Carnegie-Rochester Conference Series on Public Policy 35 (1991) 67-78
North-Holland

An empirical exploration of exchange-rate target-zones

A comment

Karen K. Lewis
University of Pennsylvania
New York University
and National Bureau of Economic Research

In this paper, Flood, Rose and Mathieson conduct an extensive empirical study of the so-called "target zone" model of exchange rates. This model delivers an intuitive message. Namely, when monetary policy keeps the exchange rate within particular bands, forward-looking expectations of this type of policy affect the behavior of exchange rates within the bands. In particular, market expectations can themselves help contain exchange rates within the target-zone.

The authors focus upon the original (Krugman (1991)) version of this model that assumes three essential conditions. First, variables that affect demand and supply for foreign exchange follow a random walk within the band. Second, intervention to influence the exchange rate occurs only at the bands. Third, intervention occurs with probability one at the bands, i.e., market participants do not expect any realignments.

The authors look for the relationship implied by this model by examining the European Monetary System primarily. They also examine evidence from other episodes such as the Bretton Woods and the Gold Standard period. For this purpose, they carry out different types of empirical analyses ranging from plots to explicit parametric tests. In the end, they find little empirical support for the prototype model and no systematic relationship across "regimes."

This exhaustive empirical study of the prototype target-zone model should influence future research in the area. The Flood/Rose/Mathieson results in-

0167-2231/91/$03.50 © 1991 – Elsevier Science Publishers B.V. All rights reserved

dicate that the strict predictions for exchange-rate behavior implied by this model do not hold up empirically. The immediate question that arises from this evidence is: where to go from here? That is, given the rejection of the target-zone model under the three assumptions described above, what are likely reasons for this rejection? Answering this question should help guide future research.

To consider this question, I will begin by taking another look at the model and evidence in the authors' paper. I will point to important assumptions that might lead one to think the model should not hold *a priori*. I will then use a couple of these assumptions to describe where I think future research could lead. First, I will discuss the empirical importance of intramarginal interventions in the EMS and how it alters the interpretation of the results found by Flood/Rose/Mathieson. Second, I will touch upon the key role of the boundary conditions in target-zone models.

A different look at the prototype model and evidence

The essence of the prototype target-zone model comes from a standard asset-market exchange-rate model, originally due to Mussa (1982). In this standard asset market model, the (logarithm of the) exchange rate depends upon fundamental variables at time t and its own expected future change as given in the Flood/Rose/Mathieson equation (1).

$$e_t = f_t + \alpha E_t(de_t)/dt \tag{1}$$

Clearly, equation (1) is a first-order differential equation. Solving equation (1) forward gives equation (2).

$$e_t = (1/\alpha)\int_t^{\infty} exp[(t-s)/\alpha]E_t(f_s)ds \tag{2}$$

Of course, the solution in (2) is vacuous without specifying a process for the fundamental variable. A popular form of the process is a Brownian motion process given in the authors' equation (3),[1]

$$df = \eta dt + \sigma dz. \tag{3}$$

Using equation (3) and Ito's Lemma, the exchange-rate equation can be written as a second-order differential equation,[2]

$$g(f) = f + \alpha\eta g'(f) + (\alpha\sigma^2/2)g''(f). \tag{4}$$

[1]See Krugman (1991), Froot and Obstfeld (1989), and Bertola and Caballero (1989) for just a few examples.

[2]This appears as equation (5) in Flood, Rose, and Mathieson.

Since this is a second-order differential equation, the solution to equation (4) is unique up to two boundary conditions. Different boundary conditions considered in the literature imply different relationships for the exchange rate as a function of fundamentals. Flood/Rose/Mathieson focus upon the initial boundary condition considered by Krugman (1991), a solution they call the "Credible Target-Zone." This boundary condition says: When the exchange rate hits the band, the market knows the authorities will intervene with certainty."

Figure 1 depicts the upper half of this solution for the case when the target-zone is symmetric. The line labeled $e^k(f)$ corresponds to the prototype Krugman (1991) solution. The exchange rate responds less than proportionally to changes in the money supply and other fundamentals because market participants know that when the exchange rate reaches $\bar{e}^k$, central bankers will intervene. As a result, the function $e^k(f)$ at $\bar{f}$ has zero slope, corresponding to no change in the exchange rate at this fundamental level. The lower graph demonstrates the assumption about intervention underlying this solution. The probability of intervention $\pi^k(f)$ equals zero everywhere except at $\bar{f}$. At this level of fundamentals, the probability is one.

Flood/Rose/Mathieson look for evidence of this "credible zone" relationship between e, the exchange rate, and f, the "fundamentals." Since f is unobserved, they construct measures of fundamentals by assuming there is no risk premium. In this case, $E_t(\Delta e_t) = i_t - i_t^*$ and equation (1) implies:

$$f_t = e_t - \alpha(i_t - i_t^*). \tag{5}$$

They then examine the relationship between the exchange rate and the constructed measure of fundamentals. For example, in their Figures 6 to 10, they present plots of these two variables for various countries and periods.

We can see the more basic underlying relationship behind these plots if we look at these results in a slightly different way. For this purpose, define the logarithm of the forward rate as fd_t. Then, by covered-interest parity, we have approximately:

$$i_t - i_t^* = fd_t - e_t \tag{6}$$

Substituting (6) into the fundamentals equation (5) implies that the authors' constructed fundamentals level can be rewritten:

$$f_t = (1+\alpha)e_t + \alpha\ fd_t. \tag{7}$$

In other words, this measure of fundamentals is just a weighted sum of the forward and spot rate. Therefore, the scatter plots given in Figures 6 to 10 are simply plots of the exchange rate against a linear combination of the forward rate and of the exchange rate itself. Clearly, when $\alpha = 0$, the

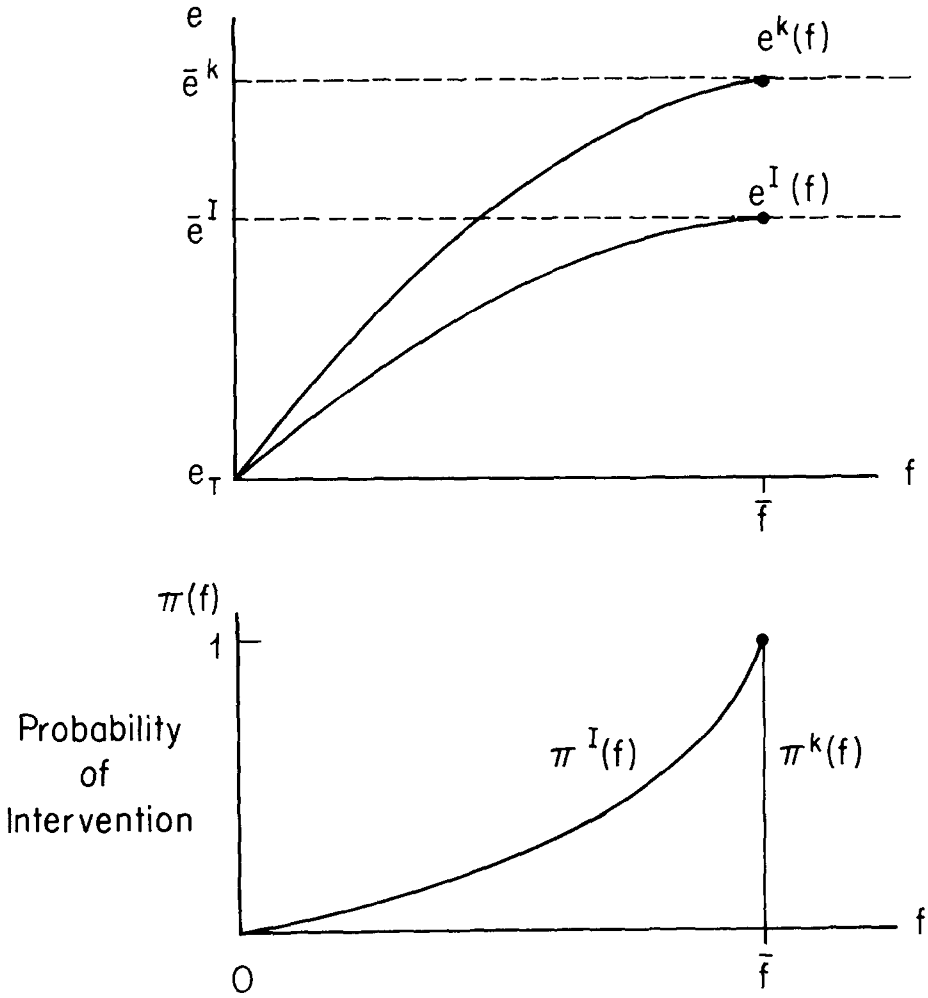

Figure 1: Target Zone With and Without Intra Marginal Intervention

relationship is a tautology that would graph the exchange rate against itself. Therefore, the exchange rate will deviate from measured fundamentals only to the extent that α deviates from zero.

To help guide intuition about this relationship, consider how spot and forward rates would be related under floating exchange rates. As observed by Frenkel and Mussa (1980), spot and forward rates tend to move together. If so, then e_t and fd_t in equation (7) should follow a common trend. Since the constructed fundamental is just a weighted sum of the exchange rate and the forward rate, we would expect to see a very linear relationship between the spot rate and this constructed variable. It is therefore comforting that Flood/Rose/Mathieson indeed find this kind of relationship for floating rates.

For some of the fixed-rates countries during some of the regimes, the relationship appears to look somewhat nonlinear, however. What does this suggest? Say, for example, that the relationship between e and f were as in Figure 1. From a purely statistical point of view, this relationship says: within the sample, exchange rates near the upper band were on-average correlated with lower forward rates than for other spot levels. A target-zone model is one possible explanation of this sample correlation.

Finally, after an exhaustive search across different regimes, different countries, different measures of α, and different types of tests, the authors find no systematic evidence of a spot-and-fundamentals relationship that looks like the prototype model in Figure 1. This leads to the next question: What went wrong? They suggest three possibilities. First, they may have bad estimates of α and therefore have improperly constructed fundamentals. Second, uncovered interest parity may not hold. Third, the model is wrong. They evaluate the first and second possibilities finding similar empirical results. Therefore, they conclude the third possibility is the answer.

I would also add a fourth possibility: there may be econometric difficulties with some of the parametric tests.[3] But overall I agree with them. I think the model is wrong. I will explain why in the process of describing where I think future research could go from here.

[3]The authors describe tests finding unit roots in exchange rates and interest differentials. They therefore first-difference these variables to obtain estimates of α in their equation (14). However, they later ask "Is there a honeymoon effect?" by regressing the *level* of the exchange rate on the *level* of the fundamental. If these variables contain unit-roots as the authors claim, then the standard errors associated with the $e : f$ slopes are incorrect. For example, in their Figure 11, it is not clear whether these coefficients would significantly differ from one if the true standard errors were depicted.

Where to go?

1. Intramarginal intervention

A key assumption in the prototype model studied by Flood/Rose/Mathieson is that central bankers only intervene at the bands. Until the exchange rate reaches the band, the growth rate of money is allowed to freely follow a random walk. As an empirical matter, however, much of the intervention within the EMS has taken place within the bands. For example, Mastropasqua, Micossi, and Rinaldi (1988) report that EMS currency intervention at the bands was worth 15.2 billion dollars in 1987. By contrast, intramarginal interventions during the same year amounted to 22.2 billion dollars worth in purchases of EMS assets and 34.3 billion dollars worth of sales. Furthermore, their numbers show that the degree of intramarginal intervention has been increasing over time, commensurate with the increased credibility of the European Monetary System.

These figures look at intervention alone. Some countries use domestic monetary policy as well to keep the exchange rate near parity. Therefore, the underlying assumption in the prototype model that monetary policy drifts until it reaches the band appears counterfactual.

How might intramarginal intervention help explain the findings by the authors? There are at least two related ways. First, Flood/Rose/Mathieson note that exchange-rate observations have been clustered within the bands, not near the bands as would be implied by the standard model. Furthermore, they argue that we should find exchange-rate histograms that become increasingly clustered around the bands for countries that have maintained more credible bands over time, such as the Netherlands. In their Figure 20, they find the opposite.

On the other hand, we would expect to observe this relationship if the Dutch engaged in intramarginal intervention. To help make the argument, I will assume that the regime is "credible" in the authors' sense that intervention occurs with certainty at the boundaries.[4] However, central bankers intervene within the band to keep the exchange rate from moving away from the targeted level. For simplicity, suppose they target an exchange rate defined as e_T and that there is no drift in fundamentals. Suppose further that when central bankers intervene, they temporarily stop the exchange rate from moving. Thus, they temporarily buy or sell currency to keep fundamentals where they were at the previous instant. This intervention policy implies that fundamentals either evolve according to equation (3), if there is no intervention, or cause fundamentals to remain unchanged so that $f(t^-) = f(t^+)$,

[4]This assumption is made purely for purposes of comparison. It can be relaxed and still generate similar implications for the empirical distribution.

if intervention occurs. Suppose further that the probability of intervention, $\pi^I(f)$, increases as the exchange rate deviates from the central parity level. Therefore, the exchange rate will either move with fundamentals with probability $(1 - \pi^I(f))$ or will momentarily stop with probability $\pi^I(f)$.

The exchange-rate solution with intramarginal intervention of this form is depicted in Figure 1 with the line labeled $e^I(f)$. For purposes of comparison, $\bar{f}$, the level of fundamentals where intervention occurs with certainty, is assumed the same as in the standard solution $e^k(f)$. As a comparison of e^I and e^k shows, intramarginal intervention implies the same basic shape as the standard case. Since intervention will occur with probability one at the same fundamentals level, both $e^k(f)$ and $e^I(f)$ have zero slope at this level. However, the upper target level implied by intramarginal intervention, $\bar{e}^I$, is lower than $\bar{e}^k$. Intuitively, since the market knows that central bankers will intervene intramarginally, the discounted present value of future fundamentals in equation (2) is lower. Therefore, the exchange rate fluctuates in a tighter band. By similar reasoning, it can be shown that, if $\bar{e}^k$ is the known upper band on the exchange rate, intramarginal intervention will imply a solution with zero slope at $\bar{e}^k$ but at a fundamentals level greater than $\bar{f}$.[5]

So far, this intramarginal intervention seems to be trading off one nonlinear form for another. The essential difference between the two solutions, however, is that central banks are acting to keep the exchange rate near its midpoint. This intervention means exchange-rate observations will be clustered in the middle of the band instead of around the bands, as the standard model would indicate.

Clearly, how much clustering of the exchange rate occurs depends upon how quickly the probability of intervention increases as the exchange rate deviates from the target. I therefore conducted Monte Carlo experiments assuming that the probability of intervention, π^I, had a logistic distribution. Beginning at the level corresponding to the target level of the exchange rate, I generated empirical distributions for the fundamentals, assuming elasticities of the odds ratio with respect to fundamentals, defined as b: $b \equiv (\%\Delta(\pi/(1-\pi)))/(\%\Delta f)$. When b is high, the probability of intervention increases quickly with departures from the target level.

Figure 2 illustrates histograms for 1000 observations of a fundamentals process that deviates from the target level in a range from -.25 to .25. The bottom histogram depicts the results without intervention. The observations are fairly evenly distributed over the entire range as would be expected from the uniform distribution. However, the observations become clustered near the midpoint with intramarginal intervention as shown in the middle and top histograms. As the elasticity of intervention, b, increases from 20 in the middle graph to 50 in the top graph, these observations become more

[5]See Klein and Lewis (1990) for more information.

concentrated at the target level.

Furthermore, when I used these fundamentals distorted by intramarginal intervention to generate exchange-rate observations, most of the exchange-rate observations were also concentrated near the central parity. These histograms look very much like the exchange-rate histograms such as the Dutch exchange-rates histograms in the Flood/Rose/Mathieson Figure 20.

To the extent that more "credibility" in exchange-rate regimes have been associated with more intramarginal intervention, these results indicate greater clustering of the exchange rate around its parity level. This relationship is precisely what the authors' Figure 20 suggests.

Intramarginal intervention also suggests an explanation of a second relationship that the authors find inconsistent with the standard target-zone model. They expect to find a relationship that looks like the standard S-shaped $e : f$ relationship when regimes are more "credible." They notice that countries such as the Dutch in their Figure 6 have exchange-rate fundamentals relationships that look more linear, if anything, than less credible countries. Similarly, since the EMS has become more credible over time, they expect the relationship between the exchange rate and fundamentals to look more S-shaped. In contrast to their expectations, the relationship appears to have become more linear over time.

However, one would expect to find greater linearity in more "credible" regimes with intramarginal intervention. As Figure 2 shows, the relationship between exchange rate and fundamentals appears fairly linear around the middle of the band. The nonlinearity is only apparent near the boundaries. Therefore, if the distribution of fundamentals and, therefore, the exchange rate is concentrated away from the bands by intramarginal intervention, the relationship would look linear.[6] More credible regimes would be associated with more observed linearity between the exchange rate and fundamentals, as Flood/Rose/Mathieson find.

Overall, the empirical evidence in the authors' paper appears to constitute a strong rejection of the target zone only when combined with the assumption of no intervention within the bands. Intramarginal intervention would appear to be an important aspect to incorporate into future empirical analysis of the model.

2. Boundary conditions and central bank behavior

The results in Flood/Rose/Mathieson also raise questions about the boundary condition. They consider the target-zone model assuming a particular set of boundary conditions for the solution of the exchange rate. In partic-

[6] See Lewis (1990) for evidence of this phenomenon on the DM and yen rate against the dollar following the Louvre accord.

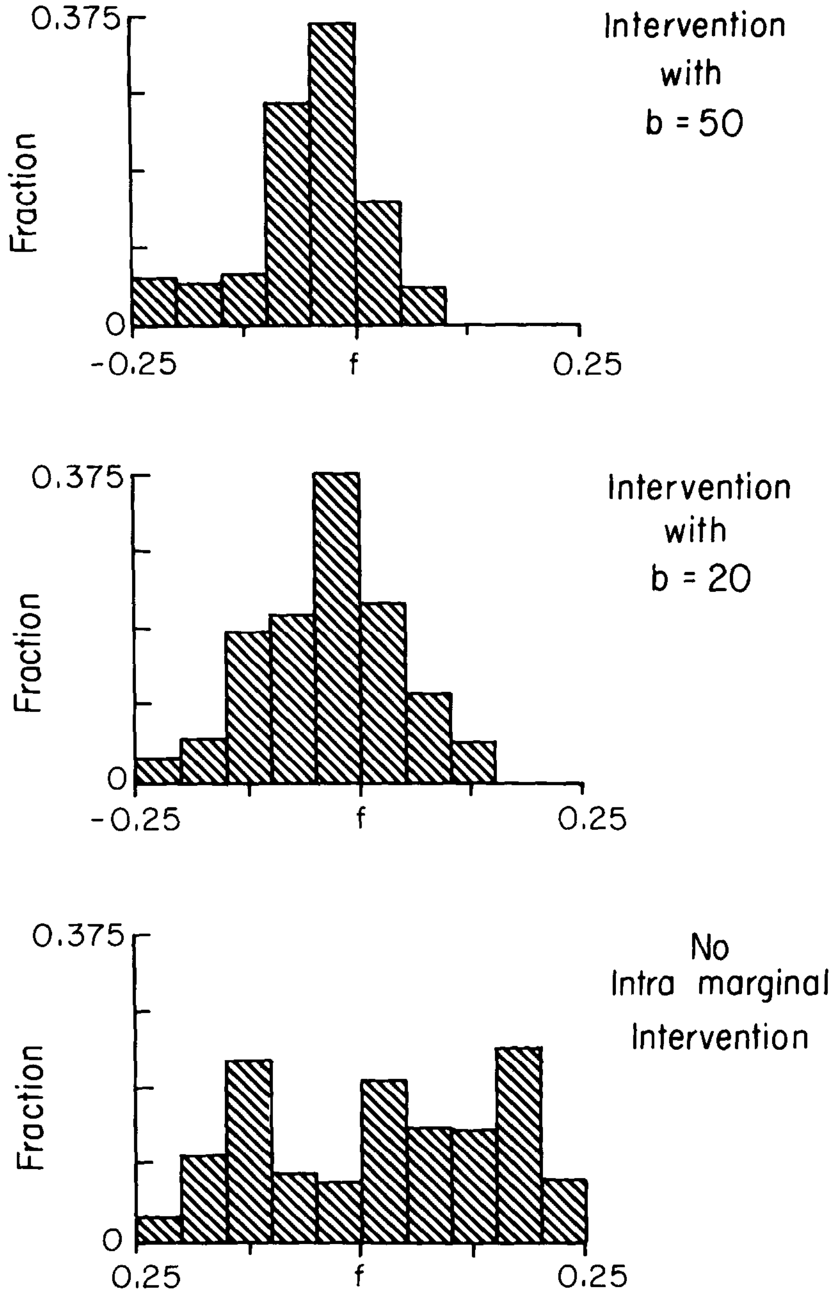

Figure 2:
Histograms of Fundamentals With and Without Intra Marginal Intervention

ular, they focus upon the case where the regime is "credible" as depicted in Figure 1. However, they also discuss the realignment exchange-rate solution considered by Bertola and Caballero (1989) that they call the "incredible" regime.[7] They claim that the model fails because they find no consistent relationship across time or countries.

After closer consideration, this empirical finding actually seems perfectly consistent with rational economic agents. Afterall, if we found that people believed a "credible" regime relationship when in reality realignments were taking place, people would have to be quite irrational. In particular, people's expectations about what would happen if the exchange rate hit the bands would likely change over time with factors such as the level of reserves, the policies of the government in power, and so forth. In this case, solving the exchange-rate problem becomes a much more difficult problem to solve. Recall that the boundary condition used to define the regime, whether credible, incredible, or floating, is based upon people believing in this boundary condition *forever*. If people's beliefs about the regime evolve over time, the simple boundary conditions considered in the target-zone literature will not be correct. For this reason, future research in this area may have to think more seriously about how the central bank behaves and the evolution of people's beliefs about these policies.[8]

Concluding remarks

After an extensive study, Flood/Rose/Mathieson find little evidence for the standard target-zone model of the exchange rate. This version of the model assumes that interventions only occur at the boundaries and that all realignments are unanticipated. Given the failure to find evidence supporting this model, future research may now move forward to consider difficult tasks such as incorporating realistically the market's expectations of official policy at bands and intramarginal intervention.

---

[7]As Obstfeld (1990) has pointed out, the inverted S-shaped relationship that arises from this model depends crucially upon special assumptions. Specifically, Bertola and Caballero (1989) assume that when the exchange rate hits the band, the probability of a realignment exceeds 1/2. Furthermore, if a realignment occurs, the exchange rate jumps to the middle of the next band but if no realignment occurs, the exchange rate jumps back into the middle of the current band. Obstfeld (1990) shows that the solution looks more like the standard S shape under plausible alternative assumptions, such as a probability of realignment less than 1/2 or a smaller jump with realignment. Thus, there are many possible "incredible regime" boundary solutions that need not have the inverted S-shape that Flood/Rose/Mathieson consider.

[8]Svensson (1990) has pioneered an interesting direction in this literature by incorporating additional state processes that jump with realignments.

References

Bertola, G. and Caballero, R., (1989). Target Zones and Realignments. Columbia University Working Paper.

Frenkel, J.A. and Mussa, M., (1980). The Efficiency of Foreign Exchange Markets and Measurs of Turbulence. *American Economic Review Papers and Proceedings*, **70**: 374-81.

Froot, K.A. and Obstfeld, M., (1989). Exchange Rate Dynamics under Stochastic Regime Shifts: A Unified Approach. *Harvard Institute of Economic Research*, Discussion Paper, **1451**.

Klein, M.W. and Lewis, K.K., (1990). Learning About Intervention Target Zones. *National Bureau of Economic Research*, Working Paper, 3674.

Krugman, P., (1991). Target Zones and Exchange Rate Dynamics. *Quarterly Journal of Economics*, **106**: 669–682.

Lewis, K.K., (1990). Occasional Interventions to Target Rates. *National Bureau of Economic Research*, Working Paper, **3398**.

Mastropasqua, C., Micossi, S., and Rinaldi, R., (1988). Interventions, Sterilization and Monetary Policy in European Monetary System Countries. *European Monetary System*, (eds.) F. Giavazzi, S. Micossi, and M. Miller. Cambridge: Cambridge University Press.

Mussa, M., (1982). A Model of Exchange Rate Dynamics. *Journal of Political Economy*, **90**: 74–104.

Obstfeld, M., (1990). Target Zones and Realignments: Comment. *National Bureau of Economic Research Conference on Asset Pricing.*

Svensson, L.E.O., (1990). The Foreign Exchange Risk Premium in a Target Zone with Devaluation Risk. *National Bureau of Economic Research*, Working Paper Series, **3466**.