

REPUTATION AND SIGNALING*

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Abstract

Static adverse selection models of security issuance show that informed issuers can perfectly reveal their private information by maintaining a costly stake in the securities they issue. This paper shows that allowing an issuer to both signal current security quality via retention and build a reputation for honesty leads that issuer to misreport quality even when owning a positive stake—the equilibrium is neither separating nor pooling. Although an issuer retains less as reputation improves, prices are u-shaped in reputation because retention is no longer a sufficient statistic for issuer private information.

Keywords: costly signaling, reputation, repeated games, asset backed securities.

JEL Classifications: G01, G21, G24, G28, D82.

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On April 16th, 2010 the Securities and Exchange Commission filed a complaint against Goldman Sachs and Co. (GS) alleging that the investment bank had misled investors in the ABACUS 2007-AC1 collateralized debt obligation (CDO). In response, GS raised at least two important points that seem to be in line with financial economic theory:

A significant point missing from the SEC's complaint was the fact that Goldman Sachs retained a significant residual long position in the transaction... We certainly had no incentive to structure a transaction that was designed to lose money.

(Palm 2010)

Nor is there any basis to suggest that Goldman Sachs would have intentionally jeopardized its own reputation and relationship with established customers and counterparties. ... Goldman Sachs had no reason to mislead anyone.

(Klapper, Tamaino, and Dunne 2010)

The “skin-in-the-game” defense could follow from the notion that informed issuers reveal their information through costly signaling by maintaining a stake in their issue and that such a stake should be a sufficient statistic for all private information (Leland and Pyle 1977, DeMarzo and Duffie 1999, DeMarzo 2005). The “reputation and relationships” defense could follow from the notion that concern for future business can lead issuers to be truthful when making public statements about their issues, such as in marketing materials or a deal prospectus. Considered separately, these defenses make some economic sense, but the literature has yet to understand the interactive effects of signaling and reputation. This paper fills that gap and shows that the interaction of reputation and signaling can cause issuers to have a larger incentive to mislead investors.

In addition to explaining opportunistic behavior by asset backed securities (ABS) issuers, combining costly signaling and reputation leads to new insights into the behavior of markets with asymmetric information. First, issuer retention can decrease with issuer reputation, indicating that costly signaling and reputation can act as substitutes. Second, the equilibrium relationship between retention and issuer reputation implies that a better reputation can decrease the probability that the issuer will truthfully reveal asset quality. As a result, equilibrium prices can be U-shaped in reputation. These insights translate into the following testable implications:

- Issuer retention decreases (increases) with good (poor) past performance of issues where good performance is characterized as performance consistent with information previously made public by the issuer.
- Prices are more sensitive to retention for low reputation issuers than for high reputation issuers.
- Prices at issuance do not perfectly reveal issuer private information, but are most informative for low and high reputation issuers.
- Prices for mid-range reputation issuers are relatively low.

To arrive at these results, I consider a model of an infinitely repeated securitization game. In each stage game, a risk neutral issuer is endowed with an asset to securitize for sale to investors. Nature chooses the type of the asset, which can be either good or bad, where asset type denotes expected future cash flow. The asset yields a cash flow one period after it is securitized. Patient risk neutral investors compete to buy the fraction of the asset that is securitized. The issuer can perfectly observe the type of the asset, but this information is not available to investors. This gives rise to lemons problem as in Akerlof (1970). The issuer publicly reports the type of the asset to the investors in a prospectus, but may lie in doing so. In addition, the issuer can signal asset type by retaining a fraction of the asset. Because the issuer is relatively impatient, such a signal is credible.

Reputation concerns arise due to asymmetric information over issuer preferences for honesty. Specifically, the issuer could be of two possible types. The honest type issuer is committed to truthfully reporting the asset type in the prospectus. In contrast, the opportunistic type issuer will choose a reporting strategy by maximizing payoffs. Both types optimally choose a fraction to retain. The issuer's reputation is the probability the investors place on the issuer being the honest type. By mimicking an honest issuer, i.e. truthfully reporting asset type in the prospectus, an opportunistic issuer's reputation can be improved and thereby reduce the lemons discount on the fraction of the asset sold to investors.

Issuer type in the model can be viewed as a proxy for the issuer's preferences over accuracy. For example, the issuer may have separate lines of business that depend on reputation in an opaque fashion. This would be the case for an investment bank with many lines of business, all of

which depend on its reputation for accuracy. Investors in one product issued by this bank (e.g., example mortgage backed securities) may not know the profitability and sensitivity to reputation of the bank's underwriting business. A bank with a highly profitable underwriting business would correspond to the honest type, while a bank with a less profitable underwriting business would correspond to the opportunistic type.

Combining costly signaling and reputation leads to multiple equilibria. The simplest class of equilibria, referred to as separating equilibria, arise when the issuer perfectly reveals the quality of assets through either retention or public report. In a separating equilibrium, the ex post performance of an asset does not yield any new information about the type of the issuer, so reputation and price remain constant. A truth telling equilibrium, a special type of separating equilibrium, obtains when the issuer's public report is credible regardless of issuer retention. I show that a truth telling equilibrium exists if and only if the issuer is sufficiently patient.

The next class of equilibria, referred to as mixed strategy equilibria, arises when the opportunistic issuer deviates from truthful reporting at least part of the time and does not perfectly reveal asset quality through retention. For certain parameter values, this type of equilibrium will Pareto dominate the separating equilibrium. In the discussion below, I characterize a particular mixed strategy equilibrium. In such an equilibrium, the issuer does not perfectly reveal asset type and the reputation of the issuer fluctuates according to the ex post asset performance. Retention then becomes a signal of the opportunistic issuer's reporting strategy. The importance of retention necessarily varies with the reputation of the issuer since, as issuer reputation increases, the strategy of the opportunistic issuer has a lower impact on price.

The mixed strategy equilibrium creates an endogenous link between issuer reputation and asset retention. As might be expected, retention decreases with issuer reputation. What is less obvious is that the opportunistic issuer will decrease the probability of truthfully reporting a bad type asset as her reputation is improved. Eventually, the opportunistic issuer "cashes in" on her reputation by reporting that a bad type asset is the good type and collects one period payoffs. This will occur even when the issuer retains a positive fraction of the asset. In addition, the mixed strategy equilibrium demonstrates that the price for reportedly good type assets may not depend monotonically on issuer reputation. For low levels of reputation, the issuer perfectly reveals asset type through retention alone, and prices will be equal to the full information case. For higher levels of reputation, the

issuer's public report is more credible and prices for reportedly good type assets will be close to the full information value of a good type asset regardless of retention.

Finally, the model establishes that although the addition of reputation to a costly signaling framework can increase issuer payoffs, it does not increase the probability of perfect information transfer. This result leads to a criticism of the claim that reputation is the core self-disciplining mechanism for markets such as ABS markets. If one ignores the possibility that issuers may signal asset quality through costly retention, reputation certainly provides some incentives for issuers of ABS to truthfully reveal their private information. However, including costly retention shows that reputation may actually diminish the issuer's incentives to truthfully reveal information. This feature of the model is appealing given that opportunistic behavior allegedly occurred in many ABS markets.

1 Related Literature

It is well known that private information can cause important distortions in markets (Akerlof 1970). As early as Myers and Majluf (1984), this idea has been applied to financial markets. If entrepreneurs know more about investment opportunities than outside investors, then the irrelevance of capital structure (Modigliani and Miller 1958) no longer holds and a particular security design may be more advantageous than another. This effect leads to the "pecking order" theory of capital structure, with managers issuing the least informationally sensitive securities (i.e. debt) first. Using model similar to that of Myers and Majluf (1984), Nachman and Noe (1994) show that debt is the optimal security design to finance investment over a very broad set of payoff distributions. For the special case of asset backed securities, Riddiough (1997) shows that a senior-subordinated security structure dominates whole asset sales when an issuer has valuable information about assets and liquidation motives are non-verifiable. However, the pecking order literature assumes an investment of fixed size, resulting in a pooling equilibrium in which no mechanism of information transmission exists between issuers and investors.

Another branch of the literature focuses on signaling mechanisms in security design. Spence (1973) showed that agents can credibly reveal information through actions with costs that depend on that information. Applying this costly signaling concept to financial markets, Leland and Pyle

(1977) introduce the notion that retention can be a credible signal of private information because it is costly, in their case because of reduced risk sharing. DeMarzo and Duffie (1999) build on this signaling mechanism and show how debt arises optimally by creating a more informationally sensitive security which the issuer can retain in order to signal private information. The addition of a signaling mechanism leads to a separating equilibrium in which all information costs are borne through signaling rather than through prices as in pooling equilibria. DeMarzo (2005) uses the signaling theory of security design to understand the benefits of pooling and tranching in the market for asset backed securities. Downing, Jaffee, and Wallace (2009) consider the case when issuers cannot choose partial retention and a market for lemons ensues. Unlike the previous literature on signaling and security design, the model I consider focuses on binary cash flow distributions for the sake of tractability. However, the key difference between this paper and previous work on signaling equilibria in security design is the inclusion of dynamic reputation effects. As a result, issuers may choose to do some signaling, but such signaling will not lead to a perfect separating equilibrium all of the time. In that sense, this paper provides a theoretical rationale for a middle ground between separating and pooling equilibria.

There has also been theoretical inquiry into dynamic signaling problems. Nöldeke and Van Damme (1990) study a multi-period version of Spence (1973). Kremer and Skrsypacz (2007) and Daley and Green (2011) study dynamic signaling games when news about quality arrives over time, the latter providing a general framework that encompasses settings with and without static adverse selection problems. In this dynamic signalling literature an agent's hidden type is permanent. In contrast, the issuer in my setting has a (stochastically) different type of asset to sell in each period.

A second strand of literature to which this paper relates is that of reputation effects in repeated games. Intuitively, agents involved in repeated games may try to attain a reputation for a certain characteristic in early stages of the game if that characteristic improves payoffs in later stages. Kreps and Wilson (1982) and Milgrom and Roberts (1982) introduce the notion that imperfect or asymmetric information about player preferences can provide such a mechanism. By observing a given player's previous actions, other agents can form beliefs about that player's type. When a player is one of two types, e.g., honest or opportunistic, such a learning process provides a means by which that player can gain a reputation. If having a reputation for being honest leads to higher equilibrium payoffs, then an opportunistic player may employ the equilibrium strategies

of the honest type. Thus, reputation can act as an effective mechanism to encourage “desirable” characteristics. This literature often assumes that the desirable type plays a mechanical strategy and does not optimize. In my model, I depart from this assumption by allowing the honest type to optimize over retention strategy. To my knowledge this is the first model to include a separate dimension of the strategy space over which the mechanical type player has as much flexibility as the opportunistic type player.

Some notable papers have applied the Kreps and Wilson (1982) concept of reputation to financial markets. Diamond (1989) shows that the possibility of acquiring a reputation for good investment opportunities can incentivize firms to choose safer investments by lowering the cost of borrowing for firms with good reputations. Diamond (1991) examines the choice between bank and public debt in the presence of reputation concerns. John and Nachman (1985) analyze the role of reputation in mitigating the problem of underinvestment induced by risky debt in the presence of asymmetric information. They show that a reputation for “good” investment policies can lead to higher prices in bond markets and hence firms will want to implement such an investment policy to attain a good reputation. Bénabou and Laroque (1992) show that when private information is not ex post verifiable, insiders have an incentive to exploit a reputation for honesty in order to manipulate securities markets for gain. Carlin, Dorobantu, and Viswanathan (2009) analyze the evolution of public trust in financial markets. These papers focus the efficacy of reputation to implement “good” behavior in financial markets without considering other mechanisms that may have similar effects. In contrast, I examine the interaction of reputation with costly signaling.

A closely related paper is Mathis, McAndrews, and Rochet (2009), hereafter MMR, which considers a reputation building model of credit rating agencies. MMR show that the reputation concerns are insufficient for imposing market discipline on rating agencies when they are impatient. Indeed, rating agencies may knowingly misreport reputation precisely when reputation is high. Although some aspects of my model resemble that of MMR, unlike MMR I allow the issuer to retain a portion of any issued security, creating an additional mechanism for the credible revelation of private information. MMR only allow the stage game payoff to depend on reputation through a single equilibrium strategy: the probability that the rating agency tells the truth. While rating agencies do not typically retain a meaningful stake in an issue that they have rated, securities issuers do. In this way, my model explores the interaction of reputation effects with a more formal

mechanism like costly retention, an interaction which is largely neglected in the literature.

Recent work by Gopalan, Nanda, and Yerramilli (2011) and Lin and Paravisini (2010) provides empirical evidence for such a link between hard incentives like costly retention and soft incentives like reputation using data on syndicated lending markets. Gopalan, Nanda, and Yerramilli show that lead arrangers who have lent to borrowers who have subsequently declared bankruptcy retain more of future syndication, syndicate fewer loans, and attract fewer participants. These findings are in line with the idea that reputation and lead arranger share can act as substitutes. Lin and Paravisini show that the capital contribution of a lead arranger in new syndicated loans increases when that arranger has previously lent to firms who subsequently commit fraud. The model they provide to motivate this finding treats reputation as an exogenously specified continuation value. In contrast, reputation in this paper arises endogenously and leads to different implications about the equilibrium transmission of information. For example, in their model, a combination of reputation effects and incentive contracting lead monitoring banks to implement high monitoring effort, whereas in my model a similar combination of reputation effects and costly retention leads the opportunistic type issuer to release false information in equilibrium. The empirical results presented by Lin and Paravisini and Gopalan, Nanda, and Yerramilli highlight the need for a better theoretical understanding of the trade-off between reputation and hard incentives like the one presented below.

2 The model

2.1 Assets, Agents and Actions

The economy is populated by an issuer and a measure of competitive investors. The issuer has a per period discount factor of $\gamma < 1$ and the investors have a per period discount factor of 1. The difference in the discount rates of the issuer and investors represents the relative impatience of the issuer and creates the gains from trade in the model. The relative impatience of the issuer could arise for a variety of reasons, including capital requirements and access to additional investment opportunities.

Time is infinite and indexed by $t = 0, 1, 2, \dots$. At the beginning of each period, the issuer is endowed with a single asset which produces a cash flow X_{t+1} at the beginning of the next period.

Each is one of two possible types. The asset is the good type with probability λ and the bad type otherwise. Good assets produce a cash flow of 1 while bad assets produce a cash flow $\ell \in (0, 1)$.¹ As will become apparent shortly, these cash flows imply perfect observability and allow for explicit characterizations of strategies and value functions.

At the start of period t , the issuer may sell a fraction $q_t \in [0, 1]$ of the asset to investors. The investors observe the quantity q_t at each date. The issuer has an incentive to sell the asset since investors are relatively patient. In practice, it may be impossible for the issuer to choose $q \in (0, 1)$ due to regulatory restrictions as considered in Downing, Jaffee, and Wallace (2009).² I consider this type of restricted signaling space as an extension to the basic model. In addition to choosing the level of retention, the issuer produces a *prospectus* that contains a report indicating the type of the asset.

2.2 Issuer Type, Reputation and Strategies

Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), reputation concerns arise due incomplete information about issuer preferences. Specifically, the issuer can be of two possible types. The *honest* type issuer always provides a truthful report and chooses a quantity of the asset to issue that maximizes expected proceeds from securitization and retained assets. In contrast, the *opportunistic* type issuer chooses both a report and quantity to maximize expected proceeds from securitization and retained assets. The formal definition of the objective functions of both types of issuer is given below in Definition 1. The reputation of the issuer is then summarized by the probability the investors place on the issuer being the honest type with an initial probability ϕ_0 . As time evolves, the investors update their beliefs about issuer type by observing the history of public information of the game, denoted \mathcal{H}_t . The investors' belief that the issuer is the honest type is thus given by

$$\phi_t = \mathbb{P}(\text{issuer is honest type} | \mathcal{H}_t).$$

¹Since the focus of the present paper is to jointly analyze the effect of costly signaling and reputation, I consider binary asset cash flows and abstract away from security design issues. For an analysis of security design under a rich set of cash flow distributions in a static setting see Nachman and Noe (1994) or DeMarzo and Duffie (1999).

²An alternative specification of the model would be to consider a different security design space. For example, the issuer could sell a claim with payoff $\min\{q_t, X_t\}$ to investors and retain a claim $\max\{X_t - q_t, 0\}$. Since such an alternative specification does not add to the richness of the results, I limit the analysis to equity claims in order to ease the exposition.

I will assume that the current quantity q_t and the reputation ϕ_t contain all the relevant information for the beliefs of the investor about the current asset type given a public report. That is, I assume that ϕ_t is a Markov state variable for the history of the game. In principle, the investors' beliefs about the current asset type could depend on the entire history of the game and in particular the path of past quantities. Because such a dependence makes the notation overly cumbersome, I do not consider it in the main text. In Appendix A, I allow investor beliefs to depend on the path of past quantities and reports and show that restricting attention to investor beliefs which are Markov in reputation does not rule out important equilibria.

The market must price the asset based on the report of asset type, the issued quantity, and the reputation of the issuer. If the issuer reveals that the asset is the bad type, then investors believe the asset is the bad type with probability one and the market will pay a price ℓ per unit for the offering. When the issuer reports that the asset is the good type, the market price per unit is given by the inverse demand curve, denoted $P(q, \phi) : [0, 1] \times [0, 1] \rightarrow [0, 1]$, which is the value investors place on an asset with issuer retention $1 - q$ and report g offered by an issuer with reputation ϕ . It is potentially costly for the issuer to choose $q < 1$ due to the impatience wedge between the issuer and investors. Accordingly, the level of retention represents a credible signaling mechanism for asset type. That is, investors should believe that an ABS with a relatively higher level of retention is backed by an asset of relatively higher quality. Thus, for each ϕ , the inverse demand curve $P(q, \phi)$ is downward sloping in q . This setup is a simplified version of the signaling mechanism of DeMarzo and Duffie (1999), with the exception that the demand curve may now depend on a report of asset quality and issuer reputation.

At every t , both issuer types form strategies conditional on their current reputation and the type of the period t asset. Before formally defining the issuer's strategy space, I make some convenient assumptions to simplify notation. Specifically, I assume that either type of issuer will always provide a truthful report when the asset is the good type and will always sell the entire asset when revealing that the asset is the bad type.³ Thus a *reporting strategy* is a function $\pi(\phi) : [0, 1] \rightarrow [0, 1]$ giving the probability of accurately reporting a bad asset's true type. A *quantity strategy* is a function

³This assumption is without loss of generality. The opportunistic type issuer would never report that a good type asset is the bad type in equilibrium since doing so would not increase reputation or instantaneous proceeds from asset sale. Thus, if the issuer reveals that the asset is the bad type, then the issuer's private value for the asset is always less than that of the investors, regardless of quantity sold, and selling the entire asset is optimal.

$Q(\phi) : [0, 1] \rightarrow [0, 1]$ giving the fraction of the asset sold to investors when the issuer reports that the asset is the good type. Recall that the set of admissible strategies for the issuer depends on issuer type. The set of admissible strategies for the opportunistic type, denoted \mathcal{A}_O , is simply the set of all possible strategy pairs defined above, whereas the set of admissible strategies for the honest type issuer is given by $\mathcal{A}_H = \{(\pi, Q) \in \mathcal{A}_O | \pi(\phi) = 1\}$. The restriction that defines \mathcal{A}_H reflects the fact that the honest type is committed to truthfully revealing a bad type asset.

To recapitulate, the timing of the game is as follows. At each date $t = 0, 1, 2, \dots$ the investors and issuer play a securitization stage game. At the beginning of a given period t , the cash flow from the asset sold on the previous date is realized and the players update the reputation of the issuer. Second, the current asset type is revealed to the issuer and who then chooses report and quantity strategies. Finally, investors buy the security at a price $qP(q, \phi)$ and the process starts anew. Figure 1 gives a timeline of the game with the sequence of actions that occur within a given period t .

2.3 Equilibrium

At any given period t the issuer maximizes the discounted expected value of proceeds from securitization plus the cash flow from retained assets. Formally an issuer with reputation ϕ has an instantaneous cash flow to an action (q, π) given by

$$U_t(q, \pi, P, \phi) = \lambda(qP + \gamma(1 - q)) + (1 - \lambda) [\pi\ell + (1 - \pi)(qP + \gamma\ell(1 - q))], \quad (1)$$

when facing the demand curve P at time t .

Definition 1. The quadruple (P, Q^H, π, Q^O) is an *equilibrium* if at all times t the following conditions are satisfied

1. The strategy of the honest type maximizes payoffs:

$$Q^H \in \arg \max_{\mathbf{q}} E_t \left[\sum_{n=t}^{\infty} \gamma^{n-t} U_n(q_n, 1, P, \phi_n) \right],$$

2. The strategy of the opportunistic type maximizes payoffs:

$$(Q^O, \pi) \in \arg \max_{\mathbf{q}, \pi} E_t \left[\sum_{n=t}^{\infty} \gamma^{n-t} U_n(q_n, \pi_n, P, \phi_n) \right],$$

3. ϕ_t is determined using Bayes rule whenever possible, and

4. Investors earn zero expected profits: $P(Q^i(\phi_t), \phi_t) = E [X_{t+1} | \phi_t, Q^i(\phi_t)]$ for $i \in \{O, H\}$.

Furthermore, an equilibrium is *separating* if $P(Q^H(\phi_t), \phi_t) = P(Q^O(\phi_t), \phi_t) = 1$.

To be clear, by using the term separating equilibrium, I am referring to equilibria that reveal the true type of the underlying *assets*, rather than the true type of the issuer. Indeed, it is impossible for the issuer to credibly reveal issuer type via a particular retention strategy in equilibrium, which will become apparent shortly. When referring to a given equilibrium as the least cost separating equilibrium, I mean the separating equilibrium that delivers the highest payoff to the issuer.

3 Equilibrium Analysis

3.1 The Game Without Reputation

To begin the analysis, I consider equilibria when the issuer is revealed to be the opportunistic type by the history of the game, so that $\phi_t = 0$. In this case, Bayes' rule implies that $\phi_s = 0$ for all $s \geq t$. In other words, a reputation of zero is an absorbing state. Thus, equilibria in this state will serve as an important input to the solution of the general case.

Before considering the repeated game, it is useful to consider equilibria of the static game. The natural restriction of Definition 1 for the static game replaces conditions (1) and (2) with a one period maximization problem. The following proposition summarizes the equilibria of the static game without reputation.

Proposition 1. *Suppose $\phi_0 = 0$ in a static game.*

- *A separating equilibrium exists and is given by $Q = \tilde{q}$, $\pi = 1$, and*

$$P(q, 0) = \begin{cases} 1 & q \leq \tilde{q} \\ \ell & q > \tilde{q} \end{cases}$$

for all $\tilde{q} \leq \hat{q} = \frac{\ell(1-\gamma)}{(1-\gamma)\ell}$. The least cost separating equilibrium is $\tilde{q} = \hat{q}$.

- *A pooling equilibrium exists and is given by $Q = 1$, $\pi = 0$, and $P(q, 0) = \lambda + (1 - \lambda)\ell$ for all $q \in [0, 1]$ if and only if $\gamma \leq \lambda + (1 - \lambda)\ell$.*

Other equilibria, in particular those with mixed reporting strategies in which $0 < \pi < 1$, may also exist. However, as will become clear when considering repeated versions of the static equilibria, mixed strategy equilibria of the static game without reputation are Pareto dominated by the least cost separating equilibrium or the pooling equilibrium, depending on the parameterization of the model.

The least cost separating equilibrium follows from the classic signaling intuition. The quantity \hat{q} is defined so that even when the market responds with a price per share of one for the quantity \hat{q} , the issuer with a bad type asset is better off selling the entire asset for a price of ℓ . At the same time, the issuer with a good type asset strictly prefers selling the quantity \hat{q} at a price per unit of one to retaining the entire asset. Such a quantity \hat{q} exists because the relative impatience of the issuer implies that retaining a fraction of the asset is more costly for the issuer when she has a good type asset than when possessing a bad type asset since the issuer is less patient than the investors.

The pooling equilibrium arises when the issuer's value for retaining a good type asset is less than the ex ante expected value of the asset to the investors. In a standard static signaling game, pooling equilibria are typically ruled out by the D1 refinement of Cho and Kreps (1987) which restricts off equilibrium beliefs. However, this refinement cannot readily be applied to the repeated game. At the same time, the pooling equilibrium of the static game only exists for parameterizations of the model in which the lemons problem is not too severe and markets could function without any means of information transfer. Specifically, a repetition of the pooling equilibrium would lead to payoffs equal to the full information case. For the remainder of the paper I will assume that parameters are such that the pooling equilibrium does not exist:

Assumption 1. The parameters of the model do not admit a pooling equilibrium of the static game: $\lambda + (1 - \lambda)\ell \leq \gamma$.

I can now consider equilibria of the repeated game for $\phi_t = 0$. Since investors' strategies were assumed to be Markovian in ϕ , there is no mechanism to make the issuer's current payoffs depend on past actions. Consequently, public reports of asset quality are no more credible than in the static version of the game. This observation leads to the following lemma.

Lemma 1. *Suppose $\phi_t = 0$. Then $\phi_s = 0$ for all $s \geq t$, and a strategy pair (Q, π) and price schedule $P(q, 0)$ are an equilibrium if and only if they are an equilibrium of the static game.*

In principal, investors could play punishment strategies in which past quantities affect beliefs about current asset types even though reputation is fixed at zero. The result of such a strategy would be that a truth telling equilibrium may emerge even though $\phi_0 = 0$. This type of equilibrium behavior is sometimes thought of as resulting from reputation, however this is not the concept of reputation I consider here. For completeness, I consider the possibility of punishment strategies

in Appendix A. With punishment strategies, the set of possible equilibria for the repeated game with no reputation would include a truth telling equilibrium, provided the parameters satisfy a given restriction. It turns out that this restriction is also a necessary condition for the existence of a truth telling equilibrium for the case with positive reputation. Thus, by assuming away punishment strategies, I have only eliminated truth telling equilibrium for the no reputation state. In particular, introducing punishment strategies does not allow for additional truth telling equilibria for the positive reputation state.

Proposition 1 and Lemma 1 imply there are multiple equilibria for the repeated game without reputation. Since the structure of equilibria with positive reputation will hinge on what equilibrium strategies obtain if the issuer's reputation falls to zero, it is necessary to have a consistent means of selecting an equilibrium in this state. Again, the D1 refinement of Cho and Kreps (1987) is not clearly applicable in the repeated setting. Instead, I rely on the fact that conditional on parameters, a single equilibrium delivers the most value to the issuer. Since investors are competitive and always earn zero profits in expectation, such an equilibrium is Pareto dominant. Under Assumption 1, the least cost separating equilibrium of Proposition 1 delivers the highest equilibrium payoffs to the issuer. Thus, I will assume the least cost separating equilibrium obtains for the no reputation state.

3.2 Reputation Dynamics and Optimization

Now that there is a fixed equilibrium for the repeated game in the no reputation state, I can proceed to solve for the dynamics of reputation in equilibrium. I start with the following important result which will be key in deriving equilibrium.

Lemma 2. *The honest issuer and the opportunistic issuer always issue the same quantity, $Q^H(\phi) = Q^O(\phi)$ for all ϕ .*

The intuition behind Lemma 2 is as follows. The opportunistic issuer and the honest issuer both value instantaneous payoffs from the securitization of a good asset identically. Moreover, the opportunistic issuer values a higher reputation weakly more than the honest issuer. Therefore, any quantity strategy which increases reputation will be at least as attractive to the opportunistic type as it is to the honest type. This implies that the quantity issued is not a credible signal of *issuer* type and cannot contain any new information about the type of the issuer. In particular, this

means that reputation is only updated during the reputation updating phase, and not during the securitization phase. This will be very useful for the analysis since it means that the issuer need not take into account the effect of quantity strategy on future reputation. In addition it simplifies the analysis of the reputation updating process.

Given Lemma 2, it is straightforward to derive the dynamics of reputation in terms of the report and ex post performance of the asset. Let $f : \{g, b\} \times \{\ell, 1\} \times [0, 1] \rightarrow [0, 1]$ denote the reputation updating function. Using Bayes' rule whenever possible, I have

$$f(g, 1, \phi) = \phi^S = \phi \quad (2)$$

$$f(g, \ell, \phi) = \phi^F = 0 \quad (3)$$

$$f(b, \ell, \phi) = \phi^B = \frac{\phi}{\phi + \pi(1 - \phi)}. \quad (4)$$

The optimization problem faced by the issuer can now be simplified given the reputation updating function and the fact that the opportunistic and honest type issuers always choose the same retention strategy. Since Lemma 2 implies that the honest type issuer and opportunistic type issuer play the same retention strategy, I drop the superscript and refer to a retention strategy as simply Q . Consider the opportunistic issuer's problem. Let $V(\phi|P)$ denote the value function of the opportunistic type issuer when facing the demand schedule P , and let $V_G(\phi|P)$ and $V_B(\phi|P)$ denote the value functions when the opportunistic type issuer faces the demand schedule P and is endowed with a good asset or a bad asset respectively. Then $V(\phi|P) = \lambda V_G(\phi|P) + (1 - \lambda)V_B(\phi|P)$, and $V_G(\phi|P)$ and $V_B(\phi|P)$ satisfy the following system of Bellman equations

$$V_G(\phi|P) = \max_{(\pi, Q) \in \mathcal{A}_O} \{ \gamma(1 - Q) + QP(Q, \phi) + \gamma V(\phi^S|P) \}. \quad (5)$$

$$V_B(\phi|P) = \max_{(\pi, Q) \in \mathcal{A}_O} \{ \pi(\ell + \gamma V(\phi^B)) + (1 - \pi)(\gamma(1 - Q)\ell + QP(Q, \phi) + \gamma V(0)) \}. \quad (6)$$

3.3 Separating Equilibria

For an equilibrium to be separating, investors must be able to perfectly infer the type of asset by observing the quantity issued and the report given. In general, such perfect inference can arise either because the quantity issued maps perfectly to the type of asset, or the loss of continuation

value from being exposed as the opportunistic type is so great that the issuer will never misreport a bad type asset. I refer to equilibria of this latter type as *truth-telling*. Specifically an equilibrium is truth telling if $Q(\phi) > \hat{q}$ for some $\phi > 0$ and $\pi(\phi) = 1$ for all $\phi > 0$. The truth-telling equilibrium is desirable in that it allows for the credible revelation of issuer private information with less issuer retention. Indeed, when a truth telling equilibrium exists with $Q(\phi) = 1$ for all $\phi > 0$, it delivers payoffs to the issuer equal to what would be received in a first best setting. However, as the following proposition shows, a restriction on parameters is needed for a truth-telling equilibrium to obtain.

Proposition 2 (Folk Theorem). *Suppose $\phi_0 > 0$. There exists a truth telling equilibrium if and only if $\gamma \geq \frac{1}{\lambda + \ell}$.*

The restriction on parameters required for the existence of a truth telling equilibrium depends on the instantaneous gains to the issuer from misreporting a bad type asset and the loss in continuation value from being identified as the opportunistic type. Suppose the investors always believe the report of the issuer. Then an opportunistic issuer may receive a price of 1 for a bad type asset for one period and be known to be the opportunistic type thereafter. Such a deviation is profitable if and only if

$$\underbrace{1 - \ell}_{\text{gain in proceeds}} \leq \underbrace{\frac{\gamma \lambda (1 - \hat{q})(1 - \gamma)}{1 - \gamma}}_{\text{loss in continuation value}}. \quad (7)$$

This restriction simplifies to $\gamma \geq \frac{1}{\lambda + \ell}$. One interpretation of this restriction is that the issuer must be sufficiently patient so as to make a loss in continuation value severe enough to provide incentives to always accurately report a bad type asset. In this way, the conditions guaranteeing the existence of a truth telling equilibrium are similar to a classic folk theorem.⁴

When the issuer is impatient, the truth telling equilibrium cannot be supported even with positive initial reputation. However, the repeated version of the least cost separating equilibrium of the game without reputation still obtains.

Proposition 3. *The least cost separating equilibrium of Proposition 1 where $P(q, \phi) = P(q, 0)$ for*

⁴Here the assumption that ϕ is a Markov state variable has some bite. If investor beliefs could depend on the path of past signals, truth telling could be supported in equilibrium without positive reputation. Moreover the parameter restriction required would be slightly weaker.

all ϕ is an equilibrium of the game for all $\phi_0 \in [0, 1)$.

The cost to the issuer to credibly reveal her information about the quality of the asset to the investors thus depends importantly on the parameters of the model. When the issuer is relatively patient, or $1 - \lambda\ell > \lambda\gamma$, information can be revealed credibly via a public report without cost, so long reputation is strictly positive. This is the truth-telling equilibrium. If the issuer is relatively impatient, information can still be credibly revealed, however, doing so requires the issuance of a quantity strictly less than one, which is costly. Figure 2 shows a partition of the parameter space highlighting the region for which truth telling is supported in equilibrium. Region I corresponds to parameters for which the truth telling equilibrium are not supported, whereas in Region II, truth telling is supported. A natural question is whether higher equilibrium payoffs for the issuer may be supported in Region I by considering mixed reporting strategies. In other words, is there an equilibrium for parameters in Region I in which the issuer achieves higher payoffs and does not always perfectly reveal the type of the asset. This possibility is considered in the next subsection.

3.4 Mixed Strategy Equilibria

In light of the previous subsection's results, I look for equilibria in which $0 < \pi(\phi) < 1$ for some $\phi \in [0, 1]$, or mixed strategy equilibria for parameters under which truth telling is not supported in equilibrium. Specifically, I impose the following assumption to rule out truth telling.

Assumption 2. The parameters of the model do not support the truth telling equilibrium: $\gamma < \frac{1}{\lambda + \ell}$.

Given this assumption, a mixed strategy equilibrium obtains in which issuer retention is an informative signal of, but not a sufficient statistic for, the issuers private information about asset type. I detail the properties of this equilibrium in the following proposition.

Proposition 4. *Suppose Assumptions 1 and 2 hold, then there exist thresholds $\underline{\phi}$, $\hat{\phi}$, $\bar{\phi}$, and an equilibrium in which the opportunistic issuer*

- *plays the separating equilibrium strategies for low levels of reputation: $Q(\phi) = \hat{q}$ and $\pi(\phi) = 1$ for $\phi \leq \underline{\phi}$,*
- *sells a larger portion of the asset than the separating quantity and misreports a bad type asset with positive probability for mid-range levels of reputation: $\hat{q} < Q(\phi) < 1$ for $\underline{\phi} < \phi < \hat{\phi}$, and $0 < \pi(\phi) < 1$ for $\underline{\phi} < \phi < \bar{\phi}$,*

- sells the entire asset and always chooses to misreport a bad type asset for high levels of reputation: $Q(\phi) = 1$ for $\phi \geq \hat{\phi}$, and $\pi(\phi) = 0$ for $\phi \geq \bar{\phi}$.

Moreover, $Q(\phi)$ is weakly increasing in ϕ and $\pi(\phi)$ is weakly decreasing in ϕ .

For completeness I include a sketch of the construction of the equilibrium given above. The reader may choose to skip this construction and proceed directly to section 3.5.

Construction of mixed strategy equilibrium. To begin the analysis, I assume a candidate equilibrium demand curve $P(q, \phi)$ is a step function of q of the following form

$$P(q, \phi) = \begin{cases} 1 & q \leq \hat{q} \\ p^*(\phi) & \hat{q} < q \leq q^*(\phi) \\ \ell & q > q^*(\phi) \end{cases} \quad (8)$$

where $p^*(\phi)$ and $q^*(\phi)$ are continuous in ϕ such that $p^*(0) = 1$ and $q^*(0) = \hat{q}$.⁵ The inverse demand curve P , depicted in Figure 3, is consistent with the investor beliefs that only an issuer with a good type asset would ever offer a quantity q less than \hat{q} , while an issuer with either type asset might offer a quantity q greater than \hat{q} but less than some level $q^*(\phi)$, and only an issuer with a bad type asset would choose a quantity q greater than $q^*(\phi)$. Given the demand curve $P(q, \phi)$, an issuer with a good asset and reputation ϕ will choose a quantity $Q(\phi) \in \{\hat{q}, q^*(\phi)\}$. To see this, observe that the issuer's proceeds are increasing over each subinterval of quantity given in the definition of the demand curve, while the continuation value is fixed. This argument implies that $q^*(\phi)$ is a natural candidate equilibrium quantity strategy given the inverse demand curve P . I denote the candidate equilibrium reporting strategy as $\pi^*(\phi)$. In addition, I let the discounted loss in issuer value associated with a drop in reputation from ϕ to 0 be denoted $L(\phi) = \gamma(V(\phi) - V(0))$. Since the candidate equilibrium strategies are known at $\phi = 0$, it is trivial to calculate

$$V(0) = \frac{1}{1-\gamma}(\lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell). \quad (9)$$

Thus deriving the value function $V(\phi)$ is equivalent to deriving the discounted loss function $L(\phi)$.

⁵By assuming that the opportunistic issuer plays the strategies that arise in the static signaling game when known to be the opportunistic type, I am explicitly forcing this derivation to yield an equilibrium consistent with Lemma 1 as issuer reputation decreases to zero.

Now that I have assumed a particular functional form for the demand schedule P , I can further simplify the maximization problem faced by the issuer. Specifically, I identify the following four conditions that must hold in any equilibrium in which the quantity strategy is $Q(\phi) = q^*(\phi)$, the reporting strategy is $\pi^*(\phi)$, and the demand schedule is given by equation (8),

$$q^*(\phi)p^*(\phi) - \hat{q} \geq \gamma(q^*(\phi) - \hat{q}), \quad (10)$$

$$q^*(\phi)p^*(\phi) - \ell \geq L(\phi^B) - (1 - q^*(\phi))\gamma\ell, \quad (11)$$

$$p^*(\phi) = \ell + \frac{\lambda(1 - \ell)}{\lambda + (1 - \lambda)(1 - \phi)(1 - \pi^*(\phi))}, \quad (12)$$

$$L(\phi) = \frac{\gamma}{1 - \gamma\lambda} [q^*(\phi)(p^*(\phi) - \gamma\bar{x}) - (1 - \gamma)(\lambda\hat{q} + (1 - \lambda)\ell)], \quad (13)$$

where $\bar{x} = \lambda + (1 - \lambda)\ell$. Inequality (10) states that an issuer with a good asset must weakly prefer the quantity strategy $Q(\phi) = q^*(\phi)$ to the strategy $Q(\phi) = \hat{q}$. Similarly, inequality (11) states that an issuer with a bad type asset must weakly prefer the retention strategy $Q(\phi) = q^*(\phi)$ and reporting strategy $\pi^*(\phi)$ to the strategy $\pi(\phi) = 1$. Equation (12) follows directly from Bayes' rule and the fact that investors earn zero profits in expectation as specified in the fourth condition of Definition 1. This equation must hold for all ϕ , so that to characterize an equilibrium, it is enough to find the quantity-price pair $(q^*(\phi), p^*(\phi))$. Finally, equation (13) follows from the maximization problem described by equations (5) and (6).

The next step in constructing a candidate equilibrium is to divide the interval $\phi \in [0, 1]$ into subintervals over which the inequalities (10) and (11) either bind, or are slack. To this end, I assume there exists $\underline{\phi}$ and $\bar{\phi}$ such that $\pi^*(\phi) = 1$ and $q^*(\phi) = \hat{q}$ for $\phi \leq \underline{\phi}$ and $\pi^*(\phi) = 0$ and for $\phi \geq \bar{\phi}$. The one-shot deviation principle implies that inequality (11) must bind whenever $0 < \pi^*(\phi) < 1$, hence it must bind whenever $\underline{\phi} < \phi < \bar{\phi}$. I assume there exist $\hat{\phi}$ such that inequality (10) binds for $\phi \leq \hat{\phi}$ and $q^*(\phi) = 1$ for $\phi \geq \hat{\phi}$. It will turn out that $\hat{\phi} \leq \bar{\phi}$, so that the problem of deriving equilibrium strategies can be broken up into the four subintervals of reputation: $[0, \underline{\phi}]$, $(\underline{\phi}, \hat{\phi}]$, $(\hat{\phi}, \bar{\phi}]$, and $(\bar{\phi}, 1]$. Figure 4 shows the proposed decomposition of the interval with the corresponding constraints on the candidate equilibrium strategies π^* and q^* .

The assumption that inequality (10) binds for $\underline{\phi} < \phi < \hat{\phi}$ amounts to restricting attention to the corners of the space of incentive compatible strategies. Once inequality (11) binds, one

can interpret inequality (10) as placing a lower bound on the set of possible equilibrium reporting strategies. For example, suppose ϕ^B is fixed and $L(\phi^B)$ is known, then the reporting strategy is a known function

$$\pi^*(\phi) = \frac{\phi(1 - \phi^B)}{\phi^B(1 - \phi)}. \quad (14)$$

In other words, for each level of reputation $\phi \in (0, \phi^B)$ there is a single reporting strategy $\pi^*(\phi)$ for which reporting a bad asset will result in an increase of reputation to ϕ^B . Moreover, since (11) binds and $L(\phi^B)$ is known, inequality (10) can be rearranged to get

$$\pi(\phi) \geq 1 - C \frac{1}{1 - \phi}, \quad (15)$$

where C is some constant which may depend on ϕ^B . Figure 5 plots equation (14) and inequality (15) and illustrates that by assuming inequality (10) binds, I am choosing the minimal admissible π^* for each ϕ^* .

With the partition of the unit interval of reputation described above and the four relations which must hold in equilibrium given by (10), (11), (12), and (13), the problem becomes one of solving a system of equations. The important caveat to this approach is that to solve the equations for equilibrium strategies at a given level of reputation ϕ , I must first know the discounted loss $L(\phi^B)$ at the level of reputation which would arise from a truthful report of a bad type asset. Since $\phi^B \geq \phi$ for all ϕ , I can solve the problem by working downwards from $\phi = 1$.

For $\phi \in [\bar{\phi}, 1]$, the equilibrium strategies are assumed to be $q^*(\phi) = 1$ and $\pi^*(\phi) = 0$. This means the equilibrium price $p^*(\phi)$ and associated discounted loss function $L(\phi)$ follow directly from equations (12) and (13). For convenience, let $L_1(\phi)$ denote the solution to (13) when $q^*(\phi) = 1$ and $\pi^*(\phi) = 0$.

For $\phi \in [\hat{\phi}, \bar{\phi})$, the equilibrium retention strategy is assumed to be $q^*(\phi) = 1$, however the equilibrium price must be calculated. I assume for the time being that $\phi^B \geq \bar{\phi}$ for all $\hat{\phi} \leq \phi < \bar{\phi}$.⁶ Thus, for $\phi \in [\hat{\phi}, \bar{\phi})$, the equilibrium price $p^*(\phi)$ of a reportedly good asset solves the equation

$$p^*(\phi) = L_1 \left(\left(\frac{(1 - \lambda)(p^*(\phi) - \ell)}{p^*(\phi) - \ell - \lambda(1 - \ell)} \right) \frac{1}{\phi} \right). \quad (16)$$

⁶This amounts to a parameter restriction, detailed in Appendix B. This assumption can be relaxed, although with a considerable amount of extra algebra.

Equation (16) follows from substituting equation (12) and the reputation updating function into inequality (11) (which must bind). Again for convenience, let $L_2(\phi)$ denote the solution to equation (13) when $p^*(\phi)$ solves equation (16) and $q^*(\phi) = 1$.

Finally, I characterize the discounted loss function $L(\phi)$, reporting strategy π^* , and quantity strategy $q^*(\phi)$ of the opportunistic type issuer for $\underline{\phi} \leq \phi < \hat{\phi}$. To do so, I construct a decreasing sequence starting at $\hat{\phi}$ such that each element of the sequence is the level of reputation for which the decision to truthfully report a bad type asset would lead to an increase in reputation to the preceding element of the sequence. Formally, let $\phi(n)$ be a sequence given by

$$\phi(0) = \hat{\phi} \tag{17}$$

$$\phi^B(n) = \phi(n-1) \tag{18}$$

I can combine (10) and (11) to get

$$q^*(\phi)p^*(\phi) + \gamma(1 - q^*(\phi)\bar{x}) = \lambda(\hat{q} + \gamma(1 - \hat{q})) + (1 - \lambda)(\ell + L(\phi^B)). \tag{19}$$

Equation (19) can be thought of as a combined incentive compatibility constraint. It states that the expected one period proceeds from playing the strategy $(q^*(\phi), \pi^*(\phi))$ is exactly equal to the expected loss from doing so. Equations (13) and (19), along with the definition of the sequence $\phi(n)$, imply

$$L(\phi(n)) = \beta^n L_2(\hat{\phi}) \tag{20}$$

where $\beta = \frac{\gamma(1-\lambda)}{1-\gamma\lambda}$. Now suppose that the values $\phi(k)$ are known for $k \leq n-1$. Then the strategy pair $(q^*(\phi(n)), \pi^*(\phi(n)))$ and the level of reputation $\phi(n)$ solve the following three equations

$$q^*(\phi(n))p^*(\phi(n)) - \hat{q} = \gamma(q^*(\phi(n)) - \hat{q}), \tag{21}$$

$$q^*(\phi(n))p^*(\phi(n)) - \ell = \beta^{n-1}L_2(\hat{\phi}) - (1 - q^*(\phi(n))), \gamma\ell \tag{22}$$

$$p^*(\phi(n)) = \ell + \frac{\lambda(1 - \ell)\phi(n-1)}{\lambda\phi(n-1) - (1 - \lambda)\phi(n)}. \tag{23}$$

Equation (21) defines the price quantity pairs such that the issuer with a good type asset is indifferent between issuing the quantity q^* at a price per unit p^* and issuing the quantity \hat{q} at a price

per unit ℓ . Equation (22) defines the price quantity pairs such that the issuer with a bad type is indifferent between issuing the quantity q^* at a price per unit p^* resulting in a reputation of zero, and issuing the quantity one, at the price per unit ℓ , resulting in a reputation of $\phi(n-1)$. Figure 6 illustrates the solution to the system of equations (21)-(23). Equations (21) and (22) both define p^* as a function of q^* . It is straightforward to show that these two functions satisfy a single crossing property and thus admit a unique solution for the quantity price pair $(q^*(\phi(n)), p^*(\phi(n)))$, as demonstrated by Figure 6(a). Similarly, equation (23) defines p^* as a function of ϕ . Again, it is straightforward to show that this function is decreasing in ϕ . Thus by setting the right hand side of equation (23) equal to the solution for $p^*(\phi(n))$ from equations (21) and (22), a unique solution for $\phi(n)$ obtains, as demonstrated by Figure 6(b).

The above argument shows how to characterize the candidate equilibrium on a sequence of levels of reputation defined by (18) up to the solution of $L(\phi)$ for $\phi \geq \bar{\phi}$. Applying the reverse of the sequence argument to values of $\phi(n) < \phi < \phi(n-1)$ yields a characterization of the candidate equilibrium for levels of reputation not lying in the above sequence. Namely, for each $\phi \in (\phi(n), \phi(n-1))$ there exists a $\tilde{\phi} \in (\hat{\phi}, 1]$ such that an issuer with current reputation ϕ will have reputation $\tilde{\phi}$ after choosing to report a bad type asset n times in a row.

The complete derivation of this mixed strategy equilibrium is detailed in the Appendix B. The key step is to show that the system of equations given above has a solution for each sub-interval described above. Once this is shown, it is simple to verify that the proposed strategies are indeed an equilibrium by using the single deviation principal for repeated games.

□

3.5 Discussion and Empirical Implications

The model admits multiple equilibria. This means that in order to understand the empirical implications of the model, one must have a consistent method of selecting among different equilibria. For static signaling games, the D1 refinement of Cho and Kreps (1987) provides such a selection criterion. However, it is not clear how to apply the D1 refinement criteria in the current dynamic setting, so I take the following simple approach. Observe that by construction, the mixed strategy equilibrium delivers at least as much per period payoff as the separating equilibrium when reputation alone is not sufficient to implement full market discipline. This observation leads directly to

the following corollary of Proposition 4:

Corollary 1. *Suppose Assumptions 1 and 2 hold, then, the mixed strategy equilibrium delivers weakly greater value to both issuer types for all levels of reputation than the separating equilibrium.*

It is also possible to construct mixed strategy equilibria that are quantitatively distinct from the equilibrium presented of Proposition 4. However, such equilibria will have qualitatively similar features that generate the same empirical predictions.

The first feature of the equilibrium of Proposition 4 that warrants discussion is the relationship between the quantity of the asset sold (or retained) by the issuer. The equilibrium quantity issued in the mixed strategy equilibrium q^* , is an increasing function of reputation. Figure 7 plots q^* versus reputation ϕ . For $\phi \leq \underline{\phi}$, reputation is too low to make misreporting a bad type asset attractive to the opportunistic underwriter, so $Q = \hat{q}$ and the equilibrium reduces to the static signaling equilibrium of Proposition 1. For $\underline{\phi} \leq \phi \leq \hat{\phi}$ the equilibrium quantity must make the opportunistic issuer indifferent, conditional on facing a good type asset, between signaling the true type of the asset by choosing the quantity \hat{q} versus issuing q^* . This implies that q^* must be a strictly increasing function of ϕ for $\underline{\phi} \leq \phi \leq \bar{\phi}$ since the equilibrium price p^* is a decreasing function of ϕ over this interval. The equilibrium quantity issued reaches one at $\hat{\phi}$. The shape of q^* leads directly to the following implication:

Implication 1. *Reputation and signaling are substitutes in the sense that the quantity an issuer retains decreases with her reputation.*

Implication 1 is useful in that it provides a testable empirical prediction. If both signaling and reputation are available as a means of credible information transfer and the market conditions are such that reputation alone does not suffice, then the amount of signaling employed by the issuer should be decreasing in reputation, conditional on the issuer's public report of asset quality. This effect directly leads to an implication on the slope of the demand curve.

Implication 2. *Prices are more sensitive to the presence of retention for low reputation issuers than for high reputation issuers.*

Prediction 2 is particularly important for empirical studies that look for evidence of signaling motives in retention. For example, Garmaise and Moskowitz (2004), do not find any evidence that sellers of commercial real estate signal private information by retaining a stake in the sold property.

To come to this conclusion, they regress price on a dummy variable for “vendor-to-buyer” financing and find no effect.⁷ However, if their sample contains many high reputation sellers, then the model presented above would predict that they should find a larger effect of retention on prices if they control for seller reputation.

The next interesting implication concerns the flow of information in equilibrium. In a standard static signaling model, the quantity issued is a perfect signal of asset quality. However, in the equilibrium presented above, quantity is a signal of issuer reporting strategy. This interpretation of Q means that given a level of reputation ϕ , the investors can infer the reporting strategy $\pi(\phi)$ of the opportunistic issuer by observing issuer retention. When investors observe a relatively large Q , they can infer that the probability the issuer will misreport a bad asset is high, and will price assets accordingly. Hence, the opportunistic issuer faces a trade-off between choosing a perfect signal of asset quality, which implies no lemons discount, and choosing an imperfect signal of asset quality, which gives the issuer the ability to sell a larger fraction of the asset to the investors. This trade-off importantly depends on the current level of issuer reputation. Figure 8 shows the equilibrium reporting strategy of the opportunistic issuer. For $\phi \leq \hat{\phi}$, an increase in reputation increases instantaneous gains to misreporting a bad type asset hence the probability the issuer will misreport increases. The issuer offsets the price impact of this increase by retaining less of the asset. The shape of π leads to the following implication.

Implication 3. *An asymmetry of information should persist ex post. That is, investors cannot infer the issuer’s private information from retention alone.*

Implication 3 implies that observing asset performance after issuance provides investors with valuable information. This is important in the context of allegation that issuers engage in fraudulent activities. If investors could perfectly infer an issuers private information from observing issued quantity, there would be no scope for fraud. However, since the mixed strategy equilibrium implies that an ex post asymmetry of information is possible, the issuer knowingly misreports asset type and fraud is feasible.

The fact that an asymmetry of information can persist post issuance leads to an interesting

⁷In many commercial real estate transactions the seller provides some financing to the buyer via the so called vendor-to-buyer financing. In this way, the seller maintains some exposure to the value of the underlying property. As such, the presence of this type of financing might indicate that the seller is trying to signal to the buyer that the property is high quality.

observation about the investors' beliefs about asset quality as it relates to issuer reputation. The opportunistic issuer's equilibrium reporting strategy implies that p^* is a U-shaped function of ϕ . Figure 9 plots the equilibrium price function. For $\phi \leq \underline{\phi}$, the opportunistic issuer never reports that a bad type asset is the good type, hence the equilibrium price for a reportedly good type asset must be one. For $\underline{\phi} \leq \phi \leq \bar{\phi}$, the equilibrium price is a decreasing function of reputation. For relatively low (high) levels of reputation, the potential gain from misreporting a bad type asset is relatively small (large) since any probability that the issuer misreports a bad type asset impacts prices more (less) at low (high) levels of reputation. Hence, in equilibrium the investors place a low probability on the issuer misreporting a bad type asset for low levels of reputation and the equilibrium price is high. For $\phi \geq \bar{\phi}$ the investors know that an opportunistic issuer will always misreport a bad type asset, but by definition the probability that investors are facing an opportunistic issuer decreases as ϕ increases, so equilibrium price increases with ϕ for $\phi \geq \bar{\phi}$. The shape of p^* leads to the following implication.

Implication 4. *The investors' beliefs about the quality of the asset is U-shaped in the issuer's reputation, conditional on the report of asset quality.*

Implication 4 highlights an important difference between my model and that of MMR. In their paper, there is no other mechanism that allows information transfer besides reputation. As a result, beliefs about the credibility of the information sender are monotonic in reputation. In my setting, the issuer may always resort to retention as a means of conveying information and credibility can depend non-monotonically on reputation. This difference follows from the important institutional differences between rating agencies and securities issuers, i.e. rating agencies have only their reputations at stake, while securities issuers can also retain risk.

The final aspect of the mixed strategy equilibrium which requires some discussion is the upper threshold $\bar{\phi}$ that represents the level of reputation past which the opportunistic issuer will “cash in” on reputation. For $\phi \geq \bar{\phi}$ equilibrium proceeds from securitization will be high regardless of the opportunistic issuer's reporting strategy, so that if the opportunistic issuer achieves a reputation $\phi \geq \bar{\phi}$, that issuer will strictly prefer to report all assets as the good type. This outcome of the model is somewhat paradoxical when considering the benefits of reputation effects for providing incentives for truth-telling. Once an opportunistic issuer has a high enough reputation the optimal strategy under the mixed strategy equilibrium calls for a complete lack of reporting discipline.

4 Extensions

4.1 Binary Signal Space

This section considers the case in which the issuer may not divide the asset and hence must sell the entire asset to investors or retain it. I maintain the definition equilibrium, with the additional restriction of the strategy space that $Q \in \{0, 1\}$. Assumption 1 implies that an the issuer with zero reputation will only sell bad type assets since there is no mechanism that allows the separation of good type assets from bad and the pooled price is lower than the issuer's value for retaining the asset. Thus, the only equilibrium of the game when reputation reaches zero is for the issuer to retain all good assets and sell all bad assets. This means that losing reputation in this setting results in a greater loss of value than when the issuer can signal private information through costly information regardless of reputation. As a result, the indivisibility of assets acts as a commitment mechanism allowing the loss of reputation to be a more powerful incentive mechanism.

First the existence of a truth telling equilibrium in this new setting is considered.

Proposition 5. *A truth telling equilibrium exists if and only if $\gamma \geq \frac{\lambda}{1-\ell}$.*

Comparing Proposition 5 with Proposition 2, it is clear that a looser restriction on parameters is required to implement truth telling when the issuer's quantity choice is restricted to all or nothing. This is precisely because reputation is a more powerful incentive mechanism in this new setting and hence the issuer can credibly commit to truthfully reporting asset type for a wider range of parameters.

It remains to characterize an equilibrium for parametrizations which do not allow the existence of a truth telling equilibrium, i.e. when $\gamma < \frac{\lambda}{1-\ell}$. The method for constructing is essentially the same as in Section 3.4. First, a particular form for the demand schedule is assumed:

$$P(\phi, q) = \begin{cases} 1 & \text{for } q = 0 \\ p^*(\phi) & \text{for } q = 1. \end{cases} \quad (24)$$

Then analogs to equations (10), (11), and (13) in this setting are

$$p^*(\phi) \geq \gamma \tag{25}$$

$$p^*(\phi) \geq \ell + \gamma(V(\phi^B) - V(0)) \tag{26}$$

$$L(\phi) = \frac{\gamma}{1 - \gamma\lambda}(p^*(\phi) - (1 - \gamma)V(0)) \tag{27}$$

for $Q = 1$. Inequality (25) states that the issuer must prefer to sell a good type asset at price $p^*(\phi)$ rather than retain it. Inequality (26) states that the issuer must prefer to misreport a bad type asset and sell it at price $p^*(\phi)$ rather than accurately report it and sell it at price ℓ . Equation (27) simply follows from equations (5) and (6). Inequalities (25) and (26) only apply to levels of reputation for which there is a positive probability the issuer misreports a bad type asset and still chooses to sell a good type asset. I assume there exists a subinterval for which (26) binds, so that the issuer misreports the bad type asset with probability strictly between zero and one. Then finding equilibrium strategies reduces to a problem of solving a system of equations in a certain number of unknowns. The following proposition summarizes the resulting mixed strategy equilibrium:

Proposition 6. *Suppose $1 - \ell > \gamma\lambda$, then there exists $\underline{\phi}'$ and $\bar{\phi}'$ and an equilibrium in which the opportunistic issuer*

- *retains good assets and sells bad assets for low levels of reputation, that is*

$$Q(\phi) = 0 \text{ for } \phi \leq \underline{\phi}',$$

- *sells both asset types and misreports a bad type asset with positive probability for mid-range levels of reputation, that is*

$$Q(\phi) = 1, \text{ and } 0 < \pi(\phi) < 1 \text{ for } \underline{\phi}' < \phi < \bar{\phi}',$$

- *sells both asset types and always misreports a bad type asset for high levels of reputation, that is*

$$Q(\phi) = 1 \text{ and } \pi(\phi) = 0 \text{ for } \phi \geq \bar{\phi}'.$$

Moreover, $\pi(\phi)$ is weakly decreasing.

Restricting of quantities to the set $\{0, 1\}$ substantially changes the equilibrium reporting strategy relative to the general case. Figure 10 compares the equilibrium reporting strategies for Propositions 4 and 6. The first difference is that there is a discontinuity in the reporting strategy for the indivisible asset case. This discontinuity exists because quantities cannot continuously adjust with

reputation to make the issuer indifferent between retaining a good asset and selling it in its entirety. Notably, the indivisibility of assets causes the issuer to refrain from sending inaccurate reports at levels of reputation for which would do if part of the asset could be retained. More formally, the reputation threshold $\underline{\phi}'$ below which the issuer never misreports under the assumption of an indivisible asset is greater than $\underline{\phi}$, the same threshold under the assumption of a fully divisible asset. Similarly, the issuer will send an inaccurate report with probability 1 for lower levels of reputation when the asset is indivisible than when the asset is divisible.

The differences between reporting strategies described above have important implications for policies that aim to reduce the probability that the issuer sends an inaccurate report. Disallowing issuer retention is actually one possible mechanism to decrease the amount of lying in equilibrium. The drawback to such a restriction is that issuer payoffs will decrease. In addition, for low levels of reputation the market will be a market for lemons.⁸ Thus the issuer, conditional on having a low initial reputation, receives a lower value for the game when the asset is indivisible than when the asset is perfectly divisible. However, this means that the cost of a loss in reputation is potentially greater when the issuer is not free to choose quantity.

If the opportunistic issuer starts with a high enough reputation, an inaccurate report will be sent at least part of the time. In this case the relationship between the probability of sending an inaccurate report and the divisibility of assets is not clear. For intermediate levels of reputation the issuer will want to sell only part of the asset to investors but is constrained to sell the entire asset and thus may lie with greater frequency. However for high levels of reputation, the threat of losing reputation becomes more powerful for the indivisible asset case because the market will revert to a market for lemons when the issuer's reputation is lost. As a result, the issuer has less of an incentive to send an inaccurate report for the indivisible asset case when reputation is high.

4.2 Risky Assets

In this section I consider a richer specification for the cash flow of assets. As above, I assume there are two types of assets: good and bad. Both asset types produce a cash flow in the next period

⁸Downing, Jaffee, and Wallace (2009) show that the “to be announced” MBS market is indeed a market for lemons. This finding follows from the indivisibility of assets and the anonymity of issuers. The implication of the current model is that such a finding can persist in a setting with reputation effects. This does not depend on the stylized type of reputation considered here and would obtain even when punishment strategies are allowed since the parameters are such that punishment strategies are not sufficient to support a truth telling equilibrium.

contained in the set $\{x^l, x^h\}$ with $x^l < x^h$. Good assets have a probability p_G of producing a cash flow x^h such that

$$p_G x^h + (1 - p_G) x^l = 1,$$

while bad assets have a probability p_B of producing x^h such that

$$p_B x^h + (1 - p_B) x^l = \ell.$$

These asset cash flows mean that the expected cash flow of the asset conditional on its type remains unchanged.

The next step is to characterize the reputation updating function in this new setting. Lemma 2 still applies since its proof does not depend on the distribution of asset cash flows. So once again current quantity choice does not affect current reputation. The equilibrium reputation updating function is thus given by Bayes rule as follows

$$f(g, x^h, \phi) = \phi^S = \frac{\phi}{1 + (1 - \phi)(1 - \pi(\phi)) \frac{1-\lambda}{\lambda} \frac{p_B}{p_G}}, \quad (28)$$

$$f(g, x^l, \phi) = \phi^F = \frac{\phi}{1 + (1 - \phi)(1 - \pi(\phi)) \frac{1-\lambda}{\lambda} \frac{1-p_B}{1-p_G}}, \quad \text{if } p_G < 1 \text{ and } 0 \text{ otherwise} \quad (29)$$

$$f(b, x, \phi) = \phi^B = \frac{\phi}{\phi + (1 - \phi)\pi(\phi)}. \quad (30)$$

The first observation is that the separating equilibrium of Proposition 3 still holds. In that equilibrium, the assets ex post cash flows do not affect the future play of the game. This is precisely because the issuer credibly reveals private information about the asset's expected cash flow via quantity choice. Therefore, no asymmetry of information about the current asset persists once the current period quantity choice is observed. As a result, earlier statements about the inability of the separating equilibrium of Proposition 3 to explain the existence of fraud do not depend on the stylized asset cash flow distribution of the previous section.

Since the assets expected cash flows have not changed, the existence of a pooling equilibrium remains unchanged as well. Specifically, Assumption 1 still implies that pooling equilibria do not exist. That assumption states that the issuer would rather retain a good type asset than sell it at a price equal to the ex ante expected cash flow of the asset.

The next task is to characterize the conditions under which a truth telling equilibrium exists for the game with risky asset cash flows. If $p_G = 1$, investors know that the issuer is the opportunistic type when a cash flow of x^l follows a report that the asset is the good type. In this case, the restriction on parameters needed for the existence of a truth telling equilibrium again places an upper bound on the discount rate of the issuer. However now the bound also depends on the probability that the bad type asset produces the cash flow x^h as is stated in the following proposition:

Proposition 7. *There exists a truth telling equilibrium if and only if $\gamma \geq \frac{1}{\lambda(1-p_B)+\ell}$*

An important feature of Proposition 7 is that the restriction required for the existence of a truth telling equilibrium is more strict for the case of risky asset cash flows than for the setting of the previous section. This is because there is still the chance an opportunistic agent will not get “caught” misreporting the bad type asset. Thus, the issuer has less of an incentive to accurately report a bad type asset when asset cash flows are risky. In reality asset cash flows are indeed risky, making the empirical observability of truth telling equilibrium less likely.

If $p_G < 1$, then a cash flow of x^l does not reveal an inaccurate report. This again decreases the issuer’s incentive to accurately report the bad type asset since even if the asset yields the cash flow x^l , the investors cannot be sure that the issuer is indeed the bad type. This leads directly to the following negative result.

Proposition 8. *If $p_G < 1$, a truth telling equilibrium does not exist .*

The intuition behind Proposition 8 is as follows. Suppose that a truth telling equilibrium did indeed exist and $p_G < 1$. Then the equilibrium reputation updating function given in equations (28) - (30) imply that $\phi^S = \phi^F = \phi^B = \phi$. Thus, a cash flow of x^l does not affect reputation and the issuer would never lose any reputation from misreporting bad type assets. At the same time, if investors believe that the issuer is following the truth telling strategy, then they must price a reportedly good type asset accordingly, leading to one period gains for the issuer as a result misreporting a bad type asset.

The non-existence of a truth-telling equilibrium for the case of risky asset cash flows suggests that other equilibria, such as the mixed strategy equilibrium of the previous section, may exist. Unfortunately, this setting will not admit a solution technique for a mixed strategy equilibrium like

the one presented in the previous section. To see this, assume that the demand schedule is given by equation (8), then $(q^*(\phi), \pi^*(\phi))$ is a natural candidate equilibrium strategy pair. However, the analog to equation (11) is

$$q^*(\phi)p^*(\phi) - \ell \geq \gamma(p_B(V(\phi^B) - V(\phi^S)) + (1 - p_B)(V(\phi^B) - V(\phi^F))) - (1 - q^*(\phi))\gamma\ell \quad (31)$$

which depends on the value function evaluated at three different levels of reputation. Accordingly, more constraints on equilibrium strategies would be needed to pin down a particular mixed strategy equilibrium.

5 Concluding Remarks

I have presented a model of an informed securities issuer that unifies signaling and reputation effects. A lemons problem arises due to asymmetric information about the quality of assets. Partial retention by the issuer is a credible signal of asset quality since the issuer is impatient relative to investors causing such retention to be costly. Imperfect information over issuer preferences induces a market reputation for the issuer. A high reputation can increase payoffs for the issuer by reducing issuer retention in equilibrium while decreasing the lemons discount relative to an identical security offered by an issuer with a low reputation. Reputation effects do not imply that an issuer will be more likely to perfectly reveal information.

The implications of this model call into question the benefits of reputation as a substitute for regulation and oversight to impose market discipline. For example, although conceptually appealing, the assertion that issuers of ABS will behave in the best interests of the wider markets as a consequence of protecting their reputations misses an important point. The benefit of having a good reputation may be due to the ability to “cash in” on a high reputation in the future. In the case of my model, an opportunistic issuer will cash in on a good reputation by misreporting bad type assets. In contrast, signaling in the absence of reputation can force issuers to reveal the true full information value of any assets underlying ABS at the cost of reducing equilibrium payoffs.

One of the key applications of the theory of costly signaling has been to understand the capital structure of ABS. Future work will be to empirically examine the implications of this paper for

mortgage backed securities (MBS) markets. Some questions that could be answered are the following. Is the size of the first loss tranche related to the reputation of the issuer of the MBS? How do the prices of the senior and mezzanine tranches depend on the size of the first loss tranche? Is that relationship altered by issuer reputation? One challenge of mapping the results of this paper into the proceeding questions is that the simple asset cash flows do not allow for rich security design problem, another topic for future work.

While I mostly discuss the model by referring to a securities issuer, the theory developed in this paper applies equally well to other important financing problems. The key features of the model are that the issuer has valuable private information, a means by which to signal that information in a single period, and a means by which to gain a reputation for accurate reports. For example, the model could refer to a venture capitalist raising funds from limited partners. Maintaining a larger stake as general partner may be necessary, if a good track record of matching investment projects with stated fund goals has not yet been established. Similarly, a private equity firm may need to put up a larger amount of capital to implement a leveraged take over if that firm does not have a long history of accurate analysis of target firm prospects. Finally, both a private equity firm and venture capitalist may at some point find it advantageous to exploit a good reputation for one period gains.

Since the model is silent on the effects of different equilibrium strategies on markets for underlying assets and collateral, like the primary mortgage market, I do not make specific policy suggestions. However, depending on the goals of a regulator, this model does offer the following advice. If the policy goal is to ensure accurate information disclosure, then a regulator should adopt a policy that encourages perfect signaling of asset quality. Providing a legal means for issuers to publicly disclose their holdings and commit to holding them to maturity could increase accurate information transmission. In contrast, if the policy goal is maximizing payoffs for issuers then reputation and signaling should both be facilitated, i.e. through a centralized repository of past deal performance and a credible means to reveal issuer holdings.

A Appendix - Path Dependent Strategies

In this appendix, I consider investor beliefs that may depend on the entire history of the game, rather than only current reputation and current quantity. The investors' demand curve at time t is then given by a function $P_t(q, \mathcal{H}_t) : [0, 1] \times \mathbb{H}_t \rightarrow [0, 1]$ where \mathbb{H}_t is the set of all possible histories of the game up to time t . The issuer then implements a reporting strategy $\pi_t(\mathcal{H}_t) : \mathbb{H}_t \rightarrow [0, 1]$ and a quantity strategy $Q_t(\mathcal{H}_t) : \mathbb{H}_t \rightarrow [0, 1]$. I maintain the assumptions that the issuer always reports a good type asset as good, and always issues the quantity one when reporting the asset is the bad type.⁹ The definition of equilibrium in this setting replaces conditions (4) and (5) of Definition 1 with

5. Investors earn zero expected profits: $P_t(Q^i(\mathcal{H}_t), \mathcal{H}_t) = E[X_{t+1} | \mathcal{H}_t, Q^i(\mathcal{H}_t)]$ for $i \in \{H, O\}$
6. An equilibrium is separating if $P_t(Q^i(\mathcal{H}), \mathcal{H}_t) = 1$.

The first result is that an equilibrium satisfying Definition 1 will satisfy the definition of equilibrium in this more general setting.

Proposition 9. *Suppose the quadruple $(\hat{P}, \hat{Q}^H, \hat{\pi}, \hat{Q}^O)$ satisfies Definition 1, then there exists an equilibrium of the game with history dependent strategies such that $P(q, \mathcal{H}_t) = \hat{P}(q, \phi)$, $Q_t^H(\mathcal{H}_t) = \hat{Q}^H(\phi)$, $\pi_t(\mathcal{H}_t) = \hat{\pi}(\phi)$, and $Q_t^O(\mathcal{H}_t) = \hat{Q}^O(\phi)$ where $\phi = \mathbb{P}(\text{Issuer is Honest Type} | \mathcal{H}_t)$*

Proof. Observe that if strategies are defined as in the proposition and

$$\phi = \mathbb{P}(\text{Issuer is Honest Type} | \mathcal{H}_t),$$

then conditions (4)-(5) of Definition 1 imply conditions (4)-(5) of this section. \square

The above proposition is not surprising. It simply states that stationary Markov equilibria are also equilibria under the more general definition. It is of greater interest to determine whether there are equilibria under the more general definition that do not obtain under Definition 1. I show that there are truth-telling equilibria under the general definition that do not exist under Definition 1, however the parameter restriction required for their existence coincides exactly with that of Proposition 2.

Proposition 10. *Suppose $\phi_0 = 0$, then there exists a truth telling equilibrium with $Q_t(\mathcal{H}_t) > \hat{q}$ for some \mathcal{H}_t if and only if $\gamma \geq \frac{1}{\lambda + \ell}$.*

Proof. Note that since $\phi_t = 0$ for all $t \geq 0$ by Bayes' Rule, it is enough to consider the strategies of the opportunistic type issuer. First suppose there exists a truth telling equilibrium given by $Q_t(\mathcal{H}_t)$ and $\pi_t = 1$. Let \tilde{Q} be defined as follows

$$\tilde{Q} = \sup\{Q_t(\mathcal{H}_t) | t \geq 0 \text{ and } \mathcal{H}_t \in \mathbb{H}_t\}. \quad (32)$$

Note that $\tilde{Q} > \hat{q}$. For all $\epsilon > 0$, there exist (t, \mathcal{H}_t) such that $\tilde{Q} - Q_t(\mathcal{H}_t) < \epsilon$ by the definition of supremum. Let (t, \mathcal{H}_t) be such a pair. The one-shot deviation principal implies that

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell \leq L_t \quad (33)$$

⁹These assumptions are without loss of generality as a variant of Lemma 2 holds in this more general setting.

where L_t is the discounted loss faced by the issuer after misreporting a bad type asset given the history \mathcal{H}_t . By the definition of \tilde{Q} , I have

$$L_t \leq \gamma \left(\frac{\lambda(\tilde{Q} + (1 - \tilde{Q}))\gamma + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q})\gamma) + (1 - \lambda)\ell}{1 - \gamma} \right) = \gamma\lambda(\tilde{Q} - \hat{q}). \quad (34)$$

But

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell = (1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}). \quad (35)$$

This implies that for all $\epsilon > 0$

$$(1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}) \leq \gamma\lambda(\tilde{Q} - \hat{q}) \quad (36)$$

which implies $\gamma \geq \frac{1}{\lambda + \ell}$ since $\tilde{Q} > \hat{q}$.

Now suppose $\gamma \geq \frac{1}{\lambda + \ell}$. I'll show that a truth telling equilibrium is given by $Q_t = 1$, $\pi_t = 1$, $P(q, H_t) = 1$ if no misreports have been made and

$$P(q, H_t) = \begin{cases} 1 & \text{if } q \leq \hat{q} \\ \ell & \text{if } q > \hat{q} \end{cases}$$

otherwise. Note that the continuation value of the issuer just depends on whether or not a misreport has been made. Let L be the discounted loss in continuation value faced by the issuer if a bad type asset has been misreported

$$L = \gamma \left(\frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q})\gamma) + (1 - \lambda)\ell}{1 - \gamma} \right) = \gamma\lambda(1 - \hat{q}). \quad (37)$$

To see that the above strategies indeed constitute an equilibrium observe that

$$\begin{aligned} L &= \gamma\lambda(1 - \hat{q}) = \gamma\lambda \frac{1 - \ell}{1 - \gamma\ell} \\ &\geq 1 - \ell. \end{aligned}$$

Thus the discounted loss in continuation value is greater than or equal to the one-shot gains from misreporting a bad type asset and the above strategies constitute a truth telling equilibrium for $\phi_0 = 0$. \square

Proposition 10 shows that although allowing for path dependent strategies does mean that a truth telling equilibrium is supportable without the type of reputation considered in this paper, the restriction on the parameters required for truth telling does not change.

B Appendix - Proofs

Proof of Proposition 1.

Separating Equilibrium: Let $\tilde{q} \leq \frac{\ell(1-\gamma)}{1-\gamma\ell}$ and

$$P(q, 0) = \begin{cases} 1 & \text{for } q \leq \tilde{q} \\ \ell & \text{for } q > \tilde{q} \end{cases},$$

then

$$\begin{aligned} qP(q, 0) + (1 - q)\gamma\ell &= \gamma\ell + (1 - \gamma\ell)q \leq \ell \\ qP(q, 0) + (1 - q)\gamma &\leq \tilde{q}P(q, 0) + (1 - \tilde{q})\gamma \\ \tilde{q}P(q, 0) + (1 - \tilde{q})\gamma &> \gamma. \end{aligned}$$

for all $q \leq \tilde{q}$. Thus $\tilde{q} = \arg \max_q \{qP(q, 0) + (1 - q)\gamma\}$ and

$$(1, 1) \in \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)qP(q, 0) + (1 - q)\gamma\ell\}$$

Thus the issuer chooses to issue \tilde{q} when in possession of a good type asset and 1 when holding a bad type asset, thus satisfying conditions (1) and (2) of Definition 1. This implies that $E[X|\tilde{Q}, g] = 1$, condition (4) is satisfied, and the strategies proposed in Proposition 1 constitute a separating equilibrium.

Pooling Equilibrium Let $\gamma \leq \lambda + (1 - \lambda)\ell$ and $P(q, 0) = \lambda + (1 - \lambda)\ell$ for all q , then $1 = \arg \max_q qP(q, 0) + (1 - q)\gamma$ and

$$(1, 0) \in \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)qP(q, 0) + (1 - q)\gamma\ell\}.$$

Thus the issuer always chooses to issue a quantity 1 and report that the asset is the good type. This implies that $E[X|\tilde{Q}, g] = \lambda$ and condition (4) is satisfied, and the strategies proposed in Proposition 1 constitute a pooling equilibrium.

Now let $\gamma > \lambda + (1 - \lambda)\ell$ and let \tilde{Q} be the quantity chosen by the issuer in a pooling equilibrium, then $P(\tilde{q}, 0) = \lambda + (1 - \lambda)\ell\gamma < \gamma = 0 \cdot P(q, 0) + (1 - 0)\gamma$ and \tilde{q} cannot be an equilibrium quantity strategy for the issuer possessing good type asset—a contradiction. Thus a pooling equilibrium does not exist. □

Proof of Lemma 1. First note that Bayes' Rule implies that $\phi = 0$ is an absorbing state, since

$$\mathbb{P}(\text{Issuer is honest} | \mathcal{H}_t) = \frac{\mathbb{P}(\text{Issuer is honest} \cap \mathcal{H}_t)}{\mathbb{P}(\mathcal{H}_t)} = 0$$

so that $\phi_t = 0$ implies that $\phi_s = 0$ for all $s \geq t$.

Now suppose (Q, π) and $P(q, 0)$ is an equilibrium of the static game and not an equilibrium of the repeated game at time t with $\phi_t = 0$. Let \hat{V} be the continuation value the opportunistic issuer receives for playing the strategy (Q, π) given the demand curve $P(q, 0)$. Then at least one of the following must be true:

- $Q \notin \arg \max_q \{qP(q, 0) + (1 - q)\gamma + \hat{V}\}$
- $(Q, \pi) \notin \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)(qP(q, 0) + (1 - q)\gamma\ell) + \hat{V}\}$
- $P(Q, 0) \neq E[X_{t+1}|Q, g]$

Any of the above conditions imply that (Q, π) and $P(q, 0)$ is not an equilibrium of the static game—a contradiction. □

Proof of Lemma 2. For convenience let $W(\phi)$ and $V(\phi)$ denote the value the honest and opportunistic issuers place on reputation ϕ respectively. Note that $V(0) = W(0)$ since $\phi = 0$ is an absorbing state and both issuer types play the separating equilibrium at $\phi = 0$. Suppose there exists $\hat{\phi}$ and equilibrium strategies Q^H and Q^O such that $Q^H(\hat{\phi}) \neq Q^O(\hat{\phi})$, then the investors would believe the issuer is the honest type upon observing a quantity equal to $Q^H(\hat{\phi})$, and would believe the issuer to be the opportunistic type upon observing a level of retention equal to $Q^O(\hat{\phi})$ when the issuer has a prior reputation $\hat{\phi}$. This implies that $P(Q^H(\hat{\phi}), \phi) = 1$. The definition of equilibrium strategies for the honest issuer thus implies that

$$Q^H(\hat{\phi}) + \gamma(1 - Q^H(\hat{\phi})) + \gamma W(1) \geq Q^O(\hat{\phi})P(Q^O(\hat{\phi}), \hat{\phi}) + \gamma(1 - Q^O(\hat{\phi})) + \gamma W(0)$$

It is straightforward to shown that $\pi(1) = 0$, which implies that $V(1) > W(1)$ so that

$$Q^H(\hat{\phi}) + \gamma(1 - Q^H(\hat{\phi})) + \gamma V(1) > Q^O(\hat{\phi})P(Q^O(\hat{\phi}), \hat{\phi}) + \gamma(1 - Q^O(\hat{\phi})) + \gamma V(0).$$

Hence, the opportunistic issuer can profitably deviate to $Q^H(\hat{\phi})$ and the strategies Q^H and Q^O cannot be played in equilibrium. □

Proof of Proposition 2. First suppose there exists a truth telling equilibrium given by $Q(\phi)$ and $\pi(\phi) = 1$, such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ implies $\phi^B = \phi^S = \phi$, so the one-shot deviation principal then states that for all ϕ such that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell \leq \gamma(V(\phi) - V(0)). \quad (38)$$

Next note that

$$\begin{aligned} Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell &= \hat{q} + (1 - \hat{q})\gamma\ell - \ell + (1 - \gamma\ell)(Q(\phi) - \hat{q}) \\ &= (1 - \gamma\ell)(Q(\phi) - \hat{q}) \end{aligned}$$

since $\hat{q} + (1 - \hat{q})\gamma\ell = \ell$. Finally, I have

$$\gamma(V(\phi) - V(0)) = \gamma\lambda(1 - p_B)(Q(\phi) - \hat{q}).$$

So it must be the case that $1 - \gamma\ell \leq \gamma\lambda$ since $Q(\phi) > \hat{q}$, which is the intended result.

Now suppose that $\gamma \geq \frac{1}{\lambda + \ell}$. I show that $\pi(\phi) = Q(\phi) = 1$ for all $\phi > 0$ defines an equilibrium. To check that these strategies constitute an equilibrium, note that they imply

$$V(\phi) = \frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} \quad (39)$$

for all $\phi > 0$ and $\phi^B = \phi^S = \phi$. This in turn implies

$$\begin{aligned} \gamma V(\phi) - V(0) &= \gamma\lambda \frac{(1 - \hat{q})(1 - \gamma)}{1 - \gamma} \\ &= \gamma\lambda \frac{1 - \ell}{1 - \gamma\ell} \\ &\geq (1 - \ell), \end{aligned}$$

so that the issuer does not have a profitable one-shot deviation and the equilibrium is verified.

□

Proof of Proposition 3. The case for $\phi_0 = 0$ is given in Lemma 1. Let $\tilde{V}(\phi)$ denote the value the issuer receives by playing the strategy $(\hat{q}, 1)$ forever. Note $\tilde{V}(\phi) = \hat{V}$ for all ϕ where \hat{V} is defined in the proof of Lemma 1, thus the proof applies to $\phi_0 > 0$ as well.

□

Proof of Proposition 4. First I provide the solution to the method of construction given in the text. Let

$$\bar{\phi} = \frac{1}{1-\lambda} \left(1 - \frac{\lambda(1-\gamma\lambda)}{\gamma(1-\gamma\lambda(1-\hat{q}))} \right) \quad (40)$$

$$\hat{\phi} = \hat{\phi}^B \left(\hat{\phi}^B - \frac{1-\gamma}{1-\lambda} \right) \quad (41)$$

where

$$\hat{\phi}^B = \frac{(1-\ell) - \lambda(1-\gamma\ell)}{(1-\lambda)(1-\ell)}. \quad (42)$$

For the remainder of the proof, I assume

$$\frac{1-\gamma\ell}{1-\ell} < \frac{1-\gamma\lambda}{\gamma(1-\gamma\lambda(1-\hat{q}))}$$

so that $\hat{\phi}^B > \bar{\phi}$. This assumption could be relaxed with only minor changes to the proof, but doing so would make the calculation of $L(\phi)$ for $\phi \in [\hat{\phi}, \bar{\phi}]$ needlessly laborious. It is straightforward to show that $\hat{\phi}$ solves equations (10)-(12) when $q^*(\phi) = 1$, $\pi^*(\phi) = 0$, and (11) binds. Similarly, it is straightforward to show that $\hat{\phi}$ solves (10)-(12) when $q^*(\phi) = 1$ and (10) binds.

For $\phi \geq \bar{\phi}$, $\pi^*(\phi) = 0$ and $q^*(\phi) = 1$, so

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi}$$

$$L_1(\phi) = \frac{\gamma\lambda(1-\ell)}{1-\gamma\lambda} \left(\frac{1}{1-(1-\lambda)\phi} - \frac{\gamma(1-\ell)}{1-\gamma\ell} \right).$$

For $\phi \in [\hat{\phi}, \bar{\phi}]$, $q^*(\phi) = 1$ and the solution to (16)

$$\pi^*(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{(1-\ell)}{L_1(\zeta^{-1}(\phi))} - 1 \right) \frac{1}{1-\phi}, \quad (43)$$

where

$$\zeta(\phi) = \frac{1}{1-\lambda} \left(1 - \frac{\lambda(1-\ell)}{L_1(\phi)} \right) \phi. \quad (44)$$

Note that ζ has a unique inverse for $\hat{\phi} \leq \phi \leq \bar{\phi}$. Using (43), I can calculate $p^*(\phi)$ and the function L_2 for $\hat{\phi} \leq \phi \leq \bar{\phi}$:

$$p^*(\phi) = \ell + L_1(\zeta^{-1}(\phi)) \quad (45)$$

$$L_2(\phi) = \frac{\gamma}{1-\gamma\lambda} \left(L_1(\zeta^{-1}(\phi)) - \frac{\gamma(1-\ell)^2}{1-\gamma\ell} \right). \quad (46)$$

Next I consider $\underline{\phi} \leq \phi < \hat{\phi}$. Let $\psi_n(\phi) = \prod_{k=0}^n \delta_k(\phi)$ where $\delta_0(\phi) = \phi$, and

$$\delta_n(\tilde{\phi}) = 1 - \left(\frac{\lambda}{1-\lambda} \right) \left(\frac{(1-\gamma)\beta^{n-1}L(\tilde{\phi})}{\gamma(1-\ell)^2\hat{q} + (\gamma-\ell)\beta^{n-1}L(\tilde{\phi})} \right) \quad (47)$$

for $n \geq 0$, then the solution to equations (21)-(23) is

$$q^*(\phi) = \hat{q} + \frac{\beta^{n-1}}{\gamma(1-\ell)} L(\psi_n^{-1}(\phi)), \quad (48)$$

$$\pi^*(\phi) = 1 - \left(\frac{\lambda}{1-\lambda} \right) \left(\frac{(1-\gamma)\beta^{n-1}L(\psi_n^{-1}(\phi))}{\gamma(1-\ell)^2\hat{q} + (\gamma-\ell)\beta^{n-1}L(\psi_n^{-1}(\phi))} \right) \frac{1}{1-\phi}, \quad (49)$$

$$p^*(\phi) = 1 - \frac{(1-\gamma)\beta^{n-1}L(\psi_n^{-1}(\phi))}{\gamma(1-\ell)\hat{q} + \beta^{n-1}L(\psi_n^{-1}(\phi))}, \quad (50)$$

$$L(\phi) = \beta^n L(\psi_n^{-1}(\phi)) \quad (51)$$

where $\phi(k) = \psi_k(\hat{\phi})$ and n is such that $\phi(n) \leq \phi < \phi(n-1)$.

To show that the proposed strategies constitute an equilibrium, I must verify that conditions (1)–(4) of Definition 1 hold. Note condition (4), that investors earn zero expected profits in equilibrium, follows by construction. To show that conditions (1)–(3) hold, I repeatedly apply the “one-shot deviation principle.” To see that the principle applies in this case, note that the game has perfect monitoring so that Proposition 2.2.1 of Mailath and Samuelson (2006) applies. Hence, to show that no profitable deviation exists for the opportunistic issuer, it is enough to examine the single deviation payoffs.

Observe that by construction the function V is the value delivered to the opportunistic issuer by playing the strategies given. Also note that $\hat{q}(1-\gamma) + \gamma > \ell$ and $\ell = \hat{q}(1-\gamma\ell) + \gamma\ell$. Thus to show that the opportunistic issuer never has a profitable one shot deviation it is sufficient to show that

$$q^*(\phi)(p^*(\phi) - \gamma) \geq \hat{q}(1-\gamma) \quad (10)$$

$$q^*(\phi)(p^*(\phi) - \gamma\ell) \geq (1-\gamma)\ell + \gamma(V(\phi^B) - V(0)), \quad (11)$$

where

$$\phi^B = \frac{\phi}{\phi + \phi(1-\phi)}.$$

Also by Lemma 2, I need only consider deviations for the opportunistic issuer, since the honest issuer will always choose the same quantity. Finally, I must show that the proposed equilibrium prices and strategies yield zero expected profits to the investors. The fact that equilibrium beliefs are given by Bayes’ Rule whenever possible is by construction.

The proof proceeds in three steps by first showing that no profitable one shot deviation exists for the opportunistic issuer over the three subintervals of reputation $[\hat{\phi}, 1]$, $[\underline{\phi}, \hat{\phi})$, and $[0, \underline{\phi})$.

Step 1: $\phi \geq \hat{\phi}$ Note that $\zeta(\phi)$ is increasing for $\phi \geq \hat{\phi}$ since $L(\phi)$ is increasing in ϕ . This implies that $p^*(\phi)$ is increasing for $\hat{\phi} \leq \phi \leq \bar{\phi}$. More over $p^*(\phi)$ is increasing for $\phi \geq \bar{\phi}$ since for such ϕ

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi}.$$

Note that $q^*(\phi) = 1$ so that

$$\begin{aligned} p^*(\phi) - \gamma &\geq p^*(\hat{\phi}) - \gamma \\ &= \ell + \frac{\lambda(1-\ell)}{\lambda + (1-\lambda)(1-\pi^*(\hat{\phi}))(1-\hat{\phi})} - \gamma \\ &= \ell + L(\zeta^{-1}(\hat{\phi})) - \gamma = \hat{q}(1-\gamma) \end{aligned}$$

by the definition $\hat{\phi}$. Thus, the opportunistic issuer does not have a profitable one shot deviation when faced with a good asset. Next note that $\phi^B = 1$ for all $\phi \geq \bar{\phi}$

$$\begin{aligned} p^*(\phi) - \gamma\ell &\geq p^*(\bar{\phi}) - \gamma\ell \\ &= \ell(1-\gamma) + \frac{\lambda(1-\ell)}{1 - (1-\lambda)\bar{\phi}} \\ &= (1-\gamma)\ell + \gamma(V(1) - V(0)) \end{aligned}$$

by the definition of $\bar{\phi}$. For $\hat{\phi} \leq \phi < \bar{\phi}$

$$\begin{aligned} p^*(\phi) - \gamma\ell &= \ell(1-\gamma) + \frac{\lambda(1-\ell)}{\lambda + (1-\lambda)(1-\pi^*(\phi))(1-\phi)} \\ &= \ell(1-\gamma) + L(\zeta^{-1}(\phi)) \\ &= (1-\gamma)\ell + L(\phi^B) \end{aligned}$$

by the definition of $\zeta(\phi)$. Thus, the opportunistic issuer does not have a profitable deviation when faced with a bad asset.

Step 2: $\underline{\phi} \leq \phi < \hat{\phi}$

By construction I have

$$p^*(\phi) = \gamma + \frac{\hat{q}(1-\gamma)}{q^*(\phi)}$$

so that $q^*(\phi)(p^*(\phi) - \gamma) = \hat{q}(1-\gamma)$, which implies that the opportunistic issuer does not have a profitable deviation when faced with a good type asset.

Now consider the case when the opportunistic issuer has a bad type asset. I'll show that $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1-\gamma)\ell + \gamma(V(\phi^B) - V(0))$ for all $\underline{\phi} \leq \phi < \bar{\phi}$. Let $I_n = (\phi_n, \phi_{n-1}]$ where ϕ_n is the sequence defined by equations (17) and (18). The argument proceeds by induction on n . For I_1 , I have

$$\begin{aligned} q^*(\phi)(p^*(\phi) - \gamma\ell) &= \hat{q}(1-\gamma) + \gamma(1-\ell)q^*(\phi) \\ &= \hat{q}(1-\gamma\ell) + L(\psi_1^{-1}(\phi)) \\ &= (1-\gamma)\ell + \gamma(V(\psi_1^{-1}(\phi)) - V(0)) \end{aligned}$$

and

$$\begin{aligned}\psi_1^{-1}(\phi) &= \frac{\phi}{\delta_1(\psi_1^{-1}(\phi))} \\ &= \frac{\phi}{\phi + \pi(\phi)(1 - \phi)} = \phi^B\end{aligned}$$

so that $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1 - \gamma)\ell + \gamma(V(\phi^B) - V(0))$ for all $\phi \in I_1$. Now assume that the equality holds for all $\phi \in I_k$ and all $k \leq n - 1$.¹⁰ This assumption implies

$$\begin{aligned}L(\phi) &= \frac{\gamma}{1 - \gamma\lambda}(\lambda(p^*(\phi)q^*(\phi) + \gamma(1 - q^*(\phi))) \\ &= +(1 - \lambda)(p^*(\phi)q^*(\phi) + \gamma(1 - q^*(\phi))\ell) - (1 - \gamma)V(0)) \\ &= \frac{\gamma}{1 - \gamma\lambda}(\lambda(\hat{q} + \gamma(1 - \hat{q})) + (1 - \lambda)(\ell + L(\phi^B))) - (1 - \gamma)V(0)) \\ &= \frac{\gamma(1 - \lambda)}{1 - \gamma\lambda}L(\phi^B) = \beta L(\phi^B)\end{aligned}$$

for all $\phi \in I_k$ and $k \leq n_1$. Thus $L(\phi) = \beta^{n-1}L(\psi_{n-1}^{-1}(\phi))$ for all $\phi \in I_{n-1}$. For $\phi \in I_n$, note that

$$\begin{aligned}\phi^B &= \frac{\phi}{\phi + \pi(\phi)(1 - \phi)} \\ &= \frac{\phi}{\delta_n(\psi_n^{-1}(\phi))} \\ &= \psi_{n-1}(\psi_n^{-1}(\phi))\end{aligned}$$

by the definition of π , δ_n and ψ_n , so that $\phi^B \in I_{n-1}$ and $L(\phi^B) = \beta^{n-1}L(\psi_{n-1}^{-1}(\phi^B))$. But $\psi_{n-1}^{-1}(\phi^B) = \psi_{n-1}^{-1}(\psi_{n-1}(\psi_n^{-1}(\phi))) = \psi_n^{-1}(\phi)$ so that

$$\begin{aligned}(1 - \gamma)\ell + L(\phi^B) &= (1 - \gamma)\ell + \beta^{n-1}L(\psi_n^{-1}(\phi)) \\ &= q^*(\phi)(p^*(\phi) - \gamma\ell).\end{aligned}$$

by the definition of q^* and p^* . Then, by induction, $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1 - \gamma)\ell + L(\phi^B)$ for all $\phi \in I_n$ for all $n \geq 1$, and the opportunistic issuer does not have a profitable deviation when faced with a bad type asset for $\underline{\phi} \leq \phi \leq \bar{\phi}$.

Step 3: $\phi < \underline{\phi}$ The fact that the opportunistic issuer does not have a profitable one shot deviation for $\phi < \underline{\phi}$ follows directly from Proposition 3. □

Proof of Proposition 6. First I construct a candidate equilibrium. As in Section 3.4, I assume there exists $\underline{\phi}'$ and $\bar{\phi}'$ so that (26) binds for $\underline{\phi}' \leq \phi \leq \bar{\phi}'$ and $\pi^*(\phi) = 0$ for $\phi \geq \bar{\phi}'$. Thus for $\phi \geq \bar{\phi}'$ it is

¹⁰This proof utilizes *complete induction* so the base case is unnecessary, but is left for clarity.

trivial to compute the equilibrium loss and price functions

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi} \quad (52)$$

$$L(\phi) = \frac{\gamma}{1-\gamma\lambda} \left(\frac{\lambda(1-\ell)}{1-(1-\lambda)\phi} - \lambda(\gamma-\ell) \right) \quad (53)$$

The next task is to calculate π^* , and subsequently p^* and L , for $\underline{\phi}' \leq \phi \leq \bar{\phi}$. Let $\psi_n(\phi) = \prod_{k=0}^n \delta_k(\phi)$ where $\delta_0(\phi) = \phi$ and

$$\delta_n(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{1-\ell}{\left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\phi) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k} - 1 \right) \quad (54)$$

for $n \geq 1$. Let a sequence be given by $\phi(n) = \psi_n(\bar{\phi}')$, note that now the first element of the sequence is $\bar{\phi}'$ rather than $\hat{\phi}$, since it is no longer necessary to consider $0 < Q < 1$. Then a candidate equilibrium is given by

$$\pi^*(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{1-\ell}{\left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k} - 1 \right) \frac{1}{1-\phi} \quad (55)$$

$$p^*(\phi) = \ell + \left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k. \quad (56)$$

$$L(\phi) = \left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k \quad (57)$$

where n is such that $\max\{\phi_n, \underline{\phi}'\} \leq \phi \leq \phi_{n-1}$ and

$$\underline{\phi}' = \min\{\phi | p^*(\phi) \geq \gamma\}.$$

The rest of the proof proceeds almost identically to the proof of Proposition 4 □

Proof of Proposition 7. First suppose there exists a truth telling equilibrium given by $Q(\phi)$ and $\phi(\phi) = 1$ such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ implies $\phi^B = \phi^S = \phi$, so the one-shot deviation principal then states that for all $\phi > 0$ such that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1-Q(\phi))\gamma\ell - \ell \leq \gamma(1-p_B)(V(\phi) - V(0)). \quad (58)$$

Next note that

$$\begin{aligned} Q(\phi) + (1-Q(\phi))\gamma\ell - \ell &= \hat{q} + (1-\hat{q})\gamma\ell - \ell + (1-\gamma\ell)(Q(\phi) - \hat{q}) \\ &= (1-\gamma\ell)(Q(\phi) - \hat{q}) \end{aligned}$$

since $\hat{q} + (1-\hat{q})\gamma\ell = \ell$. Finally, I have

$$\gamma(1-p_B)(V(\phi) - V(0)) = \gamma\lambda(1-p_B)(Q(\phi) - \hat{q}).$$

So it must be the case that $1-\gamma\ell \leq (1-p_B)\gamma\lambda$ since $Q(\phi) > \hat{q}$, which is what I needed to show.

Now suppose that $\gamma \geq \frac{1}{(1-p_B)\lambda+\ell}$. I show that $\pi(\phi) = Q(\phi) = 1$ for all $\phi > 0$ defines an equilibrium. To check these strategies constitute an equilibrium, note that they imply

$$V(\phi) = \frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} \quad (59)$$

for all $\phi > 0$ and $\phi^B = \phi^S = \phi$. This in turn implies

$$\begin{aligned} \gamma(1 - p_B)(V(\phi) - V(0)) &= \gamma\lambda(1 - p_B)\frac{(1 - \hat{q})(1 - \gamma)}{1 - \gamma} \\ &= \gamma\lambda(1 - p_B)\frac{1 - \ell}{1 - \gamma\ell} \\ &\geq (1 - \ell), \end{aligned}$$

so that the issuer does not have a profitable one-shot deviation and the equilibrium is verified. \square

Proof of Proposition 8. Suppose that there exists a truth telling equilibrium given by $Q(\phi)$ and $\pi(\phi) = 1$ such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ and $p_G < 1$ imply $\phi^B = \phi^S = \phi^F = \phi$, so the one-shot deviation principal then states that for all $\phi > 0$ that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell \leq \gamma(1 - p_B)(V(\phi) - V(0)) = 0. \quad (60)$$

a contradiction since $Q(\phi) + (1 - Q(\phi))\gamma > \hat{q} + (1 - \hat{q})\gamma\ell = \ell$.

\square

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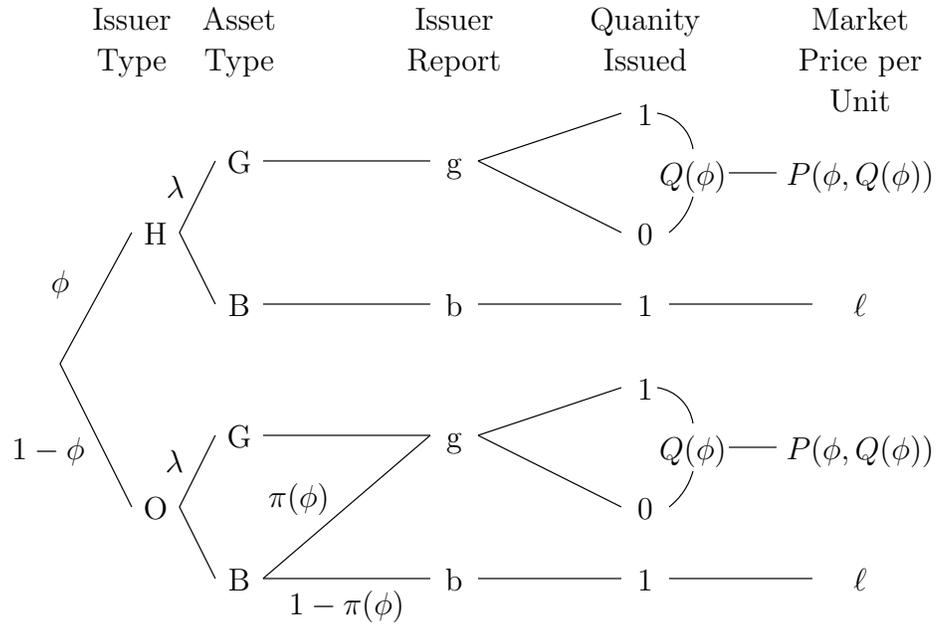


Figure 1: Game tree for the stage game. At the beginning of a given date the cash flow from the previous date's asset is realized and reputation is updated. Next, the issuer learns the type of the asset and implements a purchasing and securitization strategy (π, Q) . Finally the investors purchase the fraction of the asset Q at a price P .

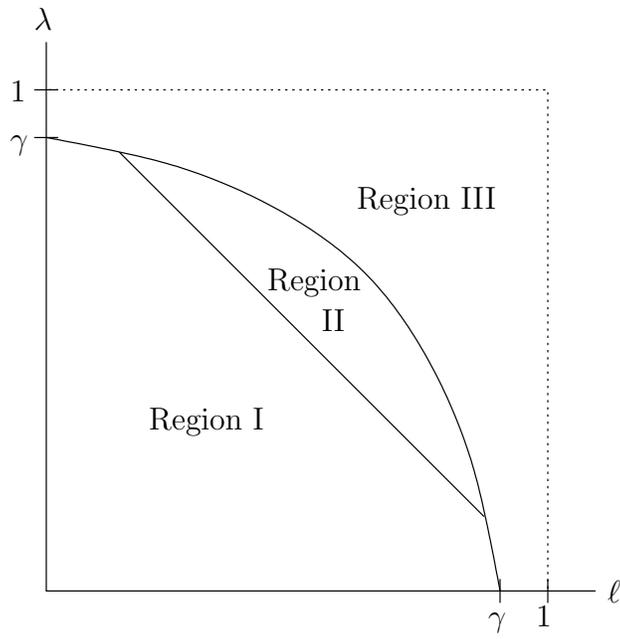


Figure 2: A partition of the parameter space of the model. Region I corresponds to parameters for which the truth telling equilibrium is not supported. Region II corresponds to parameters for which the truth telling equilibrium is supported. Region III corresponds to parameters for which Assumption 1 fails and hence is not considered.

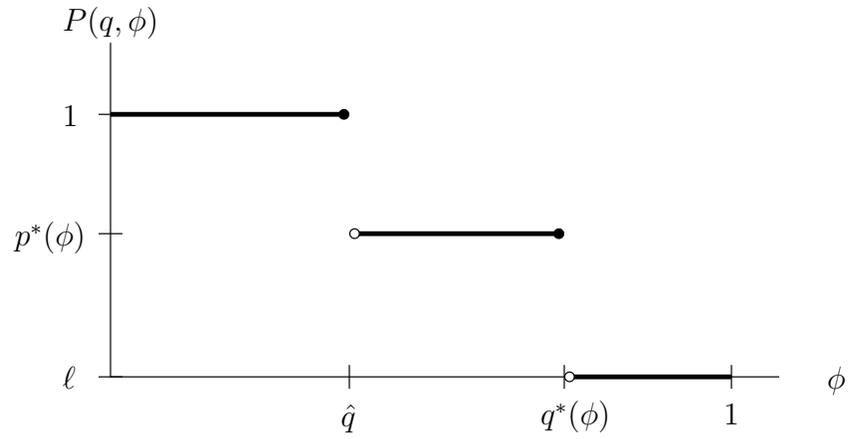


Figure 3: An example of the demand curve (8) for a given reputation ϕ . For a quantity $q \leq \hat{q}$ investors believe the asset is the good type. For $\hat{q} < q \leq q^*(\phi)$, investors believe the asset is the good type with some probability and the bad type with some probability. For $q > q^*(\phi)$ the investors believe the asset is the bad type.

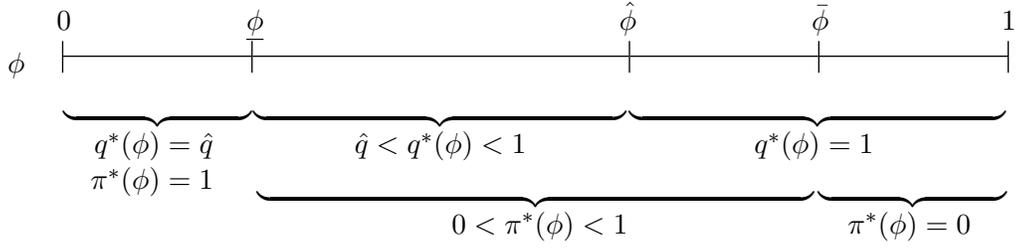


Figure 4: The interval of reputation $0 \leq \phi \leq 1$ can be decomposed into subintervals over which the candidate strategies are either known or the inequalities (10) and (11) bind. For $\underline{\phi} < \phi < \hat{\phi}$, so that $\hat{q} < q^*(\phi) < 1$, inequality (10) binds. For $\underline{\phi} < \phi < \bar{\phi}$, so that $0 < \pi^*(\phi) < 1$, inequality (11) binds.

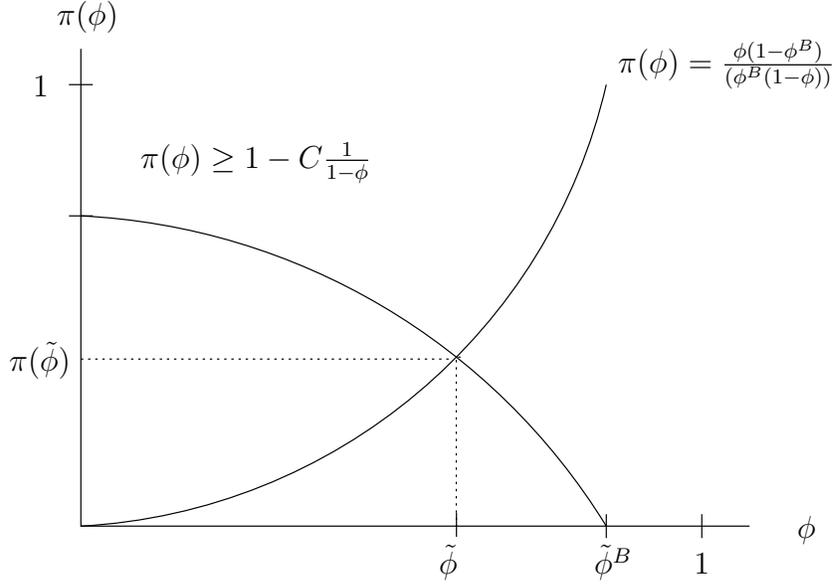
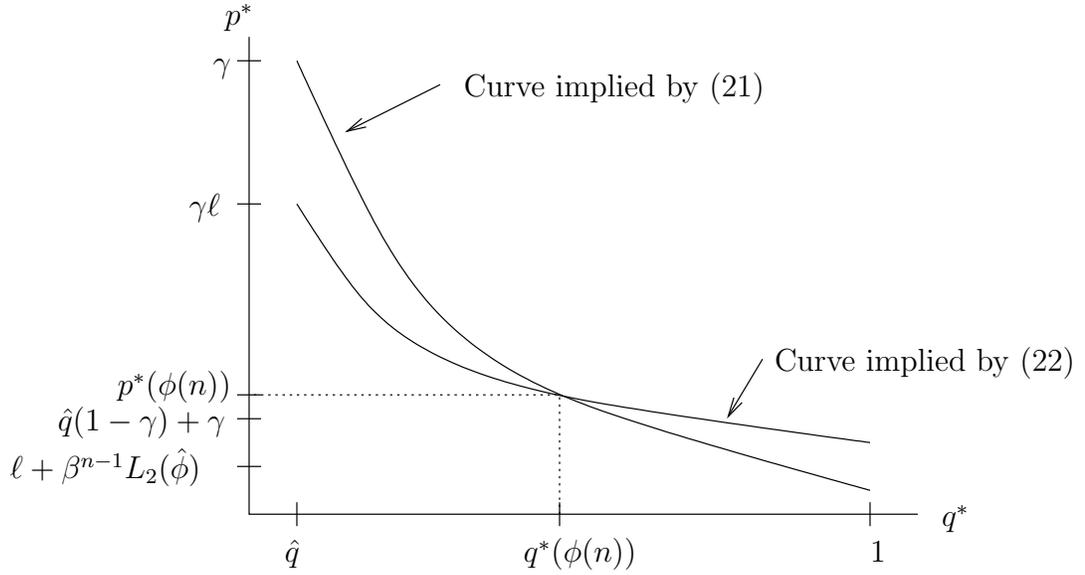
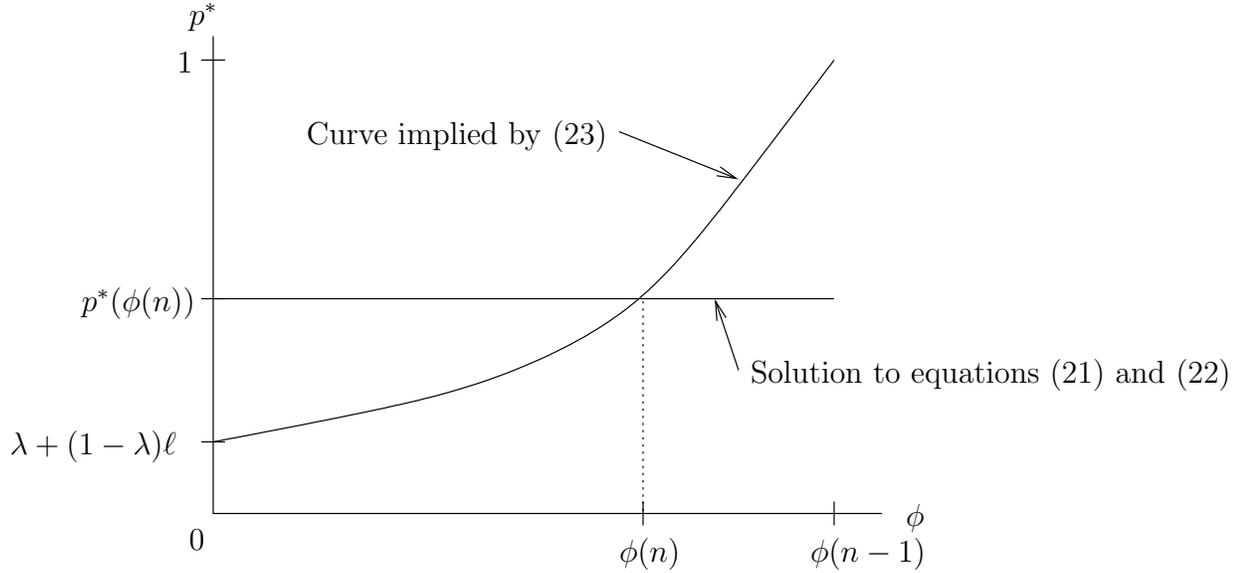


Figure 5: Plot of Inequality (15) and $\pi(\phi) = \frac{\phi(1-\phi^B)}{(\phi^B(1-\phi))}$. The downward sloping curve represents the lower bound on $\pi(\phi)$ imposed by (15). The region above this curve is the set of all pairs $(\phi, \pi(\phi))$ such that the opportunistic issuer at least weakly prefers the quantity $q^*(\phi)$ at price per unit $p^*(\phi)$ implied by the strategy $\pi(\phi)$ and reputation ϕ . The upward sloping curve is given by the definition of $\pi(\phi)$. The portion of the upward sloping curve which lies above the downward sloping curve is the set of all pairs $(\phi, \pi(\phi))$ such that inequality (15) is satisfied and inequality (11) binds, and this set may contain many points supportable by an equilibrium. To simplify the analysis, I choose the point at which the two curves intersect denoted by $(\pi(\tilde{\phi}), \tilde{\phi})$. This point gives the lowest value of π such that both inequality (15) and the definition of π are satisfied.



(a) Equations (21) and (22) in price-quantity space. The equations both define equilibrium price p^* as a function of equilibrium quantity q^* and satisfy a single-crossing property which allows for a unique solution $(q^*(\phi(n)), p^*(\phi(n)))$.



(b) The solution to (21) and (22) versus (23). Equation (23) defines p^* as a downward sloping function of ϕ , thus giving a unique solution $(\phi(n), p^*(\phi(n)))$.

Figure 6: The solution to the system of equations (21)–(22). Together, the single-crossings shown in panels (a) and (b) imply that the solution is unique.

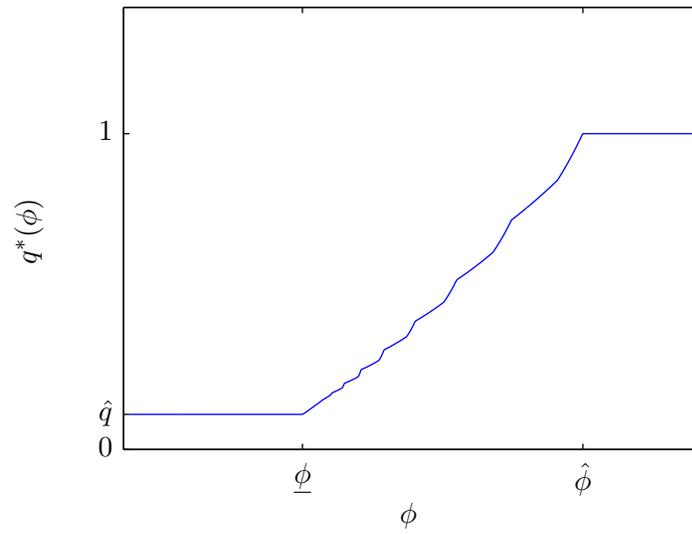


Figure 7: A plot of $q^*(\phi)$ versus ϕ . For $\phi \leq \hat{\phi}$, the issuer simply issues the quantity from the least cost separating equilibrium. For $\hat{\phi} \leq \phi \leq \hat{\phi}$, the issuer chooses an increasing quantity in her reputation. For $\phi > \hat{\phi}$ the issuer chooses to sell the entire asset.

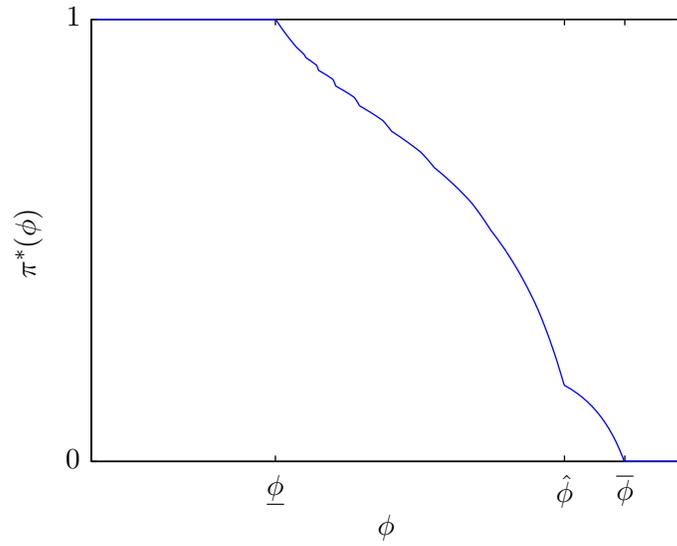


Figure 8: A plot of $\pi^*(\phi)$ versus ϕ . For $\phi \leq \underline{\phi}$ the issuer simply plays the least cost separating equilibrium and never misreports a bad type asset. For $\underline{\phi} < \phi < \hat{\phi}$ the issuer increases the probability of misreporting a bad type asset as her reputation increases. The kink at $\hat{\phi}$ arises from the constraint that quantity cannot be greater than one. Finally for $\phi > \hat{\phi}$ the issuer always misreports a bad type asset.

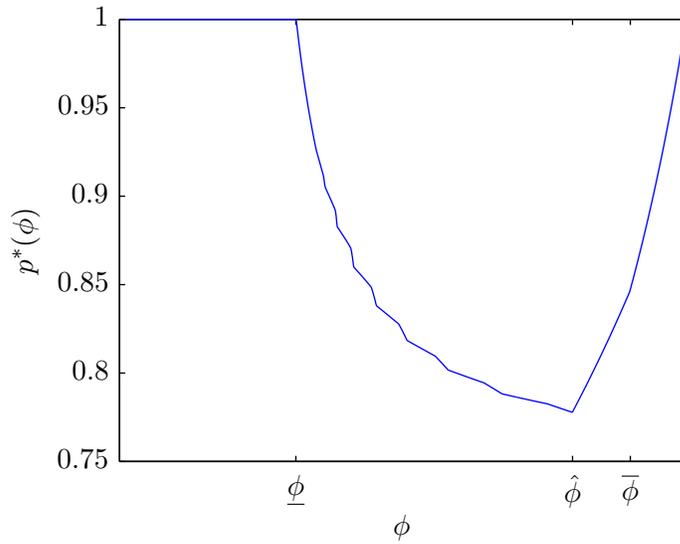


Figure 9: A plot of $p^*(\phi)$ versus ϕ . For $\phi \leq \underline{\phi}$, the opportunistic issuer never reports that a bad type asset is the bad type, hence the equilibrium price for a reportedly good type asset must be one. For $\underline{\phi} \leq \phi \leq \bar{\phi}$, the equilibrium price is a decreasing function of reputation. For relatively low (high) levels of reputation the potential gain from misreporting a bad type asset is relatively small (large) since any probability that the issuer misreports a bad type asset impacts prices more (less) at low (high) levels of reputation.

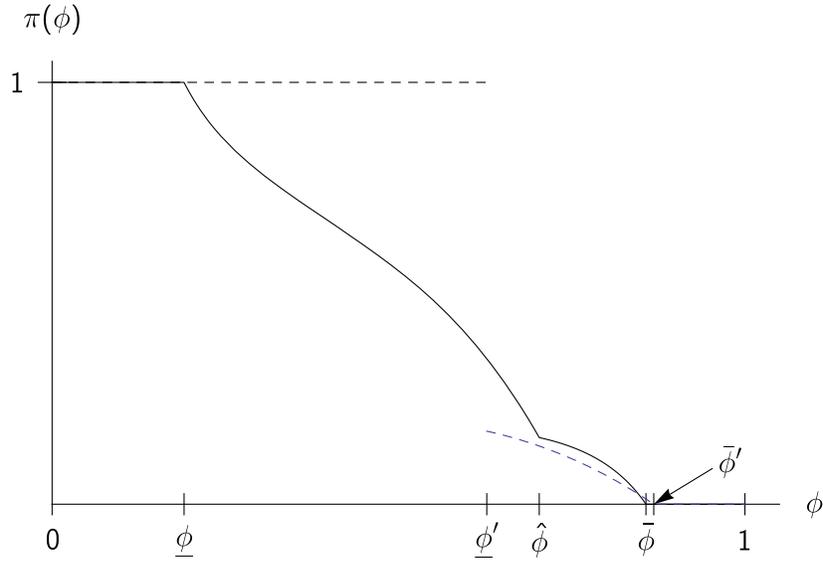


Figure 10: A comparison of $\pi(\phi)$ for the divisible and indivisible asset settings. The solid curve is the reporting strategy for the divisible setting, while the dashed curve is the reporting strategy for the indivisible setting. $\underline{\phi} < \underline{\phi}'$ implies that the indivisibility of assets causes the issuer to refrain from sending inaccurate reports at levels of reputation which would be done if part of the asset of the asset could be retained. Similarly, $\bar{\phi} < \bar{\phi}'$ implies the issuer will send an inaccurate report with probability 1 for lower levels of reputation when part of the asset can be retained than when the asset is indivisible.