
		Coefficient			Regressor mean	
Prior	Income elasticity	b_1	b_2	S.E.R.	$-(1+\alpha)\Delta\hat{\delta} - \hat{v}$	U
NC	$h = 0.4$	0.58[b,d] (0.22)	0.10[b] (0.05)	3.24	−0.46	1.91
	$h = 0$	0.18[c] (0.40)	−0.02 (0.10)	3.34	−0.50	2.21
CU	$h = 0.4$	−0.16 (0.15)	0.01 (0.01)	3.40	−0.16	−5.00
	$h = 0$	0.14 (0.13)	−0.01 (0.01)	3.42	−0.35	−0.39

[a]All data are monthly for the period October 1979 to June 1984. The left-hand-side variable is the forward prediction errors given by the difference between (the logarithm of) the current exchange rate and the previous month's forward rate for exchange in the current period. The first right-hand side is the exchange-rate forecast error implied by the money-demand component of the learning model conditional upon each prior and the assumption about the income elasticity. The second right-hand side, U_t, is the exchange-rate forecast error implied by the time-series processes for money supply and income conditional upon each prior. The NC prior is based upon an estimate using data from January 1973 to September 1979. The CU prior is a diffuse prior beginning in October 1979. The column labeled 'Coefficient' is the estimate of the parameter b. Standard errors conditional upon the initial prior are in parentheses. The column labeled 'Regressor mean' is the mean of the forward prediction errors over the period.

[b]Significantly different from zero at the 95% confidence level.

[c]Insignificantly different from one at the 90% confidence level.

[d]Insignificantly different from one at the 95% confidence level.

accounts for monthly exchange-rate prediction errors that are about −0.5%. From table 1, the average forward prediction errors for this period were about −1%. Thus, the magnitude of underprediction in dollar forecasts based upon this prior is just under one-half of the observed underprediction of the dollar's strength by the forward rate. For the 'CU' prior, the absolute value of average forecast errors is smaller, as low as 0.16%, since the market learns more quickly.

3.4. *The implied empirical forecast errors: Including other fundamentals*

Although *ex post* evidence from the fundamentals processes other than money demand did not indicate that their parameters changed, market participants may have felt uncertain about these parameters as well. This interpretation comprises the third view of learning forecast errors, as discussed in section 3.2. When market participants learn about both an increase in money demand, δ, and a change in the parameters of the time series of m and y, exchange-rate forecasts are the conditional forecast of eq. (17) based upon the time $t-1$ conditional estimates of the parameters: $\hat{\delta}_{t-1}, \hat{c}_{m,t-1}, \hat{\rho}_{m,t-1}, \hat{c}_{y,t-1}, \hat{\rho}_{y,t-1}$. Subtracting this forecast from the actual exchange rate based upon the time t parameter estimates gives

$$
\begin{aligned}
&\left(s_t - \mathrm{E}_{t-1}s_t \mid \hat{\delta}_{t-1}, \hat{\rho}_{t-1}, \hat{c}_{t-1}\right) \\
&= -(1+\alpha)\left(\hat{\delta}_t - \hat{\delta}_{t-1}\right) - \hat{v}_t \\
&\quad + (1+\alpha)\left\{\left[\frac{\Delta m_t + \alpha \hat{c}_{m,t}}{1+\alpha(1-\hat{\rho}_{m,t})}\right] - \left[\frac{\hat{\rho}_{m,t-1}\Delta m_{t-1} + (1+\alpha)\hat{c}_{m,t-1}}{1+\alpha(1-\hat{\rho}_{m,t-1})}\right]\right\} \\
&\quad - h(1+\alpha)\left\{\left[\frac{\Delta y_t + \alpha \hat{c}_{y,t}}{1+\alpha(1-\hat{\rho}_{y,t})}\right] - \left[\frac{\hat{\rho}_{y,t-1}\Delta y_{t-1} + (1+\alpha)\hat{c}_{y,t-1}}{1+\alpha(1-\hat{\rho}_{y,t-1})}\right]\right\} \\
&\quad - w_t \\
&\equiv -(1+\alpha)\left(\hat{\delta}_t - \hat{\delta}_{t-1}\right) - \hat{v}_t + U_t - w_t. \qquad (19)
\end{aligned}
$$

As eq. (19) indicates, the learning forecast errors include not only the money-demand components investigated earlier, but also the composite error arising from revisions in parameter estimates of the money and income processes, denoted U_t above, and the deviations from purchasing power parity, w_t. As long as PPP continues to follow the same process, this last disturbance does not affect the learning process and only increases the overall conditional exchange-rate variance. Therefore, w_t will be set equal to zero throughout the rest of this paper without affecting the expected behavior of forecast-error means.

To investigate the relationship between this total learning error and the forward prediction error, the errors from the money and income process, U_t, were also calculated using the NC and CU priors. Since the effects of the income process depend critically upon the income elasticity, this parameter was constrained to different estimates of h in order to study the sensitivity to this parameter.[23] For the first model, the implied errors were calculated assuming that $h = 0.4$, a value used by Meese (1986) that he reports unrejected by his model; the second assumed $h = 0$.

Table 3b reports the results of regressing the forward prediction errors on the implied money demand and U_t forecast error components. Using the NC

[23]The errors were also calculated allowing for h to be learned, giving estimates between these two extremes.

prior, the coefficient on the money-demand component is insignificantly different from one in both models, but it is also insignificantly different from zero in the $h = 0$ case. Furthermore, the regression coefficients of the U_t terms differ substantially across models. The coefficient is significantly positive in the $h = 0.4$ case, but is negative and insignificantly different from zero when $h = 0$. As in table 3a, the size of the coefficients on the money-demand component fall under the CU prior and, due to the large parameter variability, become much less precise.

To further investigate these results, regressions of the money-supply error component of U_t were conducted separately from the income-process component. Despite the assumption of the monetary model that an innovation in the U.S. relative to German money-supply process should be *positively* correlated with exchange-rate forecast errors, the implied errors arising from revisions in the Δm_t process were *negatively* correlated with the forward prediction errors.[24] Therefore, treating the money-market equilibrium condition as driven by money demand, as in eq. (12), appears to be a more appropriate empirical representation than the exogenous money-supply process specified in eq. (16). Indeed, the correlation of the two right-hand-side variables in the second regression for the $h = 0$ case is over 0.9, indicating strong multicolinearity.

Furthermore, the sample means of the U_t are on-average positive and close to 2% on a monthly rate, due primarily to the increase in U.S. monetary expansion noted in table 2. Although the U_t term treats these innovations as changes in money supply, their behavior appears more consistent with money-demand accomodation as discussed above. For the CU prior, the U_t are on-average negative and differ substantially across models due to the presence of a few large outliers in the beginning of the learning process.

Overall, the results from tables 3 have shown, based upon a prior that weights the past, that learning about changes in the strength of U.S. money demand during the early 1980's can account for roughly one-half of the forward market's systematic underprediction of the dollar's strength. Using this prior, regressions of the forward prediction errors on the implied forecast errors from the money-demand component gave coefficient estimates that were insignificantly different from one, suggesting comovements in the learning errors and the forward-market errors. However, strong correlation between the different components of the implied error indicates that future research should incorporate cross-equation correlation.

3.5. *The market's beliefs about the exchange-rate variance*

The market's beliefs about the conditional variance of their exchange-rate forecasts, as derived in eq. (10), were calculated for the money-demand

[24]The errors arising from revisions in the Δy_t process were appropriately correlated with surprise appreciations in the dollar, however.

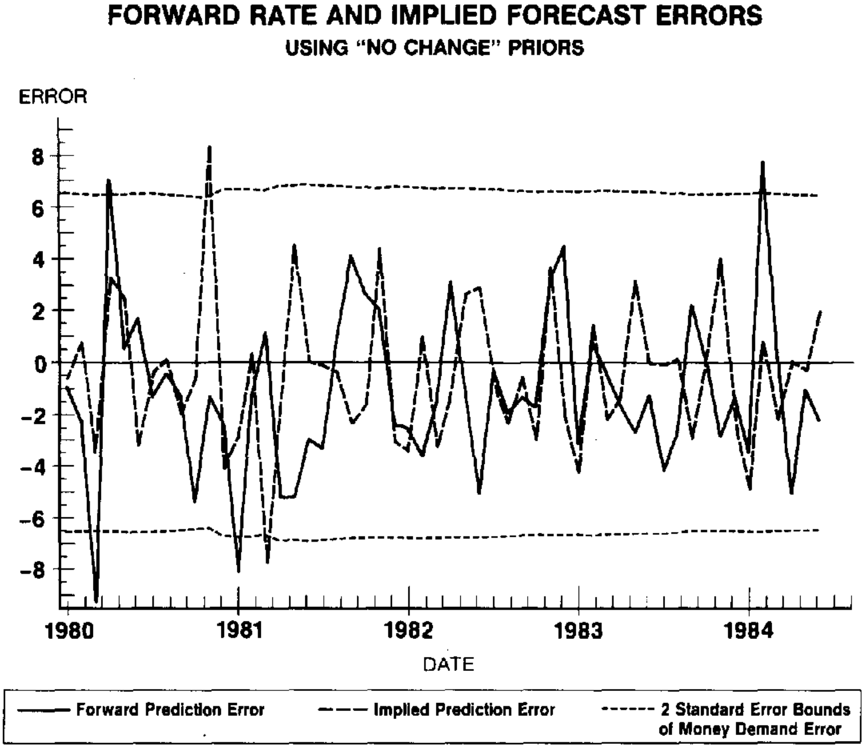

Fig. 1

component, $-(1+\alpha)(\hat{\delta}_t - \hat{\delta}_{t-1}) - \hat{v}_t$. Since the results from the previous section suggest some accomodation of the shift in money demand, the remaining discussion focuses upon money demand alone. The market's beliefs about the exchange-rate variances were calculated conditional upon the two extreme priors, 'No Change' and 'Change to Unknown'. Standard errors based upon these prior variances gave the market's 95% confidence intervals for forecast errors.[25] Figs. 1 and 2 plot the forward prediction errors against their implied forecast errors for the $h = 0$ money-demand model together with two standard errors bounds.

For the NC prior case plotted in fig. 1, the error bounds actually *increase* in the beginning of 1980, again in the volatile period in the end of 1980, and then level off near $\pm 7\%$, a range that includes much of the variation in the forecast errors. This behavior is consistent with the evolution in the market's

[25]Measuring these variances requires adjusting by the gamma function according to the degrees of freedom as in footnote 9. Since this function is difficult to compute, the estimates reported in the figures do not make this adjustment. Asymptotically, the omission is not important but will matter for small samples of the diffuse estimates case.

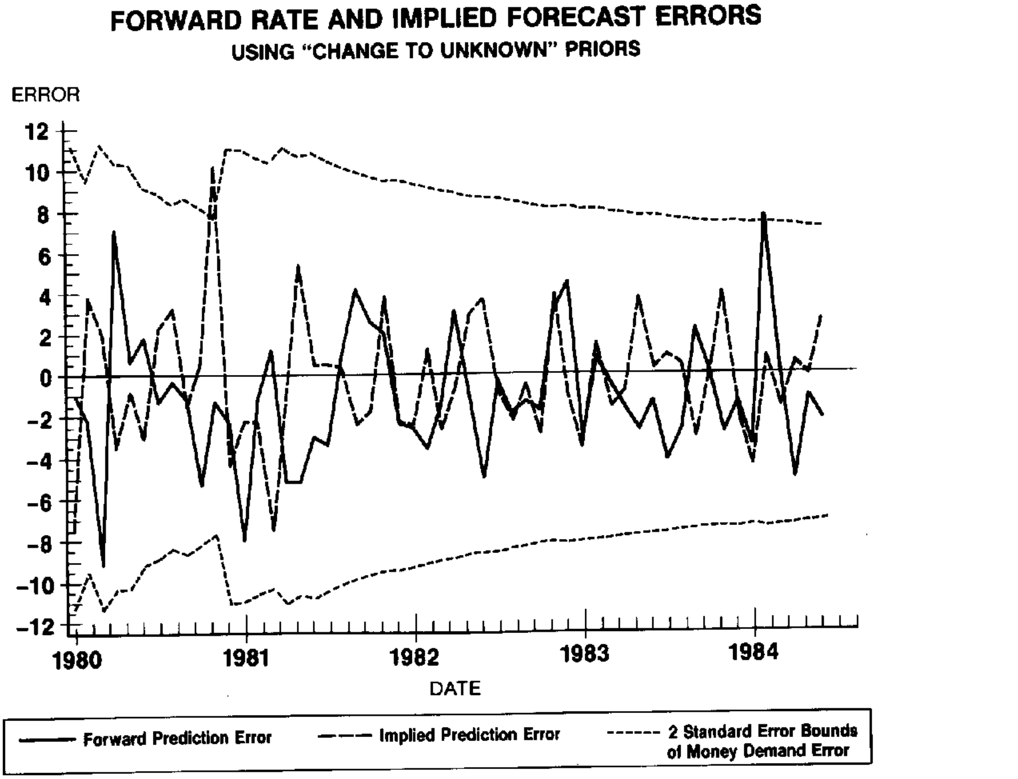

Fig. 2

exchange-rate variance estimate when the initial market estimate of the variance of money demand, ${}_{t-1}\hat{\sigma}_v^2$, is smaller than the actual variance, σ_v, as discussed in section 2.2. For the estimates of the CU prior case plotted in fig. 2, the variance of the estimates decreases initially during 1980, experiences a large upswing at the end of 1980, and generally declines thereafter. Almost all of the forecast errors are contained within these wide error bounds except for the large forward prediction error in the beginning of 1984. Thus, the market's beliefs about the exchange-rate variance under either prior includes much of the observed variation in the forward prediction errors.

4. Concluding remarks

This paper has investigated the contribution of learning about the shift in U.S. money demand during the early 1980's to systematic underpredictions of the dollar's strength by the market. A prior that weighted past parameter estimates of money demand implied exchange-rate forecasts that were significantly correlated with forward prediction errors and account for a little less than one-half of the observed underprediction in the DM–dollar rate.

A remaining issue is: how much of the systematic nature of forward prediction errors over longer time periods can be explained by learning behavior? As noted in Levich (1985), forward market underprediction of the dollar continued into 1985, beyond the period when new parameter estimates would have been learned. To explain this longer-term behavior may require a model integrating some combination of learning, anticipations of future shifts in fundamentals, and risk premia.

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