

# The U.S. Economy and Sovereign Defaults

Gill Segal\*

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## Abstract

The paper studies the term structure of interest rate spreads of government debt in emerging markets. In the data on the U.S. economy and emerging markets, one can observe that (1) the nominal term structure in the U.S. is upward sloping; (2) the slope of spread curves, defined as the difference between the defaultable yield curves of emerging market economies and the nominal yield curve of U.S. treasury bonds, is positive, but becomes negative in distressed times; (3) defaults in emerging markets tend to occur at 'bad' times for the U.S. economy, when consumption growth is low, and the volatility of fundamentals is high. I propose a dynamic model of sovereign defaults to explain the former stylized facts. I focus on Brazil as a case study. The model matches spreads in a broad context that can also account for prominent U.S. asset-pricing features, including the nominal term-structure, and the equity premium of the U.S. economy. Moreover, in the model, U.S. shocks to consumption and volatility propagate to the rest of the world, and affect international sovereign credit risk and bond prices, as also observed in the data.

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# 1 Introduction

The construction of macroeconomic models of the yield curve in the asset-pricing literature is a challenge, even for closed economies and without default risk. Taking into consideration a sovereign decision to default, which is relevant for the case of emerging economies that have experienced defaults in their past, makes the challenge particularly recognizable, because the emerging market's counter-cyclical real interest rates lead to downward sloping real yield curve. In the data, however, the nominal yield curve for the U.S. economy, as well as the yield curve for most emerging markets is upward-sloping. In the context of sovereign defaults, and the international financial markets, **consider the following three stylized facts:**

Fact (1): On average, the nominal yield curve in the U.S. is upward-sloping. Using annualized quarterly yields, for the years 1969 to 2010, the average level of a 1-year yield is 6.09%, and for a 5-year yield it is 6.79%. At the same time, the real risk free rate is 0.86%, and the equity premium is 6.33%.

Fact (2): Define the *spread curve* of an emerging market economy to be the series of spreads between the yields on the emerging economy's U.S.-dollar denominated nominal bonds and the corresponding nominal yields on U.S. treasury bonds, across different maturities. Using Bloomberg's zero-coupon fair value data for the period of 1998-2012, I estimate the intercept and slope of such curves for emerging market countries that have experienced sovereign defaults in their past, including Brazil, Argentina, Mexico, and Venezuela. I present the average spread curve statistics in table 1. Three main features arise:

1. Across these countries, the spread curve is upward sloping on average across time. The slope of each curve, defined as the difference between a 10-year spread and a 1-year spread is approximately 1.9% for all sampled countries.
2. The standard deviation of the spreads tends to decrease with the maturity of the spread, for most sampled countries.
3. The magnitude and the sign of the spread-curve's slope is time-varying. For the sampled countries, when the 1-year spread is below its 50th percentile, that is, at periods when these economies are less distressed, the slope is higher and amounts to 2.18% on average. However, when the 1-year spread is above its 50th percentile, that is, at times when the markets have a higher probability of turning into a crisis, the slope is only 1.5%. In extremely distressed times, when the 1-year spread is above its 90th percentile, both spreads increase, but the slope of the spread curve turns negative: the 1-year spread is above the 10-year spread (see also table 8).

Since the spread curve behavior of the sampled countries is similar, I shall focus on Brazil as a case-study in this paper, as Bloomberg offers the longest yields sample for that country. Specifically, the level of Brazil's spread curve is 2.9%, and its slope is 1.8%. Features (1) - (3) above are well illustrated in figure 9, which presents how the slope of the Brazilian spread curve changes over time. Mostly, the slope is positive. In times of distress and high uncertainty though, the slope becomes more volatile and turns negative. For instance, in 2002-2003 the slope becomes negative, due to high uncertainty caused by a financial and banking crisis in Brazil during that period.

Fact (3): I document in this paper an existence of a link between sovereign credit risk and the U.S. economy. I present evidence in sections 3.1 and 3.2, that:

1. During times of defaults in 44 emerging markets from 1947 to 2008, the contemporaneous economic conditions in the U.S. worsen: smoothed consumption growth drops, smoothed industrial production drops, smoothed inflation peaks, the realized volatility of these U.S. fundamentals rises, the price-dividend ratio on the U.S. market drops, the realized volatility of the market return jumps, and the risk-free rate increases significantly.
2. U.S. yields and U.S. volatility measures, such as the VIX index, predict contemporaneous and future spreads of emerging markets, with an  $R^2$  of above 50% for 10-year spreads. In particular, a very robust feature is that  $\partial spread_{t+k} / \partial VIX_t^{US} > 0$ , for annual offset  $k = 0..3$ .
3. In a VAR(1) structure that includes U.S. yields, U.S. VIX, and Brazilian spreads, the U.S. yields granger cause the Brazilian spreads, but the converse is not true.

The financial literature on asset-pricing, and the macroeconomic literature on sovereign defaults, have not tried yet to reconcile and account for the above three stylized facts jointly. This paper is aimed at studying the behavior of nominal yield curves and spread curves of emerging markets from a novel perspective. *The key novelty is the international view of explaining the above asset-pricing features together.* That is, in this paper I consider the spreads of the emerging economies in a broader context: I develop a general equilibrium consumption-based pricing model of sovereign defaults, which assumes a full two-country setup, namely, the U.S. and Brazil, and which generates realistic moments not only for the spread curve of the emerging market's (the borrower's) spread curve, but also for the real and nominal term-structure, and equity premium of the U.S. (the lender). *Another key novelty of the paper is that I document a correlation between the trend of prominent economic U.S. variables and timings of defaults in emerging markets, and manage to replicate this correlation qualitatively in the dynamic model.* An important innovative channel which seems to affect spreads in the data, as well as in the

model, is the U.S. volatility.

In my model, which builds on Aguiar and Gopinath (2006), a risk averse borrower, calibrated to Brazil's fundamentals, faces persistent income shocks and can issue only zero-coupon, one-period, dollar-denominated, nominal bonds. The borrower can default on debt at any point in time, but faces costs of doing so. The lenders, or the U.S. investors, are risk averse, and exhibit a long-run risk environment as in Bansal and Yaron (2004). The lenders are the sole buyers of the borrower's bonds, and they require risk premium for the risk they bear. The borrower is assumed to be a "small open economy" in that the borrowing and default decisions of the small open economy have no impact on the large economy, the U.S. (i.e. there is no feedback effect). But the contrary is not true: defaults occur in equilibrium at times when consumption growth in the U.S. drops, and inflation and volatility peaks, because of a positive correlation between the innovations to the borrower's and the lender's endowments, which is backed-up by the data. Even without any correlation between the endowments, Brazil defaults after a sequence of positive shocks to the U.S. expected inflation, *ceteris paribus*. The reason is that debt-burden is higher when the U.S. nominal risk-free rate is higher. Additionally, defaults occur in low-income, and high-debt times because the cost of repayments outweighs the costs of default when consumption is low. Interest rate spreads on long and short bonds compensate foreign lenders for the expected loss from future defaults. In the model, yields on bonds with maturity greater than one year are derived using their shadow price.

In order to emphasize the contribution of the paper, I shall now describe the strands of literature to which this paper relates to, and explain how this paper departs from existing work.

The paper is related to the large strand of quantitative models of sovereign defaults based on the classic setup of Eaton and Gersovitz (1981). Until recently, most papers on sovereign defaults focused on business cycle statistics, such as debt-to-output ratio, the volatility of consumption and output, the correlation between GDP and labor, and so forth. The asset-pricing implications in most of the papers in this strand are limited to matching the average spread for some fixed maturity of debt. Examples of such papers are those of Aguiar and Gopinath (2006), Arellano and Ramanarayanan (2008), and Mendoza and Yue (2011), who model equilibrium default with incomplete markets. In recent work, Chatterjee and Eyigungor (2011) and Hatchondo and Martinez (2009) show that long-term defaultable debt allows a better fit of emerging market data in terms of the volatility and mean of the country spread (for a fixed maturity) as well as debt levels. However, the mentioned papers assume that lenders are risk-neutral. By assuming so, the former papers not only ignore fact (1), but they also cannot produce the mentioned term-structure moments of spreads in fact (2). The reason is intuitive: under risk neutrality, bond spreads are linked one-to-one with average default probabilities, and the implied bond spreads are of a similar magnitude as the average default frequency, for all maturities. In fact, the models of Aguiar and Gopinath (2006), and Arellano and Ramanarayanan (2008) produce an almost

flat spread curves.

There are two particular paper in the former literature that are closely associated with my work. Borri and Verdelhan (2009), are one of the few pioneering papers to explore the benefits of incorporating risk aversion into the lender's side. They apply the general equilibrium setup of Campbell and Cochrane (1999) to price sovereign bonds and establish a link between emerging markets sovereign risk premia and the U.S. business cycle. Yet, they investigate only one-period bond prices, which are not matched in magnitude to the data, and they neglect any term structure effects, ignoring facts (1) and (2) above.

The second closely related work is that of Arellano and Ramanarayanan (2012), who attempt to explain fact (2), that is, matching key moments in Brazil's spread curve. In the benchmark version of their model, the borrower (Brazil) is allowed to issue both short term and long term debt simultaneously, but keeping the risk-neutrality assumption for the lenders. This leads to no unconditional slope. To achieve a positive slope, the setup is then extended to include an exogenous stochastic discount factor for the lenders, which is similar to those explored in affine term structure models, but is driven only by Brazil's output shocks. The explored discount factor has the property of yielding a constant risk free rate, and a flat term structure. With risk aversion, their model generates a slope of 0.6%. My current work differs from Arellano and Ramanarayanan (2012) in several ways. First, the reduced-form discount factor that they use does not offer any appealing asset-pricing implications, except for matching, by construction, a single-period risk free rate. Thus, their model may explain some features of Brazil's spread curve but at the expense of unsolving the equity premium puzzle in the U.S. of Mehra and Prescott (1985), for instance. In contrast, I use a long-run risk kernel of Bansal and Yaron (2004) and Bansal and Shaliastovich (2012), which is known to solve the risk free rate as well as the U.S. equity premium puzzle. Second, their chosen stochastic discount factor generates a flat term-structure for the U.S. economy, failing to explain stylized fact (1). Hence, their model implied spread curve is merely Brazil's yield curve shifted downward by a constant. My framework on the other hand, creates a realistic upward sloping yield curve for the U.S. economy. Consequentially, my model-implied spread curve indeed reflects the difference between two upward sloping yield curves, while reproducing the feature that the term-structure of the lenders also increases with tenor. In that sense, my model offers a better congruency with the data. Third, in the paper of Arellano and Ramanarayanan (2012), as well as in any other paper in the current default literature, both expected inflation and/or the existence of a non-zero price of expected inflation are completely ignored of. Therefore, their model-implied spreads are real, not nominal as in the data. To avoid this in congruency, I use a nominal stochastic discount factor in pricing bonds. Fourth, Arellano and Ramanarayanan (2012) assume a counter-factual full correlation between the emerging market's and the U.S. economy's endowments. My model allows for a positive but imperfect correlation, as observed in the data. Furthermore, the existence of U.S. state variables in my model, facilitates an analysis of how U.S. systematic risk propagates into emerging markets, thus accounting for stylized fact (3), which is ignored of in Arellano and Ramanarayanan

(2012). Lastly, instead of using a reduced form approach in modeling the lenders side, I develop a full general equilibrium setup.

The other strand of literature that my paper relates to is the line of structural asset-pricing models, which focuses on matching the key moments of the real and nominal yield curves in the U.S. (see among others Bansal and Shaliastovich (2012), Wachter (2006), Gallmeyer et al. (2007)). I do not attempt to contribute further to this literature, but rather to use its implications as a cornerstone for my model. Specifically, I use the setup of Bansal and Shaliastovich (2012), which extends the long-run risk model of Bansal and Yaron (2004), who use a discount factor that depends on the predictable components of consumption, inflation, and volatility. In section 3.1, I identify fluctuations in the dynamics of these state variables at times of defaults. Thus, the chosen framework enables the exploration of fact (3). Moreover, since my paper puts focus on asset-pricing implications, it is crucial that all spread moments created by my model would co-exist in an environment that can solve the risk free rate and equity premium puzzles. I shall elaborate on other reasons for selecting a long-run risk based environment in section 4.1.3.

The last branch of literature associated with this work is that which emphasizes how shocks to the U.S. propagate to the rest of the world. Among these papers are Arora and Cerisola (2001), Gelos et al. (2004), Goetzmann et al. (2001), Obstfeld and Rogoff (2010), Pan and Singleton (2008), Reinhart and Rogoff (2008), and Uribe and Yue (2006). These papers suggest that there is substantial evidence that global factors, and in particular U.S. factors, have strong explanatory power for the price of sovereign credit risk, in particular at lower frequencies, above and beyond that of country specific fundamentals. However, most of the mentioned papers offer only a regression-based analysis. In this paper, I contribute further to this literature by providing evidence for the ability of U.S. fundamentals and *volatility* to forecast defaults in emerging markets. I also show that the link between the U.S. and the emerging market's decision to default exists in a general equilibrium model.

When calibrated to U.S. and Brazilian data, the model quantitatively matches the level and slope of the U.S. yield curve, the level of Brazil's spread curve, and 40% of the positive slope of the spread curve that is seen in the data. The model closely matches the conditional average level 1-year spreads, depending on their decile. The model also generate a time-varying pattern for the slope of the spread curve, with a downward sloping curve at highly distressed times. As seen in the data, defaults in the model tend to occur when U.S. economic conditions worsen. The model misses though in explaining the volatility of the long-term spread of 10-years, due to high mean-reversion in the model. The predictability of future Brazilian spreads by current U.S. yields is preserved in the model, though to a smaller degree.

The rest of the paper is organized as follows. Section 2 describes the sources of data used to construct the U.S. and Brazilian yield curves, as well as the U.S. fundamentals. Section 3 documents

the correlation between the U.S. fundamentals and default occurrences in emerging market economies, which motivates the model presented in this paper. Next, in section 4, I present the theoretical sovereign-default model. Section 5 is dedicated to derive some closed form results of the model, and describes the numerical techniques used in its solution. Section 6 discusses how the various model parameters are calibrated. Section 7 contains the quantitative results of the theoretical model, and an explanation of the economic forces which drive them. Lastly, section 8 concludes the paper.

## 2 Data

The data for constructing Brazil's spread curve are taken from Bloomberg. Bloomberg offers for some countries a dataset named 'Bloomberg Fair Value (BFV) Yield Curve'. The curves are constructed using Bloomberg Fair Value prices, which indicate where the price of a bond should trade based on where comparably rated bonds with comparable maturities actually trade (an option-free, zero-coupon yield curve). Bloomberg uses a piecewise linear function to estimate the zero-coupon yields in the interpolation process. The BFV indices are usually offered for 1, 2, 5, 10, 15, and 20 years, and they are also separated by currency (there are indices for USD bonds only). Bloomberg offers such curves for a medium sample of countries: Argentina, Brazil, Peru, Uruguay, Venezuela, Mexico, Colombia, Turkey, Philippines, Russia, South Korea, China, and the US. The sample period is 1998-2012, and is available at daily basis. I use the zero-coupon yields of Bloomberg fair value for Brazil and the U.S. when I estimate the spread curve.

Data on default and rescheduling events in Brazil are taken from Reinhart and Rogoff (2008). Data on external debt of Brazil are taken as the levels of total long-term public and publicly guaranteed external debt outstanding at the end of each year, as reported in the World Bank's Global Development Finance Database for the years 1960-2010. Data on annual dollar GDP of Brazil are taken also from the World Bank's database for the same time-span.

For the U.S. side, data on consumption per capita chained data are taken from Bureau of Economic Analysis, NIPA Table 7.1; annual data start at 1930, and quarterly from 1947. I define real consumption level as the real expenditure on non-durable goods and services. Data on seasonally-adjusted CPI are taken from the Federal Reserve Bank of St. Louis from 1947. The real interest rate is constructed by subtracting the 12-month expected inflation from the annualized yield on the 3-month Treasury bill taken from the CRSP treasury files, starting at 1930. Daily and quarterly market returns are taken from CRSP. I obtain data on the VIX index from the CBOE starting from 1990. Growth rates are constructed by taking the first difference of the corresponding log series.

### 3 The U.S. and Defaults in Emerging Market Economies

The two-country model of sovereign default, to be presented in section 4, is predicated on a notion that defaults in emerging economies tend to happen at ‘bad’ times for the U.S. economy. That is, U.S. fundamentals and returns are both correlated with emerging market’s default decisions, and have the ability to predict them. The sections below are aimed at assessing these statements qualitatively and quantitatively.

#### 3.1 The Behavior of U.S. Economic Variables around Times of Default

Is the state of the real and nominal U.S. economy correlated with default decisions of developing economies? Can one observe a fall or an increase in U.S. economic variables at times of default? To answer these question, I collect the dates of default occurrences of 44 countries in the period 1947-Q1 to 2008-Q4 from Reinhart and Rogoff (2008). Over this time span there were 87 defaults for the sampled countries. Specifically, my main interest is in the behavior of two U.S. fundamentals at times of default: consumption growth and inflation. The behavior of the two would turn to be important in presenting the model.

Suppose that default event  $i$  occurred at time  $\tau_i$ . Let  $X$  be some U.S. economic variable. In order to see how the smoothed long-term component of  $X$  (proxy for the predictable component of  $X$ ) behaves around default event  $i$ , I define a new variable  $Y^i$  as follows: (1)  $Y_0^i = X_{\tau_i}$ , (2) for post-default window,  $t > 0$ ,  $Y_t^i = \frac{1}{t+1} \sum_{k=0}^t X_{\tau_i+k}$ , (3) for pre-default window,  $-t < 0$ ,  $Y_{-t}^i = \frac{1}{t+1} \sum_{k=0}^t X_{\tau_i-k}$ . Next, I plot the behavior of  $\bar{Y}$ , the average of all  $Y^i$ ’s across the sampled default instances. Using this methodology, I examine the behavior of the level of smoothed consumption growth, inflation, real risk-free rate, market return, and the behavior of the fundamentals’ realized volatility. I construct the smoothed  $Y$  variables at quarterly frequency, and consider a window of a year before and after default. The realized volatility of consumption growth and inflation at time  $t$  are computed by fitting the data into an AR(1) process, and averaging the squared residuals over the last four quarters ( $t-3 \rightarrow t$ ). Results are reported in figure 1.

Real consumption growth drops by 10% in its level of pre-default window, while the inflation increases by 16% in its level two months before default. This indicates that defaults in emerging markets tend to occur when economic conditions worsen for U.S. consumers. The volatility of the fundamentals also peaks around the time of defaults. The realized volatility of consumption growth, for example, increases by 15% in its level from one year before default to time of default. From a perspective that views the volatility of such fundamentals as systematic, this serves as an additional evidence for the association of defaults with bad times in the U.S. and with high uncertainty levels.



The worsened economic state at times of default in the U.S. is translated into an increase in both the risk free rate, and the market return (about 50% higher in their levels), very close to default. The results remain robust when the U.S. variables are not smoothed, or when different smoothing mechanisms are applied.

In figure 2, I repeat the same analysis only now at annual frequency, considering a window of four years before and after defaults. Since consumption growth data is only available on quarterly basis post-war, I previously restricted the sample to default occurrences after 1947. On annual frequency however, I use industrial production growth as a proxy for the U.S. cash-flow growth, which allows me to extend the span of default occurrences to the period 1930-2008. Within this expanded time frame, the sample includes a more comprehensive 117 defaults across the sampled emerging markets. The realized volatility of industrial production growth is calculated by fitting the monthly series of industrial production growth into an AR(1) process, and averaging the squared residuals over the last twelve months ( $t - 11 \rightarrow t$ ). The same procedure is done to compute the market return realized volatility. I further calculate the price-dividend ratio of the market, where the denominator includes all dividends of the past twelve months. Figure 2 offers a similar picture to the one depicted in the quarterly analysis. Industrial production and the market P-D ratio drop around defaults. The drop in the industrial production growth from four years before default to time 0 is statistically significant at a level of 5% using a t-test. The realized volatilities of industrial production growth, and of the market return peak around time 0. The difference between the volatility measures at their bottom point before default and their respective peaks at time 0 is also statistically significant. Lastly, the annual risk-free rate in the U.S. also exhibits a significant jump from four years before default to time 1.

This qualitative analysis points out that there is some degree of correlation between default timings in emerging markets, and the trend of some prominent U.S. economic variables. To put some statistical flavor into the graphical analysis of this sub-section, in the next sub-section I shall turn to project emerging markets' spreads onto U.S. yields and volatility measures.

### 3.2 Predictability of Future Spreads By U.S. Variables

Do U.S. nominal yields, and the market return's realized volatility predict future nominal yield spreads between the U.S. and emerging market economies? Table 2 provides the results of projecting contemporaneous, one-year ahead, and three-year ahead short-term (one-year) spreads between the yields of sampled emerging markets and the U.S. yields for the same maturity, on lagged U.S. nominal one-year yields, and on a lagged U.S. volatility measure. For the volatility measure, I pick the VIX index. The sampled emerging market economies are the same as in table 1, excluding Argentina due to a

very short sample. These regressions are aimed at showing that the U.S. volatility is important in explaining emerging market spreads, even after controlling for the level of yields. Since spreads are closely related to the excess return on the emerging markets' bonds, the regressions indicate whether foreign bond risk premia are driven by U.S. volatility. The regressions are performed with daily data on spreads, yields and volatility. The sample covers all trade days between June 1<sup>st</sup> 1998 to May 1<sup>st</sup> 2012. Reported t-statistics are Newey-West (HAC) corrected. Likewise, table 3 reports regression results of predicting long-term spreads (ten-years) using long-term U.S. yields and the VIX index.

The regression results indicate a high predictability level, both for short-term foreign spreads and long-term ones. The predictability tends to decrease with the predictive horizon. Across the sampled countries, the average  $R^2$  in explaining short-term spreads using U.S. yields and VIX is 46% for contemporaneous spreads, and 17% for three-year ahead short-spreads. For long-term spreads, the average  $R^2$  is 55% in contemporaneous regressions, which drops to 38% on average in predicting three-year ahead long-spreads. The loadings on the U.S. yields are positive, except for the case of Venezuela, and statistically significant. The VIX coefficients are positive in all regressions, and are significant in all contemporaneous projections, and in most long-term spreads projections. This robust pattern across the sampled emerging markets serves as evidence that (1) U.S. yields and foreign spreads tend to co-move together, especially in longer maturities; (2) The U.S. volatility has a significant explanatory power for sovereign credit risk beyond that of the U.S. yield level; (3) U.S. volatility affects the risk premia of emerging markets' bonds. The last finding is rather innovative - that is, U.S. uncertainty is important in predicting sovereign defaults. The covariation of spreads with U.S. volatility is also pronounced in table 4. The table presents the average VIX level in the U.S. at times when the Brazilian spreads are below or above the  $n$ -th percentile. The emerged pattern matches the regression results: at times of distress, when Brazilian spreads are higher, the level of the U.S. VIX increases significantly. The U.S. VIX level is almost twice as high during the most distressed times (when Brazilian spreads are above their 90<sup>th</sup> percentile), compared to the VIX level contemporaneous with the most stable times (when Brazilian spreads are below their 10<sup>th</sup> percentile). In unreported results, I find the same pattern for the VIX, conditional on other countries' spreads.

### 3.3 VAR Estimation

The former sections 3.1 and 3.2, show that there is a degree of covariation between defaults in emerging market economies and contemporaneous or lagged economic conditions in the U.S. economy. This evidence is by no means necessarily causal. It could be the case, that a macro-global shocks are the driving force behind the former results. These shock would have to be persistent though, as the covariation is not just contemporaneous. However, I would still like to persuade the reader that at least some component of this covariation is causal, or in other words, that U.S. shocks to consumption,

inflation and volatility, affect the decision of sovereigns around the world to default.

In light of this interpretation, there exists a large strand of literature which argues that U.S. shocks propagate to emerging market economies, while affecting international credit risk (see Arora and Cerisola (2001), Gelos et al. (2004), Goetzmann et al. (2001), Obstfeld and Rogoff (2010), Pan and Singleton (2008), Reinhart and Rogoff (2008), and Uribe and Yue (2006)). In fact, this interpretation is economically plausible given the pivotal role of the U.S. economy and its relative size.

Furthermore, although causality cannot be proven, in order to provide a causal taste to the previous results, I perform a VAR(1) analysis for three variables: U.S. VIX, U.S. one-year yield, and Brazilian one-year spread. The predictive offset is one year. The estimated system is presented below:

$$\begin{bmatrix} VIX_{t+1} \\ y_{t+1}^{US} \\ s_{t+1}^{Brazil} \end{bmatrix} = \begin{bmatrix} 14.1 \\ 1.4 \\ -0.3 \end{bmatrix} + \begin{bmatrix} 0.29 & 0.62 & -0.18 \\ 0.04 & 0.70 & 0.03 \\ 0.04 & 0.37 & 0.19 \end{bmatrix} \begin{bmatrix} VIX_t \\ y_t^{US} \\ s_t^{Brazil} \end{bmatrix} + \begin{bmatrix} 8.76 & 0 & 0 \\ -0.56 & 1.22 & 0 \\ 1.68 & 0.22 & 3.75 \end{bmatrix} \varepsilon_{t+1}$$

The VAR(1) analysis yields that U.S. VIX and U.S. one-year yield both separately granger cause the one-year Brazilian spread. The opposite however, is not true: the one-year Brazilian spread does not granger cause either the U.S. VIX, or the one-year U.S. yield.

From the VAR(1) analysis I also obtain impulse-response functions to 1% Cholesky shocks, in order to determine the effect of Brazilian spread shocks on U.S. economic variables, and vice versa. Figure 3a shows how the U.S. yield respond to a shock to the Brazilian spread, and how the Brazilian spread respond to a shock to the U.S. yield. The impact of a shock to the U.S. yield creates a large kink in the Brazilian spread, but the converse has a very small impact. In figure 3b ones can observe the same pattern for the U.S. VIX. A shock to the U.S. volatility creates a large positive response to the Brazilian spread, which only decays four years past the shock.

Though the above evidence is not a decisive proof of causality, it does clearly show that U.S. shocks, and in particular U.S. volatility shocks, affect the international credit market. The sovereign default model presented next in section 4 shall be built upon this premise.

## 4 Model

I turn to provide a quantitative model that attempts to match and explain the behavior of the term structure of the spread curve between Brazil and the U.S. economy. To model default, I adopt the classic framework of Eaton and Gersovitz (1981), and its recent version in Arellano and Ramanarayanan (2008) and in particular Aguiar and Gopinath (2006). I depart from the previous literature by assuming that the lenders are risk averse, instead of risk neutral. Similarly to Borri and Verdelhan (2009), I propose a two-country setup, but where the lender's side is characterized by a Long-Run Risk environment of Bansal and Yaron (2004), instead of an external habit environment of Campbell and Cochrane (2000).

The model assumes that the international assets are limited to zero-coupon, one-period, dollar-denominated nominal bonds, where one-period in the model is equivalent to one-year. There are two economies: the lender, which represent the U.S. economy, and the borrower, or the 'emerging market' economy, which represents Brazil. The borrower is assumed to be a "small open economy": the borrowing and default decisions of the small open economy have no impact on the large economy. At the beginning of each period, the borrower can borrow debt from the lender, by issuing one-period dollar-denominated bonds. These bonds are purchased, and therefore priced, only by the lender. However, there is no enforcement in paying the debt. If the small open economy refuses to pay any part of the debt that come due, it enters a state of default. Once in default, the emerging market is forced into autarky for a period of time as punishment. This captures the fact that there may be ex-post renegotiation and debt rescheduling, as discussed in Aguiar and Gopinath (2006).

The lenders supply any quantity of funds demanded by the small open economy, but require compensation for the risk they bear. The lenders cannot default. As lenders in the model are risk averse, they require not only a default-probability premium, but also a default risk premium. This risk premium can then be decomposed to risk premium due to interaction of defaults with expected consumption, with expected inflation, and with long-run volatility.

Below I provide a detailed description of the preferences and endowments of both the borrower and the lender, and conclude by specifying the decision problem faced by the lender, and the definition of an equilibrium and prices in this two-country setup.

### 4.1 Lenders (U.S.)

The lenders in the model are risk averse, and exhibit a Long-Run Risk environment as in Bansal and Yaron (2004). As opposed to the borrower's side, the lenders represent a 'big' economy, or namely the U.S. economy. This assumption is manifested in the dynamics of the U.S. cash-flows, which are

specified below: neither the borrowings of the emerging market, nor its decision to default, affect the exogenous U.S. endowment process, or its endogenous optimal wealth portfolio choice. In other words, there is no feedback effect of defaults to the lenders' consumption.

#### 4.1.1 Preferences

Following Bansal and Yaron (2004), and Bansal and Shaliastovich (2012), I assume that the U.S. representative agent's preferences over the uncertain consumption stream  $C_t$  are described by the Kreps-Porteus, Epstein and Zin (1989) recursive utility function:

$$U_t = \left[ (1 - \beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where  $0 < \beta < 1$  reflects the agent's time preferences,  $\gamma \geq 0$  is the risk aversion coefficient,  $\theta = \frac{1-\gamma}{1-\psi}$ , and  $\psi \geq 0$  is the elasticity of intertemporal substitution (IES). The logarithm of the intertemporal marginal rate of substitution in an Epstein-Zin economy is given by:

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (2)$$

where  $\Delta c_{t+1} = \log(C_{t+1}/C_t)$  is the log growth rate of aggregate consumption, and  $r_{c,t}$  is the return on all invested wealth. This return represents the return on an asset that delivers aggregate consumption as its dividends each period. Therefore, as in Bansal and Shaliastovich (2012), I assume later an exogenous process for consumption growth. To derive implications for nominal bonds, I shall further specify an exogenous inflation process  $\pi_{t+1}$ . The nominal stochastic discount factor then equals the real one adjusted by the inflation rate:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \quad (3)$$

Therefore, one can use the standard Euler equation to price nominal assets, as follows:

$$E_t \left[ \exp(m_{t+1}^{\$} + r_{t+1}) \right] = 1, \quad (4)$$

which holds for any continuous nominal return, including that on a foreign country's dollar - denominated issued bond. Note that an Euler equation in (4), with the real discount factor instead

of the nominal one, can be used to price any continuous real return, including the one on the wealth portfolio, to solve for the unobserved wealth-to-consumption ratio in the model. This then allows to write the nominal discount factor in (3) in terms of the fundamental state variables and shocks in the U.S. economy.

#### 4.1.2 Cash-Flow Dynamics

It is assumed that the conditional distribution of consumption and inflation rates varies over time. Denote by  $x_{ct}$  and  $x_{\pi t}$  the expected consumption growth and expected inflation rate in the economy, and let  $x_t = [x_{ct} \ x_{\pi t}]'$ . Predictable fluctuations in growth rates are governed by a VAR(1) process  $x_t$ , while the fluctuations in their second moments are driven by a common variance AR(1) component  $\sigma_t^{21}$ . The joint dynamics of consumption and inflation can be written as follows:

$$\Delta c_{t+1} = \mu_c + x_{ct} + \sigma_c \eta_{c,t+1}, \quad (5)$$

$$\Delta \pi_{t+1} = \mu_\pi + x_{\pi t} + \sigma_\pi \eta_{\pi,t+1},$$

$$x_{t+1} = \Pi x_t + \sigma_t \begin{bmatrix} \phi_c & 0 \\ 0 & \phi_\pi \end{bmatrix} e_{t+1}, \quad (6)$$

$$\sigma_{t+1}^2 = \sigma_0^2(1 - \nu) + \nu \sigma_t^2 + \sigma_w w_{t+1}, \quad (7)$$

where  $\eta_{c,t+1}$ ,  $\eta_{\pi,t+1}$ , and  $w_{t+1}$ , are standard uncorrelated Normal shocks,  $e_{t+1} = [e_{c,t+1} \ e_{\pi,t+1}]'$  is a vector of 2 by 1 Normal innovations which are orthogonal to each other and to former shocks.  $\sigma_c$  and  $\sigma_\pi$  are the conditional volatilities of short-run consumption and inflation shocks, while  $\sigma_t$  captures the time-variation in long-run uncertainty in the economy. The persistence matrix  $\Pi$  captures the interaction between the real and nominal economy, using the following definition:

$$\Pi = \begin{bmatrix} \rho_c & \rho_{c\pi} \\ 0 & \rho_\pi \end{bmatrix}$$

The parameters  $\rho_c$  and  $\rho_\pi$  capture the persistence of expected real growth and expected inflation, and  $\rho_{c\pi} < 0$  reflects a negative feedback effect of expected inflation on future expected consumption growth. Bansal and Shaliastovich (2012) show that this is consistent with the data, as real growth

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<sup>1</sup>This is a simplification of Bansal and Shaliastovich (2012), in which there are two separate variance components,  $\sigma_{ct}$  and  $\sigma_{\pi t}$ . I use the standard single volatility specification for parsimony.

forecast in the data loads negatively on lag of expected inflation.<sup>2</sup>

#### 4.1.3 The Role of The Long-Run Risk Environment

It is first important to stress the importance of introducing risk aversion into the lender's side of the model, as opposed to the more common assumption of risk neutrality (see Arellano and Ramanarayanan (2008); Chatterjee and Eyigungor (2011); Mendoza and Yue (2011)). Risk aversion is crucial to reproduce large enough spreads, and a positive slope for the spread curve, while keeping the equilibrium probability of default at conservative low values. With risk neutrality of lenders, the spreads for all maturities are proportional to the probability of default, implying a flat curve with a low intercept. With risk aversion however, if emerging market governments tend to default in states when foreign investors have high marginal utility, then bond prices should reflect compensation for this risk. I shall elaborate more on this point in section (4.4), when equilibrium prices are defined.

Of course the question is then whether a power utility would suffice for the purpose of pushing spreads upwards. The answer is no, however. As illustrated in Borri and Verdelhan (2009), a model where borrowers and lenders share the same constant relative risk aversion preferences does not produce a large enough spread. When using a power utility for the U.S., one must keep the risk aversion coefficient low, in order to avoid implausible and volatile risk free rate, otherwise the case of the risk free rate puzzle arises. Suppose that we therefore set the risk aversion at a maximal admissible level of 10. Furthermore, for illustration, assume that the two countries share the same default probability and the same yield volatility. Then under the power utility case, the spread equals twice the product of the risk aversion coefficient (set to 10), multiplied by the standard deviation of consumption growth (around 1.5%), the standard deviation of returns (around 13%), and the correlation between the U.S. marginal utility of consumption and return differences (around 0.3 in the data), which implies a spread of only 1.17%, about one half of the observed short-term spread of Brazil.

A good asset-pricing model for the purpose of this paper, is one that views the spreads of the emerging economy in a broader two-country context. That is, a model which explains the emerging economy's government bond prices, but without unsolving other asset-pricing puzzles, such as the equity premium puzzle (see Mehra and Prescott (1985)), or the risk-free rate puzzle. The choice of a long-run risk environment serves this exact purpose: it enables generating realistic moments for the lenders, for both the risk-free rate and the equity premium (as previously shown in Bansal and Yaron (2004)).

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<sup>2</sup>Bansal and Shaliastovich (2012) regress real growth forecast  $\hat{x}_{ct}$ , defined as the one year ahead real GDP growth rate forecast from Survey of Professional Forecasts (SPF), on its on lagged value, and lagged value of  $\hat{x}_{\pi t}$ , defined as the one year ahead inflation rate forecast from SPF, using quarterly frequency data from 1969-2010, and find a loading of -0.01 on  $\hat{x}_{\pi t}$ .

Moreover, the extended Long-Run Risk model suggested above, which introduces the effect of inflation on the real economy plays a double role in the model. First, the framework offers a full endogenous derivation of a nominal stochastic discount factor, as in (3). Hence, all model implied yields and spreads are well-paralleled to the data observed spreads, which are all constructed from nominal bonds. In comparison, the equilibrium bond prices in both Borri and Verdelhan (2009), and Arellano and Ramanarayanan (2012), are real, incongruously with the data. The suggested framework does not ignore the role of both expected inflation and an existence of a non-zero price of expected inflation. Second, since the main interest of the model is in matching the term structure of the spread curve, it is not only essential to match the term structure of the emerging market’s government bond yields, but also that of the U.S. economy. The feedback effect of expected inflation on future consumption generates quantitatively large and positive inflation premium. In turn, this enables matching key features of the nominal U.S. term structure, including the upward slope of the U.S. nominal yield curve. By alternatively assuming, like other models, that lenders are risk neutral, or have a reduced form stochastic discount factor as in Arellano and Ramanarayanan (2012), the implied nominal term structure of the U.S. is completely flat, which in turn would imply that the model’s spread curve is merely the emerging market’s yield curve shifted downwards (all emerging market’s yields would then be subtracted with the same constant U.S. yield).

## 4.2 Borrowers (Brazil)

### 4.2.1 Preferences

In the current frameworks of modeling optimal sovereign default, which are derived from the classic Eaton and Gersovitz (1981) framework, the representative agent has a CRRA preferences over the consumption good. At this point, I deviate from this common assumption. In light of the Long-Run Risk framework chosen for the lending country, I assume that the representative agent of the borrowing country also has Epstein and Zin (1989) recursive utility function, as in equation (1), over its own endogenously selected consumption stream.

The primary motivation for choosing such preferences is related to the calibration of the model for the lender’s side. As discussed in section (6), working at annual frequency necessitates calibrating the risk aversion coefficient of the U.S. to a relatively high value ( $\gamma = 25$ ). Moreover, currently all existing models of sovereign default refrain from assuming heterogeneity in the level of risk aversion between the borrower and lender. Such heterogeneity would unarguably produce a spread between the borrower and lender, but would be subject to a debate on whether this is an empirically and theoretically sound economic channel to explain default behavior and implied asset prices. Therefore, to be in-line with the current work of sovereign default, I calibrate the coefficient of risk aversion of the



borrowing country to the same value as the lending country. But now, picking a CRRA utility with a very high risk-aversion coefficient is in conflict with evidence from various studies that this coefficient is a small number, certainly less than 10. It would also imply that the borrower wishes to smooth consumption over time in such a high degree that defaults rarely happen.

A solution to this problem is choosing the Epstein-Zin preferences for the borrower, as it allows for a separation of risk-aversion from the intertemporal elasticity of substitution. This choice also keeps a full symmetry in terms of preferences between the borrowing and lending countries. Note that the preferences are defined over the same consumption good as in the lending country - there is only one consumption good in the model.

#### 4.2.2 Cash-Flow Dynamics

Following Aguiar and Gopinath (2006), I assume that the borrower receives a stochastic endowment stream,  $y_t$ . The endowment  $y_t$  is composed of a transitory component  $z_t$  and a trend component  $\Gamma_t$ :

$$y_t = e^{z_t} \Gamma_t. \quad (8)$$

The transitory (business cycle) component,  $z_t$  follows an AR(1) process around a long run mean  $\mu_z$ :

$$z_{t+1} = \mu_z(1 - \rho_z) + \rho_z z_t + \sigma_z \varepsilon_{t+1}^z, \quad (9)$$

and the trend (stochastic growth) follows around the long run growth rate mean  $\mu_g$ :

$$\begin{aligned} \Gamma_{t+1} &= G_{t+1} \Gamma_t, \\ g_{t+1} \equiv \log(G_{t+1}) &= \mu_g(1 - \rho_g) + \rho_g g_t + \sigma_g \varepsilon_{t+1}^g, \end{aligned} \quad (10)$$

where  $\varepsilon_{t+1}^z$  and  $\varepsilon_{t+1}^g$  are standard uncorrelated Normal shocks. These shocks, however, are correlated with the innovations of the lender's endowment, in a way that is specified in section (4.3). Note that a positive shock  $\varepsilon_{t+1}^g$  implies a permanent increased level of output. Aguiar and Gopinath (2006) considered each component, trend and transitory, separately. Nonetheless, as in Borri and Verdelhan (2009), I consider them together in order to get significant yield spreads and slope.

On top of receiving the endowment  $y_t$  every period, the borrowing country's representative agent receives a goods transfer from the government in a lump-sum fashion. The government has access to international capital markets, as long as the country has good credit, i.e. it is not excluded from financial markets at time  $t$  due to past default occurrence. At the beginning of period  $t$ , it can purchase  $a_{t,t+1}$  one-period zero-coupon bonds at price  $q_t$ . The quantity  $a_{t,t+1}$  is a dollar amount, and denotes the quantity of one-period zero-coupon bonds purchased at date  $t$ , which matures at date  $t + 1$ . Note that  $a_{t,t+1} \leq 0$ , which implies borrowing.  $q_t$  is the price of debt per dollar borrowed. I shall show later that  $q_t$  depends on  $a_{t,t+1}$ , the state of the economy of both the lender  $x_t$  and  $\sigma_t^2$ , and the borrower's states  $z_t$  and  $g_t$ . Hence, if the government borrows amount  $a_{t,t+1}$  at time  $t$ , it receives  $q_t a_{t,t+1}$  dollars at time  $t$ , and conditional on not defaulting, it has to repay  $a_{t,t+1}$  at time  $t + 1$ . Therefore, conditional on having a good credit score, the consumption of the borrowing country can be written as:

$$C_t = y_t - q_t a_{t,t+1} + a_{t-1,t}. \quad (11)$$

In case of default, the sovereign cannot selectively default on parts of its debt, and thus all current debt disappears. Upon default, the credit of the borrower turns bad, and it stays so for a stochastic number of periods. While the economy has bad credit, the borrower is excluded from international capital markets (i.e. reverts to financial autarky) and suffers a direct output loss of  $\delta$  percent. An economy with bad credit rating must consume its endowment. Hence, conditional on having a bad credit score, the consumption is simply:

$$C_t = (1 - \delta)y_t. \quad (12)$$

### 4.2.3 Optimal Default Decision

At the beginning of the period, the agent decides whether to default or not, assuming that it has borrowed a non-zero amount at the beginning of the last period. Let  $V^B$  denote the value function of the agent once it defaults. Following the notations in Aguiar and Gopinath (2006), the superscript  $B$  refers to the fact that the economy has a bad credit history.  $V^B$  depends on all five exogenous states in the model:  $x_t, \sigma_t^2, z_t$ , and  $g_t$  ( $x_t$  is 2 by 1). Let  $V^G$  denote the value function given that the agent decides to maintain a good credit history this period (i.e. to repay any debt issued last period).  $V^G$  depends on all exogenous states, as well as the endogenous state  $a_{t-1,t}$ . Denote the value function of an economy with assets  $a_{t-1,t}$  and access to international credit as  $V(a_{t-1,t}, z_t, g_t, x_t, \sigma_t^2)$ . The value of being in good credit standing at the start of period  $t$  with net borrowing  $a_{t-1,t}$  can be defined as

$V(a_{t-1,t}, z_t, g_t, x_t, \sigma_t^2) = \max \{V_t^G, V_t^B\}$ . At the beginning of period  $t$ , an economy in good credit standing and net assets  $a_{t-1,t}$  will default only if  $V^B(z_t, g_t, x_t, \sigma_t^2) > V^G(a_{t-1,t}, z_t, g_t, x_t, \sigma_t^2)$ .

Once the economy has bad credit score, it is “redeemed” with probability  $\lambda$  and start the next period with a good credit rating, renewed access to capital markets, and zero debt. In recursive form, we therefore have:

$$V^B(z_t, g_t, x_t, \sigma_t^2) = \left\{ (1 - \beta)((1 - \delta)y_t)^{\frac{1-\gamma}{\theta}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \quad (13)$$

where,

$$E_t U_{t+1}^{1-\gamma} \equiv \lambda E_t V(a_{t,t+1} = 0, z_{t+1}, g_{t+1}, x_{t+1}, \sigma_{t+1}^2)^{1-\gamma} + (1 - \lambda) E_t V^B(z_{t+1}, g_{t+1}, x_{t+1}, \sigma_{t+1}^2)^{1-\gamma}. \quad (14)$$

Recall that  $\delta$  is the output loss upon default, and that once the economy has bad credit score, it lives in autarky. On the other hand, if the economy does not default, and maintains a good credit score, we have:

$$\begin{aligned} V^G(a_{t-1,t}, z_t, g_t, x_t, \sigma_t^2) = & \max_{a_{t,t+1}} \left\{ (1 - \beta)((1 - \delta)c_t)^{\frac{1-\gamma}{\theta}} + \beta(E_t V(a_{t,t+1}, z_{t+1}, g_{t+1}, x_{t+1}, \sigma_{t+1}^2)^{1-\gamma})^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ & s.t. \\ & c_t = y_t - q_t(a_{t,t+1}, z_t, g_t, x_t, \sigma_t^2)a_{t,t+1} + a_{t-1,t}. \end{aligned} \quad (15)$$

### 4.3 Cross-Country Correlation

The risk premium that U.S. investors demand on Brazilian bonds should depend on the correlation between Brazil’s default decision and U.S. consumption. Therefore, although each country’s own innovations are uncorrelated, I do allow for cross-country correlation in shocks which is motivated by a correlation between the two countries’ GDP in the data. The correlation structure can be succinctly described as follows:

$$\begin{bmatrix} e_{c,t+1} \\ e_{\pi,t+1} \\ \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^g \\ w_{t+1} \end{bmatrix} \sim N \left[ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V = \begin{pmatrix} 1 & 0 & \varrho_{cz} & \varrho_{cg} & 0 \\ 0 & 1 & \varrho_{\pi z} & \varrho_{\pi g} & 0 \\ \varrho_{cz} & \varrho_{\pi z} & 1 & 0 & 0 \\ \varrho_{cg} & \varrho_{\pi g} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right]. \quad (16)$$

Denote by  $F(z, g, x, \sigma^2)$  the joint cumulative density function of all exogenous states, which is Normal under the assumption of equation (16).

#### 4.4 Bond Prices and Equilibrium Definition

Given a position of debt  $a$ , define a default set  $D(a)$  as the region of all exogenous states at which not repaying back the debt (default) is optimal:

$$D(a) = \{(z, g, x, \sigma^2) \in S : V^B(z, g, x, \sigma^2) > V^G(a, z, g, x, \sigma^2)\}, \quad (17)$$

where  $S$  is the support of all exogenous states in the model. Likewise, denote by  $ND(a)$  the no-default, or repayment region:

$$ND(a) = \{(z, g, x, \sigma^2) \in S : V^B(z, g, x, \sigma^2) \leq V^G(a, z, g, x, \sigma^2)\}. \quad (18)$$

let  $1_{ND(a)}$  be an indicator function which is equal to 1 over all exogenous states in  $s \in S$  such that  $s \in ND(a)$ .

As I show later under the model solution section, the nominal (or real) stochastic discount factor  $M^s$  can be written as a function of the current and future state variables in the U.S. economy, or explicitly as  $M^s(x', \sigma^{2'} | x, \sigma^2)$ . Because the borrower's issued bonds are only purchased by U.S. investors, and the U.S. economy is a complete market by assumption, the equilibrium price of a (nominal) one-period zero-coupon bond can be derived using the Euler equation:

$$q(a, z, g, x, \sigma^2) = E \left[ M^s 1_{ND(a)} \right] = \int_{(z', g', x', \sigma^{2'}) \in ND(a)} M^s(x', \sigma^{2'} | x, \sigma^2) dF(z', g', x', \sigma^{2'} | z, g, x, \sigma^2), \quad (19)$$

where  $ND(a)$  is the repayment set defined in (18).

The risk premium in the model comes from the interaction of the lender's pricing kernel with the default outcomes and future bond prices. To see this, note that equation (19) can be rewritten as:

$$\begin{aligned} q(a, z, g, x, \sigma^2) &= E \left[ M^{\$} \right] E \left[ 1_{ND(a)} \right] + cov(M^{\$}, 1_{ND(a)}) \\ &= \frac{1}{r_f^{\$,US}} (1 - Pr(\text{default})) + cov(M^{\$}, 1_{ND(a)}), \end{aligned} \quad (20)$$

where  $r_f^{\$,US}$  is the one period nominal U.S. risk-free rate. Equation (20) presents the role that risk-aversion plays in the model. If the lenders are risk-neutral, i.e.  $M^{\$}$  is a constant, then the covariance term in equation (20) equals zero, and the sovereign bond prices are exactly proportional to the probability of default. Under a conservatively low default probability, the implied spread would be very small in magnitude. With risk aversion however, bond prices also depend on the extent that the default decision is negatively correlated with the pricing kernel. If defaults tend to occur at bad times for the lenders (i.e. when either expected consumption growth is low, or expected inflation rate is high, or the long-run uncertainty is high), then the covariance in equation (20) is negative, the bond prices are low, and the yields are high. I elaborate more on this in section 7.4.2.

The main interest of the model is the derivation of bond prices for different maturities. Though only one-period zero-coupon bonds are traded, one can find the shadow prices of bonds of longer maturities. These prices can be attained through the iterative use of the Euler equation, and the optimal borrowing policy:

$$q^n(a, z, g, x, \sigma^2) = \int_{(z', g', x', \sigma^{2'}) \in ND(a)} M^{\$}(x', \sigma^{2'} | x, \sigma^2) \cdot q^{n-1}(a^*, z', g', x', \sigma^{2'}) dF(z', g', x', \sigma^{2'} | z, g, x, \sigma^2), \quad (21)$$

where  $n$  denotes the number of periods to maturity, and  $a^* \equiv a^*(a, z', g', x', \sigma^{2'})$  denotes the optimal debt policy at the next-period state  $(z', g', x', \sigma^{2'})$ . Note that by definition  $q(a, z, g, x, \sigma^2) \equiv q^1(a, z, g, x, \sigma^2)$ .

A *recursive equilibrium* for this economy is (i) a set of policy functions for consumption  $c^*(a, z, g, x, \sigma^2)$ , new issuances of zero-coupon dollar-denominated bond  $a^*(a, z, g, x, \sigma^2)$ , repayment set  $ND(a)$  and default set  $D(a)$ , and (ii) price functions for zero-coupon one-period debt  $q(a, z, g, x, \sigma^2)$ , such that:

1. Taking as given the bond price function  $q(a, z, g, x, \sigma^2)$ , the policy functions  $c^*(a, z, g, x, \sigma^2)$  and  $a^*(a, z, g, x, \sigma^2)$ , the repayment set  $ND(a)$  and the default set  $D(a)$  satisfy the borrower's optimization problem (13), (15) and definitions (17), and (18).
2. The bond price function  $q(a, z, g, x, \sigma^2)$  satisfies equation (19).

## 5 Model Solution and Numerical Algorithm

Since the lender (U.S.) side of the model does not depend on the small open economy's borrowings or default decisions, as the first stage of the solution I solve for a closed-form stochastic discount factor of the U.S. investors. The details are presented in the appendix. Below I present the key results.

Using the Euler condition and the dynamics of the economy, the equilibrium price-consumption ratio is log-linear in expected growth and variance states:

$$pc_t = A_0 + A_{xc}x_{ct} + A_{x\pi}x_{\pi t} + A_\sigma\sigma_t^2, \quad (22)$$

where the loadings are given by,

$$\begin{aligned} A_{xc} &= \frac{1 - \frac{1}{\psi}}{1 - \kappa_1\rho_c}, & A_{x\pi} &= \frac{\kappa_1\rho_{c\pi}A_{xc}}{1 - \kappa_1\rho_\pi}, \\ A_\sigma &= \frac{\theta\kappa_1^2}{2(1 - \kappa_1\nu)} [A_{xc}^2\phi_c^2 + A_\pi^2\phi_\pi^2] \end{aligned} \quad (23)$$

and  $\kappa_1$  is a linearization parameter.

Notice that when the IES parameter  $\psi$  is greater than one, the substitution effect dominates the wealth effect. Therefore, as in Bansal and Shaliastovich (2012), the price-consumption loading on the expected growth is positive,  $A_{xc} > 0$ , and it is negative on the expected inflation,  $A_{x\pi} < 0$ , whenever  $\rho_{c\pi} < 0$ . If also  $\gamma > 1$ , then the loading on the volatility is negative,  $A_\sigma < 0$ .

Using the price-consumption ratio, it is now possible to derive an analytical expression for the equilibrium nominal stochastic discount factor in terms of the fundamental state variables and the U.S. shocks. While the full derivation appears in the appendix, I provide here the expression:

$$\begin{aligned}
m_{t+1}^{\$} &= m_0^{\$} + m_{xc}^{\$} x_{ct} + m_{x\pi}^{\$} x_{\pi t} + m_{\sigma}^{\$} \sigma_t^2 - \lambda_c^{\$} \sigma_c \eta_{c,t+1} - \lambda_{\pi}^{\$} \sigma_{\pi} \eta_{\pi,t+1} \\
&\quad - \lambda_{xc}^{\$} \phi_c \sigma_t e_{c,t+1} - \lambda_{x\pi}^{\$} \phi_{\pi} \sigma_t e_{\pi,t+1} - \lambda_{\sigma}^{\$} \sigma_w w_{t+1},
\end{aligned} \tag{24}$$

where the loadings on the state variables appear in the appendix, and the loadings on the innovations (market prices of risk) are given by,

$$\begin{aligned}
\lambda_c^{\$} &= \gamma, & \lambda_{\pi}^{\$} &= 1, \\
\lambda_{xc}^{\$} &= (1 - \theta) \kappa_1 A_{xc}, & \lambda_{x\pi}^{\$} &= (1 - \theta) \kappa_1 A_{x\pi} & \lambda_{\sigma}^{\$} &= (1 - \theta) \kappa_1 A_{\sigma},
\end{aligned} \tag{25}$$

The nominal discount factor is log-linear in the economic states. The price of risk of inflation risk,  $\lambda_{\pi}^{\$}$ , is non-zero because of the feedback effect between expected consumption and expected inflation. When agents have preferences for early resolution of uncertainty ( $\gamma > \frac{1}{\psi}$ ), the market price of expected consumption shock is positive,  $\lambda_{xc}^{\$} > 0$ . The market price of the expected inflation is negative,  $\lambda_{x\pi}^{\$} < 0$ , whenever  $\rho_{c\pi} < 0$ , and the price of volatility risk,  $\lambda_{\sigma}^{\$}$ , is negative as well.

Equipped with the nominal stochastic discount factor expression, I can now turn to solve for the recursive equilibrium numerically, using the discrete state-space method. The numerical solution begins with discretizing the exogenous variables of the model using a discrete Markov chain. First, I discretize the  $AR(1)$  process of the volatility variable  $\sigma_t^2$  using the procedure of Rouwenhorst (1995), which is well-fitted for an almost stationary processes as  $\sigma_t^2$ , with 5 grid nodes (This small number suffices given the small variance of the process). Denote the grid of points of  $\sigma_t^2$  by the vector  $\vec{\sigma}_t^2 = [\sigma_1^2, \dots, \sigma_N^2]$ , where  $N = 5$ . I make sure that the produced grid points are all positive. Second, denote by  $s_t^{\sigma_i} = [x_{ct}^{\sigma_i}, x_{\pi t}^{\sigma_i}, z_t^{\sigma_i}, g_t^{\sigma_i}]'$ , the remaining four exogenous variables in the model, conditional on  $\sigma_t^2$  being equal to  $\sigma_t^2 \in \vec{\sigma}_t^2$ . Define  $\Phi^{\sigma_i} = \begin{bmatrix} \phi_c \sigma_i & \phi_{\pi} \sigma_i & \sigma_z & \sigma_g \end{bmatrix}'$ . In other words,  $s_t^{\sigma_i}$  evolves according to:

$$s_{t+1}^{\sigma_i} = \begin{bmatrix} 0 \\ 0 \\ \mu_z(1 - \rho_z) \\ \mu_g(1 - \rho_g) \end{bmatrix} + \begin{bmatrix} \rho_{xc} & \rho_{c\pi} & 0 & 0 \\ 0 & \rho_{x\pi} & 0 & 0 \\ 0 & 0 & \rho_z & 0 \\ 0 & 0 & 0 & \rho_g \end{bmatrix} s_t^{\sigma_i} + \epsilon_{t+1}^{\sigma_i}$$

where  $\epsilon_{t+1}^{\sigma_i} \sim N(0, (\Phi^{\sigma_i}) \cdot V_{4 \times 4} \cdot (\Phi^{\sigma_i})')$ , and  $V_{4 \times 4}$  is the upper-left  $4 \times 4$  matrix of  $V$  defined in equation (16). Now, for each  $\sigma_i^2 \in \vec{\sigma}_t^2$  I discretize the VAR(1) process  $s_t^{\sigma_i}$  using Tauchen and Hussey

(1991), and with 9 grid nodes for  $z$ , 9 grid nodes for  $g$ , 7 grid nodes for  $x_{xc}$ , and 7 grid nodes for  $x_{x\pi}$ . Note that for every  $\sigma_i^2 \in \overrightarrow{\sigma_i^2}$  I obtain a different grid  $\overrightarrow{s_i^2}$  (since the Tauchen and Hussey (1991) method generates not only a transition matrix for the process, but also a specific set of grid points). Lastly, to get the transition matrix between nodes on  $\overrightarrow{s_i^2}$  and nodes on  $\overrightarrow{s_j^2}$ , I integrate the underlying normal density over the intervals of the different nodes, and multiply by the transition probability between state  $\sigma_i$  and state  $\sigma_j$  generated by the Rouwenhorst (1995) procedure. The asset space is discretized into 200 possible values. I ensure that the limits of the asset space never bind along the simulated equilibrium paths. In total, there are 3.97 million nodes<sup>3</sup>. I then compute a discretized matrix of the stochastic discount factor, with the grid points for  $x_{ct}$ ,  $x_{\pi t}$  and  $\sigma_t^2$  and equation (24).

The next step is iterating over the value functions and prices till convergence. The solution algorithm involves the following:

1. Assume an initial price function  $q_0(a, z, g, x, \sigma)$ . I use the U.S. risk-free rate as an initial guess.
2. Use the guess  $q_0$  and an initial guess for  $V_0^B$  and  $V_0^G$  to iterate on the bellman equations (13) and (15) to solve for the optimal value functions  $V^B$ ,  $V^G$ ,  $V = \max\{V^B, V^G\}$  and the optimal policy functions.
3. For the initial guess  $q_0$ , estimate the default set  $D_0(a)$  and no-default set  $ND_0(a)$ . Next, update the price function using equation (19). Denote the new price by  $q_1$ . Using this  $q_1$  repeat steps 1-3, until the convergence of both prices and value functions.

The last step involves simulating the economy 500 times for 10,000 periods, and using these simulations to calculate the average business-cycle statistics (after HP-filtering the series, as I do with the data), and computing the steady-state distribution across all states, from which the steady-state bond prices can be derived using equation (21).

## 6 Calibration

The parameters for the U.S. side of the model are reported in table 5, panel A. The basic calibration of the lender's side relies on that of Bansal and Shaliastovich (2012), who estimate the parameters specified for the U.S. at quarterly frequency. I make some required adjustments, to match some key moments at the annual frequency. The relative risk-aversion parameter,  $\gamma$ , is estimated in Bansal and Shaliastovich (2012) to be 20.77, at the quarterly frequency. Because my model uses a lower frequency, I calibrate the parameter to a slightly higher value of 25. The inter-temporal elasticity of substitution,

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<sup>3</sup>That is  $200 \times 9 \times 9 \times 7 \times 7 \times 5$  states.



$\psi$ , is set to 2, which is close to the value estimated by Bansal and Shaliastovich (2012) of 1.86. The discount factor,  $\delta$ , in Bansal and Shaliastovich (2012) is 0.995. I adapt it to annual frequency by taking the fourth power, and setting it at 0.98.

To match growth rates, the consumption growth mean,  $\mu_c$ , is calibrated to 2%, and the inflation growth mean,  $\mu_\pi$ , is calibrated to 4%. Both of these numbers co-inside with the calibrated growth rates in Bansal and Shaliastovich (2012), as they are simply four times as high than their quarterly counterparts. The other consumption and inflation dynamics' parameters, namely the persistence matrix  $\Pi$ , and the volatility leverages  $\phi_c$  and  $\phi_\pi$ , are calibrated to approximately target the annual standard deviation of both consumption growth and inflation (1.94% and 4.09% in the data, respectively), the auto-correlation of consumption growth (0.45), the auto-correlation of inflation (0.63), the nominal one-year U.S. yield (6.08%), and the slope of the nominal yield curve (defined as the difference between a 5-year yield and a 1-year yield, or 0.7% in the data).

The volatility process parameters are set to the 'annualized parameters of Bansal and Shaliastovich (2012): The persistence is calibrated to the consumption-volatility's persistence reported in Bansal and Shaliastovich (2012), taken to the power of four; The volatility process standard deviation is set to twice the equivalent value in Bansal and Shaliastovich (2012). The resulting key moments of the annual Long-Run Risk model are reported in table 6, along with their data counterparts.

The calibration of the borrower's side is reported in table 5, panel B. I set the coefficient of relative risk aversion, and the intertemporal elasticity of substitution of the borrowers to the same level as the lender's coefficients (i.e.  $\gamma = 25$ ,  $\psi = 2$ ). The parameters governing the endowment of the borrower, are calibrated to match key moments of the trend and business cycle components of Brazil's GDP. Using the log of dollar-denominated data on Brazil's GDP between 1960-2011, which I denote by  $\log(y_t)$ , I extract the log of the trend component,  $\log(\Gamma_t)$ , by HP-filtering Brazil's log-annual GDP with a smoothing parameter of 400. The business cycle component,  $z_t$ , can then be retrieved by the difference between the original log-GDP series and the filtered series, i.e.  $z_t \equiv \log(y_t) - \log(\Gamma_t)$ . The growth component,  $g_t$ , can be obtained by taking first difference of the log of trend sequence  $\log(\Gamma_t)$ , that is,  $g_t \equiv \log(\Gamma_t) - \log(\Gamma_{t-1})$ . Equipped with observed transitory time-series  $z_t$ , I now calibrate the parameters governing its AR(1) evolution to match its mean ( $\mu_z = 0$ ), first order auto-correlation ( $\rho_z = 0.75$ ), and a variance of 0.008 ( $\sigma_z = 0.06$ ). I set  $\mu_g$  to 0.043, to match the mean growth rate of Brazil's trend. Because of the relatively short sample, I refrain from estimating the persistence of the trend component, and rather calibrate  $\rho_g$  to 0.0008, which is the taken from Tomz and Wright (2007)<sup>4</sup>. The standard deviation of the process,  $\sigma_g$ , is calibrated to 0.015, to match the variance of  $g_t$  which amounts to 0.0002.

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<sup>4</sup>Tomz and Wright (2007) calibrate the Aguiar and Gopinath (2006) model at the annual frequency for the case of Argentina. They pick  $\rho_g = 0.0008$ , and I use this value here for the case of Brazil.

The redemption probability,  $\lambda$ , is calibrated to 0.1 at the quarterly frequency in the model of Aguiar and Gopinath (2006), which implies that the economy is denied market access for 2.5 years on average. This is similar to the three years observed in the data (see Gelos et al. (2004)). At annual frequency, I calibrate this parameter to 0.4, in order to match the same average exclusion period. The additional loss of output in autarky,  $\delta$ , is set to 1%, which is close to 2% value used in Aguiar and Gopinath (2006). Lastly, I set  $\beta$  of the borrower to 0.1, in order to match the level of the 1-year spread of Brazil. As noted in Arellano and Ramanarayanan (2008) and Aguiar and Gopinath (2006), a high impatience is necessary for generating reasonable defaults in equilibrium.

Borri and Verdelhan (2009) report that the annual correlation between the GDP of the U.S. and that of emerging economies varies between -0.3 to 0.5. Specifically, the correlation between the U.S. and Brazil at quarterly frequency is 0.11 for the period of 1994-Q4 to 2008-Q3. I refrain from estimating the correlations in section 4.3, as this would require extracting the expected growth and expected inflation components at the annual frequency. With a fairly short sample, these estimates would be prone to high imprecision. Instead, I calibrate the correlation of the permanent and transitory components of Brazil with the expected growth of the U.S. to a relatively high level:  $\varrho_{cz} = 0.6$ ,  $\varrho_{cg} = 0.6$ , while the correlation of these components with expected inflation is set to a negative level:  $\varrho_{\pi g} = -0.6$ . The goal of these high positive and negative correlations is to push Brazil to default at bad times for the U.S. economy; that is, making Brazil to default when expected consumption is low, or when expected inflation is high. In turn, this makes the covariance term in equation (20), or the risk premium, to become more negative, and pushes the yields higher. Note, that in spite of the seemingly high magnitude of the correlations, the actual correlation between the total consumption growth of the U.S. and the GDP of Brazil is fairly low, since the predictable component of consumption is small in magnitude compared to the total consumption growth, and there is no correlation between the idiosyncratic innovation of consumption growth in the U.S. and Brazil's innovations.

## 7 Results

This section provides the quantitative results of the sovereign-default model described in section 4, under the calibration of section 6. Subsections 7.1, 7.2, and 7.3 contain a detailed description of the model's output and statistics, while section 7.4 is dedicated to explaining the obtained figures and numbers of the model from an economic perspective.

## 7.1 Unconditional Model Statistics

The model matches some key asset pricing features of the data. The unconditional model-implied moments, along with their data counterparts, are reported in table 7. The model generates an average short term 1-year spread of 2.9%, thus matching exactly the level of the spread curve. The average long term 10-year spread in the model is 3.7%, which is smaller than the observed level of 4.8%. In turn, this implies that the model's unconditional slope of the spread curve is about 0.8%.

While this is only 40% as high as the average observed slope, the slope level is significantly positive, and captures the well-documented feature that for emerging market economies the spread curve is upward-sloping on average. Moreover, the standard deviation of the slope in the data is 2.6%, and so the model implied slope falls within one standard deviation from its point estimate. Notice, that a model implied slope of 0.8% is higher than that generated in the model of Arellano and Ramanarayanan (2012), and that because the U.S. yield curve is also upward sloping in my framework, the model-implied slope of the nominal yield curve of Brazil is 1.8%. Other papers, with a flat term-structure for the lenders, effectively treat the slope of the emerging market's yield curve as the slope of the spread curve. Following this logic, the model fits the slope of the spread curve almost perfectly. The standard deviation of the 1-year spread in the model, 3%, is of the same magnitude as that observed in the data, 3.8%. The standard deviation of the spreads falls with the tenor in the model, as well as in the data. However, because of the strong mean-reversion of the exogenous processes in the model, the standard deviation of the spreads falls sharply with the tenor. The 10-year spread's volatility is very low, about 0.5%, while in the data it is 3.3%.

The model also yields a realistic correlation between the yields of the U.S. and Brazil. Both in the model and in the data, the correlation between a 1-year nominal U.S. yield and a 1-year nominal Brazilian yield (or spread) is negative and close to zero. For longer maturities, the correlation becomes positive and quite high in magnitude (as the yields have some degree of co-movement in the long-run). Thus, the model implied correlation between a 10-year nominal U.S. yield and a 10-year nominal Brazilian yield is 0.24, which is within one standard deviation from the data point estimate.

The average debt to GDP ratio in the model is only 5%, which is only about one-fourth of the observed ratio. This result is very similar to that reported in Arellano and Ramanarayanan (2012). Introducing risk aversion into an optimal sovereign default model tightens price schedule for all levels of debt, which in turn reduces the average stock of debt in the economy. The probability of default in the model is quite low, about 1%, while in the data it is roughly 3.8%<sup>5</sup>. Notice, that I do not target the default probability due to the short-sample. This default probability is of a smaller magnitude than the spreads. Thus, risk premia break the tight connection between average spreads and default

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<sup>5</sup>Brazil had 8 occurrences of default or rescheduling of external debt in the period between 1800-2008.

probabilities thereby addressing the so-called credit puzzle.

As in the data, consumption in the model is more variable than GDP, which is a well documented fact in emerging economies. The model also predicts that GDP is negatively correlated with short and long spreads. The magnitude of these negative correlations is smaller in the model than in the data. Nonetheless, the sample for calculating this correlation in the data is very short (annual data from 1998 to 2011).

## 7.2 Conditional Slope of The Spread Curve

The unconditional statistics of Brazil's spread curve, which are reported in table 7, suggest that on average, the spread curve is upward-sloping. However, the dynamics of the spread curve's slope in the data are time-varying, as is seen in figure 9, which shows how the level of the slope changes over time. Mostly, the slope is positive. In times of distress and high uncertainty though, the slope becomes more volatile and turns negative. For instance, in 2002-2003 the slope becomes negative, due to high uncertainty caused by the banking crisis and external default of neighboring country Argentina in March-November of 2001, and Brazil's financial crisis of 2002. Notice, that in the sampling window of Brazil's spreads, namely 1998-2012-Q2, no default or restructuring of debt occurrences were registered.

The purpose of this section is to show that the model matches the data in generating time-varying differences in the pricing of short- and long-term debt, due to movements in the risk premia and average default probabilities. To compare the spread dynamics in the model to the data, similarly to Arellano and Ramanarayanan (2012), I organize the data into deciles based on the level of the short 1-year spread. Table 8 presents the average spreads for short and long term debt across periods when the short spreads are below their 10th, 25th, 50th percentile and above their 50th, 75th, and 90th percentile. The first two columns of the table refer to the model implied spreads, and the last two columns present the data. In the model, when default is unlikely, both spreads are low, and the spread curve is upward-sloping. For instance, when the short spread is below the 50th percentile, the average short term spread is below 1%, and the average long spread is around 3%. In contrast, when the probability of default is higher (distressed times), both spreads rise, and the spread curve becomes downward-sloping: when the short term spread is above the 90th percentile, the average short term spread is 8.90%, and the average long spread is 4.69%.

Compared to the data of Brazil, the model captures the dynamics of the spread curves and in particular the conditional level of the short term spread across the different deciles. The model also generates a realistic average slope of the spread curve, for instance for the case that the short term spread is below the 50th percentile. The model misses however to capture the exact level of slope for

periods of very high or very low spreads. The reason is that the long-term spread is far less volatile in the model than in the data, due to strong mean reversion in the model.

On one hand, the fact that the endogenous probability of default in the model is mean-reverting and persistent, is a key factor in mimicking the observed dynamics of the spread curve (the probability of default is mean reverting as a result of the dynamics of debt and income). When income is low and debt is high, default is likely in the near future, so spreads are high. Long term spreads increase by less than short-term spreads because the borrower’s likelihood of repaying may raise if it receives a sequence of good shocks and reduces its debt. Although cumulative default probabilities on long-term debt are always larger than on short-term debt, the long-term spread can be lower than the short spread because it reflects lower *average* future default probability. I provide a detailed explanation on this in section 7.4.3. The effect of mean-reverting and persistent default probabilities on spread curves are the same as highlighter in Merton (1974) in the case of credit spreads for corporate debt.

On the other hand, the mean-reversion in my model creates too much persistence especially in the long spreads. This reduces the variability of the long spreads and makes it difficult for the model to match the long spread level at extremely bad or good states.

### 7.3 Predictive Regressions

Section 3.2 provides evidence that Brazil’s spread is predictable by U.S. yields, and U.S. market return’s realized volatility. In this section I shall examine whether this kind of predictability exists in the model, and to what extent. After simulating the economy for 10,000 periods, I regress future Brazilian spreads on current Brazilian spreads, current U.S. yields, and the state variable  $\sigma_t^2$ , which captures the long-run uncertainty level of the real and nominal U.S. economy. The population  $R^2$ ’s, along with the regressors’ loadings, are reported in table 9. Note further that the state variable  $\sigma_t^2$  and the U.S. VIX have different economic meanings, and different scales, and thus ex-ante I do not expect to match the data numbers reported in table 2, but rather to qualitatively obtain similar results.

The  $R^2$ ’s in predicting both short and long term spreads are quite low compared to their data counterparts reported in table 2, with a maximal  $R^2$  of 10% compared to 50% in the data. However, the model regressions share some common features with the results of table 2. First, the long term spreads are more predictable than the short ones. Second, the loadings on the U.S. yields and the U.S. volatilities in all regressions are significantly non-zero and positive, which indicates that the U.S. state variables help predicting foreign spreads, and foreign sovereign decision to default.

## 7.4 Behavior of Model Variables around Defaults

In this section I examine how the dynamics of the U.S. fundamentals, the endowment of Brazil, and the derived yields behave during a window around defaults. After solving for the optimal policy functions and bond prices, I simulate the economy 500 times for 10,000 periods, and isolate all default occurrences. I then record the level of the economic variables of the model 7 years prior to default, and 7 years after re-entry to the international financial markets. Because no defaults occurred within the sample period of Brazil's yields, I cannot compare the model-implied results to the behavior of the data at times of default. The purpose of this analysis is therefore to gain intuition about the causes of defaults in the model, and about the time-varying dynamics of prominent moments around defaults. I shall first present the stylized facts of the model, and then use those facts to explain salient features of the model.

Figure 4 shows how Brazil's economic variables change during the default window. Time 0 denotes the date of default, and time 1 denotes the date of re-entry to financial markets after the autarky penalty. Other times denote respective relative dates to the default event. Figure 4a exhibits an increase in the level of debt to output in the years prior to default, and then a reversion to the mean level after redemption. Although the increase in the debt to output is miniscule compared to the data, the model captures the feature of an increased level of debt at times of default. The increased level of debt prior to default can be explained by the sharp drop in GDP just before default, as can be seen in figure 4b. The output growth decreases around default due to a sharp fall in the trend of Brazil's GDP, as shown in figure 4d, but also due to a decline in the business cycle component, as in figure 4e. Intertemporal consumption smoothing pushes the agent to borrow more once a sequence of bad shocks to the endowment is realized, which in turn increases the ratio of debt to output, strengthens the burden of repayment, and leads to default. The drop in output is translated to a somewhat smaller decline in consumption before default as seen in figure 4c. The former figures suggest that defaults tend to happen at 'bad' times for Brazil. This is in-line with evidence found in Tomz and Wright (2007) that there is a negative relationship between domestic output and defaults.

The evolution of the lender's state variables is depicted in figure 5. Figures 5a and 5b show that the predictable component of consumption growth is hit by a large negative shock upon default, while the predictable component of inflation peaks, respectively. Interestingly, in figure 5c the volatility of the real and nominal economy in the U.S. also rises around the time of default, even though U.S. volatility shocks are independent of other shocks in the model. In light of the long-run risk model, the former three changes in the U.S. economic state suggest that defaults tend to occur also at 'bad' times for the U.S. as well. This is no coincidence in the model: the innovations to Brazil's GDP are positively correlated with the shocks to the predictable component of U.S. consumption growth, and negatively correlated with the shocks to U.S. inflation. The shape of these figures resemble the

evidence presented in section 3.1. In the data, on average, default tend to occur when the long-term component of consumption drops, and smoothed inflation and real realized volatilities peak.

Figure 6 offers an interesting insight to the time-varying dynamics of the model-implied yields and spread curve's slope. Figure 6a illustrates the time-varying sign of the Brazilian spread curve. In the pre-default window, the long-term spread is above the short-term spread, implying a positive slope for the spread curve. However, closer to default, both the short- and long- term spreads increase gradually. Getting closer to time 0, the gap between the short-term and the long-term spreads narrows. One year prior to default the order between the two swaps, making the short-term spread excessively higher than the long-term one just before default, and slope of the spread curves turns negative. I shall explain this pattern in sections 7.4.2 and 7.4.3. Contemporaneously, one can observe in figures 6c and 6b that short-term Brazilian and U.S. yields increase around times of default. In fact, one year before default, the one-year Brazilian yield becomes higher than the ten-year one. The pattern regarding the short-term U.S. yields in figure 6c is in-line with the data for the U.S. risk-free rate around times of default, as was explained in section 3.1.

What causes the yields in Brazil to increase significantly around defaults? Why is the level of the short-term yield exceeds that of the long-term one right before defaults? Figure 7 shows the evolution of the two major factors which determine the level of the Brazilian yields in the model: risk premia for default risk in figure 7a, and the average probability of default over a single period in figure 7b. The default risk premium of the short-term spread increases significantly before defaults, almost matching the risk premium of the long-term yields. Unconditionally, seven years before default, the average default probability is the same for both the one- and ten- year bonds. Getting closer to default, the average default probability of both bonds starts to increase. Yet, the *average* default probability for the one-year bond is much higher than for the ten-year bond, just one-year before default. I explain the causes for the described behavior of the average default probabilities and risk premia in section 7.4.2.

Furthermore, in order to illustrate the causality between the U.S. state variables and the likelihood of defaults in the model, I isolate occurrences of U.S. volatility jumps in the economy simulations. That is, events in which the U.S. volatility increases by 8%. I then plot in figure 7c the probability of default in the Brazilian economy around that event. Before the volatility shock, the probability of default evolves around the average probability of default in the model (0.8%). However, at the time of the volatility shock, the probability shoots to 3%. In unreported results, one can observe a similar pattern for the probability of default around times of jumps in the U.S. expected consumption growth, and around times of jumps in U.S. expected inflation. The uniqueness of the volatility channel though, is that its innovations are uncorrelated with any other shocks in the model. Thus, the shift in the probability of default is entirely endogenous.

To better understand the relative importance of the model variables to triggering default, I plot the innovations to each one of the model variables around default in figure 8. Right before default, all model variables are hit by a large ‘bad’ shock, on average. The only model variables that receive a *sequence* of ‘bad’ shocks, starting 7 years before default, are  $x_\pi$  (U.S. expected inflation), and  $g$  (Brazil’s trend). Define a ‘large bad shock’ as a realization of a shock which exceeds 1 in absolute terms (the sign determining whether the shock is good or bad depends on the specific state variable). In order of magnitude, 52% of defaults occur right after a large bad  $g$  shock; 45% of defaults occur right after a large bad  $x_\pi$  shock; 38% right after a large bad  $x_c$  shock; 34% right after a large bad  $Z$  shock; 4% right after a large bad  $\sigma_t^2$  shock. About 71% of defaults occur right after a large bad shock to a Brazilian state variable. 73.1% of defaults occur right after a large bad shock to a U.S. variable. This suggests that the Brazilian trend  $G$  is more dominant than the business cycle  $Z$  in triggering defaults. The U.S. expected inflation is the most important variable in the U.S. side to trigger defaults. It is worth mentioning that even without any correlation between the model’s shocks, Brazil would want to default after a sequence of positive shocks to the U.S. expected inflation. The reason is that debt-burden is higher when the U.S. risk-free rate is higher, and this is primarily dominated by a higher  $x_\pi$ . The above evidence is also in favor of suggesting that with correlation, the U.S. state variables are slightly more important than Brazil’s fundamentals in generating defaults.

#### 7.4.1 The story of defaults: what triggers default in the model?

The former evidence suggests that defaults happen in two stages within the model. In the first stage, or ‘warm-up’ stage, the U.S. inflation builds in the background, which is contemporaneous with increased level of borrowing. The U.S. nominal interest rate gradually increases due to higher inflation, which means that *ceteris paribus*, debt becomes more expensive. Because of the positive correlation between the U.S. expected inflation, and Brazil’s stochastic growth rate, Brazil’s trend gradually decreases. This causes Brazil to take more debt in order to smooth consumption, in spite of the increase in U.S. interest rates. The system is still stable; yet, this stage is crucial to get a positive slope for the spread curve (see section 7.4.2).

In the second stage, or ‘boom’ stage, a large ‘bad’ shock hits one of the state variable in the model. Either one of Brazil endowment variables drops significantly, which in turn makes the burden of repaying existing debt too high, given the already increased interest rate required as a consequence of the warm-up stage. Alternatively, one of the U.S. economy state variables is hit by a large bad shock. This also can make the burden of repaying existing debt too high, due to a sharp increase in the U.S. interest rates, and the already increased level of borrowing, accumulated in the ‘warm-up’ stage.



### 7.4.2 Explanation of Positive Unconditional Spread-Curve Slope

Consider a Brazilian spread at time  $t$  and maturity  $n$ , denoted by  $s_{t,n}$ . Note that:

$$s_{t,n} = y_{t,n}^{Brazil} - y_{t,n}^{US} = 1/q_{t,n}^{1/n} - y_{t,n}^{US}, \quad (26)$$

where  $y_{t,n}^{US}$  and  $y_{t,n}^{Brazil}$  denote the yields on U.S. government bonds, and Brazil's sovereign bonds respectively. The Brazilian bond price component  $q_{t,n}^{1/n}$  can be decomposed as follows:

$$\begin{aligned} q_{t,n}^{1/n} &= E[M_{t+1}M_{t+2}\dots M_{t+n}1_{ND,\{t+1\}}1_{ND,\{t+2\}}\dots1_{ND,\{t+n\}}]^{1/n} \\ &= E[M_{t+1\rightarrow t+n}1_{ND,\{t+1\}}1_{ND,\{t+2\}}\dots1_{ND,\{t+n\}}]^{1/n} \\ &= E[M_{t+1\rightarrow t+n}]^{1/n} E[1_{ND,\{t+1\}}1_{ND,\{t+2\}}\dots1_{ND,\{t+n\}}]^{1/n} + cov(M_{t+1\rightarrow t+n}, 1_{ND,\{t+1\}}1_{ND,\{t+2\}}\dots1_{ND,\{t+n\}})^{1/n} \\ &= \frac{1}{y_{t,n}^{US}} \cdot (\text{average probability of not defaulting over one period in } (t \rightarrow t+n)) + \\ &\quad (\text{average one-period default risk premium to bond price in } (t \rightarrow t+n)) + (\text{Jensen Adjustment [ignored]}). \end{aligned} \quad (27)$$

Putting together equations (26) and (27), yields:

$$\frac{\partial s_{t,n}}{\partial y_{t,n}^{US}} \approx \frac{1}{\text{average probability of not defaulting over one period in } (t \rightarrow t+n)} - 1 > 0 \quad (28)$$

The goal of this sub-section is explaining why  $s_{t,10} > s_{t,1}$ , on average. Equations (26), (27), and (28), illustrate that ceteris paribus, the  $n$ -year Brazilian spread rises when (1) the U.S.  $n$ -year yield increases; (2) the average probability of not-defaulting in a single period over the next  $n$ -years drops; (3) when the average default risk premium to bond prices over the next  $n$ -years becomes more negative (or in other words, the average default risk premium to Brazilian bonds' yields increases). How are the last three factors differ between a one-year and a ten-year bond in the model?

1. From the U.S. side of the model, we know that  $y_{t,1}^{US} < y_{t,10}^{US}$ . This is primarily due to a large positive inflation premium to the 10-year US bonds (see Bansal and Shaliastovich (2012)).
2. From figure 7b, unconditionally, the average probability of default over one-period is the same between  $(t \rightarrow t+1)$ , and between  $(t \rightarrow t+10)$ . Obviously, the *total* probability of not defaulting over  $(t \rightarrow t+10)$  is smaller than the total probability of not defaulting over  $(t \rightarrow t+1)$ . However, the average default probabilities, per single period, are similar. The average probability of not defaulting in  $(t \rightarrow t+1)$  is just  $[1 - Pr(default)]$ , which amounts to about 99% in the current calibration of the model. The average probability of not defaulting in one-period over  $(t \rightarrow t+10)$  is roughly the same, because of mean reversion. To see this, consider two possible cases: First, suppose that today, at time  $t$ , is a good state for the Brazilian economy, and the probability of not defaulting next period is 99.99%. Because of mean reversion, the economy will revert

to a median state quickly, and as it does so, the probability of no-default from one period to the next decreases. Hence, the average probability of no-default per one-period is approximately  $(99.99\% \cdot 99.5\% \cdot 99.3\% \cdot 99.1\% \cdot \dots \cdot 99\%)^{(1/10)} \approx 99\%$  (or slightly above). Second, suppose that today is a bad state, and the probability of not defaulting next period is 98%. Again, because of mean reversion the average probability of no-default is  $(98\% \cdot 98.5\% \cdot 98.7\% \cdot 98.9\% \cdot \dots \cdot 99\%)^{(1/10)} \approx 99\%$  (or slightly below). Since both of the two scenarios are equally likely, the average probability of not defaulting in one-period over  $(t \rightarrow t+10)$  is also 99%.

3. From figure 7a, unconditionally, the average default risk premium to bond prices in  $(t \rightarrow t+10)$  is much more negative than the average default risk premium to prices in  $(t \rightarrow t+1)$ . Why is this the case? The only state variables that receive a *sequence* of ‘bad’ shocks before default are  $x_\pi$  (U.S. expected inflation) and  $g$  (Brazil’s trend). All other state variables are hit with a large bad shock just right before default. If the average default risk premium to bond price in  $(t \rightarrow t+10)$  is more negative than the average default risk premium in  $(t \rightarrow t+1)$ , then that implies that if default happens in 10 years, and not next year, then in 10 years the default must occur in a worse state for the U.S. economy. Hence, the covariance between the U.S. discount factor and the no-default indicator (premium) would be more negative. This can be explained by the fact that if default does not occur next period, but occurs in 10 years, and each default occurrence is preceded by a sequence of positive shocks to U.S. expected inflation (‘warm-up’ stage), then one can expect a sequence of positive inflation shocks in the next 10 years until default. This would make expected inflation level in the U.S. in 10 years much higher than it is next year, assuming default happens in 10 years, but not next year. In turn, this translates into worsened economic conditions in the U.S. within 10 years, if default does not happen next period. Additionally, because of the negative feedback effect between expected inflation to expected consumption in the U.S., one can also predict that conditional on default occurring only in 10 years from today, the expected consumption growth in the U.S. would be much lower.

The first fact ostensibly should make  $s_{t,10} > s_{t,1}$ . However, because the average probability of no-default is close to 1 in the model, then from equation (28),  $\frac{\partial s_{t,n}}{\partial y_{t,n}} \approx 0$ . Therefore, the effect of this fact on the slope of the spread curve is marginal. The second fact should make no difference between  $s_{t,1}$  and  $s_{t,10}$ . Lastly, the third fact implies that  $s_{t,10} > s_{t,1}$ , and therefore risk premia are the *only crucial* factor that generates a positive unconditional slope in the model.

### 7.4.3 Explanation of Negative Spread-Curve Slope Before default

We know that unconditionally, the model generates a positive slope for the spread curve. Yet, as seen in figure 6a, the sign of the slope turns negative right before default (that is, close to default

$s_{t,10} < s_{t,1}$ ). What are the causes for this? there are 3 possible routes to account for the change in the slope's sign:

The first channel is the U.S. yields. If the short term U.S. yield decreases around default more than the long term U.S. yield, that would produce a flip in the sign of the spread curve's slope. however, figure 6c suggests that close to default, the U.S. short term yield increases significantly, while the U.S. long term yield increases only marginally. That pushes  $s_{t,1}$  downwards and  $s_{t,10}$  upwards, and so the U.S. yields cannot explain the variation in the sign of the slope.

The second channel is a change in the risk premia of the short- or long- term Brazilian yields. We know that unconditionally, the 10-year Brazilian yield has a much higher risk premium than the 1-year yield (see section 7.4.2 for why this is the case). Figure 7a indicates that around defaults, the default risk premium (in yield terms) for the 1 year bond goes up by a lot, almost matching the default risk premium for the 10 year bond. In turn, this fact alone should cause the slope of the spread curve to be still positive, but close to zero. Very close to default time, the default risk premium to the yield of the 1-year bond is higher, because the U.S. economy is already in a bad state, as the economy has received a sequence of bad shocks to expected inflation in the 'warm-up' stage. The reason as of why the risk premium for the 1-year bond does not exceed that of a 10 year bond, and only becomes close to it, can be explained using the same analysis of section 7.4.2.

The last channel is a fluctuation in the average probability of default. Unconditionally, the average probability of default for the 1-year and 10-year bonds are the same. But surprisingly perhaps, close to default, the average probability of default per single-period for the 1-year bond is much higher than that for the 10-year bond (see figure 7b). Trivially, the total probability of defaulting in 10 years is higher than the total probability of defaulting next year. This is not true for the average probability though. Close to default, the Brazilian economy is in a bad state. Thus, for the 1-year bond, the average probability of no-default over the next period is only 98%, and the average probability of default is 2%. For the 10-year bond however, the average probability of no-default per single-period is  $(98\% \cdot 98.5\% \cdot 98.7\% \cdot 98.9\% \cdot \dots 99\%)^{(1/10)} \approx 99\%$  because of mean reversion (the economy is expected to revert to a median state in 10 years, and consequentially, the probability of no-default from one period to the next increases gradually). Note that this calculation is done ex-ante, before any default occurs. This implies that the average probability of default is only 1% for the 10-year bond. Since the 10-year bonds are therefore "safer" *per-period on average*, the premium for default risk is lower for those bonds, compared to the 1-year bonds.

Combining the latter two channels, the model generates a downward-sloping spread curve, right before defaults occur.

## 8 Conclusion

In this paper, I have presented a dynamic model of sovereign defaults to study the behavior of prices of sovereign spread curves. I define the  $n$ -year spread for an emerging market country as the difference between the yield on a defaultable, nominal, zero-coupon bond maturing in  $n$  years issued by the country and on a zero-coupon bond of the same maturity with negligible default risk, for example U.S. treasury notes. The spread curve depicts spreads as a function of maturity. In emerging markets data, the spread curve is usually upward sloping. When spreads on short-term debt are low, long-term spreads are higher than short-term spreads. When short-term spreads rise, long-term spreads rise less. Additionally, I document that in the data, defaults in emerging markets tend to occur at ‘bad’ times for the U.S. economy. Picking Brazil as a case-study, my model simultaneously reproduces the patterns observed for the term-structure of spreads and bond prices in Brazil and in the U.S., as well as the contemporaneous U.S. fluctuations of fundamentals at times of defaults.

My main innovations in this paper were (1) developing a framework that broadly explains international asset-pricing features jointly: the behavior of the spread curve of an emerging market, which takes into account potential sovereign defaults in bond prices, as well as explaining the yield curve, the real risk free rate, and the equity premium in the U.S. economy; (2) providing further evidence on how U.S. shocks to consumption and volatility propagate to the rest of the world, and affect international sovereign credit risk; (3) pushing the macroeconomic literature on sovereign default to put more emphasis on the role of risk premia in the prices of short- and long- term sovereign bonds, and accounting for inflation in the derivation of nominal yields, in congruency with the data.

The current work can be extended in several ways. The model is limited for emerging markets that are small enough, such that neither their borrowing nor their default decision affects the U.S. budget constraint. Yet, this excludes countries such as Russia, that have experienced defaults in its past. A potential extension to the model would be to allow the borrowings to enter the U.S. endowment, i.e. account for a feedback effect. Another direction, is to consider pricing a cross-section of spread curves of different emerging market economies. Some economies, such as Uruguay, have an unconditional downward-sloping spread curve. Can this kind of heterogeneity be explained by different systematic exposure to underlying U.S. risk factors? Lastly, in unreported results I document a strong and robust correlation between the realized volatility of the industrial production in the U.S. and that in emerging market economies. The model can be extended to have two volatility processes, one of the U.S. economy and another for an emerging market, which are correlated. The question is whether volatility contagion on its own can account for the behavior of spreads, as documented in the data.

## Appendix

In this appendix I present a brief analytical solution to the long-run risk model, described for the lenders side in section 4.1. The solution rely on using a standard log-linearization of returns. Specifically, the log-linearized return on consumption claim is given by:

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1},$$

where  $pc_t$  is the log price-to-consumption ratio, and  $\kappa_0$  and  $\kappa_1$  are approximating linearization constants based on the endogenous level of price-consumption ratio in the economy. I follow Bansal, Kiku, and Yaron (2007), and Bansal and Shaliastovich (2012) to solve for these coefficients endogenously inside the model.

In equilibrium, the price-consumption ratio is linear in the expected growth and volatility factors:

$$pc_t = A_0 + A_c x_{ct} + A_\pi x_{\pi t} + A_\sigma \sigma_t^2,$$

where,

$$\begin{aligned} A_{xc} &= \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_c}, & A_{x\pi} &= \frac{\kappa_1 \rho_{c\pi} A_{xc}}{1 - \kappa_1 \rho_\pi}, \\ A_\sigma &= \frac{\theta \kappa_1^2}{2(1 - \kappa_1 \nu)} [A_{xc}^2 \phi_c^2 + A_\pi^2 \phi_\pi^2], \end{aligned}$$

and where the long-linearization coefficient  $\kappa_1$  satisfies:

$$\log \kappa_1 = \log \beta + (1 - \frac{1}{\psi}) \mu_c - \sigma_0^2 (\kappa_1 \nu A_\sigma - A_\sigma) + \frac{1}{2} \theta \kappa_1^2 A_\sigma^2 \sigma_w^2 + \frac{1}{2} \theta (1 - \frac{1}{\psi})^2 \sigma_c^2.$$

The real and nominal discount factor equal to:

$$m_{t+1} = m_0 + m_{xc} x_{ct} + m_{x\pi} x_{\pi t} + m_\sigma \sigma_t^2 - \lambda_c \sigma_c \eta_{c,t+1} - \lambda_\pi \sigma_\pi \eta_{\pi,t+1} - \lambda_{xc} \phi_c \sigma_t e_{c,t+1} - \lambda_{x\pi} \phi_\pi \sigma_t e_{\pi,t+1} - \lambda_\sigma \sigma_w w_{t+1}$$

$$m_{t+1}^{\$} = m_0^{\$} + m_{xc}^{\$} x_{ct} + m_{x\pi}^{\$} x_{\pi t} + m_{\sigma}^{\$} \sigma_t^2 - \lambda_c^{\$} \sigma_c \eta_{c,t+1} - \lambda_{\pi}^{\$} \sigma_{\pi} \eta_{\pi,t+1} - \lambda_{xc}^{\$} \phi_c \sigma_t e_{c,t+1} - \lambda_{x\pi}^{\$} \phi_{\pi} \sigma_t e_{\pi,t+1} - \lambda_{\sigma}^{\$} \sigma_w w_{t+1},$$

The discount factor parameters and market prices of risks satisfy:

$$m_0 = \theta \ln \beta - \gamma \mu_c + (\theta - 1)(-\log \kappa_1 - \sigma_0^2 A_{\sigma}(\kappa_1 \nu - 1)),$$

$$m_{xc} = -\gamma + A_c(1 - \theta)(1 - \kappa_1 \rho_c), \quad m_{x\pi} = (\theta - 1)(\kappa_1 A_c \rho_{c\pi} + \kappa_1 A_{\pi} \rho_{\pi} - A_{\pi}),$$

$$m_{\sigma} = A_{\sigma}(1 - \theta)(1 - \kappa_1 \nu),$$

$$\lambda_c = \gamma, \quad \lambda_{xc} = (1 - \theta) \kappa_1 A_{xc}, \quad \lambda_{x\pi} = (1 - \theta) \kappa_1 A_{x\pi},$$

$$\lambda_{\sigma}^{\$} = (1 - \theta) \kappa_1 A_{\sigma}, \quad \lambda_{\pi} = 0.$$

and the nominal ones are equal to the real ones, except for:

$$m_0^{\$} = m_0 - \mu_{\pi}, \quad m_{x\pi}^{\$} = m_{x\pi} - 1, \quad \lambda_{\pi}^{\$} = \lambda_{\pi} + 1.$$

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## Tables and Figures

Table 1: Average Spread Curves Across Countries

	Maturity (years)	Level (%)	Std. Dev.	When 1-year spread is above below $n$ th percentile	
				<50th	$\geq$ 50th
Argentina	1	4.22	8.28	1.12	7.32
	10	6.29	3.45	4.65	7.93
Mexico	1	0.98	1.01	0.46	1.64
	10	2.44	1.39	1.75	3.30
Colombia	1	1.79	1.51	0.65	2.58
	10	4.51	2.34	2.96	5.57
Venezuela	1	6.74	5.33	3.62	12.21
	10	8.18	4.36	5.59	12.73
Brazil	1	2.91	3.86	0.97	4.89
	10	4.75	3.37	2.78	6.72

The table shows summary statistics on the spread curves of selected emerging market economies. The data used to construct the table are taken from the zero-coupon Bloomberg Fair Value yields, available at Bloomberg's database for the years 1998-2012. The second and third columns present the average and standard deviation of the 1-year (short-term) spread and the 10-year (long-term) spread across the sample periods. Each spread at a given point in time is calculated by subtracting the U.S. zero-coupon fair value yield from the emerging market economy's fair value yield, for the same maturity. The last two columns present the average level of the short and long spreads, only across the periods in which the short spread is above or below the corresponding column percentile.

Table 2: Predictability of Short Sovereign Spreads By U.S. Yields and VIX

Dept. Variable	Regressors		$R^2$
	$\beta(y_{1,t}^{US})$	$\beta(VIX_t)$	
<b>A. Brazil:</b>			
$s_{1,t}^{Brazil}$	0.13 (0.74)	0.21 (2.30)	0.17
$s_{1,t+1}^{Brazil}$	0.55 (3.18)	0.09 (1.52)	0.08
$s_{1,t+3}^{Brazil}$	1.10 (2.69)	0.09 (1.82)	0.20
<b>B. Colombia:</b>			
$s_{1,t}^{Colombia}$	0.13 (2.19)	0.12 (6.73)	0.50
$s_{1,t+1}^{Colombia}$	0.28 (4.74)	0.04 (1.52)	0.16
$s_{1,t+3}^{Colombia}$	0.43 (4.52)	0.03 (1.70)	0.27
<b>C. Mexico:</b>			
$s_{1,t}^{Mexico}$	0.07 (1.06)	0.08 (5.06)	0.43
$s_{1,t+1}^{Mexico}$	0.16 (2.07)	0.01 (1.42)	0.10
$s_{1,t+3}^{Mexico}$	0.06 (1.01)	0.002 (0.19)	0.02
<b>C. Venezuela:</b>			
$s_{1,t}^{Venezuela}$	-1.71 (-8.66)	0.23 (3.99)	0.76
$s_{1,t+1}^{Venezuela}$	-0.71 (-1.73)	0.19 (2.32)	0.25
$s_{1,t+3}^{Venezuela}$	1.40 (1.40)	0.01 (0.07)	0.2

The table shows in each row the results of regressing the dependent variable in the first column on a short-term (1y) U.S. yield and on U.S. VIX index. I denote by  $y_{m,t}^{US}$  the nominal yield on a U.S. zero-coupon bond with maturity  $m$  at time  $t$ . I denote by  $s_{m,t}^X$  the spread between the yield on a zero-coupon bond of country X, maturing in  $m$  periods, and the corresponding yield on U.S. zero-coupon bond:  $s_{m,t}^X = y_{m,t}^X - y_{m,t}^{US}$ . The data on the yields and the VIX are daily, and covers the period 1998-Q2 to 2012-Q1, except for Venezuela for which data begin at 2001-Q1. Note that  $t+1$  refers to one year ahead. Because of overlap, I use Newey-West (HAC) standard errors.

Table 3: Predictability of Long Sovereign Spreads By U.S. Yields and VIX

Dept. Variable	Regressors		$R^2$
	$\beta(y_{10,t}^{US})$	$\beta(VIX_t)$	
<b>A. Brazil:</b>			
$s_{10,t}^{Brazil}$	2.32 (6.74)	0.23 (2.79)	0.38
$s_{10,t+1}^{Brazil}$	3.41 (8.96)	0.16 (2.52)	0.49
$s_{10,t+3}^{Brazil}$	3.79 (5.83)	0.14 (3.29)	0.56
<b>B. Colombia:</b>			
$s_{10,t}^{Colombia}$	1.42 (7.80)	0.13 (4.61)	0.49
$s_{10,t+1}^{Colombia}$	1.86 (10.73)	0.09 (2.72)	0.55
$s_{10,t+3}^{Colombia}$	2.08 (12.06)	0.06 (3.00)	0.64
<b>C. Mexico:</b>			
$s_{10,t}^{Mexico}$	0.69 (4.94)	0.11 (6.62)	0.57
$s_{10,t+1}^{Mexico}$	0.98 (5.22)	0.06 (3.21)	0.47
$s_{10,t+3}^{Mexico}$	0.68 (6.18)	0.01 (0.73)	0.34
<b>C. Venezuela:</b>			
$s_{10,t}^{Venezuela}$	-2.72 (-6.20)	0.21 (7.48)	0.76
$s_{10,t+1}^{Venezuela}$	-2.10 (-2.96)	0.18 (2.26)	0.38
$s_{10,t+3}^{Venezuela}$	0.07 (0.08)	-0.001 (-0.04)	0.005

The table shows in each row the results of regressing the dependent variable in the first column on a long-term (10y) U.S. yield and on U.S. VIX index. I denote by  $y_{m,t}^{US}$  the nominal yield on a U.S. zero-coupon bond with maturity  $m$  at time  $t$ . I denote by  $s_{m,t}^X$  the spread between the yield on a zero-coupon bond of country X, maturing in  $m$  periods, and the corresponding yield on U.S. zero-coupon bond:  $s_{m,t}^X = y_{m,t}^X - y_{m,t}^{US}$ . The data on the yields and the VIX are daily, and covers the period 1998-Q2 to 2012-Q1, except for Venezuela for which data begin at 2001-Q1. Note that  $t+1$  refers to one year ahead. Because of overlap, I use Newey-West (HAC) standard errors.

Table 4: Conditional Volatility Levels

1y Brazil's spread percentile	U.S. VIX
< 10	14.192
< 25	16.259
< 50	18.555
$\geq 50$	26.728
$\geq 75$	29.219
$\geq 90$	30.909

The table reports conditional volatility statistics, depending on the level of the short-term spread in Brazil. The data used are daily VIX indices for the years 1998-2012. Column two presents the average level of volatility, in the data, only across the periods in which the short spread is above or below the percentile in the first column.

Table 5: Model Calibration

Variable	Notation	Value
<b>A. Lenders:</b>		
Preferences:		
Relative-risk aversion	$\gamma$	25
Intertemporal elasticity of substitution	$\psi$	2
Discount factor	$\beta$	0.98
Consumption Dynamics:		
Mean Consumption Growth	$\mu_c$	0.02
Persistence Expected Consumption	$\rho_c$	0.09
Feedback of Inflation on Consumption	$\rho_{c\pi}$	-0.48
Idiosyncratic Standard Deviation	$\sigma_c$	0.02
Volatility Leverage	$\phi_c$	1
Inflation Dynamics:		
Mean Inflation	$\mu_\pi$	0.04
Persistence Expected Inflation	$\rho_\pi$	0.72
Idiosyncratic Standard Deviation	$\sigma_\pi$	0.01
Volatility Leverage	$\phi_\pi$	1.44
Volatility Dynamics:		
Mean Volatility	$\sigma_0^2$	0.01
Persistence	$\nu$	0.96
Standard Deviation	$\sigma_w$	1e-4
<b>B. Borrowers:</b>		
Preferences:		
Relative-risk aversion	$\gamma$	25
Intertemporal elasticity of substitution	$\psi$	2
Discount factor	$\beta$	0.1
Default Penalty:		
Reentry Probability	$\lambda$	0.4
Output loss	$\delta$	0.01
Endowment - Trend:		
Mean	$\mu_g$	0.043
Persistence	$\rho_g$	0.0008
Standard Deviation	$\sigma_g$	0.015
Endowment - Business Cycle:		
Mean	$\mu_z$	0
Persistence	$\rho_z$	0.75
Standard Deviation	$\sigma_z$	0.06
Correlation With U.S.:		
Between Brazil's GDP shocks and U.S. predicted consumption shocks	$\varrho_c$	0.6
Between Brazil's GDP shocks and U.S. expected inflation shocks	$\varrho_\pi$	-0.6

The table presents the calibrated values for the different model parameters. All values refer to annual frequency. For explanations on the chosen values see section 6.

Table 6: U.S. Selected Moments - Data and Model

	Data	Model
<b>Consumption:</b>		
Mean	1.93	2.00
Standard Deviation	2.16	2.15
Auto-Correlation	0.48	0.45
<b>Inflation:</b>		
Mean	3.61	4.00
Standard Deviation	4.09	2.40
Auto-Correlation	0.63	0.72
$Corr(\pi, \Delta c)$	-0.11	-0.24
<b>Nominal Bond Yields:</b>		
1y yield	6.09	6.08
3y yield	6.52	6.46
5y yield	6.79	6.87
<b>Real Bond Yields:</b>		
real 1y risk-free rate	0.86	1.80

The table shows key properties of U.S. real consumption, inflation, risk-free rate, and nominal term structure in the data and their long-run risk model counterparts. The model moments are computed using the analytical closed-form solution presented in part 5. Data on consumption and inflation are annual and covers 1930-2010. Data on U.S. yields are second-month-of-the-quarter observations of quarterly yields from 1969 to 2010.

Table 7: Unconditional Model Statistics

	Model	Data
<b>Nominal Spread Curve:</b>		
$E[1y \text{ spread}]$	2.95	2.91
$E[10y \text{ spread}]$	3.66	4.75
$\sigma(1y \text{ spread})$	3.02	3.86
$\sigma(10y \text{ spread})$	0.50	3.37
$\text{Corr}(1y \text{ spread}, 1y \text{ US yield})$	-0.004	-0.045
$\text{Corr}(10y \text{ spread}, 10y \text{ US yield})$	0.243	0.417
<b>Business Cycle:</b>		
$Pr(\text{default})$	0.84	3.80
$\sigma(c)/\sigma(\text{output})$	1.02	1.1
Debt/output	4.60	16.09
$\text{Corr}(\text{output}, 1y \text{ spread})$	-0.0427	-0.48
$\text{Corr}(\text{output}, 10y \text{ spread})$	-0.1890	-0.68

The tables presents unconditional (average steady-state) model statistics for the nominal spread curve and the business cycle properties of Brazil. Data of nominal spreads are taken from the zero-coupon Bloomberg Fair Value yields for the years 1998-2012. The default probability in the data is calibrated to 3.8, as Brazil defaulted 8 times in the past since year 1800 (longest sample available), at years: 1898, 1902, 1914, 1931, 1937, 1961, 1964, and 1983. Data on Brazil's annual real dollar output and consumption are taken from the World Bank Database for the years 1960-2010. The debt-to-output ratio here is computed as the series DT.DOD.DPPG.CD from the World Bank Database divided by the GNP, as in Chatterjee and Eyigungor (2011).

Table 8: Conditional Spread Curve Statistics

1y spread percentile	Model		Data	
	1y spread	10y spread	1y spread	10y spread
<10	0.02	3.18	0.30	1.78
<25	0.13	3.21	0.63	1.80
<50	0.52	3.33	0.97	2.78
$\geq 50$	5.35	4.01	4.89	6.72
$\geq 75$	7.40	4.21	7.44	8.65
$\geq 90$	8.90	4.69	12.42	11.72

The table contains a comparison between the model and the data on the statistics on the spread curves of Brazil. The data used to construct the table are taken from the zero-coupon Bloomberg Fair Value yields for the years 1998-2012. Columns two and three, as well as columns four and five, present the average level of the short and long spreads, in the model and in the data, respectively, only across the periods in which the short spread is above or below the percentile in the first column.

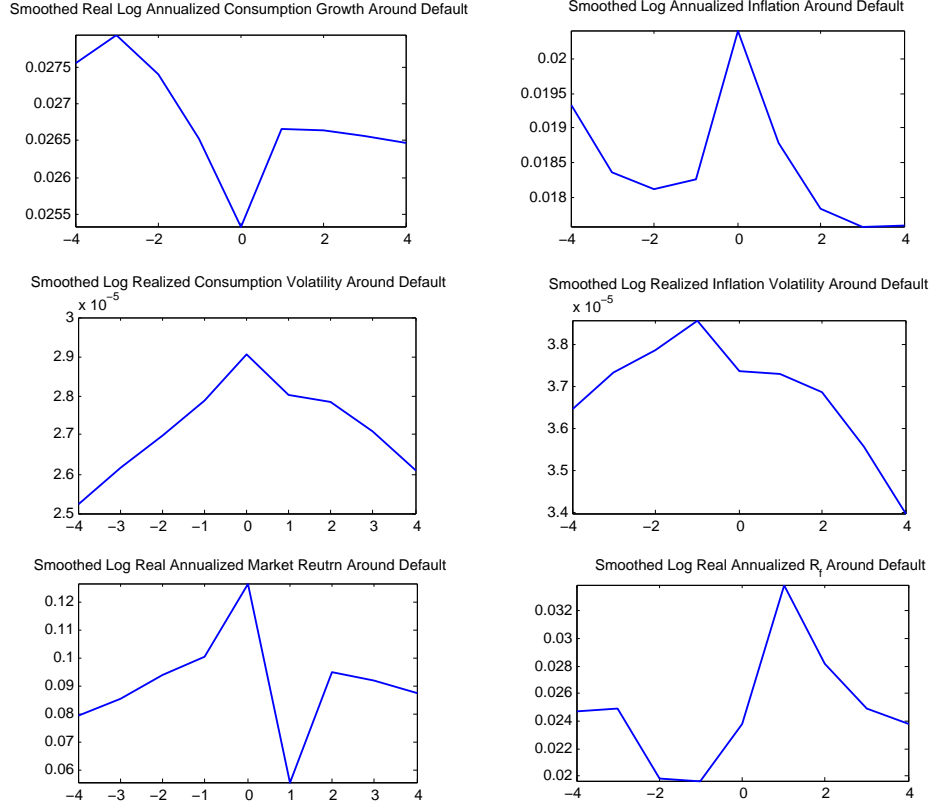
Table 9: Model Implied Predictability of Brazilian Spreads

Dept. Variable	Regressors		$\beta_1$	$\beta_2$	$R^2$
	$X_1$	$X_2$			
$s_{1,t}^{Brazil}$	$y_{1,t}^{US}$	$\sigma_t^2$	0.0062	0.9653	0.06
$s_{1,t+1}^{Brazil}$	$y_{1,t}^{US}$	$\sigma_t^2$	0.0417	0.9594	0.019
$s_{1,t+3}^{Brazil}$	$y_{1,t}^{US}$	$\sigma_t^2$	0.0221	0.8867	0.009
$s_{10,t}^{Brazil}$	$y_{10,t}^{US}$	$\sigma_t^2$	0.0956	0.1434	0.10
$s_{10,t+1}^{Brazil}$	$y_{10,t}^{US}$	$\sigma_t^2$	0.1875	0.2287	0.075
$s_{10,t+3}^{Brazil}$	$y_{10,t}^{US}$	$\sigma_t^2$	0.0919	0.2559	0.022

The table shows in each row the results of regressing in the model, the dependent variable in the first column on the two independent variables in the second and third columns. As in table 2, I denote by  $y_{m,t}^{US}$  the nominal yield on a zero-coupon U.S. bond with maturity  $m$  at time  $t$ . I denote by  $s_{m,t}^{Brazil}$  the spread between the yield on a zero-coupon bond of Brazil, maturing in  $m$  periods, and the corresponding yield on U.S. zero-coupon bond:  $s_{m,t}^{Brazil} = y_{m,t}^{Brazil} - y_{m,t}^{US}$ . Time  $t + 1$  refers to one year ahead. The data used in the regressions are based on simulating the economy for a long-sample. Thus, all coefficients and  $R^2$ 's are population values.

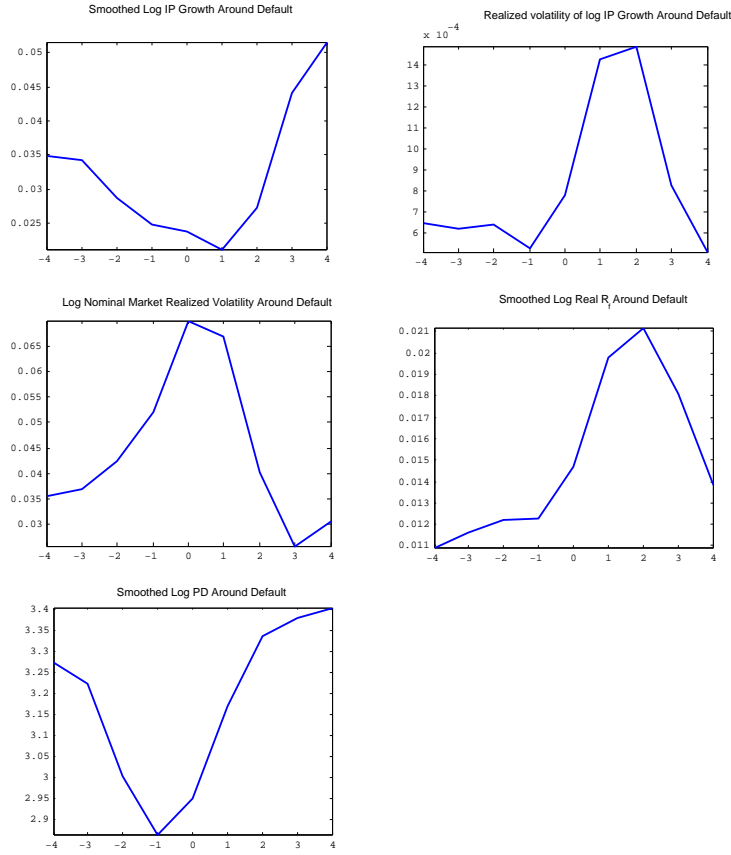


Figure 1: U.S. Fundamentals and Returns Around Times of Defaults - Quarterly Analysis



The figure presents the evolution of selected U.S. variables averaged across periods of defaults in emerging market economies. The default occurrences of the emerging market economies cover 44 countries in the period 1947-Q1 to 2008-Q4, and includes 87 default events. Time 0 refers to the time of default. Time  $+t$  refers to  $t$  quarter after default, while time  $-t$  refers to  $t$  quarters prior to default.

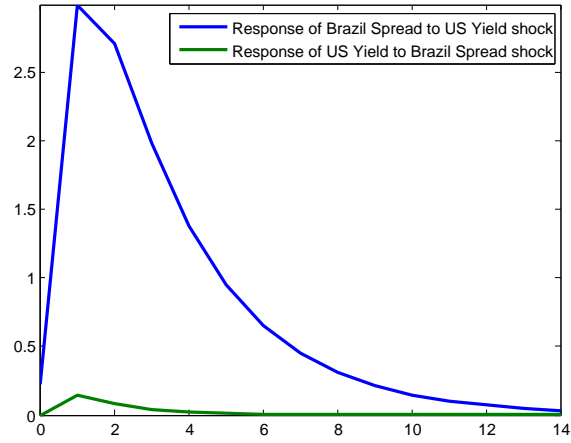
Figure 2: U.S. Fundamentals and Returns Around Times of Defaults - Annual Analysis



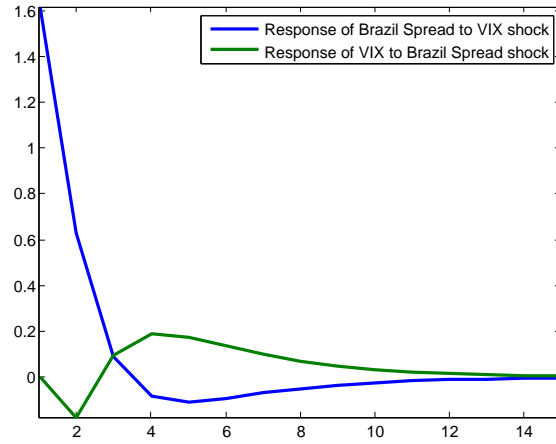
The figure presents the evolution of selected U.S. variables averaged across periods of defaults in emerging market economies. The default occurrences of the emerging market economies cover 44 countries in the period 1930 to 2008, and includes 117 default events. Time 0 refers to the time of default. Time  $+t$  refers to  $t$  years after default, while time  $-t$  refers to  $t$  years prior to default.

Figure 3: Impulse-Response Functions

(a) Brazil spread versus U.S. yield



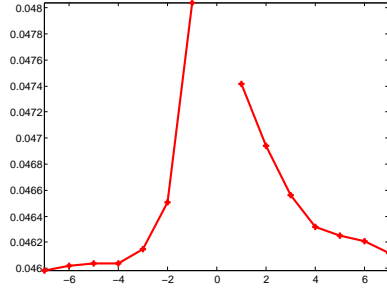
(b) Brazil spread versus U.S. VIX



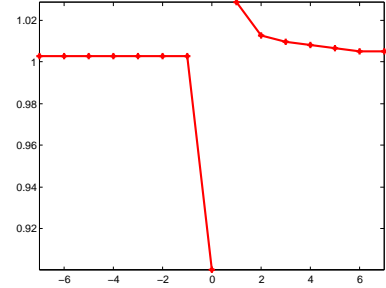
The figures presents imulse resoponse-functions based on the following VAR(1) estimation presented in section 3.3.

Figure 4: Brazil Model Statistics Around Default

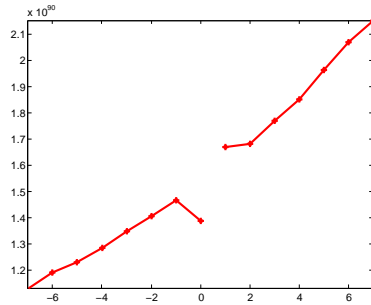
(a) Debt-to-Output ( $a_{t,t+1}/y_t$ )



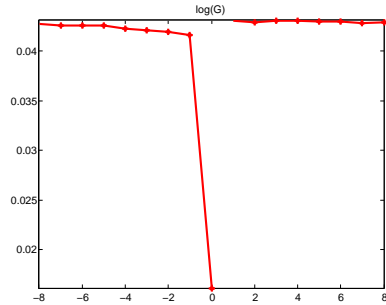
(b) Output Growth



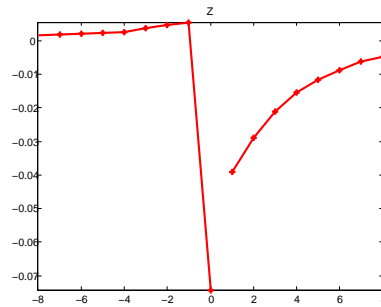
(c) Consumption (untrended)



(d) Trend (G)

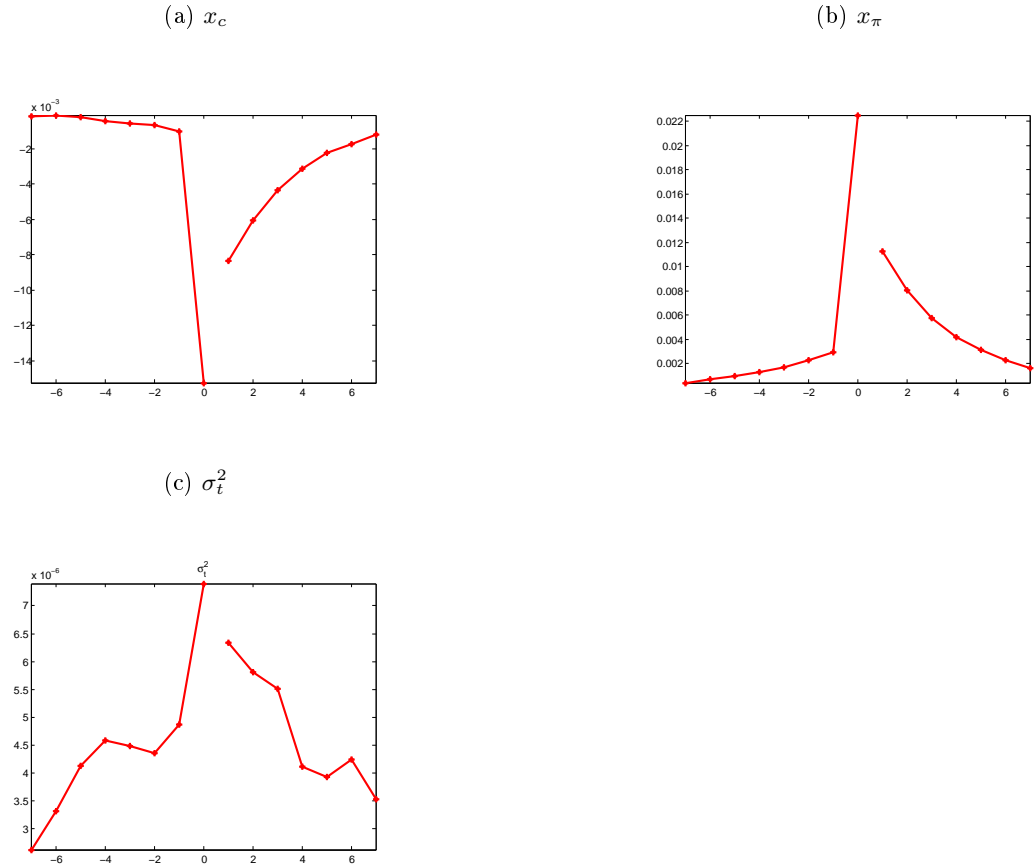


(e) Business Cycle (Z)



The figure shows the dynamics of selected Brazilian variables around times of default in the model. Time 0 refers to the time of default event. Time 1 refers to the first period of re-entry to financial markets. All results are based on simulating the economy 1000 years for 500 times.

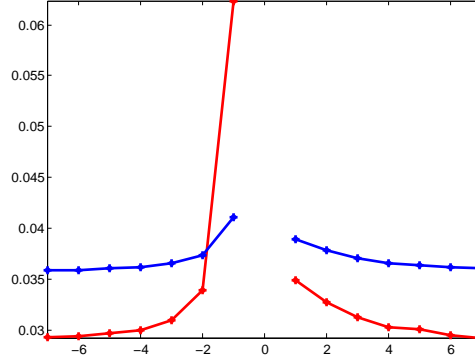
Figure 5: U.S. Model Variables Around Default



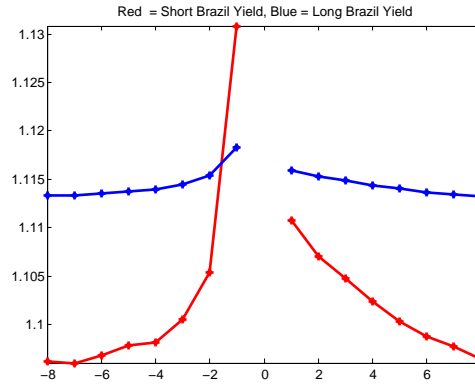
The figure presents the dynamics of selected U.S. variables around times of default in the model. Time 0 refers to the time of default. Time 1 refers to the first period of re-entry to financial markets. All results are based on simulating the economy 1000 years for 500 times.

Figure 6: Yields and Spreads Around Default

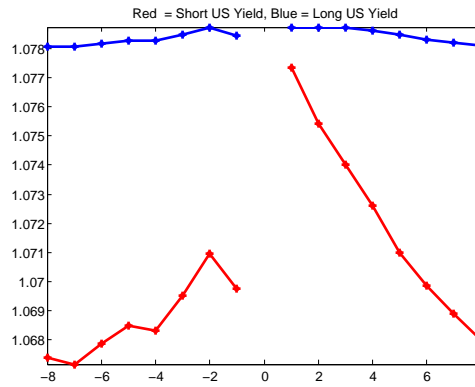
(a) Brazil Spreads (Net): Red - 1y spread, Blue - 10y spread



(b) Brazil Yields (Gross): Red - 1y yield, Blue - 10y yield



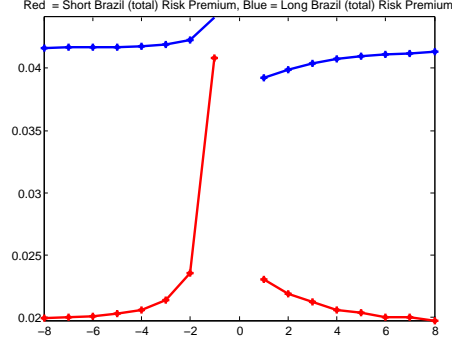
(c) U.S. Yields (Gross): Red - 1y yield, Blue - 10y yield



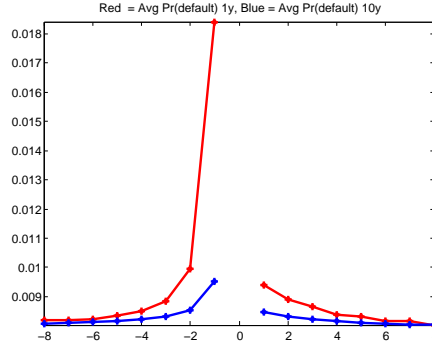
The figure shows the dynamics model-implied yields and spreads around times of default in the model. Time 0 refers to the time of default event. Time 1 refers to the first period of re-entry to financial markets. All results are based on simulating the economy 1000 years for 500 times.

Figure 7: Decomposing Yields Around Times of Default

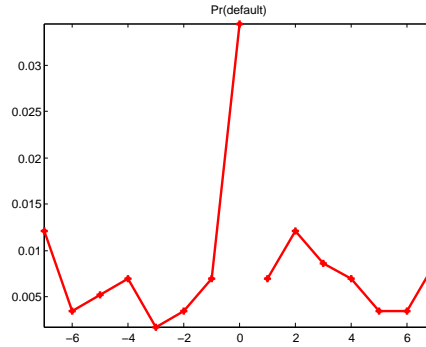
(a) Default Risk Premia to Brazilian Yields: Red - 1y spread, Blue - 10y spread



(b) Average probability of default per-single period: Red - 1y bond, Blue - 10y bond



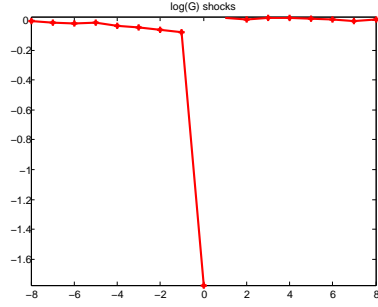
(c) Average probability of default around a shock of 8% to U.S. Volatility



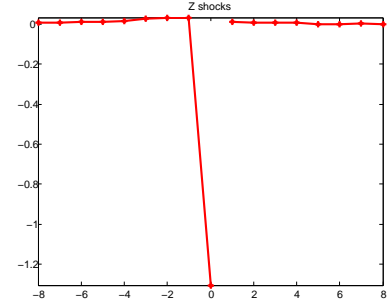
The first two figures shows the dynamics model-implied default risk premia to Brazilian yields and the average probability of default for the short- and long- term bonds around times of default in the model. Time 0 refers to the time of default event. Time 1 refers to the first period of re-entry to financial markets. The last figure shows the average probability of default around times of volatility shift in the model. Time 0 refers to the time of an event in which the U.S. volatility jumps by 8%. All results are based on simulating the economy 1000 years for 500 times.

Figure 8: Shocks to Exogenous Model State Variables Around Default

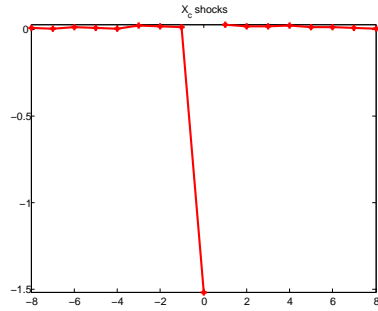
(a) Shocks to Brazil's GDP trend)



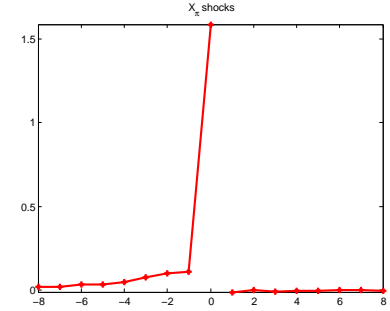
(b) Shocks to Brazil's GDP business cycle)



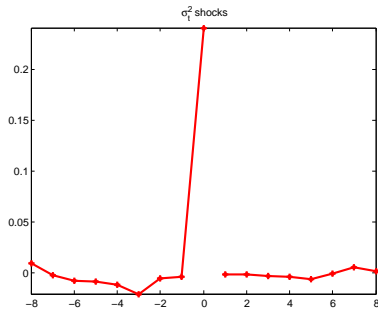
(c) Shocks to U.S. expected consumption Growth



(d) Shocks to U.S. expected consumption Growth



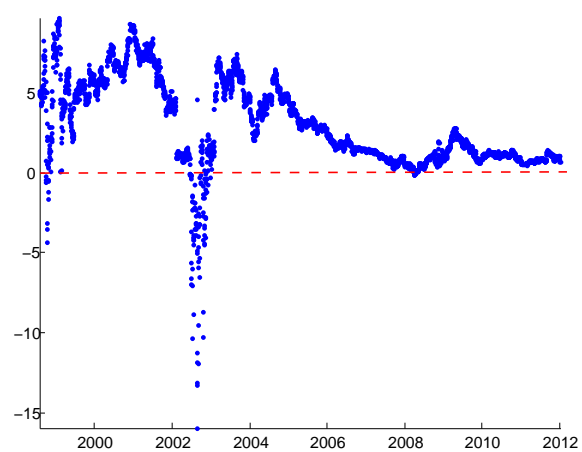
(e) Shocks to U.S. Volatility



The figure shows the dynamics of shocks to the model state variables around times of default in the model. All shocks have a standard Normal distribution. Some of the innovations are correlated (see section 4.3 for details). Time 0 refers to the time of default event. Time 1 refers to the first period of re-entry to financial markets. All results are based on simulating the economy 1000 years for 500 times.



Figure 9: The Slope of Brazil's Spread Curve over Time



The figure presents the slope of Brazil's spread curve over the period 1998-2012, in the data. The slope is defined as the difference between a 10-year spread and a 1-year spread in each period.