

# Uncertainty as Commitment

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## Abstract

When governments lack the ability to commit to ‘no bailouts’ policies, it can create incentives for banks to take excessive risks, potentially generating more crises. We argue that *government uncertainty* about the nature of shocks when facing distressed banks has the potential to relax this moral hazard problem. Under *government uncertainty*, in order to learn more about the shocks and then take the appropriate action, the government may want to delay rescuing banks. This leads to *strategic restraint*, as banks endogenously restrict the riskiness of their portfolio relative to their peers in order to avoid being the worst performers. From the perspective of these novel forces, we analyze how the government can optimally exploit uncertainty to avoid endogenous systemic events.

## 1 Introduction

Few would disagree that bailouts are socially costly. Yet, in the 2008-09 financial crisis, the U.S. government used a variety of instruments to bail out, on unprecedented scale, many financial institutions that were extremely exposed to systemic risk. In fact, government bailouts of banking institutions have been a constant fixture of the economic history of the U.S. dating as far back as 1800s<sup>1</sup>.

An extensive literature, most recently represented by Farhi and Tirole (2012), studies the potential explanation of these phenomena based on moral hazard behavior of banks. Lying at the heart of the argument is the time inconsistency of no-bailout policies of the government.

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<sup>1</sup>In 1857, in the *Livingston vs. Bank of New York* case in which the courts ordered that ‘the mere fact of suspension of specie payments when it is general is not of itself sufficient proof of fraud or injustice’, officially sanctioning suspensions of specie payments in case of an aggregate shock.

Ex-ante, policymakers may prefer to commit no bailouts, such that endogenously banks discipline their own exposure, and in effect no crises occur. However, it is always in the government's best interest to bail out when a systemic event actually happens, and there is not enough liquidity in the system for the market to rescue the banks in distress. Hence, without commitment, banks have no incentive to avoid exposing themselves to events in which all banks have problems at the same time.

Casual observation of the most recent and prior financial crises points to two prevailing features, unmodeled in previous literature. First, when banks start showing trouble, rarely do they do it all simultaneously. Second, policymakers are usually uncertain whether they are facing just an isolated event of distress, which can be solved internally in the financial system through mergers and acquisitions, or a more systemic event, in which valuable projects can be lost. We call this situation *government uncertainty*. For example, when U.S. policymakers decided not to bail out Lehman Brothers on September 14, 2008, for example, allowing the company to file for bankruptcy in the hopes that another company would take over, they were criticized for putting the financial system on the brink of a collapse. However, this decision may have been dictated by uncertainty about the nature of the underlying problem and by the hope that the financial system were able to solve the problem without relying to a costly public intervention.<sup>2</sup> Examples can be drawn from more distant past as well. In the case of the bailout of Continental of Illinois Bank and Trust Company in 1984, the then FDIC chairman William Isaac talked about the 'Best estimates of our staff, *with the sparse numbers we had at hand*, were that more than 2,000 banks might be threatened or brought down by a Continental collapse'<sup>3</sup>, which to us points to the uncertainty surrounding the decision whether or not to bail out the bank, given the informational and time limitations in that situation.

In this paper, we show that *government uncertainty* about the nature of the shock when observing banks in distress has the potential to relax the time inconsistency of policymakers that leads to endogenous systemic events. Intuitively, in order to learn about the nature of the underlying shock – which has the benefit of avoiding a potentially costly and unnecessary intervention – the government may want to *delay bailout* and let the first bank(s) in distress fail. That makes the relative performance of banks' portfolios crucial for individual banks – neither wants to be first in line for government help – making leverage choices strategic substitutes. We call this effect *strategic restraint*, as banks endogenously restrict the riskiness

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<sup>2</sup>For a thorough discussion of the timing of events and evidence on the government announcements of the possibility of bailout during the most recent crisis, see Kelly, Lustig, and Van Nieuwerburgh (2011).

<sup>3</sup>Emphasis added by us.

of their portfolio relative to their peers in order to avoid being the worst performers.

We build a theoretical model in order to formally investigate the role of *government uncertainty* and *strategic restraint* in an economy without government commitment. To this end, we build on the model of Farhi and Tirole (2012) by introducing both idiosyncratic and aggregate risks and a natural sequentiality in banks showing signs of trouble. Instead of focusing on the coordination that generates collective moral hazard, as in their paper, we focus on the substitutability that uncertainty introduces and that has the potential to eliminate any form of moral hazard, regardless of its collective nature. In the benchmark model, banks borrow short-term to finance illiquid projects with uncertain timing of payoffs. Either aggregate or idiosyncratic shocks may hit banks, in which case they need refinancing to continue the projects. High levels of short term debt allow banks to invest in large projects, but at the same time hinder their ability to refinance the project when a shock hits.

A central authority, which we call the *government*, maximizes total welfare and has, as an instrument, an interest rate policy that affects the cost of all banks' refinancing. The costs of an intervention, which we call a *bailout*, are given by the costs of an implicit transfer of surplus from consumers to bankers. The benefits of bailout are given by bringing banks' projects to fruition at full scale, increasing output. The government only observes whether a bank that needs refinancing doesn't have enough liquidity to do so, at the moment that the bank runs out of refinancing channels available in the market. It doesn't, however, observe whether the situation was caused by an idiosyncratic or an aggregate event (*government uncertainty*). If the shock is idiosyncratic, other banks have enough liquidity to take over the distressed bank, and no intervention is needed. If the shock is aggregate, no bailout implies the projects are lost.

The central result we derive is identifying the conditions under which *government uncertainty* leads to time-consistent outcomes under no commitment. In the model, under sufficient *government uncertainty*, the government will not bail out the first distressed bank it sees, which introduces competition among banks not to be the worst performing bank. This Bertrand-type competition for the relative position, which we call *strategic restraint*, has the potential to eliminate the banks' moral hazard. In such case, the only outcome that can be sustained as an equilibrium is the one which is ex-ante optimal under full commitment, but here it is instead due to delay of bailout triggering *strategic restraint*.

Given this result, questions arise as to the effect of regulation and other features of the environment on the nature of the government uncertainty and strategic restraint forces. What is the effect of securitization on the equilibrium of the economy? How do the results depend on the relative sizes of banks and the number of banks in the market? What about

the possibility of contagion? We address these questions in our setup.

In our first set of results, we study the role of securitization in making bailouts time inconsistent. In principle, securitization allows for diversification that eliminates idiosyncratic shocks, which lessens government uncertainty. We show that a stronger result holds in our benchmark model: any level of securitization introduces a difference in the time of distress of banks facing idiosyncratic shocks and banks facing aggregate shocks, since in the former case has more cash due to securitization. Through that difference in the timing of distress, the government can infer the type of shock that hits the economy, and hence there is no government uncertainty for *any* level of securitization.

This is certainly an extreme result, and is driven by the fact that bank *leverage ratios* are perfectly observable by the government and perfectly map into time of distress. In an extended setup, we consider additional, i.i.d. shocks to the asset position of the bank, which hit after the refinancing shocks and affect banks' cash positions and their capacity to continue the projects. Since the only observable to the government is the size of the bank's project at the time of distress, these shocks introduce back government uncertainty in case of observing a bank in distress. In the extended setup we show that there is an optimal, nonzero cap on the level of securitization which preserves government uncertainty and strategic restraint forces and leads to ex-ante optimal outcomes. This cap on securitization depends on the variance of the asset position shocks; a larger variance allows for more securitization without inducing systemic events since it becomes harder for the government to infer the state from the timing of distress by banks, which renders securitization less powerful in reducing government uncertainty.

These results point to an unnoticed effect of securitization – it hinders the benefits of government uncertainty and strategic restraint, making potential endogenous systemic events more likely. This suggests a new rationale for a cap on securitization which is implied by our extended framework.

In our second set of results, we study the effect of bank size heterogeneity on time-inconsistency. We find that the bigger the relative size of a bank, the lower is the probability that it can be acquired by any other bank in case of distress, and hence the higher the chance that it will be bailed out, even if it is the first bank in distress. Hence, asymmetrically large banks will take excessive risks, while other, smaller banks, will expose themselves slightly less than the large bank. In this case, the 'too big to fail' problem shows up in our setting in a different way than in the literature, since large banks become a shield for smaller banks to coordinate and to take excessive risk. Large banks generate a negative externality in the economy by inducing excessive leverage by small banks, and a potential systemic event.

The third set of results explores the role of industry concentration from the perspective of our novel forces. In particular, we show in a simplified version of our setup that the incentives to bail out the first bank in distress are decreasing with the number of banks in the industry. In the full setup, we show that there is a cutoff number of banks above which delay of bailout is always optimal.

In our last set of results, we allow the possibility of contagion and study its effects on intervention delays. We show that the possibility of contagion reduces the importance of government uncertainty and hence makes attaining time-consistent outcomes less likely.

**Related Literature** Our paper contributes to the literature on time-inconsistency of government policies that target the financial sector, and is most closely related to Acharya and Yorulmazer (2007), Pasten (2011) and especially Farhi and Tirole (2012). These papers stress the time-inconsistency of policymakers, which generate collective moral hazard. It is well understood that the policy of not bailing out a ‘too big to fail’ bank is time-inconsistent – even if the government announces a no-bailout policy, after the bank indeed shows trouble, it is in the best interest of the government to bail it out<sup>4</sup>. This literature argues that even in the absence of ‘too big to fail’ banks, banks can coordinate their positions because, even when it is time-consistent not bailing out a single bank, it may still be time-inconsistent not bailing out the whole financial sector. Our work argues that the difficulty of the government in distinguishing between idiosyncratic and systemic problems when banks start showing distress may introduce a strategic force that counteracts the coordination required for a collective moral hazard to arise.

Farhi and Tirole (2012) exploit a modification of the setting of Holmström and Tirole (1998) to show that private leverage choices are strategic complements – when all banks engage in maturity mismatch, authorities have little choice but to intervene, making it profitable for a single bank to adopt a risky balance sheet, correlating their risk exposures. In this setting, we take this reasoning to the extreme and allows for banks being willing to take an excessive exposure regardless of the behavior of other banks, and purely motivated by lack of commitment. We additionally allow for takeover of one bank by another, and introduce the possibility of idiosyncratic shocks and imperfect observability of the type of shock by the government at the time of deciding whether to bail out or not the first bank showing problems. Delay by the government makes banks’ choices *strategic substitutes*, providing a natural force for restraining leverage choices. This result highlights the difficulty of sustaining coordination and correlation of risk exposures.

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<sup>4</sup>See an extensive discussion in Stern and Feldman (2004).

Acharya and Yorulmazer (2007) develop a model of ‘too-many to fail’ in an environment in which bank takeover is possible, as well as technologically inferior government takeover of banks. When many banks fail, the government finds it ex-post optimal to bail out some or all failed banks. When few banks fail, failed banks can be acquired by the successful banks. This gives banks incentives to herd and increases the risk that many banks may fail together. We also allow for healthy banks to take over distressed banks, and assume that the government is as bad as households at running banks. In our model, the ‘wait and see’ strategy of the government has the additional gain of providing information to the government about the nature of the failure, which creates *strategic restraint* and destroys the possibility of herding.

Our work is also related to the recent paper of Cukierman and Izhakian (2011), who study an environment with model uncertainty and max-min behavior of investors, and show that uncertainty about policymakers’s actions can induce sudden financial collapses. In our case, the government is confused about the state of the economy, which generates uncertainty about its actions after the first bank gets into trouble.

In another related strand of the literature, Green (2010) and Keister (2011) argue that bailouts may be optimal, because in the absence of bailouts the economy may hoard excessive liquidity. In a similar vein, Cheng and Milbradt (2010) show that bailouts can instill confidence on credit markets. While these papers characterize the ex-ante optimal level of liquidity that policymakers would like to attain if having commitment, ours shows that government confusion can implement those allocations even in the absence of commitment.

Some papers, such as Allen and Gale (2000), raise the issue of contagion (as opposed to correlation) as the main driving force pushing the government towards bailouts. Dell’Ariccia and Ratnovski (2011), for example, show that, when a bank’s success depends on both its idiosyncratic risk and the overall stability of the banking system, bailouts insure banks from contagion and reduce their incentives to take risk. In our work the possibility of contagion across banks reduces the effectiveness of uncertainty in relaxing time-inconsistency since it makes the ‘wait and see’ strategy more costly. Hence, when contagion is possible, coordination and systemic problems are more likely.

In what follows, we first set up a reduced form analytical example in order to illustrate the main forces behind our arguments. Then, we provide a micro-founded model of these forces, based on Farhi and Tirole (2012). We analyze the model under full information, as a frictionless benchmark for our results, and then proceed to the imperfect information environment and our main result. Then, we discuss policy implications.

## 2 Simple Example

Consider an economy with two banks,  $N = 2$  which are endowed with projects of size 1, and choose costly preventive cash holdings  $c$ . If the project is lucky, it pays  $Y > 1$  at time  $t = 1$ , and if it is unlucky, it needs additional cash to continue and pays  $cY$  in period 2. The expected payoff for the bank is  $cY - c$  if the project needs refinancing and  $Y - c$  if it is successful. Without loss of generality, we restrict  $c \in [0, 1]$ . This implies the firm always loses  $c$ , but it is useful to save a fraction  $c$  of the project in case of refinancing.

The project may need additional refinancing for two reasons: (i) an aggregate shock that affects *both* banks and happens with probability  $(1 - \alpha)$  or (ii) an idiosyncratic shock that affects a single bank, with iid probability  $(1 - \lambda)$ . This implies both projects need refinancing with probability  $P_2 = 1 - \alpha + \alpha(1 - \lambda)^2$ , one project needs refinancing with probability  $2P_1 = 2\alpha\lambda(1 - \lambda)$ , and no projects need refinancing with probability  $P_0 = \alpha\lambda^2$ .

In case the project(s) need refinancing, the government may decide to bail out banks if they don't have enough cash to continue full scale (i.e.  $c < 1$ ). The cost of the bailout is proportional to the size of the transfer,  $(1 - c)$ , and is equal to  $(1 - c)T$ ,  $T > 1$ . Importantly, we assume that the bailout is *undirected*, i.e. once offered, all banks can use the transfer. The banks with the healthy projects gain consumption  $(1 - c)$  from a transfer of  $(1 - c)T$ , so it is a social loss ( $T > 1$ ). This will become important when the government cannot distinguish an aggregate shock from an idiosyncratic shock.

The timing of events is as follows. First, the government announces a policy (bailout or no bailout), contingent on the realized state of the economy. Then, banks choose cash  $c$ . At  $t = 1$ , nature moves and the exogenous state of the economy is realized. Under full information, all agents observe the state of the economy simultaneously. After that, under non-commitment, the government has a chance to change the policy. Under commitment, payoffs are realized following the initial announcement.

Under imperfect information, the government doesn't observe the realized state, but only observes whether a bank is unable to refinance the full size of the project (*bank distress*), i.e. it can only observe market outcomes but not the underlying state. If both banks need refinancing, the government observes banks' distress in a sequence determined by their cash holdings – those with lower cash show up earlier. In case of a tie, a coin is flipped to determine the order. After each such signal, the government can choose to enact a bailout. Once a bailout is in place, however, it cannot be reversed.

## 2.1 Full Information without Commitment

Under no commitment, the government can change its announcement after observing the actual state. Hence we solve the problem by backward induction, as the initial announcement becomes effectively irrelevant.

Note first that the government never wants to provide a bail out if only one bank is in trouble. This is because the healthy bank's payoff is  $Y$  at  $t = 1$  and hence it has the cash to take over the rest of the distressed bank's project. We assume it makes a take-it-or-leave-it offer: it pays  $(1 - c)$  to reap the full benefit equal to  $(1 - c)Y$ , which is always profitable for  $Y > 1$ . For  $T > 1$  it is also socially optimal, and so under full information, there is no intervention under idiosyncratic shocks.

When both banks need refinancing, the no-bailout social payoff is  $2(cY - c)$  and the social payoff of a bailout is  $2(Y - c - (1 - c)T)$ . The government trades off an additional cost of  $(1 - c)T$  for the extra payoff equal to  $(1 - c)Y$ . Hence, under no commitment, the government always bails out banks if

**Assumption A** *Saving projects is socially optimal:  $Y > T$ .*

What will the banks do, knowing that? The bank's ex-ante payoff from choosing  $c$  under bailout (only when both fail) when the other bank is choosing  $c'$  is

$$V(c) = P_0(Y - c) + P_1(Y - c + (1 - c')(Y - 1)) + P_1c(Y - 1) + P_2(Y - c),$$

with derivative equal to

$$-P_0 - P_1 + P_1(Y - 1) - P_2 = P_1Y - 1.$$

The above condition trades off the marginal benefit of increasing reinvestment,  $Y$ , with the cost of carrying cash, 1. The benefit happens to be useful when the bank is about to be taken over, which happens with probability  $P_1$ , but the cost is incurred always. We make the following assumption

**Assumption B** *Without commitment, banks prefer not to hold cash:  $P_1Y < 1$ .*

This assumption guarantees that under no commitment the unique equilibrium is zero cash holdings by the banks and a bailout in the aggregate state.

## 2.2 Full Information with Commitment

In this section, we assume the government has a commitment device which makes the initial announcement of policy binding and credible. As argued above, the government never wants to announce a bailout in the state when only one bank fails. Hence, we will consider two commitment policies: (i) bailout in the state when both banks fail, and (ii) no bailouts ever.

When the government commits to bailing out bank in case of an aggregate shock, the optimal response of banks is  $c = 0$ , since it is effectively as the case without commitment, and ex-ante welfare is

$$W^{ea} = 2\{P_0Y + P_1(2Y - 1) + P_2(Y - T)\}.$$

When the government commits not bailing out banks ever, the value for a bank is

$$\widehat{V}(c) = P_0(Y - c) + P_1(Y - c + (1 - c')(Y - 1)) + P_1c(Y - 1) + P_2(cY - c),$$

with derivative

$$-P_0 - P_1 + (P_1 + P_2)(Y - 1) = (P_1 + P_2)Y - 1.$$

This condition trades off the marginal benefit of increasing reinvestment with the marginal cost. Reinvestment needs also arise in the case of an aggregate shock. Then

**Assumption C** *With commitment to no bailouts, banks prefer to hold cash:  $(P_1 + P_2)Y > 1$ .*

Under Assumption C, the optimal response of banks to a government's commitment of not bailing out is to choose cash holdings  $c = 1$ . hence, the ex-ante welfare of committing to a non-bailout policy is

$$\widehat{W}^{ea} = 2(Y - 1).$$

The optimal commitment strategy of the government prescribes commitment to no bailouts when the following assumption holds

**Assumption D** *It is ex-ante optimal to commit to no bailout:  $P_1 + P_2T > 1$ .*

Assumption D weighs the costs of providing refinancing under the two regimes, such that  $\widehat{W}^{ea} < W^{ea}$ . The cost of committing to no bailouts is equal to the social cost that each bank carries a unit of cash in all states. The cost of committing to bailouts is twofold. On the one hand, there is a social cost of a unit of cash to save a single project, which happens with probability  $P_1$ . On the other hand, it costs  $T > 1$  per bank to society to save projects in case of an aggregate shock.

Lemma 1 summarizes the results of this section.

**Lemma 1** *Under Assumptions A - D, and full information:*

(i) *The unique equilibrium without commitment is  $c = 0$  and bailouts in the presence of an aggregate shock.*

(ii) *The unique equilibrium with commitment is  $c = 1$  and no bailouts ever.*

## 2.3 Imperfect Information

Below, we relax the assumption of full information. The government only observes whether a given bank is in distress (is not able to continue operations if it does not obtain new cash). Banks show distress in sequence, with the bank with the lowest cash showing distress first.

When the government observes the first bank in distress, it does not know whether the second bank will also be in distress (hence both need refinancing) or not (in which the second bank has enough resources to recover the project of the first bank at a lower social cost). In particular, after one bank shows distress, the government updates the probability that both banks need refinancing to

$$P'_2 = \frac{P_2}{P_1 + P_2} \equiv \frac{P_2}{1 - \alpha\lambda} > P_2. \quad (1)$$

Intuitively, upon observing a bank in distress, the government updates up the probability of an aggregate shock, such that  $\alpha' = \frac{\alpha - \alpha\lambda}{1 - \alpha\lambda} < \alpha$ , and then  $\gamma = 1 - \alpha' + \alpha'(1 - \lambda) = \frac{P_2}{1 - \alpha\lambda}$ .

In case both banks are in distress, the welfare from providing a bailout is

$$2[Y - c - (1 - c)T]$$

and the welfare from *delaying* action, down scaling the first bank's project but optimally recovering the second bank's project is,

$$cY - c + Y - c - (1 - c)T.$$

If the first bank is in distress is not bailed out, a second bank in distress sends a precise signal to the government of the presence of an aggregate shock, such that  $P''_2 = 1$ . Given Assumption A it is naturally optimal in this case for the government to bail out the second bank.

In case only the individual bank showing distress is in need of refinancing, the welfare from providing a bailout is

$$Y - c - (1 - c)T + Y - c + (1 - c)(1 - T),$$

where the last term is the unnecessary utility transfer to the healthy bank. The welfare from delaying action is

$$Y - c - (1 - c) + Y - c.$$

Given these payoffs and the posterior probability of an aggregate shock, the expected gain from learning through delay is equal to the difference in expected welfare from no bailout and bailout, and is given by

$$2(1 - P'_2)(T - 1)(1 - c) - P'_2(Y - T)(1 - c). \quad (2)$$

The second part of the equation is the cost of delay, which is losing the distressed project of the first bank in case of an aggregate shock. The first part is the cost of giving unnecessary transfers to both banks in case bailout of an idiosyncratic shock, and is given by two parts, (i) the cost of providing an unnecessary bailout to the bank in distress, which implies using a technology that transforms  $T$  into  $Y$  instead of a takeover, which transforms 1 into  $Y$ ; the difference is equal to  $(T - 1)(1 - c)$  and (ii) the cost of providing an unnecessary utility transfer to the healthy bank, equal to  $(T - 1)(1 - c)$ .

Hence, this expression captures the static and dynamic net gains of delay. The static part is not making a mistake with the first distressed bank. The dynamic part is equal to the value of learning the true state of the world and not providing welfare reducing transfer to the healthy bank unnecessarily.

The *delayed bailout condition* for the government therefore is

$$2(1 - P'_2)(T - 1) - P'_2(Y - T) > 0. \quad (3)$$

Under equation (3), the government will delay the bailout to learn the true state of the world, and therefore the first bank in distress will always be taken over or fail. Condition (3) crucially depends on the probability  $P'_2$  that the government assigns to the aggregate shock. If  $P'_2 = 1$ , i.e. the government is sure that the shock is aggregate, then (3) is never satisfied and the government always bails out the first bank. If, on the other hand,  $P'_2 = 0$ , the government is certain that the shock is idiosyncratic and will never bail out the first bank.

In general, there is a cutoff  $\bar{P}$ , such that for  $P'_2 < \bar{P}$ , the government always decides to learn the true state through delay and does not bail out the first bank in trouble. This cutoff

is the updated belief  $P_2'$  that satisfies equation (3) with equality, such that

$$\bar{P} = \frac{1}{1 + \frac{(Y-T)}{2(T-1)}} \quad (4)$$

When  $P_2' < \bar{P}$ , there is a discontinuous difference in terms of a bank's payoff from being the first versus being the second bank in distress in the presence of an aggregate shock. This discontinuous difference arises from the difference from not being versus being bailed out. If the sequence of showing distress depends on the cash holdings of the bank, the value for Bank 1 of choosing the same cash holding as Bank 2,  $c = c'$  is

$$V(c = c'|c') = P_0(Y - c) + P_1(Y - c + (1 - c')(Y - 1)) + P_1c(Y - 1) + P_2(1/2(Y - c) + 1/2(cY - c))$$

and the value of deviating and choosing  $\hat{c} = c' + \varepsilon$  is

$$V(\hat{c} = c' + \varepsilon|c') = P_0(Y - \hat{c}) + P_1(Y - \hat{c} + (1 - c')(Y - 1)) + P_1\hat{c}(Y - 1) + P_2(Y - \hat{c})$$

The *strategic restraint* condition says that the gain from the deviation is positive

$$\Delta V = P_2Y \frac{(1 - c)}{2} - \varepsilon > 0. \quad (5)$$

It is strictly satisfied for small enough deviation ( $\varepsilon \rightarrow 0$ ) for any  $c < 1$  and equal to zero at  $c = 1$ . The results of this section are summarized in Lemma 2

**Lemma 2** *Under Assumptions A - D, no commitment and imperfect information:*

(i) *If  $P_2 > (1 - \alpha\lambda)\bar{P}$ , the unique equilibrium is  $c = 0$  and bailouts in the presence of both aggregate and idiosyncratic shocks.*

(ii) *If  $P_2 \leq (1 - \alpha\lambda)\bar{P}$ , the unique equilibrium is  $c = 1$  and no bailouts ever.*

In what follows, we provide a micro-founded model of the forces studied in our analytical example. We embed the imperfect observability of the state by the government in a standard model of outside liquidity demand, building on Farhi and Tirole (2012) and Holmström and Tirole (1998).

### 3 Benchmark with Full Information

The model environment is an extension of Farhi and Tirole (2012), with several important modifications. First, we introduce two types of shocks, aggregate and idiosyncratic, and

allow for imperfect information about the nature of the shock. Second, we allow for a non-degenerate timing of events, in which banks which have higher leverage ratios show distress earlier (endogenized later). Third, we admit the possibility of bank takeover by other banks.

This section extends the previous simpler case by introducing micro foundations for borrowing, leverage and the cost of bailout through non targeted interest rates.

The specifics of the economy are as follows. Time is continuous and finite,  $t \in [0, 2]$ . Shocks in the economy hit only at date  $t = 1$ , and for the rest of time the economy is deterministic. There are three types of agents in the economy: two banks, a continuum of households and a government. Banks (banking entrepreneurs) borrow short-term to finance illiquid projects which either pay off at  $t = 1$  or need refinancing and pay off at  $t = 2$ . A bank's project needs refinancing because of an aggregate shock (both banks need refinancing) or an idiosyncratic shock (only one of the two banks needs refinancing). Households in the model are risk neutral providers of loans to banks. There is no discounting. The government maximizes total welfare, using the interest rate policy and taxes on households as the only instruments.

### 3.1 Banks

There are  $N = 2$  banking entrepreneurs (banks hereafter) in the economy, whose objective is to maximize their net worth. At date 0, they choose to finance an investment project, financed by their own initial assets  $A$  and funds borrowed from the households. Financing the project of size  $i$  implies a constant outflow of  $idt$  during the project. The payoff from the investment project consists of two parts. First, there is a deterministic payoff  $\pi i$  at date 1, independent of the state. The second part is uncertain: either it returns amount  $(\rho_0 + \rho_1)i$  at time  $t = 1$ , or it requires additional financing to keep it running, and then returns amount  $(\rho_0 + \rho_1)j$  at time  $t = 2$ , where  $j$  is the amount refinanced. A project requires additional financing either because an aggregate shock hit the economy ('crisis state'), which happens with probability  $(1 - \alpha)$ , or an idiosyncratic shock hit a particular bank, which happens with probability  $1 - \lambda$ , and is independent of the realization of the aggregate shock.

Bank borrowing is non-contingent<sup>5</sup>. In particular, at  $t = 0$  banks promise to repay amount  $b$  per unit of investment independently of the realized state at date 1. Under limited liability and risk-neutrality and competitiveness of the lenders (households), the date-0 borrowing,  $i - A$ , has to be equal to the repayment,  $bi$ , which determines the size of the initial

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<sup>5</sup>This assumption does not affect any of the results. If bank borrowing only has to be repaid in the good state, the optimal level of investment will increase, but the liquidity choice considerations will remain.

investment to be  $i = A/(1 - b)$ . The cash available at time  $t = 1$  for reinvestment purposes is equal to  $c = (\pi - b)$  per unit of investment  $i$ . By the non-contingent nature of the promise  $b$ , it has to be true that  $b \leq \pi$ .

Here, as in our analytical example, the reinvestment scale is going to depend on the amount of cash carried at  $t = 1$ , which in turn solely depends on the initial leverage choice of the bank,  $i/A$ . In particular, at  $t = 1$ , a project either pays off in full, in which case no action is taken by the bank, or may need additional financing and there is a second round of borrowing. The bank enters the period with cash holdings  $ci = (\pi - b)i$ , and levers this amount in order to finance additional investment  $j$  in the project, with the restriction that the reinvestment cannot increase the size of the project, i.e.  $j \leq i$ . The second period payoff in this case is  $(\rho_0 + \rho_1)j$ , of which, crucially,  $\rho_0 j$  is pledgeable to the lenders. If the required market rate of return on bank lending is  $R$ , then the pledgeability condition introduces the following condition on maximal bank lending in period  $t = 1$

$$R(j - ci) = \rho_0 j, \quad \text{which implies} \quad j = \min \left\{ \frac{c}{1 - \rho_0/R}, 1 \right\} i. \quad (6)$$

The following assumptions guarantee that our problem is economically well defined. First, Assumption 1 guarantees that investment is finite:

**Assumption 1** *Finite investment:*  $\pi < 1$ .

Without this assumption, the deterministic part of the payoff would be enough to attract any amount of investment. Second, Assumption 2 guarantees that banks need to borrow to continue the project at full scale in case of refinancing needs (first inequality), and that banks invest all of their cash in the project in period  $t = 1$  since refinancing guarantees positive profits (second inequality).

**Assumption 2** *Binding pledgeability and profitable refinancing:*  $\rho_0 < 1$  and  $\rho_0 + \rho_1 > 1$ .

Finally, we assume that maintaining projects to completion is ex-ante socially optimal, using definitions of probabilities from the previous section.

**Assumption 3** *Projects are socially beneficial:*  $\underbrace{\pi + \rho_0 + \rho_1}_{\text{social payoff}} - \underbrace{(1 + P_1 + P_2)}_{\text{expected resource cost}} > 0$ .

Finally, projects that have been successful at  $t = 1$  have an alternative use between  $t = 1$  and  $t = 2$ . It is possible to rerun those projects at the original scale  $i$  to generate a

deterministic payoff to the bank of  $\hat{\rho}$  per unit of investment at  $t = 2$ , with  $\rho_0 < \hat{\rho} < 1$ . If a bank rerun a successful project at the moment  $t > 1$ , then the maximum scale available at that time is just  $(2 - t)\hat{i}$ . This implies that rerunning projects is inefficient (they require a unit of investment for a return  $\hat{\rho} < 1$ ). Furthermore banks are not willing to rerun projects unless the cost of doing it is less than  $\hat{\rho}$ .

## 3.2 Households

Households who are born at  $t \in \{0, 1\}$  consume at  $t + 1$  and are risk neutral, with utility given by  $U_t = x_{t+1}$ . They are endowed with assets  $S_t$  when born, allocate their savings between cash (storage), bank lending and government bonds, all which yield a return of 1 (pinned by return to storage). The return on their savings is consumed in period  $t + 1$ . Since the generation born at date 0 experiences no shock, their date-1 consumption is just equal to  $S_0$ .

The date-2 consumption for the generation born at  $t = 1$  is equal to the return on cash holdings,  $S_1$ , and government taxes,  $X$ , which will serve to finance potential bailouts. That is,  $x_2 = S_1 + T$ , where  $T = \rho_0 j - (j - ci)$  is the tax used to finance below-market rate borrowing to banks. It is the difference between  $\rho_0 j$ , the maximal return that banks can promise on the project (due to limited pledgeability) and the amount borrowed,  $j - ci$ .

## 3.3 Government

The government is benevolent and maximizes welfare, which is equal to the weighted sum of the bank and household surpluses. The policy instrument at the government disposal is an interest rate policy which determines the cost of borrowing to the banks, i.e.  $R(t)$ ,  $t \in [0, 2]$ , and taxes on households. Without loss of generality<sup>6</sup>, we will assume that the policy interest rate takes only two values: (i) a no intervention *market rate* of 1 and (ii) a *bailout rate* of  $R = \rho_0$ . At the bailout rate, banks are able to finance any amount of reinvestment even with zero cash holdings, because the policy rate is exactly equal to the pledgeable amount  $\rho_0$ .

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<sup>6</sup>This restriction gives us a natural way of modeling banks' distress - it is when they run out of money (i.e. cash holdings go to zero), and cannot continue the project unless intervention or takeover take place. For a more general set of policies (i.e.  $R > \rho_0$ ), intervention would have to take place earlier, and for some strictly positive level of cash holdings of a bank. In a more general environment, a number of forces can break the government's indifference between different ways of generating the same average interest rate towards backloading intervention. For example, if there is a chance of a stochastic shock that nullifies the bank's distress, it would generate a strictly positive option value of waiting until the last possible moment before intervention. We do not explicitly incorporate these forces here, but we view our restriction on policies as motivated by such considerations.

As in the motivating analytical example of Section 2, the temptation of introducing the bailout interest rate is going to be operational only in the state when both banks need refinancing, since in the case only one bank does, the other bank can take it over and it is optimal for the government to set  $R = 1$ . Denote the belief of the government<sup>7</sup> that both banks need refinancing as  $p$ . Then, the decision of the government is going to be a binary one: whether or not to introduce the bailout interest rate, given belief that both banks are in need of refinancing  $p$  and after observing at least one bank running out of cash at time  $t$  (in case of asymmetric cash holdings, the other bank will follow suit). We summarize it by a function  $\Pi(t, p)$ ,  $t \in [1, 2]$  which takes values in  $\{0, 1\}$ .

For the purposes of the banks' optimization, it is going to be crucial what is the *earliest* time that the government is willing to introduce a bailout under aggregate state.

**Definition 1** *The **earliest bailout time**  $t^*$  is the minimum time of government bailout when the probability that both banks need refinancing is  $p$  and the government observes a bank with zero funds, i.e.*

$$t_p^* = \min\{t | \Pi(t, p) = 1\}. \quad (7)$$

When it does not generate confusion, we just call  $t^*$  the case of  $t_1^*$ , when there is full information about the aggregate state happening. We are restricting the set of government policies  $\Pi$  to ones that guarantee that  $t^*$  is well defined.

We will refer to a situation when the bailout interest rates are introduced as a *government bailout*. The costs of a government bailout are the taxes needed to finance below-market lending, which introduce a reallocation of resources from households to bankers. Additionally, to capture the costs of using an untargeted bailout instrument, we assume that the healthy bank in the case of an idiosyncratic shock can borrow at the same bailout interest  $R = \rho_0$ . Why would healthy bank like to borrow? Because healthy banks can borrow at  $\rho_0$  and invest in an additional socially inefficient project that produces  $\hat{\rho} > \rho_0$  per unit of investment. Hence, a cost of bailing out a bank in the presence of an idiosyncratic shock is that not only that it prevents successful banks from taking over, but also induces them to invest in socially inefficient projects.

### 3.4 Timing

The timing of events is as follows:

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<sup>7</sup>Clearly it is going to be nondegenerate only under imperfect information.

At  $t = 0$ , the government announces a policy of bailout or no bailout  $\Pi(t)$ , as a function of the time the first bank shows distress (i.e. zero cash). A bailout policy at  $\bar{t}$  means that the government introduces the interest rate  $\rho_0$  in case both banks are hit by a negative shock and the first bank ran out of money at  $t = \bar{t}$ . A no-bailout policy means that it keeps the market interest rate of 1.

At  $t = 1$ , both banks suffer a refinancing shock with probability  $1 - \alpha + \alpha(1 - \lambda)^2$ , only one bank suffers a refinancing shock with probability  $2\alpha\lambda(1 - \lambda)$ , and no bank suffers a refinancing shock with probability  $\alpha\lambda^2$ . If no bank suffers the shock, the payoffs are realized and the game ends. If only one bank suffers the shock, the successful bank has enough resources to take over the project (under symmetry), and hence the projects are financed full scale by takeover. If both banks suffer the shock, each bank chooses borrowing levels and reinvestment scale, which depends on the policy of the government. At the point when potentially a bank is forced to downscale if bailout is not enacted. The government decides whether to bail out or not (under no commitment), or the announced policy response is realized (under commitment).

### 3.5 Full Information and Commitment

In this section we assume that the government has the ability to commit to its announcement at the beginning of time. We first solve the optimal reaction of banks given a policy announcement and then we compute the optimal announcement of the government.

**The period-1 problem of the bank** At  $t = 1$ , a project either pays off in full, in which case no action is taken by the bank, or may need additional financing and there is a second round of borrowing. There are two possible cases to consider: (i) both banks need refinancing or (ii) only one bank needs refinancing.

In case only one bank needs refinancing (idiosyncratic shock), we allow for the healthy bank to make a take-it-or-leave-it offer for the assets of the distressed bank. The price paid is equal to the value of the continued investment that the distressed banker can guarantee by having access to the market (i.e. borrowing from households at rate 1). Under full information, there will never be any intervention on the side of the government in such case (from the social point of view takeover is always better than bailout), and hence the distressed bank in such state will always get taken over if it cannot refinance full scale. The value to the distressed bank is the value associated with refinancing at the maximum scale of  $j = ci/(1 - \rho_0)$  at rate  $R = 1$  (from equation (6)). In a symmetric equilibrium, healthy banks always have enough resources to take over, and by Assumption 2 they always find

takeover profitable.

In case both banks need refinancing, the *maximum* amount they are able to borrow at the market rate  $R = 1$  allows them to refinance a fraction  $c/(1 - \rho_0)$  of the project (by equation (6)), in which case they run out of cash at time

$$\bar{t}(c) - 1 = c/(1 - \rho_0). \quad (8)$$

If the policy of the government is not to bail out any bank in need of refinancing at any time below  $t \leq \bar{t}(c)$ , banks' projects are downscaled to  $j = ci/(1 - \rho_0)$ .

If the policy of the government is such that the *earliest bailout time* is  $t^* \leq \bar{t}(c)$ , then the optimal choice of banks is to minimize borrowing at the market rate 1 and maximize it at the bailout rate  $\rho_0$ . They do it by lowering the repayment promise at  $t = 1$ , to  $d < \rho_0$ , in which case the constraint on borrowing (6) is

$$j - ci = dj,$$

which implies

$$j = \frac{c}{1 - d}i.$$

For a given earliest bailout time of  $t^*$ , the optimal  $d$  is such that  $j/i + 1 = t^*$ , i.e.

$$d = 1 - \frac{c}{t^* - 1}. \quad (9)$$

By construction, if  $t^* = \bar{t}$ , then  $d = \rho_0$ .

**The period-0 problem of the bank** At  $t = 0$ , the bank chooses the cash  $c$  carried to  $t = 1$ , conditional on the government's policy and the choice of refinancing described above. The value function of the bank as a function of the cash choice  $c$  depends on whether  $\bar{t}(c)$  is larger or smaller than the government's policy  $t^*$ . In particular,

$$V(c) = \begin{cases} V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t} - 1)]i & \text{if } \bar{t}(c) < t^* \\ V_s(c) + P_2[c + \rho_0 + \rho_1 - (t^* - 1) - \rho_0(2 - t^*)]i & \text{if } \bar{t}(c) \geq t^* \end{cases} \quad (10)$$

where

$$V_s(c) = (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}$$

is the expected value when no bank suffers a shock (with probability  $P_0$ ) or when only one bank suffers a shock (with probability  $P_1$ ), which are naturally independent on the

government's policy,  $t^*$ . Recall that, since  $ci = (\pi - b)i$ , and  $i = \frac{A}{1-b}$ , cash can be rewritten as  $ci = (\pi - 1)i + A$ . Also, the value of taking over another bank,  $V_{TO}$  only depends on the other bank's choice of  $i'$  and  $c'$ ,  $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$ .

The kink in the value function is generated by the policy of the government and the payoffs in case of an aggregate shock (with probability  $P_2$ ). When  $\bar{t}(c) < t^*$ , there is no bailout. This implies banks refinance as much as possible with interest rate  $R = 1$ . From the previous discussion, we know banks can refinance up to a fraction  $j = (\bar{t}(c) - 1)i$  of the full project size, using its cash  $ci$ . In this case, the payoff for the bank is  $ci$  plus the returns  $(\rho_0 + \rho_1)j$  minus the cost of refinancing at a cost 1 per unit of reinvestment. Since  $j - ci = \rho_0 j$ , the value function in this case can be rewritten simply as  $V_s(c) + P_2 \rho_1 j$ .

In contrast, when  $\bar{t}(c) \geq t^*$ , there is bailout, which implies banks can borrow at a rate  $\rho_0$  at time  $t^*$ . This implies banks refinance as much as possible with interest rate  $R = 1$ . In this case banks will refinance at full scale, a fraction  $(t^* - 1)i$  at a cost of 1 per unit of refinancing and the rest (a fraction  $(2 - t^*)i$  at a cost  $\rho_0$  per unit of refinancing. Naturally in this case, the gains are given by  $(\rho_0 + \rho_1)i$ , for the full project.

Proposition 1 below establishes that the optimal cash choice  $\bar{c}$  of banks is a level that allows them to refinance fully in case of an aggregate shock, given government policy summarized by  $t^*$ , but not high enough to refinance fully in case of an idiosyncratic shock (i.e. no bailout policy). It relies on two natural assumptions that give our problem economic bite in terms of the government effect on leverage choices. First, Assumption 4 below assures that banks care about refinancing scale  $j$ , i.e. when faced with a tradeoff of increasing investment  $i$  by sacrificing reinvestment  $j$ , they choose not to sacrifice reinvestment scale. It guarantees that banks will always choose  $\bar{c}$  such that  $\bar{t} \geq t^*$ , (as defined by (8)).

**Assumption 4** *Banks care about reinvestment scale:*

$$(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} (1 - \pi) < 0.$$

This condition is given by the derivative of the value function when  $\bar{t}(c) < t^*$  with respect to  $i$ . The first term on the left is the benefit of decreasing  $c$  given by the increased size of the project. This benefit is obtained when there is no shock ( $P_0$ ) or when the other bank needs refinancing only ( $P_1$ ). The second term is the payoff lost due to downscaling  $j$ , which happens when there both ( $P_2$ ) or this bank ( $P_1$ ) needs refinancing. When Assumption 4 is violated, banks sole objective is to maximize  $i$ , independent of the government policy.

Second, in Assumption 5 we impose that if the government provides a bailout, i.e.  $t^* < 2$ , then it is not optimal for banks to carry a cash level such that the implied  $\bar{t} > t^*$ . This

implies the cash banks hold is high enough to refinance fully at the market rate, otherwise the commitment problem and moral hazard is not operational in this economy.

**Assumption 5** *Promise of a bailout increases leverage:*

$$(P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1 \frac{\rho_1}{1 - \rho_0} (1 - \pi) - P_2 > 0$$

This condition is given by the derivative of the value function when  $\bar{t}(c) \geq t^*$  with respect to  $i$ , and evaluating it at the more stringent condition,  $t^* = 2$ . The first term on the left is the benefit of decreasing  $c$  due to higher  $i$ , now, compared with Assumption 4, accrued also in case both banks fail (there is no change in reinvestment scale  $j$  if  $\bar{t} > t^*$ ). The second term is the cost of downsizing of project, now only incurred when an idiosyncratic shock pushes the bank to fail. The third term hands in for the cost of foregone consumption of extra liquidity in case of a bailout.

**Proposition 1** *Under Assumptions 1-5, given government policy  $\Pi(t, p)$ , the optimal choice of cash is characterized by*

$$c^* = (1 - \rho_0)(t^* - 1),$$

where  $t^* \in [1, 2]$  is the earliest bailout time.

**Proof** In appendix.

Given Proposition 1, and the solution to the bank's maximization problem,  $c^*(t^*)$ , the only characteristic that matters for welfare in terms of choosing a policy rule  $\Pi(t, p)$ , is the earliest bailout time  $t^*$ , which under commitment is like choosing  $c^*$  directly from the set  $[0, 1 - \rho_0]$ . We will therefore express welfare in terms of the cash choice of banks. Given the weight on banks in the welfare function  $\beta$ , the ex ante welfare is  $W^{ea}(c) = \beta V(c) + U_0 + U_1$ , which, ignoring constants, can be expressed as

$$W^{ea}(c) = \beta[\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2]2i(c) - (1 - \beta)P_2((1 - \rho_0) - c)2i(c). \quad (11)$$

where  $i(c) = \frac{A}{1 - \pi + c}$  and  $ci(c) = (\pi - 1)i(c) + A$ .

For  $\beta = 1$ , i.e. equal weights in the welfare function, the fact that banks are subsidized does not change welfare per se, because utility is transferrable one to one. In that case, the government only cares about output, and wants to transfer resources ex-ante from households to bankers. In that case the optimal government policy is  $t^* = 1$  and hence  $c^* = 0$ . On the other hand, the lower the  $\beta$ , the less weight the welfare function puts on producing output, since households gain nothing from it.

Definition 2 defines equilibrium under commitment and Proposition 2 below characterizes equilibrium outcomes as a function of  $\beta$ .

**Definition 2 (Commitment Equilibrium)** *A symmetric equilibrium of the economy under commitment is a cash level  $c^*$  and policy of the government  $\Pi(t, p = 1)$ , such that  $c^*$  is the optimal response of the banks to policy, i.e. it maximizes (10) given  $\Pi(t, p = 1)$ , and  $\Pi(t, p = 1)$  is such that  $c^*$  maximizes welfare (11).*

**Proposition 2 (Optimal Policy with Commitment)** *Given other parameters, define*

$$\beta^* = \frac{P_2(2 - \rho_0 - \pi)}{(\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2) + P_2(2 - \rho_0 - \pi)} < 1.$$

*Then the following is true:*

(i) *For  $\beta < \beta^*$ ,  $\frac{dW^{ea}(c)}{dc} > 0$  for all  $c \in [0, 1 - \rho_0]$ . In this case, the equilibrium (welfare maximizing) cash level is  $c^* = 1 - \rho_0$ , which corresponds to welfare maximizing policy choice of no bailout:  $t^* = 2$ , i.e.  $\Pi(t, p) \equiv 0$ .*

(ii) *For  $\beta > \beta^*$ ,  $\frac{dW^{ea}(c)}{dc} < 0$  for all  $c \in [0, 1 - \rho_0]$ , and the equilibrium (welfare maximizing) cash level is  $c^* = 0$ , which corresponds to a policy of immediate bailout, i.e.  $t^* = 1$  and  $\Pi(t, 1) \equiv 1$  (zero otherwise).*

(iii) *For  $\beta = \beta^*$ , the equilibrium government policy is indeterminate.*

### 3.6 Full Information and Lack of Commitment

In this section we assume that the government has no ability to commit to its initial announcement of the bailout policy. This makes the initial announcement irrelevant for the banks' incentives and the eventual outcomes of the economy – only the ex-post optimal actions of the government are going to be credible actions. The banks internalize the government's optimal ex-post actions in their optimization problem, effectively making them first-movers.

First, we study the optimal reaction of the government when observing a bank in trouble, through maximizing the *interim welfare function*,  $W^{in}$ , evaluated at time  $\bar{t}(c)$ , after the shock at date 1 already hit, and all relevant banks' choices had already been made<sup>8</sup>.

In this case, as under commitment, there is never a bailout if only one bank needs refinancing. When both banks need refinancing and run out of cash at  $\bar{t}$ , in a symmetric

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<sup>8</sup>For a given level of cash, welfare only depends on the total amount refinanced. Therefore, it doesn't make a difference whether we consider welfare at time  $\bar{t}$  or  $t = 1$  after the shock hit.

allocation, they need to borrow  $(2 - \bar{t})i(c)$  to refinance the rest of their projects to full size. If the government decides *not* to make the switch to  $R = \rho_0$ , i.e. there is no bailout, the projects cannot continue and are downsized to  $(\bar{t} - 1)i(c)$ . Given that, the interim welfare function of a *bailout* at  $\bar{t}$  when facing banks in distress (out of cash), is

$$W^{in} = \beta(\rho_0 + \rho_1 - 1)2(2 - \bar{t}(c))i(c) - (1 - \beta)(1 - \rho_0)2(2 - \bar{t}(c))i(c), \quad (12)$$

where  $i(c) = \frac{A}{1 - \pi + c}$ . If the government does not bail out, the projects cannot continue and the interim value function is just  $W^{in} = 0$ .

**Definition 3 (Non-Commitment Equilibrium)** *A symmetric equilibrium without commitment is a cash choice of banks  $c^*$  and a policy of the government  $\Pi(t, p = 1)$  such that given the banks' choice of cash, the policy  $\Pi(t, p = 1)$  is the best response, i.e. it maximizes interim welfare  $W^{in}$  (12), and hence the cash choice of banks maximizes (10) given the best response of the government to both banks' cash choices.*

Effectively, this implies that under no commitment banks choose the time of bailout  $t^*$  to maximize equation (10).

The best response of the government is to introduce a bailout if the value defined in (12) is positive, which gives *the bailout temptation condition*

$$\beta \geq \beta^{**} \equiv \frac{1 - \rho_0}{\rho_1}. \quad (13)$$

Under (13), the government enacts a bailout for any  $\bar{t} < 2$ , and in particular in the case when banks hold no liquidity, i.e.  $c = 0$  and  $\bar{t} = 1$ . Additionally, even if only a single bank chooses such strategy, it will be bailed out in the state when both banks need refinancing. Individual banks know that they can unilaterally force the government's hand, and since they prefer to hold as little cash as possible, the unique non-commitment equilibrium is when all banks hold zero cash.

**Proposition 3 (Optimal Policy without Commitment)** *Under the bailout temptation condition (13), the unique equilibrium is characterized by banks choosing  $c^* = 0$  (no liquidity held to face aggregate shocks), and the government intervening by bailing out those banks in case of an aggregate shocks.*

In what follows, we focus on the parameter space subset under which it is ex-ante optimal for governments to commit not bailing out banks in the aggregate state, but it is ex-post optimal for them to bail out banks in such a state. This restriction implies

**Assumption 6** *Inefficient excessive leverage:*  $\beta^{**} < \beta < \beta^*$ .

This assumption makes our model interesting, since it implies that when commitment is not a possibility, there is excessive inefficient leverage in the economy, with large projects but no liquidity to refinance in case both banks fail. Intuitively this assumption is more likely to be fulfilled with low  $P_2$  or relative low  $\rho_1$  or relative high  $\pi$  with respect to  $\rho_0$ .<sup>9</sup>

## 4 Imperfect Information Environment

The previous section distinguished between two possible scenarios unraveling at  $t \geq 1$ : one in which just one bank is in distress and there is enough liquidity in the banking system to take over its projects; the other in which only government intervention can save projects. In contrast to the previous section, here we assume that the government cannot perfectly observe which one of these two scenarios played out.

In particular, at some time  $1 \leq t < 2$ , the government may observe a bank in distress: not having any liquidity to continue the project. In such case, the government has to decide whether to bail out – introduce the low interest rate  $\rho_0$  immediately, or do nothing, in which case the remainder of the project gets lost<sup>10</sup> if it is not taken over. An important consideration in evaluating the latter possibility is that *there may not be enough liquidity in the system*, i.e. both banks may be in distress. The posterior probability of both banks in distress, conditional on one bank in distress, is given by  $P'_2$  in equation (1).

In what follows, we derive conditions under which the government optimally chooses delay – for the purpose of learning the true state – after observing the first bank in distress, which gives rise to a *delayed bailout condition*. After that, we show that delayed bailout gives banks incentives to strategically limit their leverage choices, a force we call *strategic restraint*.

### 4.1 Delayed Bailout

Let  $y = \beta\rho_1 - \beta(1 - \rho_0)$  be the social gains per unit of investment from a private takeover of a bank in distress by another bank,  $x = \beta\rho_1 - (1 - \rho_0)$  be the social gains per unit of

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<sup>9</sup>There are several ways this assumption may not be fulfilled. If  $\beta$  is either lower or higher than both cutoffs  $\beta^*$  and  $\beta^{**}$ , then lack of commitment does not introduce any inefficiency, rendering the problem irrelevant. If  $\beta^{**} > \beta > \beta^*$ , it is ex-ante optimal for governments to commit to bail out banks in the aggregate state, but it is ex-post optimal for them not bailing out banks in such a state. In this last case there is inefficient excessive liquidity and the strategic restraint mechanism we highlight in the next sections are not effective to eliminate such an inefficiency.

<sup>10</sup>An important assumption is that a bank that discontinues the project due to a missing a flow of needed reinvestments, cannot restart it at a later date if finding additional funds.

investment from a public takeover of a bank in distress (bailout), and  $\hat{x} = \beta(\hat{\rho} - \rho_0) - (1 - \rho_0)$  the social gains per unit of investment from banks running an inefficient project with bailout money – a ‘fake’ bailout. Under condition (13), we have  $y > x > 0 > \hat{x}$ .

In case of bailing out the first bank interim welfare is

$$W^{in}(B) = [(1 - P'_2)(x + \hat{x}) + P'_2(2x)](2 - \bar{t})i \quad (14)$$

In case of *not* bailing out the first bank, interim welfare is

$$W^{in}(NB) = [(1 - P'_2)y + P'_2x](2 - \bar{t})i \quad (15)$$

Ex-post, the government decides delay the bailout for the purpose of learning if the expected welfare gain from potentially letting the first bank in distress fail is non-negative, which gives the *delayed bailout condition*

$$[(1 - P'_2)(y - x - \hat{x}) - P'_2x](2 - \bar{t})i \geq 0. \quad (16)$$

The first term in (16) is the net cost of making a bailout when it is not needed (which happens with probability  $1 - P'_2$ ), equal to the cost of a transfer of funds from consumers to each of the banks: one of the bank to refinance the project in distress and the other to inefficiently rerun the successful project. We assume that the distressed bank can choose a refinancing method, which means that a bailout effectively prevents takeover. The second term is the net gain from bailing out the first bank: saving its project to full scale financed by taxes, given that otherwise those projects are lost (with probability  $P_2$ ). Under the *delayed bailout condition*, delay is the optimal non-commitment action of the government when facing the first bank in distress.

If the probability of an aggregate shock is zero (this is  $P'_2 = 0$ ), then with probability one some other bank is able to acquire the bank in distress, and since transferring resources from households is costly, there is always delay. In contrast, if the aggregate shock is guaranteed (that is,  $P'_2 = 1$ ), the government bails out the first bank in trouble under  $x > 0$  which is equivalent to the bailout temptation condition (13).

Summarizing, there exists a cutoff  $\bar{P}_2$ , defined by (16) holding with equality, such that government does not bail out the first bank showing distress if  $P'_2 < \bar{P}_2$ , and bails out otherwise.  $\bar{P}_2$  is given by

$$\bar{P}_2 = 1 - \frac{x}{y - \hat{x}}. \quad (17)$$

Clearly, as long as the *bailout temptation condition* holds, then  $x > 0$  and the cutoff  $\bar{P}_2$  takes an intermediate value between zero and one. The bigger the incentive for bailout, i.e. the bigger the  $x$ , the smaller the cutoff and hence bailout action is taken for a larger set of parameters. Parameters which help to satisfy the *delayed bailout condition* (larger  $P_2'$ ) are lower  $\beta$ , lower  $\rho_1$  and lower  $\hat{\rho}$ .

If the delay bailout condition (16) does not hold, then the first bank in distress is bailed out and equation (10) is the value function of banks. In this case the definition of equilibrium is the same as in Definition 3. Effectively the problem is the same that under full information and no commitment as long as  $P_2' > \bar{P}_2$  and the first bank in distress is bailed out for sure.

In contrast, if the delay bailout condition (16) is satisfied, the first bank in distress is not bailed out, but the second bank in distress is. This implies that the banks value functions become.

$$V(c) = \begin{cases} V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t} - 1)]i & \text{if } \bar{t}(c) < \bar{t}(c') \\ V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t} - 1)]i + \frac{1}{2}P_2\rho_1(2 - \bar{t})i & \text{if } \bar{t}(c) = \bar{t}(c') \\ V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t} - 1)]i + P_2\rho_1(2 - \bar{t})i & \text{if } \bar{t}(c) > \bar{t}(c') \end{cases} \quad (18)$$

In this case, since the first bank in distress is not bailed out, the choice of a single bank is not simply when to enter in distress if it is first, which is an individual choice and captured by  $t^*$  in equation (10), but when to enter in distress if it is second, which is a choice that depends on the action of the other bank. This is why, in the definition of the value functions the cutoffs are determined by the action of the other bank.

Specifically, if  $\bar{t}(c) < \bar{t}(c')$ , the bank is not bailed out, since it is the first bank in distress. If  $\bar{t}(c) > \bar{t}(c')$ , the bank is always bailed out, since it is the second bank in distress. If  $\bar{t}(c) = \bar{t}(c')$ , there is a tie breaking rule that assigns a probability one half the bank is the first in showing trouble and it is not bailed out and a probability one half the bank is the second in showing trouble and it is bailed out.

In the above specification, cash choices  $c$  transmit precisely into the time of distress according to (8), making infinitesimal deviations able to break the tie-break. This is certainly an extreme specification. In Section 4.3 we consider an additional i.i.d ‘noise’ shock to the asset position of the bank at  $t = 1$ , but after the refinancing decisions have been made. It makes the time of running out of cash stochastic and we show that the results of this section are a limiting case as the volatility of the shock converges to zero.

**Definition 4 (Non-Commitment Equilibrium with Delay)** *A symmetric equilibrium without commitment in case of delay (condition (16) holds) is a policy  $\Pi(t, p)$  and the cash*

choice of banks  $c^*$ , such that  $\Pi(t, P'_2) = 0 \forall t$  after observing the first bank in distress, and  $\Pi(t, 1) = 1 \forall t$  after observing the second bank in distress and the cash choice of banks  $c^*$  is such that given the other bank's choice of cash, each bank maximizes (18).

## 4.2 Strategic Restraint

Below, we study the ex-ante optimal cash choice problem of a bank (say, Bank 1),  $c_1$ , taking as given the choices of the other bank (Bank 2),  $c_2$ . We consider the case in which  $P'_2 < \bar{P}_2$ , i.e. bailout is delayed. We want to ask if it is optimal for Bank 1 to deviate from the symmetric strategy  $c_1 = c_2$  (which implies  $\bar{t}(c_1) = \bar{t}(c_2)$ ). What is going to be crucial in the incentives to deviate is how any deviation  $c_1 \neq c_2$  affects the probability that the bank is the first one showing distress, and hence the one failing under government policy.

A marginal deviation upwards from  $c_1 = c_2$  (i.e. carrying slightly *more* liquidity), has the benefit of increasing discretely the probability of a bail out, at the cost of downsizing the project from  $i(c_2)$  to  $i(c_1) < i(c_2)$ . However, for marginal changes, the first effect dominates, and Bank 1 always has an incentive to deviate as long as

$$\frac{1}{2}P_2\rho_1[2 - \bar{t}(c_2)]i(c_2) > 0, \quad (19)$$

which holds for all  $\bar{t}(c_2) < 2$ . The fraction  $1/2$  is the change in the probability of being bailed out, which is multiplied by the probability of two banks being in distress and the benefit of financing the project until completion. Equation (19) is a *strategic restraint condition*.

### Proposition 4 (Equilibrium: No Commitment and Government Uncertainty)

If  $P'_2 < \bar{P}_2$ , then the unique symmetric equilibrium is  $c^* = 1 - \rho_0$  (i.e.  $\bar{t}(c^*) = 2$ ), which coincides with the optimal solution under commitment. The equilibrium policy of the government is  $\Pi(t, P'_2) = 0 \forall t$  after observing the first bank in distress, and  $\Pi(t, 1) = 1 \forall t$  after observing the second bank in distress.

The statement of the proposition follows from applying the *strategic restraint condition* to all cases in which the *delayed bailout condition* holds. In all such cases, the value of being the second bank in distress is discontinuously higher than the value of being the first, and by a Bertrand-style undercutting argument, banks will want to deviate from a symmetric strategy in order to avoid being the first. At  $\bar{t} = 2$ , there is a corner solution and no longer incentives to deviate, since banks can self-finance completely.

In the next subsection we extend this argument, asking what are the limits for the Bertrand mechanism to play a role. In particular, what if a small deviation in cash holding

does not guarantee a bank being the second one showing distress, but only changes the probability in a continuous way?

### 4.3 Continous Distribution

Here we consider an extension of the benchmark model which adds an additional i.i.d. cost shock. Specifically, we consider a shock to the cash position of the bank,  $h \sim \mathcal{N}(0, \sigma_h^2)$ , that hits at  $t = 1$ , after the refinancing shock has been realized (either aggregate or idiosyncratic). The cash available for refinancing in the case of an idiosyncratic shock is then

$$c(h)i = (\pi - b)i + hi,$$

where, as before the shock  $h$  enters proportionally. Cash maps into time of distress in the following way

$$t(h|\bar{t}) = \begin{cases} 1 & \text{if } \frac{h}{1-\rho_0} < -(\bar{t} - 1) \\ \bar{t} + \frac{h}{1-\rho_0} & \text{if } -(\bar{t} - 1) < \frac{h}{1-\rho_0} < (2 - \bar{t}) \\ 2 & \text{if } (2 - \bar{t}) < \frac{h}{1-\rho_0} \end{cases} \quad (20)$$

where  $\bar{t}$  is the expected time of distress in case of a shock. Given the distribution of  $h$ ,  $t(h)$  is distributed according to following density  $f(t|\bar{t})$

$$f(t|\bar{t}) = \begin{cases} \Phi\left(-\frac{1-\rho_0}{\sigma_h}(\bar{t} - 1)\right) & \text{for } t = 1 \\ \phi\left(\frac{1-\rho_0}{\sigma_h}(t - \bar{t})\right) & \text{for } 1 < t < 2 \\ 1 - \Phi\left(\frac{1-\rho_0}{\sigma_h}(2 - \bar{t})\right) & \text{for } t = 2 \end{cases} \quad (21)$$

where  $\Phi$  denotes the standard cumulative normal distribution and  $\phi$  denotes the standard density of the normal distribution.

The value of bank 1 from deviating from a symmetric strategy  $\bar{t}$  is

$$V^1(t_1|\bar{t}) = [P_2\rho_1[t_1 - 1 + \eta(t_1|\bar{t})(2 - t_1)] + (P_0 + P_1)(\rho_0 + \rho_1 + (1 - \rho_0)(t_1 - 1)) + P_1\rho_1(t_1 - 1)]i(t_1) + V_{TO}$$

where

$$i = \frac{A}{1 - \pi + (t_1 - 1)(1 - \rho_0)}. \quad (22)$$

and  $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$ , as defined above, where  $i'$  and  $j'$  are the choices of the

other bank, and then irrelevant for the maximization problem of bank 1.

The derivative of  $V^1$  with respect to  $t_1$  is

$$\begin{aligned} V_{t_1}^1(t_1|\bar{t}) &= [(P_1 + P_2)\rho_1 + (P_0 + P_1)(1 - \rho_0)][i(t_1) + (t_1 - 1)i_{t_1}] \\ &\quad + P_2\rho_1[\eta(t_1|\bar{t})(2 - t_1)i_{t_1} + (\eta_{t_1}(2 - t_1) - \eta(t_1|\bar{t}))i(t_1)] \\ &\quad + (P_0 + P_1)(\rho_0 + \rho_1)i_{t_1} \end{aligned}$$

where

$$\begin{aligned} i_{t_1} &= -\frac{A(1 - \rho_0)}{(1 - \pi + (t_1 - 1)(1 - \rho_0))^2} \\ \eta(t_1|\bar{t}) &= Pr(t_1 > \bar{t}) = \Phi\left(\frac{(1 - \rho_0)}{\sigma_h}(t_1 - \bar{t})\right) \end{aligned}$$

and

$$\eta_{t_1} = \left(\frac{(1 - \rho_0)}{\sigma_h}\right) \phi\left(\frac{(1 - \rho_0)}{\sigma_h}(t_1 - \bar{t})\right)$$

Rearranging terms

$$\begin{aligned} \frac{A}{i^2(t_1)} V_{t_1}^1(t_1|\bar{t}) &= \overbrace{(P_1 + P_2)\rho_1(1 - \pi) - (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1)(1 - \rho_0)}^{C_1} \\ &\quad + P_2\rho_1[-\eta(t_1|\bar{t})(2 - t_1)(1 - \rho_0) + (\eta_{t_1}(2 - t_1) - \eta(t_1|\bar{t}))(1 - \pi + (1 - \rho_0)(t_1 - 1))] \end{aligned}$$

Assume full information and no bailouts ever, such that  $\eta(t_1|\bar{t}) = 0$  and  $\eta_{t_1} = 0$ . In this case, under Assumption 4, constant  $C_1$  is positive and banks always want to increase cash reserves (i.e. decrease leverage), since by increasing  $t_1$  conditional on the bank 2's policy  $\bar{t}$ , bank 1 can increase its expected profits.

In contrast, assume full information and bailouts guaranteed, such that  $\eta(t_1|\bar{t}) = 1$  and  $\eta_{t_1} = 0$ . In this case, under Assumption 5, we can rewrite the derivative as

$$\begin{aligned} \frac{A}{i^2(t_1)} V_{t_1}^1(t_1|\bar{t}) &= -(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1)(1 - \rho_0) + P_1\rho_1(1 - \pi) - P_2\rho_1(1 - \rho_0) \\ &< P_2(\rho_0 + \rho_1 + \pi - 2)(1 - \rho_0) - P_2\rho_1(1 - \rho_0) \\ &= P_2(1 - \rho_0)(\rho_0 + \pi - 2) < 0. \end{aligned}$$

and banks always want to reduce cash reserves (i.e. increase leverage), since by decreasing  $t_1$  conditional on the certainty of bailout, bank 1 can increase its expected profits.

Hence, even for a constant  $\eta$  – which would prevail if the variance of the  $h$  shocks is big – there exists a cutoff value  $\bar{\eta}$ , such that, for all  $\eta < \bar{\eta}$ , banks would like to reduce leverage

(increase  $t$ ) to avoid having to downscale a project<sup>11</sup>.

In the general case,

$$\frac{A}{i^2(t_1)} V_{t_1}^1(t_1|\bar{t}) = \overbrace{(P_1 + P_2)\rho_1(1 - \pi) - (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1)(1 - \rho_0)}^{C_1 > 0} + P_2\rho_1 \left\{ \underbrace{\eta_{t_1}(2 - t_1)(1 - \pi + (1 - \rho_0)(t_1 - 1))}_{\text{increasing chance of a bailout}} - \underbrace{\eta(t_1|\bar{t})(1 - \rho_0 + 1 - \pi)}_{\text{reducing size of the project}} \right\}$$

Strategic restraint means that the above derivative is positive, hence there are individual incentives to deviate by increasing  $t_1$ , or reducing leverage. In other words, increasing  $t_1$  above  $\bar{t}$  increases enough the chance of a bailout to justify the reduction in the size of the project. We focus on symmetric strategies, and consider deviations evaluated at  $t_1 = \bar{t}$ , which implies  $\eta(\bar{t}|\bar{t}) = \frac{1}{2}$  and  $\eta_{t_1} = \frac{(1-\rho_0)}{\sigma_h}\phi(0)$ .

If  $\sigma_h = \infty$ ,  $\eta_{t_1} = 0$ , and the bank cannot change the probability of showing problem first by reducing the leverage. In this case we will assume that  $V_{t_1}^1(\bar{t}|\bar{t}) < 0$ ,

$$\frac{A}{i^2(t_1)} V_{t_1}^1(t_1|\bar{t}) = Z \equiv C_1 - \frac{P_2\rho_1}{2}(2 - \rho_0 - \pi) < 0$$

This assumption guarantees that even when the probability of being bailed out in case of an aggregate shock is one half, bank 1 does not want to reduce leverage, and hence there is no strategic restraint.

For  $\sigma_h < \infty$ , a reduction in leverage serves the purpose of increasing the probability of not being the first in trouble and to being bailed out in case of an aggregate shock, which is captured by  $\eta_{t_1}$ . The smaller  $\sigma_h$ , the larger the marginal increase in such probability. At the extreme, when  $\sigma_h = 0$ ,  $\eta_{t_1} = \infty$ , and we obtain the benchmark assumption in the text without noise of the time of distress as a functions of the leverage.

**Proposition 5** *There exists  $\underline{\sigma}_h$  such that for all  $\sigma_h < \underline{\sigma}_h$ , the equilibrium is unique and characterized by  $c^*(\sigma_h) = (1 - \rho_0)(t^*(\sigma_h) - 1)$ , where*

$$\underline{\sigma}_h \equiv -\frac{P_2\rho_1(1 - \pi)(1 - \rho_0)\phi(0)}{Z},$$

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<sup>11</sup>Effectively, this is creating a situation in which banks perceive the probability they are going to be bailed out under aggregate shock like an exogenous coin flip, but in this situation the ‘coin flip’ is time consistent – as opposed to actually flipping a coin by the government, which is not.

and  $t^*(\sigma_h)$  solves

$$\frac{A}{i^2(t^*)} V_{t^*}^1(t^*|t^*) = Z + \eta_t P_2 \rho_1 (2 - t^*) (1 - \pi + (1 - \rho_0)(t^* - 1)) = 0,$$

where

$$\eta_t = \frac{(1 - \rho_0)}{\sigma_h} \phi(0).$$

Furthermore,  $t^*(\sigma_h)$  is increasing in  $\sigma_h$  and equilibrium cash holdings converge to the commitment outcome when the volatility of the shock goes to zero, i.e.

$$\lim_{\sigma_h \rightarrow 0} c^*(\sigma_h) = 1 - \rho_0.$$

**Proof.** For a given  $\sigma_h$ , strategic restraint for symmetric strategies  $t$  is equivalent to requiring a positive value of

$$\frac{A}{i^2(t)} V_t^1(t|t) = Z + \eta_t P_2 \rho_1 (2 - t) (1 - \pi + (1 - \rho_0)(t - 1)), \quad (23)$$

where

$$\eta_t = \frac{(1 - \rho_0)}{\sigma_h} \phi(0).$$

(i) For the first part of the proposition, we want to show that for all  $\sigma_h < \underline{\sigma}_h$ , (23) has a unique root in the interval  $t^*(\sigma_h) \in [1, 2]$ , and is positive for  $t < t^*(\sigma_h)$ . To see that, note that (23) is quadratic, and strictly negative and decreasing at  $t = 2$ . Second, under our definition of  $\underline{\sigma}_h$ , (23) is positive at  $t = 1$ . That means that it has exactly one root in  $[1, 2]$ , equal to  $t(\sigma_h)$  above, and that the strategic restraint ((23) > 0) is satisfied for all  $t < t(\sigma_h)$ .

(ii) For the limiting result of the proposition, note that the function (23) is decreasing in  $\sigma_h$ , and hence  $t^*(\sigma_h)$  is an increasing function. Furthermore, as  $\sigma_h \rightarrow 0$  then  $\eta_t \rightarrow \infty$  and  $\bar{t}^* \rightarrow 2$ . ■

Proposition 5 says that in the more general case which gives a continuous distribution of the distress times  $\eta$ , a similar result holds to our benchmark case. In particular, *if banks have sufficient control over their distress time*, i.e. if  $\sigma_h$  is low enough, then they have an incentive to restrict their leverage choices in order to avoid being the first bank in distress. In the limiting case of  $\sigma_h \rightarrow 0$ , the extended model converges asymptotically to our benchmark model.

## 5 Policy Implications

In the previous section, we showed that *government uncertainty* together with *strategic restraint* has the potential to relax time-inconsistency problems and to attain efficient, commitment-like, outcomes. Below, we analyze the impact on these two crucial conditions of securitization, industry concentration proxied by the number of banks, the asymmetry in bank size, and the possibility of contagion.

### 5.1 Securitization

This section analyzes how the information problem of the government and the incentives of the banks change with the level of securitization in the economy. We model securitization as banks diversifying the pool of projects they run by holding claims on other banks' projects. In particular, the level of securitization will be summarized by the fraction  $s$  of the project that each bank exchanges with the other bank. Each bank is then responsible for financing the fraction of each project it has a claim on. High level of securitization in the economy has the benefits of giving more diversification of cash inflows to banks, which results in less need for accessing the market for additional funds. The downside from the welfare point of view is that it reduces the level of uncertainty of the government when facing a single bank under distress, making bailouts more likely, *ceteris paribus*.

First, we show that in our benchmark model, *any* level of securitization completely removes government uncertainty (Proposition 6). Then, we extend our benchmark setup to include idiosyncratic shocks to liquidity position (as opposed to the refinancing need). In the extended setup, we show that government uncertainty remains for a set of positive securitization levels, and derive a cap on securitization that maintains the forces of strategic restraint and commitment-like outcomes.

Consider first the benchmark model. For *any* level of securitization  $s > 0$ , the time at which distress occurs differs between idiosyncratic and aggregate shocks, sending a clear signal of the source of distress to the government. Recall that  $\bar{t}$ , defined by (8), is the time of distress of banks under an aggregate shock, in which securitization does not affect the liquidity of distressed banks. Let  $\hat{t}$  be the time of distress of a bank under an idiosyncratic shock, where the condition for refinancing at rate 1 is given by

$$(\hat{t} - 1)(1 - s)i - [s(\rho_0 + \rho_1 + c) + (1 - s)c]i = (\hat{t} - 1)(1 - s)\rho_0 i.$$

Then,

$$\hat{t} - 1 = \frac{c + s(\rho_0 + \rho_1)}{(1-s)(1-\rho_0)} > \bar{t} - 1 \equiv \frac{c}{(1-\rho_0)} \quad (24)$$

where  $ci = (\pi - 1)i + A$  as defined above. An immediate implication is

**Proposition 6 (Securitization)** *For any level of securitization  $s > 0$ , the non-commitment equilibrium set coincides with the non-commitment equilibrium set under full information.*

The above proposition is very intuitive: any level of securitization introduces a difference in the time of distress between the states of the world when one bank fails and the state of the world when both fail. The reason is that in the first case, the distressed bank has more cash, because it got a payoff from the claim on the successful project. However, it follows that the government can infer which state occurred from the timing of distress, destroying uncertainty and restoring full information<sup>12</sup>.

*Discussion* The above result depends crucially on the ability of the government to perfectly observe all the banks' choices, in particular, the leverage choice  $i(c)/A$ , which determines the time of distress when both banks need refinancing. Introducing any level of heterogeneity which is unobserved ex-post, will make the amount of securitization which still preserves government confusion strictly positive. Below, we consider an extension of the basic setup in which that is the case.

### 5.1.1 Extended setting

We extend the previous setting along the lines in subsection 4.3. Consider a shock to the cash position of the bank,  $h \sim \mathcal{N}(0, \sigma_h^2)$ , that hits at  $t = 1$ , after the refinancing shock has been realized (either aggregate or idiosyncratic). The cash available for refinancing in the case of an idiosyncratic shock is then

$$c(h)i = ci + s(\rho_0 + \rho_1)i + h(1-s)i$$

Notice that the shock to the asset position,  $h$ , is proportional to the size of the project that needs to be refinanced.

Define the time of distress given an aggregate shock as  $t^a(h)$  such that  $(t^a(h) - 1)i$  is the amount of the investment that can be refinanced at interest rate 1, and it is given by

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<sup>12</sup>Mechanically, if the government can observe  $i$  and knows the initial assets of the bank  $A$ , then it can infer  $\bar{t}$ . Whenever the actual time of distress is higher than  $\bar{t}$ , then the government is certain intervention is not needed, as only one bank's projects need refinancing.

$t^a(h) = t(h)$  in equation (20), and the density of the aggregate time of distress is given by  $f^a(t|\bar{t}) = f(t|\bar{t})$  in equation (21).

Define the time of distress given an idiosyncratic refinancing shock as  $t^i(h)$ , where  $(t^i(h) - 1)i$  is the amount of the investment that can be refinanced at interest rate 1, equal to  $c(h)/[(1 - \rho_0)(1 - s)]$ . We can write  $t^i(h)$  as function of  $\bar{t}$  as

$$t^i(h|\bar{t}) = \begin{cases} 1 & \text{if } \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} < -(\bar{t} - 1) \\ \frac{\bar{t}-s}{(1-s)} + \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)} + \frac{h}{1-\rho_0} & \text{if } -(\bar{t} - 1) < \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} < (2 - \bar{t}) - s \\ 2 & \text{if } (2 - \bar{t}) - s < \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} \end{cases} \quad (25)$$

Given the distribution of  $h$ ,  $t^i(h)$  is distributed according to following density  $f^i(t|\bar{t})$

$$f^i(t|\bar{t}) = \begin{cases} \Phi\left(-\frac{1-\rho_0}{\sigma_h} \frac{(\bar{t}-1)}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)}\right) & \text{for } t = 1 \\ \phi\left(\frac{1-\rho_0}{\sigma_h} \left(t - \frac{\bar{t}-s}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)}\right)\right) & \text{for } 1 < t < 2 \\ 1 - \Phi\left(\frac{1-\rho_0}{\sigma_h} \frac{(2-\bar{t}-s)}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)}\right) & \text{for } t = 2 \end{cases} \quad (26)$$

Since the mean of  $t^i(h)$  (this is, setting  $h = 0$ ) is equal to  $\frac{\bar{t}-s}{(1-s)} + \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)}$ . Clearly, when  $s = 0$ ,  $f^a(h) = f^i(h)$ . When  $s > 0$ ,  $f^i(1|\bar{t}) < f^a(1|\bar{t})$  and  $f^i(2|\bar{t}) > f^a(2|\bar{t})$ , which implies it is more likely to see aggregate shocks earlier than idiosyncratic shocks. This is important for the inference problem.

Bayes rule on the probability of an aggregate shock after observing a bank in distress at time  $t$  gives

$$P(Agg|t) = \frac{P(t^a(h) = t|Agg)P_2}{P(t^a(h) = t|Agg)P_2 + P(t^i(h) = t|Id)P_1}.$$

For  $s = 0$ , this probability collapses to  $P'_2$  in the benchmark case and hence if in the benchmark case government uncertainty and strategic restraint give a unique optimal equilibrium, then there exists an  $s > 0$  for which it is still true. In particular, there exists a cutoff  $\bar{s} > 0$  such that for all  $s < \bar{s}$ , the government uncertainty still guarantees the achievement of an optimal commitment equilibrium outcome. In what follows we derive the condition that pins down  $\bar{s}$ . The government will decide not bailout the first bank in distress as long as the probability of aggregate shocks is smaller than the cutoff probability that induces bailout  $\bar{P}_2$  from equation (17). This is when

$$P'_2 = \frac{\frac{f^a(t)}{f^i(t)} \frac{P_2}{P_1}}{1 + \frac{f^a(t)}{f^i(t)} \frac{P_2}{P_1}} \leq \bar{P}_2, \quad (27)$$

Note that, when  $s = 0$ ,  $f^a(t) = f^i(t)$ , and  $P'_2$  is the one obtained in the benchmark without securitization,  $P'_2 = \frac{P_2}{P_1 + P_2}$ . Additionally, for  $s > 0$ , the likelihood ratio under normality is declining in  $t$ . A sufficient condition for (27) to hold generally is that it holds at  $t = 1$ , which gives

$$\left( \frac{\bar{t} - s}{(1 - s)} + \frac{s(\rho_0 + \rho_1)}{(1 - s)(1 - \rho_0)} \right)^2 - \bar{t}^2 \leq 2\sigma_h^2 \ln \left( \frac{\bar{P}_2}{(1 - \bar{P}_2)} \frac{P_1}{P_2} \right). \quad (28)$$

Since the left hand side of (28) is strictly increasing in  $s$  and the right hand side is a constant, there is a strictly positive  $\bar{S}$  that is the minimum between  $1/2$  (the maximum possible securitization) and the value of  $s$  that satisfies (28) with equality. Any cap on securitization lower than or equal to  $\bar{S}$  guarantees the optimal non-commitment outcome.

Finally, it is straightforward to see that  $\bar{S}$  is weakly increasing in  $\sigma_h^2$ . There is a  $\sigma_h^2$  large enough such that  $\bar{S} = 1/2$  and full securitization does not prevent uncertainty to implement the commitment outcome. In contrast, if  $\sigma_h^2 = 0$ , we are back in the benchmark case.

## 5.2 Number of banks

In what follows we study the government incentives to bail out the first bank in distress, for the case of  $N = 2$  and  $N = 4$  for illustration purposes, and then extend the analysis to an arbitrary number of banks  $N$ . We derive the sufficient conditions for decreasing incentives to bail out the first bank in distress as the number of banks goes up, and then provide a general result for  $N$  sufficiently large.

**Two Banks** We can express the delayed bailout condition (16) for two banks as

$$x - (1 - p_{2,2})y + p_{2,1}\hat{x} \leq 0, \quad (29)$$

where  $p_{2,2} = P'_2$  and  $p_{2,1} = 1 - p_{2,2}$ .

Conditional on a first bank showing distress, the government is more likely to delay the bigger is the difference between the value of takeover and the value of a bailout  $x - y$ , and the more socially costly it is to provide a transfer to healthy banks, i.e. the lower  $\hat{x}$  is.

Below, we derive the above condition for the case of four banks and then move on to the general case of  $N$  banks.

**Four Banks:** Conditional on a first bank showing distress, define the probability that

no other bank can take it over as

$$p_{4,4} = \frac{(1 - \alpha) + \alpha(1 - \lambda)^4}{1 - \alpha\lambda}.$$

$p_{4,4}$  is the probability of all four banks in distress conditional on one bank in distress. Let also  $p_{4,3}$  be the probability of three banks in distress conditional on one bank in distress,  $p_{4,2}$  the probability of two banks in distress conditional on one bank in distress,  $p_{4,1}$  the probability of just one bank in distress conditional on one bank in distress<sup>13</sup>.

The expected social value of governments bailing out the first bank in distress is

$$\begin{aligned} p_{4,4}4x + p_{4,3}(3x + \hat{x}) + p_{4,2}(2x + 2\hat{x}) + p_{4,1}(x + 3\hat{x}) = \\ x + p_{4,4}3x + p_{4,3}(2x + \hat{x}) + p_{4,2}(x + 2\hat{x}) + p_{4,1}3\hat{x} \end{aligned}$$

The expected social value of the government not bailing out the first bank in distress is

$$p_{4,4}EJ^f + p_{4,3}(y + EJ^{to}) + p_{4,2}(y + EJ^{to}) + p_{4,1}y = (1 - p_{4,4})y + p_{4,4}EJ^f + (p_{4,3} + p_{4,2})EJ^{to},$$

where  $E(J^f)$  is the expected social value after a second bank shows distress and previous bank was not taken over, and  $E(J^{to})$  is the expected social value of seeing a second bank in distress after the first one was taken over. Both of these encompass the option of the government deciding to start bailing out banks in distress.

The government does not bail out the first bank in distress if

$$x - (1 - p_{4,4})y + p_{4,4}3x - p_{4,4}EJ^f + p_{4,3}(2x + \hat{x}) + p_{4,2}(x + 2\hat{x}) - (p_{4,2} + p_{4,3})EJ^{to} + 3p_{4,1}\hat{x} \leq 0,$$

where the last term  $3p_{4,1}\hat{x} < 0$  is the cost of allowing successful banks to restart the project inefficiently in case of premature bailouts, in case there is only one bank in need of refinancing.

In the above expression,  $EJ^f$  is just equal to  $3x$ , as in the case of no takeover, the government is certain that the shock is aggregate and hence there will be no takeovers. This makes it always optimal to start bailing out immediately after observing a no-takeover market outcome. Canceling terms, the above condition becomes

$$x - (1 - p_{4,4})y + p_{4,3}(2x + \hat{x}) + p_{4,2}(x + 2\hat{x}) - (p_{4,2} + p_{4,3})EJ^{to} + 3p_{4,1}\hat{x} \leq 0. \quad (30)$$

The first term is analogous to the two-bank case. The next three capture the difference

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<sup>13</sup>Given respectively by  $(\alpha(1 - \lambda)^3\lambda)/(1 - \alpha\lambda)$ ,  $(\alpha(1 - \lambda)^2\lambda^2)/(1 - \alpha\lambda)$  and  $(\alpha(1 - \lambda)\lambda^3)/(1 - \alpha\lambda)$ .

between the social welfare of a certain policy of bailouts and the social welfare from delay, keeping the option of introducing the bailout later. Hence, we know that

$$(p_{4,2} + p_{4,3})EJ^{to} \geq p_{4,2}(x + 2\hat{x}) + p_{4,3}(2x + \hat{x}).$$

The last term, as before, captures the cost of making transfers to healthy banks in case of only one bank in trouble.

In general, comparing the two-bank and four-bank cases, there is clear monotonicity in the first two terms, but the monotonicity of the second term is much harder to demonstrate. Below, we focus on a simpler case in which delayed bailout condition holds for two banks also under  $\hat{x} = 0$ , i.e.

**Assumption 7** *Delayed bailout for  $\hat{x} = 0$ :  $x - (1 - p_{2,2})y \leq 0$ .*

Under Assumption 7, we also have that the delayed bailout condition (30) is satisfied, due to the fact that  $p_{4,4} < p_{2,2}$  and hence delay under two banks implies delay under four banks.

**Generalization to N Banks:** Using the same definitions as above we can extend the reasoning to an arbitrary number  $N$  of banks in the system.

$$p_{N,N} = \frac{(1 - \alpha) + \alpha(1 - \lambda)^N}{1 - \alpha\lambda}$$

The expected social value of governments bailing out the first bank in distress is

$$x + x \sum_{k=2}^N p_{N,k}(k-1) + \hat{x} \sum_{k=2}^N p_{N,k}(N-k) + (N-1)p_{N,1}\hat{x}.$$

The expected social value of governments not bailing out the first bank in distress is

$$(1 - p_{N,N})y + p_{N,N}EJ_N^f + EJ_N^{to} \sum_{k=2}^{N-1} p_{N,k},$$

where  $EJ_N^f$  and  $EJ_N^{to}$  are defined analogously to the four bank case.

The government does not bail out the first bank in distress if

$$x - (1 - p_{N,N})y + x \sum_{k=2}^N p_{N,k}(k-1) + \hat{x} \sum_{k=2}^N p_{N,k}(N-k) - p_{N,N}EJ_N^f - EJ_N^{to} \sum_{k=2}^{N-1} p_{N,k} + (N-1)p_{N,1}\hat{x} \leq 0. \quad (31)$$

As before,  $EJ_N^f = (N - 1)x$  and

$$EJ_N^{to} \sum_{k=2}^{N-1} p_{N,k} \geq x \sum_{k=2}^N p_{N,k}(k - 1) + \hat{x} \sum_{k=2}^N p_{N,k}(N - k).$$

Hence, under Assumption 7, the delayed bailout condition for  $N$  banks, (31), is satisfied as well, as all the other terms are negative. This is summarized in the following Lemma:

**Lemma 3** *If Assumption 7 holds, i.e. if delayed bailout condition with  $\hat{x} = 0$  holds for  $N=2$ , then it holds under arbitrary number of banks  $N$ , i.e. (31) is also satisfied.*

Clearly, Assumption 7 is a sufficient but not necessary condition for delay as long as  $\hat{x} < 0$ , i.e. it is more restrictive than (29). However, under the necessary condition (29), the discussion above implies that for  $N$  large enough, the value of delaying is going to outweigh the value of immediate bailout<sup>14</sup>, which is summarized in the following proposition:

**Proposition 7** *If (29) holds, then there exists  $\bar{N} > 2$  such that for all  $N > \bar{N}$ , (31) is satisfied, i.e. bailout is delayed.*

### 5.3 Asymmetric Bank Sizes

What are the effects of the banking size distribution on governments' time consistency and their likelihood of attaining efficient outcomes? In this section, we analyze the case of asymmetric bank sizes. In particular, we modify the benchmark setup by assuming that Bank 1 has higher initial assets than Bank 2, i.e.  $A_1 > A_2$ .

Now Bank 2 may not have enough funds to take over Bank 1 in case of idiosyncratic distress. Specifically, Bank 2's cash availability, which potentially can be used to refinance Bank 1's project, is equal to  $(\rho_0 + \rho_1 + (1 - \rho_0)(t_2 - 1))i_2$ . Hence, the reinvestment scale in case of a takeover is

$$I = \min \left\{ \frac{(\rho_0 + \rho_1 + c_2)i_2}{1 - \rho_0}, (2 - \bar{t}_1)i_1 \right\}.$$

The delayed bailout condition becomes

$$(1 - P_2') [yI - \hat{x}(2 - \bar{t}_1)i_2] \geq x(2 - \bar{t}_1)i_1 \quad (32)$$

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<sup>14</sup>The result is a straightforward implication of the fact that  $p_N$  is decreasing in  $N$  and that  $x - y < 0$ .

Hence the cutoff in the asymmetric bank case is

$$\bar{P}_2 = \frac{y \left( \frac{I}{(2-\bar{t}_1)i_1} \right) - \hat{x} \left( \frac{i_2}{i_1} \right) - x}{y \left( \frac{I}{(2-\bar{t}_1)i_1} \right) - \hat{x} \left( \frac{i_2}{i_1} \right)} \quad (33)$$

As can be seen, when  $A_2/A_1$  goes to zero,  $i_2/i_1$  also goes to zero. This implies that there is a level of asymmetry large enough such that  $\bar{P}_2 = 0$  and the government always bailout the large bank in distress, regardless of the updated belief about the probability the second bank is successful or not. In contrast, as  $A_2/A_1$  goes to one,  $\bar{P}_2$  converges to the original cutoff shown in equation (17). Any level of asymmetry makes the cutoff smaller, such that delay is more difficult to occur.

If condition (32) holds, Bank 1 has no incentive to restrain leverage, and Bank 2 can take advantage of that by restraining leverage just a little bit more than Bank 1. The large bank becomes a ‘shield’ for the small bank to engage in inefficient levels of leverage. This points to a new and unique external effect of ‘too big’ banks in our framework.

## 5.4 Contagion

The results of our paper can also be analyzed in the context of contagion. To address this phenomenon, we modify the benchmark model’s stochastic structure. In particular, we introduce a probability  $\chi$  that the situation is *contagious*, i.e. failure of one bank will *trigger* the need for refinancing of the other bank. This can be interpreted as an increase in the probability of an aggregate shock from  $P_2$  to  $P_2 + \chi$ .

If the contagion shock is independent of the other refinancing shocks, then it simply modifies the probability that there won’t be enough liquidity in the system if the bank is allowed to fail. It now becomes

$$\hat{P}'_2 = \frac{P_2 + \chi}{P_1 + P_2 + \chi} > P'_2.$$

where  $\hat{P}'_2$  is the updated probability in case of acknowledging the possibility of contagion. Hence, the possibility of contagion increases the belief of the government of the need for immediate bailout and makes attaining the commitment outcome more difficult to hold. This is, it is possible that  $\hat{P}'_2 > \bar{P}_2$  and the government bails out the first bank in distress, even when, without contagion  $P'_2 < \bar{P}_2$  and the government would decide not to bail it out.

*Discussion.* Clearly, this is a very crude way of modeling contagion. First, with

only two banks, the government never learns anything about the possibility of a contagious situation from observing a bank in distress. To introduce interesting dynamic learning effects under contagion, we need a minimum of three banks in the economy. Then, observing two failures in a row gives additional information to the government about the potential for facing a contagious problem. Second, we do not model at all the micro-foundations of contagion, which potentially may be important for both evaluating the welfare costs of delay or bailout, as well as for how the probabilities change with the inflow of new information to the government. In ongoing work, we are pursuing these extensions.

## 6 Conclusions

In this paper, we investigate the role of imperfect observability of the state of nature by the government in a situation where the government has to decide on whether to bail out a failing bank or not. We show that such friction, i.e. *government uncertainty*, can give incentives for the government to delay the bailout action in order to learn about the nature of the underlying shock. Crucially, such delay introduces a discontinuity in the payoffs of the banks between being the last bank to not receive the bailout and the first bank to be bailed out. Banks compete, in Bertrand-style fashion, for the relative performance, giving rise to an endogenous, strategic *deleveraging*, which we call *strategic restraint*.

We show that the novel forces we model drastically change the equilibrium outcomes in the economy. In a standard model of banking and liquidity choice à la Farhi and Tirole (2012), under mild conditions and under no commitment, modeling *government uncertainty* and the ensuing *strategic restraint* moves the equilibrium of the economy from the maximal leverage, worst from a welfare standpoint, to a *unique* constrained optimal equilibrium in which the optimal level of leverage obtains.

From the perspective of our key innovations, we offer insights into the optimal level of securitization, the effect of industry concentration and asymmetry of bank sizes in the industry. The literature has identified the time-inconsistency of governments' policies as an important justification for macro-prudential regulation, i.e. overseeing banks' activities directly. However, historically regulators have been incapable to design macro-prudential regulation that prevents crises without choking off growth. This project sheds light on the design of policies and regulations that affect variables such as securitization, the number and size of banks or the nature of lending facilities, in order to impose lower burdens to preventive regulation by relaxing time-inconsistency in the first place and inducing optimal self-regulation.

Finally, our paper points to the question of whether it is possible for governments to design political structures that delay bailouts decisions, or regulatory standards that maintain an optimal level of uncertainty? Contrary to the common view that information and speed of action are desirable characteristics of policymakers, it may be the case that banks' perception about whether policymakers have those properties may induce suboptimal outcomes.

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## A Proof of Proposition 1

Fix  $t^*$  and consider  $c$  such that  $\frac{c}{1-\rho_0} < T^* - 1$ . The bank's value function on this part of the domain is

$$V(c) = P_2(c + (\rho_0 + \rho_1 - 1)(\bar{T}(c) - 1))i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}$$

where

$$j = c/(1 - \rho_0)i$$

$V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$  and

$$i = \frac{A}{1 - \pi + c}.$$

The derivative of the value function with respect to  $c$  is

$$V'(c) \frac{(1 + c - \pi)^2}{A} = -(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) + (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} (1 - \pi) \underbrace{>}_{\text{Assumption 4}} 0.$$

The interpretation of this result is that whenever leverage is too high to refinance fully under interest rate 1 (i.e.  $c$  is too low), it is optimal for the bank to increase cash holdings to assure fuller refinancing scale.

For  $1 > \frac{c}{1-\rho_0} > T^* - 1$ , the the bank always refinances fully on the market and the value function is

$$V(c) = P_2(c + \rho_0 + \rho_1 - (\bar{T}^* - 1) - \rho_0(2 - T^*))i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}.$$

Taking a derivative with respect to  $b$  gives

$$\begin{aligned} V'(c) \frac{(1 + c - \pi)^2}{A} &= P_1 \frac{\rho_1}{1 - \rho_0} (1 - \pi) - (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) + P_2(\rho_0 + (1 - \rho_0)(T^* - 1)) \\ &< P_1 \frac{\rho_1}{1 - \rho_0} (1 - \pi) - (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) + P_2 \underbrace{<}_{\text{Assumption 5}} 0. \end{aligned}$$

Hence, on this part of the domain, it is optimal to decrease cash holdings and increase leverage.

Third for  $\frac{c}{1-\rho_0} > 1$ , the value of the bank is

$$V(c) = (P_2 + P_1)(c + \rho_1 + \rho_0 - 1)i + P_0(c + \rho_0 + \rho_1)i + P_1V_{TO},$$

with derivative

$$\begin{aligned} V'(c) \frac{(1 + c - \pi)^2}{A} &= -(P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1 - 1) - (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) = \\ &= -\{(\rho_0 + \rho_1 + \pi - 1) - (1 + P_1 + P_2)\} \underbrace{<}_{\text{Assumption 3}} 0. \end{aligned}$$

Therefore, here also it is optimal to decrease cash. This completes the proof. ■