From Private-Belief Formation to Aggregate-Vol Oscillation

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Abstract

What are the quantitative implications of learning and informational asymmetries, for generating fluctuations in aggregate and cross-sectional volatility over the business-cycle? I propose a model that relies on informational channels, for the endogenous amplification of the conditional volatility in macro aggregates and of cross-sectional dispersion during economic slowdowns, in a homoscedastic-shock environment. The model quantitatively matches the fluctuations in the conditional volatility of macroeconomic growth rates, while generating realistic real business-cycle moments. Consistently with the data, shifts in the correlation structure between firms are an important source of aggregate volatility. Up to 80% of the conditional aggregate volatility fluctuations are attributed to fluctuations in cross-firm correlations. Correlations rise in downturns due to a higher weight that firms place on public information, which causes their beliefs, and policies, to comove more strongly. In the data, correlations rise at recessions in spite of a contemporaneous increase in cross-sectional volatility, as the average between-firm covariance spikes more than dispersion does.

Keywords: Endogenous volatility, learning, asymmetric information, business cycle
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1 Introduction

What are the quantitative implications for aggregate and cross sectional volatility fluctuations, that are induced by Bayesian learning and informational asymmetry? This study provides a micro-founded model that endogenously generates time-varying volatility for aggregate growth rates, via imperfect information channels, while generating realistic unconditional real business-cycle moments. Macroeconomic volatility in this paper is built in a bottom-up approach. The model suggests that endogenous oscillations in the correlation between firms' policies are an important source for aggregate volatility fluctuations. In recessionary periods, correlations rise as a result of stronger reliance on public information. Higher between-firm correlation is translated into higher aggregate volatility. Both learning *and* asymmetric information are crucial to generate economically significant fluctuations in aggregate volatility. Empirically, correlations do rise at bad times, in spite of an increase in cross-sectional variation of real variables (dispersion), since the average between-firm covariance rises in recessions more than dispersion does.

The importance of this study lies in the growing body of literature in macroeconomics and finance, which stresses the pivotal role of higher volatility in hindering economic recovery, growth, and asset-prices. Specifically, consider the following stylized facts regarding the time-varying behavior of volatility, and its implications:

<u>Fact (I)</u>: Aggregate and cross-sectional volatilities are stochastic:

a. The conditional volatility of real aggregate macroeconomic variables, such as output and investment growth, rises in economic downturns. Quarterly GDP growth has about 35% more conditional volatility in NBER recessions (Bloom (2013)); Consumption growth's volatility increases by 30% in bad times (This paper).

b. The cross-sectional dispersion of real quantities produced by firms is countercyclical. Firms' output growth, and employment growth, are negatively correlated with detrended GDP (see e.g. Bachmann and Bayer (2013)).

c. The average correlation between firm-level real-variables (e.g. output, investment), increases in economic slowdowns (This paper). This fact complements the more established notion that the average correlation amongst stock-returns significantly increases in recessions (see e.g. Moskowitz (2003), Krishnan, Petkova, and Ritchken (2009)).

Fact (II): An increase in the volatility of macroeconomic fundamentals has an adverse effect on the real and financial economy; in particular -

a. Volatility reduces investment and economic activity due to a real-option effect (see e.g. Dixit and Pindyck (1994); Bloom (2009)), or due to a rise in the cost of capital (see e.g. Christiano, Motto, and Rostagno (2010); Arellano, Bai, and Kehoe (2010); and Gilchrist, Sim, and Zakrajsek (2014)).

b. Volatility lowers asset valuations, raises risk premia and increases return volatility (see e.g. Bansal, Khatchatrian, and Yaron (2005); Drechsler and Yaron (2011); Bansal, Kiku, Shaliastovich, and Yaron (2014)).

The studies that examine the impacts of macro-volatility on the economy, that is, explore fact (II) above, treat the shocks to volatility, detailed in fact (I), as exogenous and independent of other fundamentals. Differently put, the evolution of stochastic volatility is traditionally modeled using an exogenous process. Yet, as illustrated in this study, a perfect-information neo-classical growth model without exogenous stochastic volatility, generates only a negligible increase in the conditional volatility during downturns. This raises a gap, to uncover the economic forces that lead the volatility of aggregate fundamentals to fluctuate and rise in recessions.

This study proposes a theory for the endogenous emergence of stochastic macro volatility, in an environment of only homoscedastic first-moment shocks. It is thus aimed at *quantitatively* explaining facts (I.a) - (I.c), while generating realistic business-cycle unconditional moments. By doing so, the work helps to bridge the gap between the econometric findings of fact (I), and the macroeconomic and financial literature of fact (II). I demonstrate that learning is quantitatively important to understand the dynamics of volatility and correlations over the business-cycle[1].

The model presented in this paper relies on five main ingredients: (1) Existence of a mass of atomistic firms; (2) The aggregate TFP shock is latent, but can be fully recovered with a continuum of signals; (3) Firms use Bayesian learning to update their belief about the current TFP level, and by doing so they rely on both public and private information; (4) Higher economic activity at the firm level, helps the firm to learn about the unobserved TFP, by endowing it with more signals; and (5)

[1]Notably, more than one explanation is plausible for the observed behavior of volatility. The economic forces described in this work should be viewed as a significant source of macro volatility fluctuations, among possible others.

It takes a lag of one period to publish any macroeconomic quantity, including any public information about the aggregate TFP.

In a nutshell, the economic narrative of the paper is as follows. Each period, the firm receives two signals regarding the aggregate latent TFP shock. The first is the firm's own privately observed output, and the second is the *lagged* aggregate TFP, which serves as a public signal. On one hand, firms produce using capital and labor. The productivity of each hour of labor is subject to an unobserved, homoscedastic, and idiosyncratic labor efficiency shock, that captures the effect of time-varying tiredness, motivation, and focus on human capital. Consequentially, every hired hour of labor provides an idiosyncratic signal, with fixed precision on the aggregate state. Thus, firms that hire more labor, have a better ratio of signal to noise. In bad times, firms choose to reduce their rented working-hours, due to decreased profitability. Reducing the amount of labor drops the precision of the firm's private idiosyncratic signal (output) in recessionary periods. On the other hand, as all firms are atomistic, a central-household that observes all firms' outputs can become perfectly informed about the underlying TFP by the end of the period. Publishing this recovered aggregate TFP *in a lag*, is equivalent to a signal on today's TFP with fixed precision, as all shocks are homoscedastic. As a result, in bad times, firms place more weight on public information, and less on their idiosyncratic information, when constructing their beliefs on the current state of the economy. This generates a greater comovement in the beliefs of firms in economic slowdowns, and hence, a larger comovement in their investment and labor policies. The higher degree of correlation amongst firms in bad times increases the volatility of aggregate quantities, such as output and consumption.

When calibrated at quarterly frequency to match the unconditional moments of consumption and output growth rates, the learning model amplifies the oscillations in the conditional volatility of aggregate growth rates, while a no-learning model, similar to the neo-classical firm problem, produces only minuscule changes in the conditional volatility. In the learning environment, consumption growth's volatility rises in bad periods (when TFP growth is low) by 29% in the model versus 32% in the data, while it falls in good times (when TFP growth is high) by 20% and 14% in the model and in the data, respectively. For comparison, in the no-learning environment, the volatility of aggregate consumption fluctuates by merely 3% in bad times. Similar results are obtained for other macro growth rates. Capital's growth volatility increases in bad times by 57% and by 56%, in the model and in the data.

I further establish that the movements in the conditional volatility of aggregates, capture shifts in the average conditional covariation between firms, as all firm-specific volatility is effectively diversified away at the aggregate level. While this claim holds exactly in the model in which firms are atomistic, I show that it roughly holds in the data as well, in spite of the fact that empirically some firms are non-atomistic (see e.g. Gabaix (2011)).

Since aggregate volatility amounts to the average covariation between firms, I can then decompose aggregate volatility. The fluctuation in *aggregate* volatility between bad and normal periods is equal to the fluctuation in *firm-level* volatility multiplied by the fluctuation in the average between-firm *correlation*. In the no-learning model, the small fluctuations in the conditional volatility of aggregates, are shown to be driven by small changes in the conditional firm-level volatility, while the average correlation between firms is approximately constant. By contrast, in the learning model, the fluctuations in the conditional volatility of aggregates, are largely due to shifts in the conditional correlation between firms, that rises in bad times. The average correlation between firms' outputs increases by 32% in bad times, and drops by 29% in good times. For all aggregate growth variables, about 80% to 90% of the increase in the conditional aggregate volatility in bad times is attributed to an increase in the conditional correlations.

The endogenous fluctuations in the average correlation in the learning model stem from two main model ingredients: (1) Bayesian leaning, with time varying gains, and (2) Informational asymmetry. To show this, I shut down each of those channels separately. Namely, I solve a modified learning model in which the labor noise shocks are aggregate rather than idiosyncratic, thus eliminating informational asymmetries between firms. In addition, I solve an alternative model in which the weights that firms place on the public and private signals are fixed, and do not vary with the actual gain (non-Bayesian learning). Both of the modified models lack the ability to produce significant volatility fluctuations. Thus, a rise in (belief) uncertainty in bad times, as some earlier works feature, is not sufficient to produce enough *realized* volatility at the aggregate level.

Lastly, I show that the cross-sectional variation in the model, or dispersion, is also countercyclical, as is also the case empirically. The correlation of output growth dispersion with TFP is negative in the model and in the data. This is a result of a rise not only in aggregate volatility, but also in firm-level volatility in bad times, in the model. Ostensibly, an increase in dispersion seems to contradict a rise in the expected correlation between firms. I reconcile the two by showing that if the average between-firm covariation, increases in magnitude more than dispersion does in downturns, the average correlation increases as well. In the data, the rise in the average covariation in bad times ranges between 19% to 55%, while dispersion rises by less.

The rest of the paper is organized as follows. Section 2 offers a discussion of related literature. In section 3, I provide the economic model. Section 4 presents the data and the econometric methodology used to construct the conditional volatilities and correlations. In Section 5, I report the model calibration and its implications for unconditional business-cycle moments. Section 6 discusses the main results of this paper: the implications of learning for the fluctuations in the aggregate conditional volatility, both in the model and empirically, and its decomposition into firm-level volatility and average correlation. The section also presents the implications of the learning model for cross-sectional volatility, and establishes the robustness of the results. Section 7 provides concluding remarks.

2 Related Literature

This study relates to several strands of literature. The closest studies that my work is relates to, are theoretical macroeconomic models that aim at explaining why objects like *uncertainty*, *volatility* and *dispersion* vary over time, both at the firm-level, and at the macro level. A growing number of recent papers attempt to endogenize *uncertainty*, mainly in centralized economies, over the business cycle (that is, why the volatility of agents' *beliefs* over the state of the economy increases in recessions). In Van Nieuwerburgh and Veldkamp (2006), procyclical learning about productivity generates countercyclicality in firm-level uncertainty that may relate to countercyclical movements in asset prices.[2] Fajgelbaum, Schaal, and Taschereau-Dumouchel (2013) also endogenize uncertainty level, and link it to economic activity via learning: higher uncertainty about the fundamental discourages investment, which in turn results in fewer signals about the fundamental, thus keeping uncertainty levels high, which discourages investment further. Similarly, Orlik and Veldkamp (2013) show that a Bayesian forecaster who revises model parameters in real-time, experiences countercyclical uncertainty shocks, even if the underlying process is homoscedastic. This occurs as the agent is more confident in predicting the future when growth is normal, while sudden "unfamiliar" events in recessions make it harder for the forecaster to

[2]Related work to Van Nieuwerburgh and Veldkamp (2006) includes Ordoñez (2013). Ordoñez (2013) argues that the speed of boom and busts depends on the financial system of the country. In his work however, beliefs are only public and the state of the economy is the volatility of productivity. Thus, volatility is exogenously stochastic, while this work features homoscedastic volatility.

make predictions. *A key difference* between this paper and the former works, is that I focus on *volatility*, or in other words, the time-varying predictable variation of *realized* quantities, while the former discuss *uncertainty* levels, that is, the forecasting error squared is time-varying. The former works do not predict that firms' actual policies are necessarily becoming more volatile, or that the correlation between firms' policies is fluctuating.

Other recent works endogenize *firm-level volatility*, or *dispersion*, in good versus bad periods. Bachmann and Moscarini (2011) show that downturns offer the opportunity for firms to drastically alter their pricing policy, or to "experiment", allowing them to better learn their firm-specific demand function. This experimentation, mainly performed to decide whether to exit the market, is the driver of *cross-sectional dispersion* in the prices of firms. In Decker, DErasmo, and Boedo (2013), first moment TFP shocks enable firms to expand to more markets and expose firms to an increased number of market-specific shocks, which reduces firm-level volatility by diversification. Related, Tian (2015) also endogenizes productivity dispersion over the business-cycle. These works focus on *(micro) cross-sectional volatility.* They do not explicitly examine whether this micro volatility feeds into higher aggregate quantities. In contrast, this work propagates the notion that an important source of aggregate volatility is not merely an increase in individual firms' idiosyncratic volatility, but rather an increase in the correlation between individual firms' policies.

Some related papers explicitly discuss aggregate volatility, which is the main focus of this work. One contributor to aggregate volatility may come from Governments and Central Banks. Pastor and Veronesi (2012) argue that policy becomes more volatile during recessions because policy makers wish to experiment. In economic downturns, politicians are drawn to experiment as they attempt to boost growth. While this explanation directly feeds to macro volatility, it differs from the bottom-up approach of the decentralized economy, taken in this paper. Gabaix (2011) shows that idiosyncratic firm-level fluctuations can explain a significant portion of aggregate shocks, when some firms are non-atomistic or "granular". His study however, focuses on unconditional aggregate volatility, not on its cyclical behavior. Kelly, Lustig, and Van Nieuwerburgh (2013) endogenize firm level volatility (dispersion) using a different framework than mine: consumer-supplier business networks, with some implications for aggregate volatility. More related, Ilut, Kehrig, and Schneider (2013) show that when hiring decisions respond more to bad signals, due to ambiguity about the level of noise, both aggregate conditional volatility and dispersion of labor growth are countercyclical. A similar idea is used in the context of stock return correlations, in the works of Ribeiro and Veronesi (2002), and Ozsoy (2013). As opposed to my quantitative study, that embeds learning in a real business cycle environment, the work of Ilut *et al.* (2013) is mostly qualitative. My work also employs a different learning mechanism. The papers of Thesmar and Thoenig (2004), Comin and Philippon (2006), and Comin and Mulani (2006), also target aggregate volatility by trying to explain the so-called "great moderation" in the volatility of aggregate returns and output (see Stock and Watson (2003)). However, these works target the ostensible trend in aggregate volatility, while they do not generate fluctuations of aggregate volatility over the business cycle.

The second body of works related to this paper are studies discussing the social value of public information, starting with the influential work of Morris and Shin (2002). The work of Amador and Weill (2012), shows that increasing public information slows down learning in the long run, and may reduce welfare. While aggregate volatility fluctuates in their model, their stylized framework exhibits a hump shape for volatility over time, that converges to zero in the long run, and does not explain why volatility increases in recessionary periods.

Related, Angeletos and La'O (2013) show that even without aggregate TFP shocks, sunspot public shocks that purely affect agents' belief about the state of the economy, without altering the underlying technology or preferences, termed "sentiments", create aggregate fluctuations. While their framework highlights that public information can serve as an important source of aggregate fluctuations, it produces fixed volatility for aggregate output. A closer work of Angeletos, Iovino, and La'O (2011) demonstrates, in a comparative static manner, that more precise public information reduces dispersion, but can increase the volatility of aggregate output. In contrast, in my work the precisions of the signals is time-varying, allowing to obtain stochastic volatility. In addition, my work is quantitative in nature, and targets objects that are absent from the former works, such as investment rate and capital growth. My work complements these works in that my focal point is different. I harness the use of time-varying weights on public and private information to obtain aggregate volatility that varies over the business cycle.

The third branch of studies my paper is related to, are econometric papers that document that macro volatility, micro volatility (or cross-sectional volatility), and also correlations, rise in recessions. Bloom (2013) documents that industrial production growth, based on GARCH models, has about 35% more conditional volatility in recessions. In the context of stock returns, Bloom (2013) and Bekaert, Hoerova, and Lo Duca (2013), report that the VIX level is countercyclical, and increases by 58% in recessions. Other meaures of macro uncertainty also increase in bad times. Jurado, Ludvigson, and Ng (2013) use monthly economic series in a system of forecasting equations and look at the implied forecasting errors. They find a sharp increase in recessionary periods, and in particular, in the Great Recession. The works of Higson, Holly, and Kattuman (2002), Döpke, Funke, Holly, and Weber (2005), Jorgensen, Li, and Sadka (2012), Kehrig (2011), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Bachmann and Bayer (2013), provide extensive evidence that cross-sectional variance, or dispersion, is also highly countercyclical, for various economic outcomes including output growth, sales growth, employment growth, earnings growth, and Solow residuals. Investment-rate dispersion however, seems to be procyclical, as pointed by Bachmann and Bayer (2014). Moskowitz (2003), in the context of stock returns, uses a multivariate-GARCH approach to show that conditional correlations exhibit significant time variation, increase during recessions, and were extremely large during the 1987 stock market crash. Similarly, Krishnan *et al.* (2009) use average realized correlations of stock returns, and show that it significantly rises in recessions. My work contributes to these findings by empirically showing that the correlation of *fundamentals*, such as investment-rate and output growth, increases in recessionary periods, and explains fluctuations in aggregate volatility of fundamentals.

The last strand of papers my work relates to are macroeconomics and asset-pricing works that stress the importance of aggregate volatility in explaining business-cycle fluctuations, economic growth and risk premia. Bloom (2009) shows that increased volatility, measured via VIX, leads to an immediate drop in output and investment growth rates as firms delay their investment decisions. The work of Fernandez-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011) discusses uncertainty in an open-economy context, showing that higher volatility lowers domestic investment. Other works argue that higher volatility increases the cost of capital, or credit spreads, hence makes investment more costly (see e.g. Christiano *et al.* (2010); Arellano *et al.* (2010); and Gilchrist *et al.* (2014)). Basu and Bundick (2012) rely on nominal rigidities to show that both consumption and investment can drop in response to volatility shocks. Other works rely on alternative economic forces which can yield a positive relationship between volatility and investment. These channels include precautionary savings, time-to-build, or investment irreversibility (see e.g. Abel and Eberly (1996); Bar-Ilan and Strange (1996); Gilchrist and Williams (2005); Jones, Manuelli, Siu, and Stacchetti (2005); Malkhozov (2014); and Kung and Schmid (2014)). Importantly, these papers treat volatility shocks as exogenous, while in this paper I treat volatility as an endogenous object.

3 Model

This section describes the theoretical framework that generates stochastic aggregate volatility in a homoscedastic world. The economy is comprised of a mass of firms, indexed by $i \in [0,1]$, and one representative household, who owns all firms and consume their dividends. Below I describe the problem faced by firms, the household, and a definition of an equilibrium in this setup.

3.1 Aggregate Productivity

Aggregate productivity, denoted by G_t, evolves as geometric random walk with time varying drift. Specifically, $G_{t+1} = G_t \cdot g_t$, where

$$g_t = (1-\rho_g)g_0 + \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t},$$

and where $\varepsilon_{g,t} \sim N(0,1)$[3]. Notice that the conditional volatility of aggregate productivity growth is constant. Further, note that g_t is the gross-growth rate of productivity, and I assume that the mean growth rate $g_0 > 1$ is sufficiently large, in comparison to the volatility of the shock σ_g such that g_t is always positive.

It is assumed that aggregate productivity is a latent variable. This is also the case in the real world: total factor productivity is unobserved, but can be recovered by observing real aggregate macroeconomic growth rates. Both firms and the household learn about the current and past levels of productivity from publicly and privately observed signals. All information regarding the productivity shock is obtained from real (noisy) economic outcomes. As explained later, all agents become perfectly informed about any lagged level of aggregate productivity, but there is uncertainty regarding the current period's productivity growth g_t.

3.2 Firms

Each firm is operated by a manager. The firm operates on an island. As a result, all aggregate quantities, including aggregate productivity level, become observable to the firm in a lag of one period. This assumption parallels to the real world, in the sense that aggregate quantities are usually published in some lag. Specifically, at the beginning of every period t, the manager of the firm gets an input from its owner (the

[3]Notice that G_t is predetermined.

household): last period's aggregate productivity growth g_{t-1} [4]. This assumption is consistent with the availability of data in reality: the San-Fransisco Federal Reserve Bank, for instance, publishes a TFP time-series, in a lag of one-quarter[5]. In return, the firm ships back to the owner its current period dividend after producing and investing.

Firms produce output using capital and labor. Firm i has a stock of capital $k_{i,t}$, and rented labor inputs (measured in time-units, or hours) $l_{i,t}$.

Capital evolves according to:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + \Lambda(\frac{I_{i,t}}{k_{i,t}}),$$

where δ is the depreciation rate, and $I_{i,t}$ is the investment level at period t. The capital adjustment cost function Λ is specified as in Jermann (1998): $\Lambda(\frac{I_{i,t}}{k_{i,t}}) = \frac{\alpha_1}{1-\frac{1}{\zeta}}(\frac{I_{i,t}}{k_{i,t}})^{1-\frac{1}{\zeta}} + \alpha_2$. The parameter ζ represents the elasticity of the investment rate with $\zeta \to \infty$ representing infinitely costly adjustments. The parameters α_1 and α_2 are set such that there are no adjustment costs in the deterministic steady state.

Labor to be used in period t is rented in the period $t-1$ for a wage w_t per unit of time (hour). The wage exogenously grows at the same rate as aggregate productivity, and is given by $w_t = w \cdot G_{t-1}$ [6]. Adjusting the labor force, requires a non-pecuniary adjustment cost, and is given by $\Phi_L(l_{i,t}, l_{i,t+1}) = G_t \cdot \frac{\kappa}{2} \cdot (l_{t+1} - l_t)^2$. These adjustment costs capture, in a reduced form manner, the costs induced by the friction of search. In the absence of consumption smoothing in a risk-neutral setting, this adjustment cost is vital to make labor growth, and hence output and consumption growths, sufficiently persistent.

Firms also face two idiosyncratic shocks. First, firms revenue is affected by an observed demand shock $z_{i,t}$, that evolves according to an AR(1) process:

$$z_{i,t} = (1 - \rho_z)z_0 + \rho_z z_t + \sigma_z \varepsilon_{i,z,t},$$

[4] Alternatively, the firm's input is the aggregate productivity *level* of period $t-1$. This is an equivalent assumption, as the productivity level of time $t-2$ is already in the information set of the firm at time t. Dividing aggregate productivity level of time $t-1$ with that of time $t-2$ yields g_{t-1}.

[5] The quarterly TFP data relies on Basu, Fernald, and Kimball (2006) and Fernald *et al.* (2012).

[6] As labor is hired in period $t-1$, I specify a wage that incorporates only time $t-1$ information. The reason that labor is pre-hired in my setting is that otherwise, one could potentially learn with certainty the current level of productivity growth g_t simply by observing the current labor wage. By making labor predetermined, the current wage reflects merely g_{t-1}, which is already known to the firm at time t.

where the innovation is conditionally homoscedastic, and $\varepsilon_{i,z,t} \sim N(0,1)$. The second idiosyncratic shock, $\varepsilon_{i,l,t}$, is a shock to the efficiency of labor. It is assumed to be a latent i.i.d. shock across time and across firms, with $\varepsilon_{i,l,t} \sim N(0,1)$. This conditionally homoscedastic shock captures disturbances to the efficiency of the labor force, that are unobserved to the firm, such as time-varying levels of focus, tiredness and motivation that may affect a human resource.

The production technology of firm i at time t is therefore given by:

$$y_{i,t} = G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha} (g_t l_{i,t} + \sigma_l \varepsilon_{i,l,t})^{\nu-\alpha}, \tag{3.1}$$

where $\nu \in (0,1)$ is the the total returns to scale. This specification is similar to that used in Van Nieuwerburgh and Veldkamp (2006), but augmented to support labor and growth. It is a reduced-form production function that captures a very basic notion: bigger firms who acquire more labor, and have a higher economic activity, have a higher loading on the aggregate TFP growth g_t, and enjoy a preferable signal to noise ratio, as illustrated next. This assumption can be motivated explicitly by breaking the total labor time stock $l_{i,t}$ into operating time-units (hours), each of which provides another signal on the aggregate TFP shock. Below I outline briefly a microfounded explanation for the emergence of such a production function.

Suppose that each firm operates by hiring its labor force to work for $l_{i,t}$ hours. For motivational purposes think of $l_{i,t}$ as discrete. The productivity of the labor force, per hours $\ell \in [1..l_{i,t}]$, varies. As mentioned earlier, this assumption captures the effect of time-varying tiredness, or motivation. Specifically, in every hour ℓ, the labor force productivity, in labor efficiency units, is $g_t + \sigma_l \eta_{i,\ell,t}$, where g_t is the aggregate shock of the labor augmenting technology, and $\eta_{i,\ell,t}$ is an idiosyncratic efficiency shock, independent over firms and hours, and distributed $N(0,1)$. All $\eta_{i,\ell,t}$ shocks are latent, and so is g_t.

By integrating over all hours, the firm's total labor input, in efficiency units can be written as: $g_t l_{i,t} + \sigma_l \sqrt{l_{i,t}} \varepsilon_{i,l,t}$, where $\varepsilon_{i,l,t} \sim N(0,1)$. To make sure that all shocks are explicitly homoscedastic, I choose to solve a version of the model in which the labor efficiency is simply $g_t l_{i,t} + \sigma_l \varepsilon_{i,l,t}$, as specified in equation (3.1)[7].

It is assumed that neither the current aggregate productivity growth g_t, nor the additive idiosyncratic productivity shock to labor $\varepsilon_{i,l,t}$ is observed by the firm. Firms learn about the state of the economy, that is on g_t, by receiving two types of signals.

[7]Solving a version of the model in which $\varepsilon_{i,l,t}$ is pre-multiplied by $\sigma_l \sqrt{l_{i,t}}$ yields quantitatively very similar results.

The first signal, is the firm's own privately observed idiosyncratic output. Rewriting the firm's output as a signal on g_t, one obtains:

$$s_{i,t} = \frac{1}{l_{i,t}}\left(\frac{y_{i,t}}{G_t^{1-\alpha} z_t k_{i,t}^{\alpha}}\right)^{\left(\frac{1}{\nu-\alpha}\right)} = g_t + \frac{\sigma_l}{l_{i,t}}\varepsilon_{i,l,t}. \tag{3.2}$$

Thus, the precision of the private signal is $l_{i,t}^2 \sigma_l^{-2}$, which is time varying and increases with the amount of labor the firm rents. This assumption makes some intuitive sense: bigger firms have better access to information due to more operating branches, and access to different segments of the market. The firm's output can be written in terms of the observed signal: $y_{i,t} = G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha}(s_{i,t} l_{i,t})^{\nu-\alpha}$.

The second signal, which is analyzed next, is the publicly observed level of lagged aggregate productivity growth g_{t-1}. It is assumed that as the firm lives on an island, the only information its manager observes is what is shipped by its owner. In other words, each firm can observe aggregate productivity, sent from its owner to the island, and all other aggregate quantities or prices without restriction, in a lag of one period.

With a mass of firms, the assumption that aggregate consumption growth, or output growth are fully observed by the firm in a one-period lag, is equivalent to assuming that g_{t-1} is observed in a lag with certainty. The intuition is that at the aggregate level, all idiosyncratic shocks are diversified, thus fully revealing aggregate productivity growth. Aggregate consumption growth at time t, for instance, would be a function of g_t and the distribution of capital and labor. Assuming that the distribution of capital becomes known to the manager in a lag, once the distribution of resources is fixed, consumption is monotonically increasing with g_t. Hence, one can find a one-to-one mapping between aggregate consumption growth level, and g_t, conditioning on the distribution of capital and labor. The conclusion is therefore that observing aggregate real quantities in a lag does not provide any further information on today's g_t, beyond observing g_{t-1} directly, which is sent to the firm at the beginning of period t.

By equation (3.1), g_{t-1} can be perceived as a public signal on g_t with fixed precision, where the mean of the signal is $(1-\rho_g)g_0 + \rho g_{t-1}$ and the precision is σ_g^{-2}, by the assumption of homoscedastic shocks. As firms cannot obtain any information on the current level of g_t that is not contained in g_{t-1}, this public signal determines the *common* prior for all firms on g_t, at the beginning of the period.

At the beginning of each period, the firm first produces using its capital and labor stocks that are predetermined in the last period. Then, using the public signal g_{t-1}, and using its own private idiosyncratic signal (its output, or alternatively $s_{i,t}$), it forms a posterior belief on what today's level of g_t is. Using this belief, the firm picks its level of next period capital $k_{i,t+1}$, that is, the firm chooses its investment level, and also hires its next period labor force, $l_{i,t+1}$.

The private and the public signals the firm obtains can be collapsed into one posterior belief, that weights the private and the public information with their respective relative precisions. By Bayes rule, the weight the firm will put on the private signal $s_{i,t}$, and on the public signal, are given by:

$$w_{private,i,t} = \frac{l_{i,t}^2 \sigma_l^{-2}}{l_{i,t}^2 \sigma_l^{-2} + \sigma_g^{-2}}; \quad w_{public,i,t} = 1 - w_{private,i,t}. \tag{3.3}$$

In bad times, when aggregate TFP growth is smaller, $l_{i,t}$ is on average smaller as firms optimally choose to scale down, invest less, and hire less labor. Consequentially, expression (3.3) demonstrates that the firm puts more weight on the public information in recessions, and less on its own idiosyncratic signal. Thus, posterior beliefs are becoming more correlated among firms in recessions, triggering a higher correlation between the policies of firms, and contributing to a higher aggregate volatility.

The manager is trying to maximize the firm's value, given his own public and private information. The information set of the manager at the beginning of the period t, right after producing, is given by: $k_{i,t}$, the firm's capital, $l_{i,t}$, the firm's labor, $s_{i,t}$, the productivity signal obtained from the firm's private output, g_{t-1}, the public signal, and lagged level of aggregate productivity G_{t-1}. Given this information set, the manager solves the following maximization problem:

$$
\begin{aligned}
V_{i,t}(k_{i,t}, l_{i,t}, G_{t-1}, g_{t-1}, s_{i,t}, z_{i,t}) \quad = \quad & max_{k_{i,t+1}, l_{i,t+1}} \quad G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha} (s_{i,t} l_{i,t})^{\nu-\alpha} - w_t l_{i,t} - I_{i,t} \\
& \quad -\Phi_L(l_{i,t}, l_{i,t+1}) \\
& \quad +\beta E_t[V_{i,t+1}(k_{i,t+1}, l_{i,t+1}, G_t, \hat{g}_t, s_{i,t+1}, z_{i,t+1})] \\
& \text{s.t.} \\
& k_{i,t+1} = (1-\delta)k_{i,t} + \Lambda(\frac{I_t}{k_t}) \\
& \Lambda(i) = \frac{\alpha_1}{1-\frac{1}{\zeta}}(i)^{1-\frac{1}{\zeta}} + \alpha_2 \\
& \Phi_L(l_{i,t}, l_{i,t+1}) = G_t \frac{\kappa}{2}(l_{i,t+1} - l_{i,t})^2 \\
& w_t = G_{t-1} w \\
& V_{i,g,t} = [\frac{1}{\sigma_g^2} + \frac{l_{i,t}^2}{\sigma_l^2}]^{-1} \\
& \mu_{i,g,t} = V_{i,g,t}[\frac{1}{\sigma_g^2}((1-\rho_g)g_0 + \rho_g g_{t-1}) + \frac{l_{i,t}^2}{\sigma_l^2}(s_{i,t})] \\
& \hat{g}_t = \mu_{i,g,t} + \sqrt{V_{i,g,t}} \varepsilon_{i,\mu,t}; \quad G_t = G_{t-1} g_{t-1} \\
& s_{i,t+1} = [(1-\rho_g)g_0 + \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}] + \frac{\sigma_l}{l_{i,t+1}} \varepsilon_{l,t+1} \\
& \varepsilon_{i,\mu,t} = (g_t - \mu_{i,g,t})/V_{i,g,t} \sim N(0,1),
\end{aligned} \tag{3.4}
$$

where $\mu_{i,g,t}$ and $V_{i,g,t}$ are the posterior mean and variance (uncertainty) of the belief on g_t. $\hat{g}_t$, the stochastic belief on g_t, is defined as $\hat{g}_t = \mu_{i,g,t} + (g_t - \mu_{i,g,t}) = \mu_{i,g,t} + \sqrt{V_{i,g,t}} \varepsilon_{\mu,i,t}$, where $\varepsilon_{\mu,i,t} \sim N(0,1)$. When computing the continuation value, the manager uses his belief $\hat{g}_t$ to project the evolution of all variables that are contingent on g_t, including future aggregate and private signals.

3.3 Household

There is one infinitely lived representative household in the economy, that holds all firms, and exerts utility from a consumption stream of C_t. It is assumed that the household is risk neutral. The time discount rate of the household is β. The household derives income from dividend payments from its diversified portfolio of corporate stocks.

After firms produce, and ship back their dividend to the household, the representative household gets to observe the output of all firms, comprising together a mass of signals $\{s_{i,t}|i \in [0,1]\}$ on g_t with finite precisions. As a result, the household becomes perfectly informed about aggregate productivity growth by the end of period t, and consequentially, she sends the recovered g_t to the managers at the beginning of period $t+1$. In other words, fully learning the value of g_t by the household occurs at the end of the period. This assumption captures the notion that collecting the data from a mass of individual firms, and analyzing it to extract productivity growth requires some time and effort. This assumption also ensures that the household information regarding the fundamentals, at the beginning of period t when prices are set, is not better than that of the managers who operate on the islands. The information set of the household, at the beginning of period t is therefore any aggregate real quantity shipped back from the firms, including aggregate output, capital, and labor growth rates, and lagged aggregate productivity.

3.4 Equilibrium

An equilibrium is comprised of capital and labor policies for each firm $i \in [0,1]$, $k^*_{i,t+1}$ and $l^*_{i,t+1}$, and firm valuations $V_{i,t}$, such that:

1. Given the information set of the manager, the policies k^*_{t+1} and l^*_{t+1} solve the firm problem in (3.4).
2. Markets clear: aggregate consumption satisfies, $C_t = \int_{i\in[0,1]} y_{i,t} - I^*_{i,t}$.
3. The valuation of a firm i is given by $V_{i,t}$.

4 Data and Volatility Measures

4.1 Data

I collect both annual and quarterly data on real macroeconomic aggregate growth rates, from 1946 to 2013. Annual time-series are used for calibration purposes, while the higher frequency quarterly time-series are used for the construction of aggregate volatility measures. While some aggregate time-series span longer into pre-war era, I use only postwar data to ensure that all aggregate time-series correspond to the same time span, given the availability of the data. Consumption and output data come from the Bureau of Economic Analysis (BEA) NIPA tables. Consumption corresponds to the real per capita expenditures on non-durable goods and services and output is real per capita gross domestic product. Quarterly time-series are seasonally adjusted. Data on capital and investment are taken from the Flow of Funds for all private non-financial corporate businesses. Capital corresponds to total assets, and investment corresponds to total capital expenditures[8]. CPI data are taken from the Federal Reserve Bank of St. Louis. The real per-capita growth rate of capital, is computed by dividing capital by the mid-point population estimate from NIPA tables, and subtracting inflation obtained from CPI data. Annual and quarterly Data on Average Weekly Hours of Production per worker are taken from the Bureau of Labor Statistics (BLS). Data on TFP growth are obtained from the San-Fransisco Federal Reserve Bank. All aggregate growth time-series, including investment to capital ratio, are in log form.

To obtain cross-sectional data, for the purposes of constructing cross-sectional volatility and between-firm correlation measures, I use quarterly Compustat data. To construct a cross-sectional menu of assets, I group Compustat firms into industry portfolios. I choose to work with industry portfolios, instead of firm-specific data, as this reduces the amount of noise and measurement error in each individual asset time-series, and mitigates biases that may result from entry and exit of firms. Notice further, that there are no shifts of individual firms between portfolios over time. Industry portfolios are formed using the SIC code definitions as in Fama-French Data Library, for 38 industry portfolios. I exclude financial and utility industry firms from the sample, and hence, left with 31 industry portfolios. I use sales, capital expenditures, and total assets as proxies for firms' output, investment and capital. Industry levels of output, investment, and capital are therefore defined as the sum of the total sales, capex, and assets levels, for all firms within the industry at time t. Industry sales, total assets, and capital expenditures time-series begin at 1966-Q1, 1975-Q1, and 1985-Q1, respectively. Prior to these starting dates, some portfolios, or all, have missing observations. All industry time-series end at 2013-Q4. As data are quarterly, they exhibits strong seasonality. I remove seasonality from industry level time-series, by using X-12-ARIMA filter at the quarterly frequency. The real growth rates of the seasonally adjusted time-series are then computed by subtracting the quarterly inflation rate.

[8]Though there are other suitable variables to measure investment, the use of capital expenditures allows better comparison to Compustat data in which capex is also available.

4.2 Measurement of Aggregate and Cross-Sectional Conditional Volatilities

To measure the conditional volatility of an aggregate time-series, in the data or in the model, we first need to specify the information set of the econometrician at time t. To ensure the construction of the conditional volatility in the data is identical to the construction procedure within the model, I assume that the information available to the econometrician is the same as the information set available to the household at time t. Differently put, our household, who collects the data from individual firms, and publishes aggregate quantities, is the econometrician.

Let ΔX_t be the *log* growth of some aggregate time-series ($\Delta X_t = log(\frac{X_t}{X_{t-1}})$). To compute the conditional volatility $V_t(\Delta X_{t+1})$, I follow two steps. First, I remove the conditional *mean* of the time-series by projecting future ΔX_{t+1} on a set of time t predictors $\mathbf{Z}_t$:

$$\Delta X_{t+1} = b_0 + b_x' \mathbf{Z}_t + \varepsilon_{x,t+1}, \tag{4.1}$$

where $\varepsilon_{x,t+1}$ captures the conditionally demeaned, or innovation time-series of ΔX_t. Second, I project future squared innovations on their own lag, and the same set of time t predictors $\mathbf{Z}_t$:

$$\varepsilon_{x,t+1}^2 = \nu_0 + \nu_x' [\varepsilon_{x,t}^2, \quad \mathbf{Z}_t] + error, \tag{4.2}$$

and take the *fitted value* of the projection above as the ex-ante conditional volatility of ΔX_{t+1}, that is, $V_t(\Delta X_{t+1}) = \nu_0 + \nu_x' [\varepsilon_{x,t}^2, \quad \mathbf{Z}_t]$. In the benchmark implementation of the above procedure, both in the model and in the data, the set of the benchmark predictors Z_t includes real aggregate log output growth ΔY_t, real aggregate log capital growth ΔK_t, real aggregate log labor growth ΔL_t, real aggregate log investment to capital ratio I/K_t, and the log lagged productivity growth rate. This information set is equivalent to all aggregate variables that are observed by the household at the beginning of period t. Although this information set is log-linear in the underlying state variables, I find that it maximizes the Akaike Information Criterion of projection (4.1), and the results are robust to the inclusion of higher order powers of the underlying aggregate state variables.

Similarly, let $\Delta x_{i,t}$ be the *log* growth of some single-firm (indexed by $i \in [1, ..., N]$) time-series (or alternatively, some single-industry portfolio i time-series in the data), where N is the number of individual assets in the sample. To measure the conditional volatility of a one-firm i time-series, I follow a similar procedure. At the firm stage, I remove the conditional mean of the one-firm time-series by projecting future one-firm growth rates on their own lag and the set of predictors $\mathbf{Z}_t$:

$$\Delta x_{i,t+1} = b_{i,0} + b'_{i,x}[\Delta x_{i,t}, \quad \mathbf{Z}_t] + \varepsilon_{i,x,t+1}. \tag{4.3}$$

Next, the ex-ante conditional one-firm i volatility $V_t^{one-firm}(x_{i,t+1})$ is the fitted value of the predictive projection:

$$\varepsilon^2_{i,x,t+1} = \nu_{i,0} + \nu'_{i,x}[\varepsilon^2_{i,x,t}, \quad \mathbf{Z}_t] + error. \tag{4.4}$$

Measuring the conditional covariation between two firms' time-series, $\Delta x_{i,t}$ and $\Delta x_{j,t}$ ($i, j \in [1, .., N]$), involves a two-stage procedure, consistently with the conditional volatility measurements. First, the conditional mean is removed from $\Delta x_{i,t}$ and $\Delta x_{j,t}$, by applying the projection (4.3) twice: once for firm i, and once for firm j. The first stage provides two demeaned (innovation) time-series $\varepsilon_{i,x,t}$ and $\varepsilon_{j,x,t}$. Second, I project the interaction of future firm i and firm j shocks, $\varepsilon_{i,x,t+1}\varepsilon_{j,x,t+1}$, on its own lagged value, and the set of predictors $\mathbf{Z}_t$:

$$\varepsilon_{i,x,t+1}\varepsilon_{j,x,t+1} = c_0 + c'_x[\varepsilon_{i,x,t}\varepsilon_{j,x,t}, \quad \mathbf{Z}_t] + error, \tag{4.5}$$

The ex-ante conditional covariation, is obtained from the fitted value of the above projection: $COV_t(\Delta x_{i,t+1}, \Delta x_{j,t+1}) = c_0 + c'_x[\varepsilon_{i,x,t}\varepsilon_{j,x,t}, \quad \mathbf{Z}_t]$.

Lastly, the dispersion of a growth variable Δx, at time t, is directly computed as the *cross-sectional* variance of $\{\Delta x_{i,t} | i = 1..N\}$, that is: $DISP_t(\Delta x_t) = V_n(\Delta x_{i,t})$. The residual (or ex-post) dispersion of a growth variable is defined as the cross-sectional variance of the innovations $\{\varepsilon_{i,x,t} | i = 1..N\}$ at time t, or: $DISP_t(\varepsilon_{x,t}) = V_n(\varepsilon_{i,x,t})$.

5 Calibration and Unconditional Moments

5.1 Parameter Choice

Table 1 reports the parameters that I use for the benchmark calibration of the model, under risk neutrality. The model is calibrated at a quarterly frequency. Some choices of the production parameters are dictated by standard choices in macroeconomics. I set the degree of returns to scale to $\eta = 0.9$ consistent with Basu and Fernald (1997) and Gomes, Kogan, and Yogo (2009). The elasticity of capital input is $\alpha = 0.22$, generating a capital share of output $\frac{\alpha}{\nu}$ of approximately 25%, and a share of labor of 75%. I select a depreciation rate of capital to be a conservatively standard value of $\delta = 2\%$, or an effective rate of 8.2% at an annual frequency, consistently with the annual depreciation rate of capital in the data. This depreciation rate yields an annual investement-to-capital ratio of about 10%, which is comparable with the data.

The key parameters that affect the learning ability are the standard deviations of aggregate productivity and noise shocks. The standard deviation of aggregate productivity shock determines the amount of prior uncertainty a manager has regarding today's level of productivity growth g_t. I set the standard deviation of aggregate productivity shock at a relatively high value for quarterly frequency, of $\sigma_g = 0.02$. Calibrating this parameter at lower, more conservative value, reduces the ability of the model to amplify conditional volatilities via learning, as the prior uncertainty becomes too small to provoke a significant impact.

The standard deviation of aggregate productivity is given by $\sqrt{\sigma_g^2/(1-\rho_g^2)}$, where ρ_g is the autocorrelation parameter of aggregate productivity. To ensure this standard deviation is not too high (in the presence of high σ_g), I then need to pick a relatively small value for ρ_g. I set $\rho_g = 0.5$, which then implies a standard deviation of 2.3% for aggregate productivity. While this standard deviation is still high, setting ρ_g at significantly lower values then implies uncrealistically low autocorrelations for real growth rates in the model.

Since the autocorrelation parameter is now relatively low, while the standard deviation of aggregate productivity is large, I smooth consumption and output growth using the adjustment costs parameters. I set the adjustment cost parameter of capital to $\zeta = 1.2$, comparably with $\zeta = 0.8$ in Kung and Schmid (2014). The adjustment cost of labor is set to $\kappa = 7$. These adjustment costs facilitate targeting the standard deviation of output growth, and the autocorrelation of consumption and output growth rates. Notably, the adjustment costs for labor are quite large. I introduce this adjustment cost, to target the autocorrelation of consumption. It is crucial in the absence of consumption smoothing in a risk neutral setup.

The standard deviation of the labor efficiency (noise) shock σ_l, governs the posterior uncertainty a manager has regarding today's level of productivity growth g_t. Consequentially, this parameter governs the amplification of the conditional volatility of real quantities in bad times. Naturally, a choice of noise close to zero yields no amplification at all, as we are back in a perfect information case. I pick $\sigma_l = 0.265$, to target the increase in consumption's conditional volatility in bad times.

I set the (gross) growth of aggregate productivity g_0 to 1.005, ensuring that annual consumption growth is approximately 2%. In a risk neutral setup, the discount rate parameter β must satisfy $\beta g_0 < 1$, to ensure that the detrended value function is a contraction. I therefore pick a value of $\beta = 0.994$. This implies an annual real risk free rate of slightly above 2%.

As wages are exogenously specified, I set the (detrended) wage for labor as a numéraire, with $w = 1$. Lastly, the idiosyncratic demand shock parameters $\sigma_z = 0.01$ and $\rho_z = 0.9$ are set to approximately match the correlation between output and investment-rate dispersions with the business cycle (that is, with TFP growth).

5.2 Model Numerical Solution and Implications for Unconditional Aggregates

I solve the model using a second order perturbation method, as in Judd (1998)[9]. To solve the model, I detrend the growing model variables by the lagged value of the stochastic productivity trend. Details regarding detrending the firm problem, are provided in the Appendix. I simulate the model at the quarterly frequency for 100,000 quarters, after truncation to remove dependence on initial values. I simulate a cross section of 10,000 firms, to ensure that all idiosyncratic shocks are diversified at the aggregate level. Aggregate model-implied level time-series, of capital, labor, output, consumption and investment, are obtained by averaging the respective firm-level quantities over all firms.

To facilitate the comparison between the benchmark model (with Bayesian learning) and the data, I also solve a version of the model without any learning. This no-learning model specification is identical to the learning model. Namely, the production function including the labor efficiency shock, the evolution of capital, and the adjustment costs are the same, except for the fact that the firm *knows* every period the true value of g_t (zero prior and posterior uncertainty). The calibration used for the no-learning model is identical to that used for the benchmark learning model, and is specified in Table 1.

I report the model-implied unconditional moments of aggregate consumption, output, labor, and capital log-growth rates and the log aggregate investment-to-capital rate, versus their empirical counterparts in Table 2. The simulated quarterly model-

[9]Solving the model using third-order perturbation method yields similar results.

implied time-series is time-aggregated to form annual observations, to be compared with the annual data.

For the most part, the moments implied from the model with learning, are close to or match their empirical estimates. The growth rates of aggregate consumption, capital, and output are all roughly 2% in the model and in the data. Log investment-rate is slightly higher in the model than in the data (-2.31 in the model versus -2.91 in the data). The model-implied volatilities align generally well with the data. The volatility of output growth is about 3% in the model and in the data. The volatilities of labor growth and investment-rate and close to their empirical counterparts. Consumption growth has excess volatility in the model (3.1% and 1.4% in the model and data, respectively). However, in the long same of 1930-2012, consumption growth's volatility is 2.2%, and the upper-bound of its volatility 90%-confidence interval of 2.6%, which is much closer to the model. Capital growth is less volatile in the model than the data, due to the effect of adjustment costs, that compensate for the lack of consumption smoothing.

The learning model implied autocorrelations of consumption, output and labor growth fall into the data 90%-confidence intervals. Labor growth is much more persistent in the model at the annual frequency, yet at the quarterly frequency this problem vanishes. In the model, the quarterly auto-correlation of labor is 0.11, and in the data the quarterly auto-correlation of labor growth is 0.23 with a confidence interval of [0.048, 0.419]. Likewise, capital growth is overly persistent in the model. However, the upper-bound of the 90%-confidence interval for quarterly capital growth autocorrelation is 0.75, which is closer to the model quarterly autocorrelation of 0.94. In all, the model is capable of producing reasonable unconditional aggregate moments, in-light of the absense of risk-aversion.

While I do not target any moment implied by the no-learning model (this model bears the same calibration as the learning model for comparative reasons), the no-learning model produces similar moments to the learning model. The volatilities in the no-learning model are slightly higher. This makes intuitive sense: in the no-learning model, all firms share the same belief on the state of the economy, or aggregate TFP growth. As "beliefs" in the no-learning model are perfectly correlated, this increases the correlation between firms policies, in-comparison to the learning model in which beliefs are heterogeneous. As a result of a higher unconditional correlation between firms, aggregate volatilities are higher.

6 Results

This section illustrates the implications of the learning model for aggregate and cross-sectional volatilities, in a risk-neutral environment. In section 6.1, I show how the learning model is capable of amplifying fluctuations in the conditional volatility of aggregate growth rates, while the no-learning model, produces minute changes in the conditional volatility, which is the main result of this study. Sections 6.2 and 6.3 are dedicated to decompose the aggregate volatility movements into firm-level volatility and cross-sectional correlation fluctuations. I demonstrate the importance of the endogenous, time-varying correlation channel to produce endogenous shifts in aggregate volatility. Next, section 6.4 provides evidence that it is the combination of Bayesian learning, along with asymmetric information, that is responsible for the countercyclical correlation between the growth rates of firms, in-line with the model's economic narrative. Section 6.5 explains how non-linearities in the measurement of volatility, can produce small fluctuations in the measured conditional volatility under a homoscedastic environment. This section illustrates that the no-learning model is isomorphic to a constant conditional volatility world. Section 6.6 demonstrates that dispersion in the learning model is by large countercyclical, in spite of an increase in the conditional correlations in bad times, and reconciles the two. Finally, section 6.7 deals with the robustness of the results.

6.1 Implications of Learning for Aggregate Conditional Volatility

The learning model is capable of generating fluctuations in the conditional volatility of aggregates, that are much larger than those produced by a no-learning model, and are also comparably close to the magnitude of fluctuations observed in the data. Table 3 demonstrates this claim. The table shows by how much the conditional volatility of macroeconomic variables of interest, increases or decreases, in bad times compared to normal periods. Likewise, the table shows the fluctuations in the conditional volatility in good times compared to normal ones. Bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The table presents the volatility fluctuations induced by quarterly data from the learning benchmark model, as well as from a no-learning model, and empirical estimates of the fluctuations in the data.

In the data, the conditional volatility of real macroeconomic variables is clearly counter-cyclical. For all variables, including output and consumption growth, the volatility is higher (lower) in bad (good) times, in comparison to normal times. For all variables, except for the investment rate, the rise (drop) in the conditional volatility in bad (good) times is significantly above (below) zero, as can be seen from the confidence intervals. The magnitude of the positive (and significant) fluctuations in volatility in bad times ranges from an increase of 30% to 56%. Specifically, the estimated increase in output's (GDP) conditional volatility in bad times is about 30%. This figure aligns well with Bloom (2013), who finds that quarterly GDP and industrial production growth, has about 35% more conditional volatility in NBER recessions.

In the learning model, almost all of the oscillations in the conditional volatility in good and bad times for the variables of interest, fall into the empirical 90% confidence intervals. For some variables the fluctuations induced by the model are very close to the data point-estimates. For example, capital's growth volatility rises in bad times by 57% and 56% in the model and in the data, while it drops in good times by 41% and 47% in the model and the data. Consumption growth's volatility increases in bad times by 29% in the model versus 32% in the data, and it falls in good times by 20% and 14% in the model and the data, respectively. For the investment rate, the model tends to overstate the fluctuations in volatility, compared to the data. In the learning model, the magnitude of the positive fluctuations in volatility in bad times ranges from an increase of 29% to 58% [10].

By contrast, the no-learning model-implied volatility oscillations are muted, not only in comparison to the learning model, but also in comparison to the data. The positive fluctuations in volatility during bad times range from an increase of 1.8% to 4.3%, outside the data confidence intervals. Similarly, the fluctuations in good times are mixed in sign, and range from -1.5% to 0.76%.

Two questions arise. First, and most importantly, what triggers the large volatility fluctuations in the learning model? This questions is addressed in the following sections 6.2 - 6.4. Second, why are the volatility fluctuations in the no-learning model very small, and yet, non-zero? I provide an answer in section 6.5.

[10] The learning model is also capable of generating fluctuations in the conditional volatility of aggregate labor growth. For example, the conditional volatility of aggregate labor growth rises by 63% in the model, and by 75% in the data.

6.2 Implications of Learning for Average Between-Firm Conditional Covariation

The fluctuations in the conditional volatility of aggregates in the model, reported in Table 3, capture movements in the average conditional covariation between firms.

To see this, notice that if X is an aggregate variable, x_i is a firm level (single-firm indexed by i) variable, and N is the number of firms in the cross-section, then:

$$\begin{aligned} V_t(X_{t+1}) &= V_t(\frac{1}{N}\sum_{i=1}^{N} x_{i,t+1}) \\ &= \frac{1}{N^2}\left(\sum_{i=1}^{N} V_t(x_{i,t+1}) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} COV_t(x_{i,t+1}, x_{j,t+1}).\right) \end{aligned} \tag{6.1}$$

Denote the average conditional one-firm volatility as $\overline{V_t^{one-firm}}(x_{i,t+1}) = \overline{\sigma_{ii,t}^2}$ (all firms are ex-ante identical), and the average conditional covariation as $\overline{COV_t}(x_{i,t+1}, x_{j,t+1}) = \overline{\sigma_{ij,t}}$. Then, the expression in (6.1) can be written as:

$$V_t(X_{t+1}) = \frac{1}{N^2}\left(N\overline{\sigma_{ii,t}}^2 + N(N-1)\overline{\sigma_{ij,t}}\right). \tag{6.2}$$

With a mass of atomistic firms, $N \rightarrow \infty$ and $V_t(X_{t+1}) \rightarrow \overline{\sigma_{ij,t}}$ [11]. That is, the aggregate volatility equals the average between-firm covariation. This claim therefore implies that the aggregate volatility oscillations in Table 3, are driven by fluctuations in the conditional covariation.

While this claim is straightforward algebraically, I provide direct evidence that this claim holds in the model. I construct a measure of the changes in the (average) conditional pairwise covariation between firms in the model (and data). The methodology of constructing the pairwise covariation is described in section 4.2.

Table 4 shows by how much the conditional between-firm pairwise covariation of variables of interest, increases or decreases, in bad times, and in good times, compared

[11] Equation (6.1) is an approximation when the aggregate variable X is a growth rate, not a level. The exact decomposition for growth rates is as follows:

$$V_t(\Delta X_{t+1}) = V_t(\sum_{i=1}^{N} w_{i,t}\Delta x_{i,t+1}),$$

where $w_{i,t} = \frac{x_{i,t}}{\sum_{j=1}^{N} x_{j,t}}$. Hence, the aggregate volatility of a growth rate converges to the average "value-weighted" covariation between-firms. When firms are atomistic, this equals approximately to the "equal-weighted" covariation between-firms.

to normal periods. As in the previous section, bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The table presents the covariation fluctuations from the learning model, and empirical counterparts.

In the model, the changes in the covariation as reported in Table 4, coincide with the fluctuations in aggregate volatility reported in Table 3. All oscillations are identical, up to the units digit. Notice that the fluctuations in Table 3 are based on aggregate time-series only, while fluctuations in Table 4 are computed using firm-level data only. This exercise demonstrates that the methodology used in this study to measure the unobserved ex-ante conditional volatility and covariation satisfy equation 6.2. In unreported results, I verify that the conditional aggregate volatility fluctuations in the no-learning model, are also identical to the oscillations in covariations.

In the data, the fluctuations in the conditional covariations are countercyclical: covariation rises in bad times, and drops in good times. Perhaps surprisingly, the fluctuations in the empirical pairwise covariations, are very close in magnitude to the fluctuations in the empirical aggregate volatility. For example, the conditional volatility of aggregate output in the data rises by 29.8% in bad times, while the increase in the average covariation between firms' outputs in those periods is 28.9%. Similarly, the empirical conditional volatility of aggregate capital growth, and the conditional covariation of capital growth rates, rise by 56.2% and 55.8%, respectively. Given that in the data some firms are non-atomistic, as illustrated in Gabaix (2011), the similarity of the figures is non-trivial.

What causes the conditional covariation to rise in bad times, and drop in good times? The next section provides an answer.

6.3 Aggregate Volatility Decomposition: Implications for Average Conditional Correlations

Sections 6.1 and 6.2 show that the conditional aggregate volatility is countercyclical in the learning model, due to an increase in the conditional covariation between firms in bad times. In this section, the aggregate volatility (or average covariation) is decomposed into firm-level volatility and average between-firm correlation. This decomposition yields that:

A. In the model without learning, the fluctuations in the conditional volatility of aggregates (or alternatively, in the conditional between-firm covariation), are purely due to small changes in the conditional one-firm volatility. The average correlation between firms is fixed.

B. In the model with learning, the fluctuations in the conditional volatility of aggregates (or alternatively, in the conditional between-firm covariation), are largely due to shifts in the conditional correlation between firms, that rises in bad times.

Let x_i be a firm-level variable. Denote, as before, the average conditional one-firm volatility as $\overline{V_t^{one-firm}(x_{i,t+1})}$, and the average conditional correlation between firms as $\overline{CORR_t(x_{i,t+1}, x_{j,t+1})}$. Using equation (6.2), the volatility of the aggregate variable X can be decomposed as:

$$V_t^{agg}(X_{t+1}) \approx \overline{COV_t(x_{i,t+1}, x_{j,t+1})} = \overline{V_t^{one-firm}(x_{i,t+1})} \cdot \overline{CORR_t(x_{i,t+1}, x_{j,t+1})}.$$

As a consequence, the oscillation in aggregate volatility between bad and normal times is equal to the fluctuation in firm level volatility multiplied by the fluctuation in the average between-firm correlation, between bad and normal times:

$$\frac{\overline{V_t}(\cdot|\text{Bad})}{\overline{V_t}(\cdot|\text{Normal})} \approx \frac{\overline{V_t^{one-firm}}(\cdot|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot|\text{Normal})} \cdot \frac{\overline{CORR_t}(\cdot|\text{Bad})}{\overline{CORR_t}(\cdot|\text{Normal})}. \tag{6.3}$$

A similar decomposition can be made for good versus normal period oscillations. Thus, if the fluctuations in the aggregate volatility are *very close* to those in the one-firm volatility, there are no fluctuations in the conditional correlation. However, if the fluctuations in aggregate volatility *differ* from the one-firm volatility movements, this indicates shifts in the conditional correlation between firms.

Tables 5 and 6 respectively show by how much the average one-firm conditional volatility, and the average between-firm correlation of variables of interest, fluctuate in bad times and in good times compared to normal periods. As before, bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The tables present oscillations induced by model-implied quarterly data from the learning benchmark model, and from a no-learning model.

Comparing the figures of Table 3 and Table 5 in the no-learning case, reveals that the fluctuations in the aggregate and one-firm volatility are small and roughly the same. As a result, the fluctuations in the average conditional between-firm correlations are minuscule, as illustrated in Table 6.

In contrast, comparing Tables 3, 5 and 6 in the learning case, demonstrates that the one-firm volatility fluctuations are *amplified* by a counter-cyclical movement in the conditional correlation between firms. The conditional correlation between firms' outputs rises by 32% in bad times, and drops by 29% in good times. For investment rate, the conditional correlation increases (drops) by 27.5% (28.8%) in bad (good) times. In fact, for output growth, about 90% of the increase in the conditional aggregate volatility in bad times is attributed to an increase in the conditional correlations. For capital growth and the investment rate, the oscillation in the conditional correlation explains about 80% of the contemporaneous increase in the aggregate volatility. These numbers are comparable to the findings of Veldkamp and Wolfers (2007), who decompose (unconditional) aggregate volatility into sector specific volatility, and comovement of sectors, and attribute about 80% of aggregate volatility to the comovement term.

6.4 The Role of Bayesian Learning and Asymmetric Information for Correlation Fluctuations

The fluctuations in the conditional correlations between firms, that drive the conditional aggregate volatility in the learning model, are a result of the Bayesian learning and Asymmetric information: in the bad states, firms put more weight on public (common) information, and less on private (idiosyncratic) information. An increase in the correlation between the posterior belief of firms, triggers policies that comove more, and making aggregate growth rates more volatile. The Tables in this section provide evidence in support of these claims.

First, I solve a modified learning model, having the same calibration as the benchmark learning model, but in which the (noise) shocks to labor efficiency, $\varepsilon_{i,l,t}$ are *aggregate* shocks. In other words, the shocks $\varepsilon_{i,l,t}$ are i.i.d over time, but the same over all firms, and hence can be denoted by omitting the i index as $\varepsilon_{l,t}$. Now, privately observed signals $s_{i,t}$, obtained from firms' output, all have the same ex-port bias (per unit of labor), driven by the aggregate shock $\varepsilon_{l,t}$. Thus, in this model, both the lagged value of productivity growth g_{t-1}, *and* the signal $s_{i,t}$ obtained from firms' output, are driven by public-common information shocks. Importantly, there is still learning: the ex-ante and ex-post uncertainties about g_t are positive, and the weights on the private and public signals are still time-varying with the amount of rented labor. Yet, as no signal is idiosyncratic, shifts in the weights placed on the public and the private signals should not trigger significant changes in the average correlation between firms' posterior (or policies), as there are no effective informational asymmetries.

The results of the no-informational asymmetries model, for aggregate volatility and average correlation fluctuations in bad and good times compared to normal periods, are shown in Table 7. As conjectured, the correlation fluctuations are all close to zero. As a result, the fluctuations in aggregate volatility are small, and all range between 0.3% to 0.6% in absolute value. Notice that the speed of learning in this model is procyclical, in a similar fashion to the model of Van Nieuwerburgh and Veldkamp (2006), yet without asymmetric information, the model is not capable of producing significant fluctuations in *volatility*.

Second, suppose the learning model is altered such that there are both public signals (g_{t-1}) and private signals ($s_{i,t}$, driven by idiosyncratic shocks), but learning is not Bayesian. That is, I fix the gains (the weights) on the public and private signals at their steady state values. The posterior mean $\mu_{i,g,t}$ and variance $V_{i,g,t}$ on g_t satisfy:

$$
\begin{aligned}
\mu_{i,g,t} &= \overline{w}_{public}((1-\rho_g)g_0 + \rho_g g_{t-1}) + \overline{w}_{private}\left(s_{i,t}\right), \\
V_{i,g,t} &= \left\{\sigma_g^{-2} + l_{ss}^2 \sigma_l^{-2}\right\}^{-1},
\end{aligned}
$$

where:

$$
\overline{w}_{public} = \frac{\sigma_g^{-2}}{\sigma_g^{-2} + l_{ss}^2 \sigma_l^{-2}}, \quad \overline{w}_{private} = 1 - \overline{w}_{public},
$$

and where l_{ss} is the steady-state level of labor (ex-ante, it is identical for all firms). In this model, there is still learning (posterior uncertainty is positive), and there is still asymmetric information, hence belief heterogeneity. However, since the weight on public common information is fixed, in bad times firms *do not* place, by construction, more weight on common information. Consequentially, the correlation between firms should not fluctuate. Table 8 demonstrates that this is indeed the case. The correlation oscillations between good and bad times versus normal periods are minuscule, and thus, aggregate volatility fluctuations are small. The fluctuations in the conditional aggregate volatility are quite close to the no-learning case, as reported in Table 3. For instance, aggregate output volatility increases in bad times by 4.4% and 4.3% in the Non-Bayesian learning and No-learning models, respectively, while the volatility drops by 1.5% and 2.1% in good times in these two models, respectively.

Importantly, the oscillations in aggregate volatility in the No-Information Asymmetry model or in the Non-Bayesian learning models, should not coincide precisely with the no-learning model results: in both cases there is still some posterior uncertainty that can deviate the results from the exact full-information case. These two alternated learning model illustrate the importance of two separate model ingredients: (1) Bayesian leaning, with time varying gains, and (2) Informational asymmetry.

Next, I solve the benchmark learning model (with Bayesian learning and Asymmetric information), but calibrated with different standard deviation for the noise labor efficiency shock (changing σ_l). All other model parameters are calibrated as in the benchmark calibration outlined in Table 1. Panel A of Table 9 presents the results for four noise levels: $\sigma_l \in \{0.3, 0.265 \text{ ``benchmark-level''}, 0.2, 0\}$. Intuitively, the less noise (smaller σ_l), the closer the model is to the no-learning case, and the amplification effect on aggregate volatility induced by average correlation fluctuations become smaller. Aggregate volatility and average correlation fluctuations, in good and bad times, monotonically decrease in absolute value with the noise level. In the case where $\sigma_l = 0$, the private signal is perfectly revealing of the fundamental. As a consequence, the results for the learning and no-learning models coincide, despite different model first-order-conditions in the two cases, as shown in Panel B of Table 9.

6.5 Volatility Fluctuations in the No-Learning Model: Falsification Tests

Table 3 shows that the oscillations in the conditional volatility of aggregates between good and bad states, in the no-learning model are very small, yet non-zero. This section demonstrates that the small changes in the aggregate conditional volatility in the no-learning model, are a result of some non-linearities in the econometric construction of the conditional volatility, mainly the usage log-growth rates, and the usage of squared residuals in realized-volatility construction. The conclusion is that the no-learning model results *do not differ* from results that one would expect to find in a homoscedastic world.

It is hard to isolate a single source of non-linearity that generates small fluctuations in the aggregate volatility in a no-learning environment. To deal with this issue, I use a "falsification" test. I verify that constant conditional volatility processes, having the same unconditional moments as model-implied aggregate variables, yield the same minuscule fluctuations in the conditional volatility, when using the *econometric* methodology for the construction of volatility, as described in section 4.2.

Specifically, let $log(X_t)$ be some log aggregate time-series induced from the model. I calibrate a process $\tilde{X}_t$ of the form:

$$\tilde{X}_t = (1-\rho_x)x_0 + \rho_x \tilde{X}_{t-1} + \beta_g(g_{t-1} - g_0) + \beta_{g,2}(g_{t-1} - g_0)^2 + \sigma_x \varepsilon_{g,t} + \sigma_{x,2}(\varepsilon^2_{g,t} - 1),$$

where g_{t-1} is the lagged value of productivity growth, and $\varepsilon_{g,t}$ are the shocks to productivity used in the model simulation. The process $\tilde{X}_t$ is calibrated such that the process has the same mean, same standard deviation, same skewness, same correlation with TFP growth, and same correlation with TFP growth squared, as the original model-implied exponentiated (level) time-series X_t process[12]. Then, I construct the conditional volatility fluctuations for $log(\tilde{X}_t)$ time-series, under the econometric methodology of section 4.2, while including the lag of $log(\tilde{X})$ as a predictor.

Notice that: (1) the process $\tilde{X}_t$ has, by construction, constant conditional volatility; (2) the shocks to $\tilde{X}$ depend on the shocks to the aggregate TFP only, and are taken from the model simulation (the only aggregate shock in the model is ε_g); (3) the process can depend on the state g in a linear, and non-linear fashion.

The results for aggregate volatility fluctuations, against the no-learning model, are reported in Panel A of Table 10. Two main features arise. First, the fluctuations in the volatility for the no-learning model time-series and for the matched homoscedastic processes are quite similar. Second, the fluctuations reported for the matched homoscedastic processes are small yet non-zero. Non-zero results can arise as I apply the log function on the Gaussian process $\tilde{X}$, and as the residuals of $log(\tilde{X})$ are squared in the volatility construction. Both of these are non-linear operations, that introduce some small skewness, which is manifested in small volatility movements. In unreported results, I notice that when I do not use log-growth rates, or use absolute residuals (as opposed to squared residuals) in the volatility construction, the oscillations in the conditional volatility are even smaller.

Next, I repeat the same "falsification" test for the learning model. For each log aggregate time-series $log(X_t)$ given by the learning model, I calibrate a matched process $\tilde{X}_t$, that has the same unconditional moments as X_t, but constant conditional volatility. Panel B of Table 10 shows that in this case, the fluctuations in the volatility of $log(\tilde{X}_t)$ are tiny in comparison to the learning model-implied volatility changes. This fact provides further evidence that the learning model results are not spurious.

Finally, I consider another source of non-linearity that stems from within the model: the decreasing returns to scale technology ($\nu < 1$). Intuitively, a higher ν

[12]If $log(X_t)$ is a log aggregate growth time-series, X_t is a gross-growth time-series.

implies a closer to linear production function, which then attenuates the non-linearity and the fluctuations in the conditional volatility, both in a learning and a no-learning environments. In unreported results, I find that increasing (decreasing) ν reduces (amplifies) the reported conditional volatility movements.

6.6 Implications for Cross-Sectional Dispersion

The volatility implications discussed in the previous sections referred to the conditional volatility, which is the *predictable* variation of future shocks. Another concept of volatility is the *cross-sectional* variance, or dispersion. This section shows that dispersion in the model is counter-cyclical, in spite of an increase aggregate volatility and between-firm correlations. While ostensibly, an increase in dispersion seems to contradict a rise in expected correlation between firms, I reconcile the two in the data. When the average between-firm covariation increases more than dispersion does, correlations increase too, as is also the case empirically.

It is a well-known established fact, that the dispersion of real economic outcomes, such as output growth and earnings growth, is countercyclical (see among others Döpke *et al.* (2005), Bachmann and Bayer (2013), Bloom *et al.* (2012), and Bachmann and Bayer (2014)). There are a few exceptions in the data. Recently, the work of Bachmann and Bayer (2014) showed that the dispersion of investment rate is procyclical.

I construct a time-series of dispersion for log output and capital growth, and investment-rate in the model using the methodology detailed in section 4.2. To measure the amount of cyclicality of dispersions, I correlate each dispersion time-series with the business-cycle, namely, productivity growth. A negative correlation indicated counter-cyclical dispersion. The results for the learning and for the no-learning models, along with empirical counterparts are reported in table 11.

Empirically, output growth and investment-rate dispersions exhibit a small amount of countercyclicality (their dispersion correlates with productivity by -0.03 and -0.05, respectively). Capital growth dispersion is procyclical, as this is consistent with the finding of Bachmann and Bayer (2014).

In the learning model, all real growth rates exhibit almost the same amount of slight countercyclicality. The correlation of investment-rate dispersion with TFP growth is -0.04 in the model and in the data. Output growth's dispersion is slightly more countercyclical in the model than in the data.

The evidence presented for the countercyclicality of dispersion, coincides with the fluctuations in the one-firm volatility in the model. Dispersion, as discussed below, is a measure of the idiosyncratic one-firm volatility (in the limit, and under certain assumptions, the two are the same). Since one-firm volatility in the learning model rises in bad times, and drops in good times, (see Table 5) it explains the model-implied dispersion behavior.

The claim that the between-firm correlations rise in bad times, seems to be, at first glance, at odds with a simultaneous increase in cross-sectional dispersion. The two can be reconciled.

Let $\{x_{i,t}\}_i$ be a cross-section of some variable x time-series. Denote by $\{\varepsilon_{i,x,t}\}_i$ the cross-section of demeaned times-series of x, or equivalently, the shocks to x_i. In the model, the average predictable correlation of $\{\varepsilon_{i,x,t+1}\}_i$ at time t increases in bad periods. Does this contradict a greater cross-sectional dispersion?

To put some structure into the answer, suppose further that one can find some factor structure for the demeaned (shocks) time-series. In other words, assume:

$$\varepsilon_{i,x,t} = \beta_i F_t + e_{i,t},$$

where $e_{i,t}$ and $e_{j,t}$ are independent for $i \neq j$. Assume that $VAR_t(e_{i,t+1}) = \sigma^2_{e,t} \quad \forall i$, and that $VAR_t(F_{t+1}) = \sigma^2_{F,t}$. Here, for simplicity, I assume a single common-factor, F_t, in explaining the residuals of x.

One can write the conditional correlation between the innovations of firms i and j, as follows:

$$\begin{aligned}
CORR_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) &= \frac{\beta_i \beta_j VAR_t(F_{t+1})}{\sqrt{\beta_i^2 VAR_t(F_{t+1}) + VAR_t(e_{i,t+1})}\sqrt{\beta_j^2 VAR_t(F_{t+1}) + VAR_t(e_{j,t+1})}} \\
&= \frac{1}{\sqrt{1 + \frac{\sigma^2_{e,t}}{\beta_i^2 \sigma^2_{F,t}}}} \frac{1}{\sqrt{1 + \frac{\sigma^2_{e,t}}{\beta_j^2 \sigma^2_{F,t}}}}
\end{aligned}$$

As an approximation (or by ignoring β heterogeneity, and denoting the average β as $\overline{\beta}$), the average pairwise correlation can be expressed as:

$$
\begin{aligned}
\overline{CORR}_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) &= \frac{1}{1 + \frac{\sigma^2_{e,t}}{\overline{\beta}^2 \sigma^2_{F,t}}}, \\
&= \frac{1}{1 + \frac{\sigma^2_{e,t}}{\overline{COV}_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1})}}.
\end{aligned}
$$

Using a result from Garcia, Mantilla-Garcia, and Martellini (2011), the dispersion of the residuals $DISP(\varepsilon_{x,t}) = VAR_n(\varepsilon_{i,x,t})$, is a consistent measure of the idiosyncratic volatility $\sigma^2_{e,t} = VAR_t(e_{i,t+1})$, assuming that a factor structure that satisfies the standard arbitrage pricing-theory (APT) assumptions exists. The dispersion provides a consistent measure of idiosyncratic volatility, *without* a need to know the actual underlying factor structure.

Suppose that dispersion rises in recessions. The above decomposition reveals that if the average pairwise between-firm covariation, $\overline{COV}_t$, which equals to the variation of the underlying factors, $(\overline{\beta}^2 \sigma^2_{F,t})$, increases more than residual-dispersion, $(DISP(\varepsilon_{x,t}) = \sigma^2_{e,t})$ does in bad times, the average correlation increases too in bad times.

The last claim holds in the data, as shown in Table 12. The rise in average covariation in bad times, ranges between 19% to 55%, while dispersion rises by no more than 26%. For all variables, dispersion's increase is always less than the point estimate increase of covariation. Moreover, dispersion even drops in bad times for some variables.

6.7 Robustness

The main result of this study is the ability of the learning model to yield oscillations in the conditional aggregate volatility that are comparably close to the magnitude of volatility fluctuations in the data, and are of a much larger scale than those induced from a no-learning model. I show in this section that this main result is robust to altering some of the benchmark implementation choices.

Throughout the previous sections, I defined the business cycle (namely, good, normal, and bad times) using the TFP growth variable percentiles. I consider other economic outcomes (that vary procyclically) for the business-cycle definition. Table 13 reports the aggregate volatility fluctuations, for the learning model, the no-learning model, and empirically, when bad, normal and good times are defined as the 0-25th, 25-75th, and 75-100th percentiles of *aggregate output growth*. Both in the learning model and empirically, the implied fluctuations in the conditional volatility are quite close to the benchmark result of Table 3. The magnitude of the positive (negative) fluctuations in the conditional volatility in bad (good) times tend to be larger (smaller) in absolute value when output growth is used to define the cycle, compared to TFP growth. For the no-learning model, the volatility fluctuations are still far less pronounced in comparison to the learning model.

Another modification to the definition of good and bad periods are the percentile breakpoints. In Table 14, I still define the business-cycle using TFP growth, but I use more extreme definitions for good and bad times. Good (bad) times, are periods in which TFP growth is between its 90-100th (0-10th) percentiles. Normal times, are periods in which TFP lies between the 10-90th percentiles. The Table shows that, as expected, this modification amplifies, in absolute terms, the changes in the conditional volatility both in the model and in the data. The no-learning model results are largely unchanged. Most of the learning model-implied volatility oscillations still fall into the empirical 90%-confidence intervals.

Lastly, the results are also robust to the predictors used to demean aggregate and firm-level times series, and obtain the ex-ante volatility time-series. In the benchmark specification, I use a log-linear set of predictors $\mathbf{Z}$. I show in Table 15, that when the squared-values of the variables in $\mathbf{Z}$ are also added as additional predictors, the model-implied results are almost identical. In the data, adding the non-linear predictors tend to increase the volatility fluctuations in bad times. Empirically, it also causes volatility to increase by a small amount in good times (though volatility in general is still counter-cyclical, not U-shaped). In unreported results, I verify that no-single predictor in the set of predictors $\mathbf{Z}$ is responsible for producing the observed behavior, by dropping a different single predictor each time, and confirming that there are no significant changes in the results.

7 Conclusion

The volatility of aggregate fundamentals, such as output and consumption growth, is time-varying and increases in recessions. Recent work in macroeconomics and finance has shown that this volatility is important for recession duration and asset-pricing: it inhibits investment and recovery, and depresses assets' valuation-ratios. Traditional models that examine the impact of volatility on the real and financial economy treat aggregate volatility as an exogenous object. This paper attempts to fill the gap in our understanding of macroeconomic volatility, by proposing a theory of how aggregate volatility arises endogenously in a decentralized economy. The theory suggests that the correlation structure between firms is an important source of macro volatility. When firms do not observe the state of the economy, they learn about it from public information, whose precision is constant over time, and from private idiosyncratic information - their own output, that become more noisy in recessions as firms scale down and produce less information. Consequentially, the correlation in the beliefs of firms about the state of the economy rises in recessions, as firms scale down and put more weight on public information. As a result, the policies of firms become more correlated, contributing to a rise in aggregate volatility.

The study produces some important *quantitative* results. In the learning model, the conditional volatility of aggregate output rises when TFP growth is low (bad times) by 43%, and it drops when TFP growth is high (good times) by 32%. These numbers fall into the 90%-confidence intervals for volatility fluctuations in the data. Likewise, aggregate consumption's volatility increases by 30% in bad times, in the model and also empirically. The main economic force behind these fluctuations are endogenous shifts in the average between-firm correlations. The average correlation between firms' outputs and investment-rates rises (drops) by about 30%, in absolute value, in bad (good) times. Consquentially, about 80% of the increase in the conditional aggregate volatility of total output growth, and other macro-quantities, during slowdowns is attributed to an increase in the conditional correlations. Without Bayesian learning, or when all information is symmetric between firms, the oscillations in the correlations over time are minute, and are therefore translated to very small fluctuations in the conditional volatility of aggregates.

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A Appendix

To detrend the firm problem in 3.4, I divide the following quantities by the lagged trend level:

$$\tilde{k}_{i,t} = \frac{k_{i,t}}{G_{t-1}}; \quad \tilde{I}_{i,t} = \frac{I_{i,t}}{G_{t-1}}; \quad \tilde{\Phi}_{i,t} = \frac{\Phi_{i,t}}{G_{t-1}}; \quad \tilde{V}_t = \frac{V_t}{G_{t-1}}.$$

This allows re-writing the firm problem in a stationary form, as follows:

$$\begin{aligned}
\tilde{V}(\tilde{k}_{i,t}, l_{i,t}, g_{t-1}, s_{i,t}, z_{i,t}) \quad = \quad & max_{\tilde{k}_{t+1}, l_{t+1}} \quad g_{t-1}^{1-\alpha} z_{i,t} \tilde{k}_{i,t}^{\alpha} (s_{i,t} l_{i,t})^{\nu-\alpha} - w \cdot l_{i,t} - \tilde{I}_{i,t} \\
& \qquad -\tilde{\Phi}_L(l_{i,t}, l_{i,t+1}) \\
& \qquad +\beta g_{t-1} E[\tilde{V}(k_{i,t+1}, l_{i,t+1}, G_t, \hat{g}_t, s_{i,t+1}, z_{i,t+1})] \\
& g_{t-1}\tilde{k}_{i,t+1} = (1-\delta)\tilde{k}_{i,t} + \Lambda(\frac{\tilde{I}_t}{\tilde{k}_t}) \\
& \Lambda(i) = \frac{\alpha_1}{1-\frac{1}{\zeta}}(i)^{1-\frac{1}{\zeta}} + \alpha_2 \\
& \tilde{\Phi}_L(l_{i,t}, l_{i,t+1}) = g_{t-1}\frac{\kappa}{2}(l_{i,t+1} - l_{i,t})^2 \\
& V_{i,g,t} = [\frac{1}{\sigma_g^2} + \frac{l_{i,t}^2}{\sigma_l^2}]^{-1} \\
& \mu_{i,g,t} = V_{i,g,t}[\frac{1}{\sigma_g^2}((1-\rho_g)g_0 + \rho_g g_{t-1}) + \frac{l_{i,t}^2}{\sigma_l^2}\,(s_{i,t})] \\
& \hat{g}_t = \mu_{i,g,t} + \sqrt{V_{i,g,t}}\varepsilon_{i,\mu,t} \\
& s_{i,t+1} = [(1-\rho_g)g_0 + \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}] + \frac{\sigma_l}{l_{i,t+1}}\varepsilon_{l,t+1} \\
& \varepsilon_{i,\mu,t} = (g_t - \mu_{i,g,t})/V_{i,g,t} \sim N(0,1)
\end{aligned} \tag{A.1}$$

Tables and Figures

Table 1: **Benchmark Calibration**

Parameter	Symbol	Value
Depreciation rate of capital	δ	2%
Discount factor	β	0.994
Aggregate Productivity:		
Growth rate	g_0	1.005
Autocorrelation of aggregate productivity	ρ_g	0.5
Standard deviation of shock	σ_g	0.02
Idiosyncratic Demand Shock:		
Mean of shock	z_0	1
Autocorrelation of idiosyncratic component	ρ_g	0.9
Standard deviation of shock	σ_z	0.01
Production:		
Returns to scale	ν	0.9
Elasticity of capital input	α	0.22
Adjustment cost for capital	ζ	1.2
Adjustment cost for labor	κ	7
(Detrended) wage	w	1
Standard deviation of labor efficiency shock	σ_l	0.265

The Table presents the benchmark calibration of the learning model, at the quarterly frequency.

Table 2: **Unconditional Aggregate Annual Moments**

	Model with Learning			Model without Learning			Data				
	Mean	Std. Dev.	AR(1)	Mean	Std. Dev.	AR(1)	Mean	Std. Dev.	AR(1)		
ΔC	0.019	0.031	0.384	0.019	0.044	0.305	0.019	0.014^a	0.341	[0.07,	0.62]
ΔK	0.019	0.013	0.950	0.019	0.014	0.952	0.021	0.036	0.369^b	[0.18,	0.56]
ΔY	0.019	0.035	0.315	0.019	0.044	0.286	0.017	0.030^c	0.234	[-0.02,	0.49]
I/K	-2.315	0.135	0.936	-2.313	0.138	0.941	-2.916	0.150	0.646	[0.35,	0.94]
ΔL	-0.000	0.024	0.416	-0.000	0.015	0.819	0.000	0.018	-0.296^d	[-0.45,	-0.15]

The Table shows model-implied moments and data counterparts, for log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, log aggregate labor growth ΔL, and log aggregate investment rate I/K. Columns two to four present the mean, standard deviation and autocorrelation implied from the benchmark model with learning. The next three columns show similar moments implied from an identical model, with full information and no-learning, and under the same calibration. All model moments are based on quarterly simulated data that is time-aggregated to annual observations. The left-most columns present the empirical counterparts. The numbers in brackets present the 90%-confidence intervals of the data AR(1) coefficients. The data figures are estimates using annual observations from 1946-2013. All growth rates in the data are real and per-capita. Data on capital and investment correspond to total assets and capital expenditures from the Flow of Funds for all private non-financial firms. Data on output correspond to GDP. Data on labor correspond to Average Weekly Hours of Production per Worker from BLS.

[a] In a long sample of 1930-2013, the standard deviation is 0.022.
[b] The quarterly auto-correlation of capital growth in the data is 0.651 with a confidence interval of [0.550,0.753]. In the model, the quarterly auto-correlation is 0.945.
[c] In a long sample of 1930-2013, the standard deviation is 0.049.
[d] The quarterly auto-correlation of labor growth in the data is 0.234 with a confidence interval of [0.048,0.419]. In the model, the quarterly auto-correlation is 0.107.

Table 3: **Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle**

	Model with Learning		Model without Learning		Data		
	$\frac{\overline{V_t}(\cdot\mid\text{Bad})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\mid\text{Good})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\mid\text{Bad})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\mid\text{Good})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\mid\text{Bad})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$		$\frac{\overline{V_t}(\cdot\mid\text{Good})}{\overline{V_t}(\cdot\mid\text{Normal})}-1$	
ΔC	29.00%	-20.23%	2.91%	-0.47%	32.51%	[16.53%, 48.50%]	-13.74%	[-24.35%, -3.14%]
ΔY	42.73%	-32.91%	4.32%	-1.49%	29.88%	[15.99%, 43.77%]	-12.87%	[-23.36%, -2.39%]
ΔK	57.88%	-41.53%	1.79%	0.76%	56.22%	[28.66%, 83.77%]	-47.83%	[-60.80%, -34.87%]
I/K	58.69%	-42.41%	3.44%	-1.10%	4.24%	[-9.01%, 17.48%]	-7.35%	[-19.18%, 4.48%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b'_x[\mathbf{Z}_t] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, V_t is the fitted value of the following projection: $\varepsilon^2_{x,t+1} = const + \nu'_x[\varepsilon^2_{x,t}, \mathbf{Z}_t] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90%-confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 4: **Fluctuations in the Average-Conditional (Pairwise) Covariation between Firms with the Business-Cycle**

	Model with Learning		Data					
	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$		
ΔY	43.82%	-34.05%	28.96%	[-2.98%,	60.90%]	-28.44%	[-53.03%,	-3.85%]
ΔK	58.53%	-41.88%	55.85%	[28.41%,	83.29%]	-36.42%	[-53.71%,	-19.13%]
I/K	59.39%	-42.81%	19.55%	[-2.17%,	41.28%]	-7.64%	[-28.22%,	12.94%]

The Table presents by how much, in percentages, the conditional between-firms average covariation of variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{COV}_t(\cdot|Period)$ refers to the between-firm pairwise conditional covariation at time t, $\{COV_t(\cdot_i, \cdot_j)\}_{i,j}$, of a firm-level variable specified in the left-most column, averaged between all firm tuples (i, j) in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The average *pairwise* conditional covariation of a variable at time t, is constructed using three steps. First, the conditional mean is removed for each firm time-series separately by projecting $x_{i,t+1} = const + b'[x_{i,t}, \mathbf{Z}_t] + \varepsilon_{i,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, the ex-ante covariation $COV_t(x_{i,t+1}, x_{j,t+1})$ between firm i and firm j is computed as the fitted value of the following projection: $\varepsilon_{i,t+1}\varepsilon_{j,t+1} = const + c'[\varepsilon_{i,t}\varepsilon_{j,t}, \mathbf{Z}_t] + error$. Third, the average ex-ante covariation at time t is the average of $\{COV_t(x_{i,t+1}, x_{j,t+1})\}_{i,j}$ for all tuples (firms i, firm j) in the cross-sectional sample. Columns two and three present the fluctuations of average pairwise covariation in bad and good periods, induced by quarterly simulated data from the learning model. The two left-most columns present the empirical counterparts, along with 90% confidence intervals in brackets. The empirical estimates are based on the average covariation between quarterly data of 31 assets (industry portfolios). Likewise, the model results are based on a sub sample of 31 firms. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time-series end at 2013-Q4.

Table 5: **Fluctuations in the Average-Conditional Volatility of One-Firm with the Business-Cycle**

	Model with Learning		Model without Learning	
	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Good})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Good})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$
ΔY	8.09%	-4.96%	3.13%	-1.52%
ΔK	23.22%	-17.82%	1.79%	0.61%
I/K	24.46%	-19.09%	3.44%	-1.24%

The Table shows by how much, in percentages, the average conditional volatility of firm-level variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{V^{one-firm}}_t(\cdot|Period)$ refers to the conditional one-firm volatility of a firm-level variable specified in the left-most column, $\{V_t^{one-firm}(\cdot)\}_i$, averaged between all firms i in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t^{one-firm}(x_{i,t+1})$ of some one-firm variable x_i at time t is constructed using two projections. First, the conditional mean is removed by projecting $x_{i,t+1} = const + b'_{i,x}[x_{i,t}\mathbf{Z}_t] + \varepsilon_{i,x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, $V_t^{one-firm}(x_{i,t+1})$ is the fitted value of the following projection: $\varepsilon^2_{i,x,t+1} = const + \nu'_{i,x}[\varepsilon^2_{i,x,t}, \mathbf{Z}_t] + error$. The average one-firm volatility $V_{i,t}^{one-firm}$ at time t is the average of all $\left\{V_t^{one-firm}(x_{i,t+1})\right\}_i$ for firms i in the cross-sectional sample. Columns two and three (four and five) present the fluctuations of one-firm volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. I use a sub-sample of 31 firms in the model for the construction of the table, for consistency with the empirical analysis.

Table 6: **Fluctuations in the Average-Conditional Correlation between Firms with the Business-Cycle**

	Model with Learning		Model without Learning	
	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$
ΔY	32.04%	-29.41%	0.95%	0.11%
ΔK	28.13%	-28.86%	0.02%	0.10%
I/K	27.50%	-28.82%	0.03%	0.09%

The Table presents by how much, in percentages, the conditional between-firm average correlation of variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm pairwise correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i,j) in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The fluctuations in correlations are derived using the decomposition: $\frac{\overline{V_t}(\cdot|\text{Period})}{\overline{V_t}(\cdot|\text{Normal})} = \frac{\overline{V_t^{one-firm}}(\cdot|\text{Period})}{\overline{V_t^{one-firm}}(\cdot|\text{Normal})} \cdot \frac{\overline{CORR_t}(\cdot|\text{Period})}{\overline{CORR_t}(\cdot|\text{Normal})}$, where $Period \in \{Bad, Good\}$, where V_t denotes the conditional aggregate volatility, and $\overline{V^{one-firm}}_t$ denotes the conditional one-firm volatility, averaged over all firms i in the cross-section. The construction of the aggregate and one-firm volatility fluctuations are detailed in Tables 3 and 5.

Table 7: **Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: A Model with No Informational Asymmetries**

	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$
ΔY	1.73%	3.66%	0.27%	0.66%
ΔK	1.26%	0.35%	-0.04%	0.04%
I/K	2.01%	-0.82%	-0.04%	0.03%

The Table presents a summary of results implied from a learning model, having the same calibration as the benchmark learning model, but in which the (noise) shocks to labor efficiency are *aggregate* shocks, making all information symmetric. The left (right) panel shows by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm (pairwise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The construction of aggregate volatility and average correlation fluctuations are the same as detailed in Tables 3 and Table 6.

Table 8: **Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: A Model with Fixed Weights on Private and Public Signals (Non-Bayesian Learning)**

	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$
ΔY	4.45%	-2.18%	1.07%	-0.53%
ΔK	0.62%	1.60%	0.54%	0.77%
I/K	2.18%	-0.54%	0.00%	0.93%

The Table presents a summary of results implied from a learning model, having the same calibration as the benchmark learning model, but in which weights on the public signal, and on the private signal in the posterior updating equation are fixed to their steady state values, making leaning non-Bayesian. The left (right) panel show by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm (pairwise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i,\cdot_j)\}_{i,j}$, averaged between all firm tuples (i,j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The construction of aggregate volatility and average correlation fluctuations are the same as detailed in Tables 3 and Table 6.

Table 9: **Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: Comparison Between Different Noise Levels**

Panel A: Results for A Learning Model - Different Noise Levels

	$\sigma_l = 0.3$				$\sigma_l = 0.265$ "Benchmark"			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations		Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$
ΔY	46.25%	-37.80%	37.14%	-35.26%	42.73%	-32.91%	32.04%	-29.41%
ΔK	90.41%	-76.05%	36.74%	-64.09%	57.88%	-41.53%	28.13%	-28.86%
I/K	91.83%	-77.22%	36.46%	-65.17%	58.69%	-42.41%	27.50%	-28.82%
	$\sigma_l = 0.2$				$\sigma_l = 0$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations		Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$
ΔY	12.08%	-3.74%	8.33%	-2.86%	4.13%	-1.22%	1.61%	0.51%
ΔK	39.65%	-10.82%	24.02%	-4.23%	1.78%	0.78%	0.02%	0.10%
I/K	40.01%	-12.95%	22.80%	-4.89%	3.43%	-1.09%	0.04%	0.08%

Panel B: Results for A No-Learning Model - No-Noise Model

	$\sigma_l = 0$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$
ΔY	4.13%	-1.22%	1.61%	0.51%
ΔK	1.78%	0.78%	0.02%	0.10%
I/K	3.43%	-1.09%	0.04%	0.08%

The sub-panels in Panel A present a summary of results implied from learning models, identical to the benchmark learning model, but calibrated with different standard deviation for the noise labor efficiency shock. All other parameters in the model are calibrated as in the benchmark calibration outlined in Table 1. Panel B shows a summary of results implied from a no-learning, but calibrated with *zero* standard deviation for the labor efficiency shock. In each sub-panel the two-left (right) most columns show by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR}_t(\cdot|Period)$ refers to the between-firm (pairwise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The construction of aggregate volatility and average correlation fluctuations are the same as detailed in Tables 3 and Table 6.

Table 10: **Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Comparison Between Model-Implied Data and Matched Data from Simulated Constant Conditional Volatility Processes**

	Panel A: Falsification-test for the No-Learning Model			
	No-Learning Model		Simulation of Constant Conditional Volatility Process	
	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$
ΔC	2.91%	-0.47%	4.01%	-1.45%
ΔY	4.32%	-1.49%	3.89%	-1.33%
ΔK	1.79%	0.76%	1.73%	0.83%
I/K	3.44%	-1.10%	1.49%	1.18%
	Panel B: Falsification-test for the Learning Model			
	Learning Model		Simulation of Constant Conditional Volatility Process	
	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$
ΔC	29.00%	-20.23%	2.06%	1.16%
ΔY	42.73%	-32.91%	2.09%	0.82%
ΔK	57.88%	-41.53%	0.58%	0.68%
I/K	58.69%	-42.41%	0.69%	0.49%

The Table shows by how much, in percentages, the conditional volatility of aggregate variables fluctuates in bad and good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods in which the TFP growth is between its 0-25-th, 25-75th and 75-100th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the average conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Columns two and three present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the no-learning model, in Panel A, and from the learning model, in Panel B. Each row in columns four and five presents the volatility fluctuations, induced by a simulated constant conditional volatility process, having the same unconditional moments as the model-implied time-series, for the aggregate variable specified in the left-most column. In Panel B (Panel A), the simulated processes imitate aggregate time-series from the (no-) learning model. Specifically, for each model-implied aggregate log variable $log(X)$, denoted in the left-most column, I calibrate a process $\tilde{X}_t$ of the form: $\tilde{X}_t = (1-\rho_x)x_0 + \rho_x\tilde{X}_{t-1} + \beta_g(g_{t-1}-g_0) + \beta_{g,2}(g_{t-1}-g_0)^2 + \sigma_x\varepsilon_{g,t} + \sigma_{x,2}(\varepsilon^2_{g,t}-1)$ such that the process $\tilde{X}_t$ has the same mean, standard deviation, skewness, and same correlations with TFP growth and TFP growth squared, as the original (exponentiated) model-implied X_t process. The volatility fluctuations are then reported for the $log(\tilde{X}_t)$ process. The measurement of the conditional volatility for both model-implied and matched homoscedastic processes, is the same, and detailed in Table 3.

Table 11: **Correlation between Dispersion and the Business-Cycle**

	Model with Learning	Model without Learning	Data	
	$CORR(DISP_t(\cdot), g_t)$	$CORR(DISP_t(\cdot), g_t)$	$CORR(DISP_t(\cdot), g_t)$	
ΔY	-0.0635	-0.0945	-0.0303	[-0.1232, 0.0625]
I/K	-0.0447	0.0316	-0.0499	[-0.2302, 0.1304]
ΔK	-0.0125	0.0823	0.1275	[-0.0849, 0.3399]

The Table shows the correlation between the dispersion of variables in the cross-section and the business-cycle, measured via TFP growth g_t. The correlation between the dispersion and TFP growth is reported for firms' output growth ΔY, capital growth ΔK, and investment-to-capital ratio I/K. For some real (growth) variable x, the dispersion at time t is computed as the cross-sectional variance of $\{x_{i,t}\}_i$ for all firms i in the cross-sectional sample, or $VAR_n(x_{i,t})$. The notation $DISP_t(\cdot)$ refers to the cross-sectional dispersion of a variable specified in the left-most column. The second (third) column reports the results using the (no-) learning model-implied data. The fourth column shows empirical counterparts, along with a 90% confidence interval in brackets. The empirical estimates are based on a sample of 31 assets (industry portfolios). Likewise, the model results are based on a sub-sample of 31 firms. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time series end at 2013-Q4.

Table 12: **Empirical Fluctuations in the Average-Conditional Between-Firm Covariation, and in Residual Dispersion, with the Business-Cycle**

	Average-Conditional Covariation						Residual Dispersion					
	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{DISP_t}(\cdot\|\text{Bad})}{\overline{DISP_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{DISP_t}(\cdot\|\text{Good})}{\overline{DISP_t}(\cdot\|\text{Normal})}-1$		
ΔY	28.96%	[-2.98%,	60.90%]	-28.44%	[-53.03%,	-3.85%]	-21.41%	[-52.55%,	9.74%]	-41.48%	[-65.55%,	-17.42%]
I/K	19.55%	[-2.17%,	41.28%]	-7.64%	[-28.22%,	12.94%]	18.95%	[-35.68%,	73.57%]	31.37%	[-42.15%,	104.90%]
ΔK	55.85%	[28.41%,	83.29%]	-36.42%	[-53.71%,	-19.13%]	26.26%	[-80.42%,	132.95%]	71.61%	[-45.61%,	188.84%]

The Table shows empirically by how much, in percentages, the conditional between-firm average covariation of variables, and the cross-sectional residual-dispersion, fluctuates in bad and good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods in which the TFP growth is between its 0-25-th, 25-75th and 75-100th percentiles, respectively.

The notation $\overline{COV}_t(\cdot|Period)$ refers to the between-firm pairwise conditional covariation at time t of a firm-level variable specified in the left-most column, $\{COV_t(\cdot_i,\cdot_j)\}_{i,j}$, averaged between all firm tuples (i,j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The notation $\overline{DISP}_t(\cdot|Period)$ refers to the cross-sectional residual-dispersion at time t, of a variable specified in the left-most column, that is time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$.

The average *pairwise* conditional covariation of a variable at time t, is constructed using three steps. First, the conditional mean is removed for each firm time-series separately by projecting $x_{i,t+1} = const + b'[x_{i,t}, \mathbf{Z}_t] + \varepsilon_{i,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, the ex-ante covariation $COV_t(x_{i,t+1}, x_{j,t+1})$ between firm i and firm j is computed as the fitted value of the following projection: $\varepsilon_{i,t+1}\varepsilon_{j,t+1} = const + c'[\varepsilon_{i,t}\varepsilon_{j,t}, \mathbf{Z}_t] + error$. Third, the average ex-ante covariation at time t is the average of $\{COV_t(x_{i,t+1}, x_{j,t+1})\}_{i,j}$ for all tuples (firms i, firm j) in the cross-sectional sample. The residual-dispersion of a variable is defined as the cross-sectional variance of the shocks to x_t, $\{\varepsilon_{i,t}|i = 1..N\}$, at time t, or $DISP_t(\varepsilon_t) = V_n(\varepsilon_{i,t})$.

Columns two and three (four and five) present the fluctuations of average pairwise covariation (residual-dispersion), computed by quarterly empirical observations. The brackets show the 90%-confidence intervals. The empirical estimates are based on data of 31 industry portfolios. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time series end at 2013-Q4.

Table 13: **Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Defining the Cycle using Output Growth**

	Model with Learning		Model without Learning		Data	
	$\frac{\overline{V}_t(\cdot\|\text{Bad})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V}_t(\cdot\|\text{Good})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V}_t(\cdot\|\text{Bad})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V}_t(\cdot\|\text{Good})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V}_t(\cdot\|\text{Bad})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V}_t(\cdot\|\text{Good})}{\overline{V}_t(\cdot\|\text{Normal})}-1$
ΔC	37.99%	-14.06%	2.39%	-1.89%	48.82% [31.63%, 66.01%]	-8.14 [-18.59%, 2.32%]
ΔY	58.35%	-22.30%	5.51%	-4.95%	37.75% [23.12%, 52.38%]	-8.79 [-19.09%, 1.52%]
ΔK	82.83%	-28.39%	-0.12%	0.60%	61.75% [36.43%, 87.06%]	-65.25 [-73.87%, -56.64%]
I/K	81.92%	-27.86%	1.92%	-1.28%	20.42% [4.16%, 36.68%]	-0.32 [-12.28%, 11.64%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which aggregate output growth (ΔY) is between its 0-25-th, 50-75-th, and 75-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. Columns two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90%-confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 14: **Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Defining the Cycle using Alternative Percentiles**

	Model with Learning		Model without Learning		Data			
	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$		$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	
ΔC	44.02%	-22.93%	4.28%	-0.15%	69.58%	[52.06%, 87.10%]	-21.02	[-34.26%, -7.79%]
ΔY	64.70%	-38.72%	6.38%	-1.20%	58.65%	[43.10%, 74.19%]	-22.74	[-36.39%, -9.10%]
ΔK	85.34%	-47.25%	2.67%	1.58%	98.88%	[46.45%, 151.32%]	-67.73	[-86.78%, -48.68%]
I/K	86.23%	-48.70%	4.96%	-1.02%	18.59%	[0.22%, 36.95%]	-5.07	[-19.17%, 9.03%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which TFP growth is between its 0-10-th, 10-90-th, and 90-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b_x'[\mathbf{Z}_t] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, V_t is the fitted value of the following projection: $\varepsilon^2_{x,t+1} = const + \nu_x'[\varepsilon^2_{x,t}, \mathbf{Z}_t] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90% confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 15: **Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: using Nonlinear Predictors**

	Model with Learning		Model without Learning		Data					
	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$			$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})}-1$		
ΔC	16.55%	-14.96%	3.39%	0.26%	42.50%	[23.94%,	61.06%]	1.90	[-8.96%,	12.77%]
ΔY	24.60%	-25.46%	4.95%	-1.21%	35.63%	[12.41%,	58.86%]	4.38	[-9.30%,	18.07%]
ΔK	52.05%	-32.06%	2.11%	1.74%	92.68%	[51.36%,	134.01%]	-8.30	[-22.77%,	6.16%]
I/K	48.50%	-33.08%	3.97%	-0.40%	7.45%	[-0.48%,	15.37%]	10.06	[2.77%,	17.34%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which TFP growth is between its 0-25-th, 50-75-th, and 75-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b_x'[\mathbf{Z}_t, \mathbf{Z}_t^2] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors, and $\mathbf{Z}_t^2$ is the set of benchmark predictors squared. Second, V_t is the fitted value of the following projection: $\varepsilon^2_{x,t+1} = const + \nu_x'[\varepsilon^2_{x,t}, \mathbf{Z}_t, \mathbf{Z}_t^2] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90% confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.