Market Implied Costs of Bankruptcy *

Johann Reindl
BI Norwegian Business School

Neal Stoughton
WU-Vienna University of Economics and Business

Josef Zechner
WU-Vienna University of Economics and Business

March 2017

Abstract

This paper examines bankruptcy costs using market prices of equity and put options during the financial crisis. Our approach avoids the usual selection bias and does not require the optimal tradeoff theory of capital structure to hold. While the average bankruptcy cost is about 20%, we find wide variation across and within industries. These are related positively to asset volatility, growth options, and labor intensity and negatively to tangibility, size, weak corporate governance and entrenched management. Using our results we also find strong support for the tradeoff theory.

1 Introduction

Bankruptcy costs, along with the tax advantage of interest deductibility, are one of the two key determinants in the tradeoff theory of capital structure. This theory – which has been at the forefront of finance research over the last 50 years – hypothesizes that bankruptcy costs are to be weighed against the advantage of interest deductibility of corporate debt in determining an *optimal* capital structure. While a lot of progress has been made with respect to estimating the corporate tax advantage of debt, the magnitude and cross-sectional distribution of bankruptcy costs have only recently attracted substantial interest. A main obstacle to obtaining good empirical estimates of bankruptcy costs is the selection bias implicit in samples of

*This paper has been presented at the University of Alberta, the University of Hong Kong, HKUST, the Goethe University Frankfurt the Frankfurt School of Management, the University of Oklahoma, the University of Zürich, the European Finance Association, the European Winter Finance Conference, the Financial Research Association, and the IDC Rothschild Ceasarea Conference. We appreciate the helpful comments of Charlotte Ostergaard, Rudiger Frey, Oyvind Norli, Jean-Charles Rochet, Alexander Schandlbauer and Toni Whited, members of the seminar audiences and discussants Patrick Bolton, Murray Carlson, Egor Matveyev and Martin Schmalz.

bankrupt firms. As pointed out by Andrade & Kaplan (1998) in an important study, bankruptcy costs and the probability of bankruptcy are likely to be negatively correlated. Using a simulated economy where firms are assumed to behave according to the tradeoff theory, Glover (2016) shows that this selection bias is likely to be substantial.

Thus, we require an unconditional sample of firms in order to obtain unbiased estimates of bankruptcy costs. This can be achieved in principle by backing out bankruptcy costs implicit in observable prices or accounting data of non-bankrupt firms. Such an insight has first been utilized by Glover (2016). Intuitively, Glover's paper estimates the level of bankruptcy costs which induces a firm to choose the observed leverage ratio if it optimizes leverage according to the tradeoff model. The resulting bankruptcy cost estimates are shown to be significantly higher than most estimates reported in the literature, often between 40% and 45%. One byproduct of this procedure is that a negative relation between leverage and bankruptcy costs is built in. As a consequence, whenever a firm's leverage choice is influenced by factors other than taxes and bankruptcy costs, the resulting estimates will be biased. Another difficulty with this approach is that it cannot provide evidence on the key question whether the tradeoff theory holds empirically. To see this, consider a firm with low leverage. The estimation approach will attribute the low leverage to high bankruptcy costs while in fact a firm could have chosen a low leverage ratio for other reasons. It may have valuable growth options and may therefore want to prevent debt-overhang, or it may have a labor-intensive production technology, or a high operating leverage, or a large amount of off-balance sheet liabilities. In these cases, the firm may actually face low bankruptcy costs but still choose a low leverage. Backing out bankruptcy costs via this type of structural tradeoff model therefore biases the estimates, since it would always indicate high bankruptcy costs in these cases. In these cases, bankruptcy costs are not reflected in a firm's leverage choice but they will show up in its security prices.

Our paper applies a novel approach to estimating bankruptcy costs, which does not impose an optimal capital structure tradeoff. Bankruptcy cost estimates are thereby extracted exclusively from security prices, using a general pricing model that specifies how tax-shields and bankruptcy costs are incorporated into prices while taking the firm's existing liabilities and maturity structure as given. In doing so, we do not impose any specific optimizing behavior by the firm. Bankruptcy cost estimates obtained in this way can therefore be used to test whether firms choose their leverage ratio in accordance with the tradeoff theory.

Compared to Glover (2016) we obtain much lower bankruptcy cost estimates on average: around 20% of the value of assets. Further we find wide variation across industries and within industries. Finally, we even observe negative bankruptcy costs for some firms. This is a result that clearly cannot be obtained when applying the tradeoff model in the estimation procedure but is consistent with evidence from actual bankruptcies (Andrade & Kaplan, 1998; Davydenko et al., 2012). We believe these occur as a result of non-shareholder value maximizing behavior, for instance due to managerial agency considerations or else other (hidden) non-debt liabilities such as pension or health care obligations.

A key advantage of our approach is that we can provide the first direct evidence that the tradeoff theory of capital structure holds. As previously mentioned, this is because we *do not* derive the bankruptcy cost estimates using a structural model that assumes the tradeoff theory holds, or even that equityholders determine the optimal time of bankruptcy. Using a sample of S&P 500 firms during the financial crisis, we find that firm-specific bankruptcy cost and asset volatility estimates explain 46% of the cross-sectional variation in leverage ratios *by themselves* and remain highly significant and economically important if we include a large set of additional variables commonly used in leverage regressions.

Ideally one would use the market prices of debt instruments to infer bankruptcy costs, since they are (residual) claimholders in the event of bankruptcy. This, however, is complicated by the lack of clean market prices for corporate debt. Also, firms' debt frequently consists of different components with significant heterogeneity due to contractual differences. This means that different classes of debt, such as secured and unsecured debt, usually reflect very different amounts of bankruptcy costs. To obtain consistent bankruptcy cost estimates from debt instruments, it is therefore necessary to analyze the market prices of all outstanding types of liabilities. Since large components of corporate liabilities, such as bank debt, are not traded, this is not feasible. All these criticisms also apply to credit default swaps (CDS) since a CDS contract typically specifies a particular reference obligation.

The cleanest set of market prices that could potentially be used to extract bankruptcy costs, are those related to a firm's equity. This approach is frustrated by the fact that, without further refinancing, the costs of bankruptcy are not reflected in equity prices, since they are not borne by equityholders ex post. However, in a more realistic situation, where firms face continued refinancing needs, equity prices will reflect bankruptcy costs, even in the absence of any new equity issues. To see this, consider a firm that wishes to roll over its maturing debt by issuing new debt with the same face value and the same coupon rate. Of course the market value of the new debt will in general not equal the required redemption payment to the old debtholders. If the difference is positive, it can be paid out to equityholders as a dividend; if negative, it must be financed via a reduced dividend or a new share issue. Under this scenario, bankruptcy costs are reflected in the market value of the new debt and therefore in the net distribution to the equityholders. Since the ex-ante equity price reflects future debt refinancings, it therefore must incorporate bankruptcy costs.

This is the essence of our approach. We use a pricing model based on Leland (1994) or Leland (1998) that allows for debt refinancing to back out bankruptcy costs from equity securities. We do not rely solely on common equity prices but augment our estimation procedure through the observation of equity put option prices. An out-of-the-money put option can be seen as a CDS surrogate since its price is very sensitive to future distress states in which bankruptcy costs are likely to be a significant determinant of stock prices. This leads to a significant improvement in the accuracy with which bankruptcy costs can be estimated. In doing so, the paper derives put option prices for this structural model of debt refinancing.

Including put prices in the estimation procedure is particularly important to obtain accurate estimates of the asset volatility and the default threshold, which are two important parameters of the structural model we consider. This is so since different combinations of these two parameters can be consistent with a particular observed equity price. Using put options in the estimation procedure solves this identification problem since their prices are affected differently by the default threshold than equity prices. A higher default threshold affects the equity price negatively but the put price positively. [1]

Our bankruptcy cost estimates exhibit considerable between and within industry variation.[2] To understand the determinants of bankruptcy costs and check whether our estimates are reasonable, we relate them to firm characteristics. We find that bankruptcy costs are strongly positively related to the underlying asset volatility and negatively to firm size, asset tangibility, and brand and patents. The last two variables capture how transferable a firm's assets might be in bankruptcy. We find that market to book ratios are positively correlated with bankruptcy costs, which provides strong support for the hypothesis that growth options are lost in bankruptcy. Similarly, bankruptcy costs are higher for firms with more labor or skill intensive production. We also find specific evidence that firms might benefit from bankruptcy. Bankruptcy can be profitable for firms that have weak corporate governance, an entrenched management, employ their assets less efficiently than their industry peers, or have defined benefit pension plans in place. Finally, bankruptcy costs are lower for assets that can be repossessed more easily.

Moreover, we explore the determinants of leverage ratios via a cross-sectional analysis. When we include our estimates of bankruptcy costs we improve the explanatory power in the cross-section considerably over the previous literature. Our direct measure of bankruptcy costs is negatively related to leverage, which provides considerable support for the tradeoff theory of capital structure. Also, the asset volatility estimates show up strongly in the cross-sectional relationship as having a negative effect on leverage.

Finally, our method is also extended to provide estimates of hidden liabilities, which are either off the balance sheet, or difficult to measure, such as health care liabilities or employee labor *legacy* contracts. We find considerable cross-sectional variation here as well.

The literature on bankruptcy costs has a long history. One important approach looks at direct costs of firms that have gone bankrupt. Weiss (1990) evaluates 37 Chapter 11 bankruptcies between 1980 and 1986 and finds direct costs of bankruptcy average 3.1% of the book value of debt plus the market value of equity. Ang

[1] Existing papers get around the problem by assuming that only equityholders can decide upon default, which leads to an endogenously determined default threshold. We find it very important not to assume that firms only default when it is ex-post optimal for equityholders given the prevalence of technical default conditions in bond and loan contracts which are precisely there to preserve value. Because of their limited liability, equityholders would in general let the firm value deteriorate much more before declaring bankruptcy than debtholders would. Incorrectly specifying the default decision leads to biased bankruptcy cost estimates.

[2] A high within industry variation is also a characteristic of the cross-section of leverage ratios and has been described as puzzling by the existing literature (see, e.g., Lemmon et al., 2008; Graham & Leary, 2011).

et al. (1982) report bankruptcy costs of 7.5% of total liquidation value of assets for 86 liquidations between 1963 and 1979. However, for small firms bankruptcy fees might wipe out 100% of the assets. Bris et al. (2006) consider 300 cases of mostly smaller nonpublic firms between 1995-2001. They find that in 68% of Chapter 7 cases, the bankruptcy fees exceeded the entire estate.

A series of papers have also attempted to measure indirect bankruptcy costs. One difficulty lies in distinguishing actual distress costs from the economic factors ultimately responsible for pushing the firm into difficulty. Altman (1984) deals with this by comparing expected profits to actual profits for the 3 years prior to bankruptcy. He finds an average cost of 10% of firm value measured just prior to bankruptcy. Combined direct and indirect costs average 16.7% of firm value for this sample. Andrade & Kaplan (1998) consider 31 firms that have become financially distressed after a management buyout or a leveraged recapitalization between 1980 and 1989 but were not economically distressed. They find costs of financial distress between 10% and 20% of firm value. These estimates are used by Almeida & Philippon (2007) to calculate the ex-ante value of distress costs by multiplying them by the risk neutral default probabilities obtained from CDS spreads. These ex ante estimates amount to an average of 4.5%. Elkamhi et al. (2012) point out that estimates by Andrade & Kaplan (1998) should be applied to ex-post asset values at the time of bankruptcy. They therefore extend this approach using a structural model, which allows them to map the ex-post bankruptcy cost percentages to ex-ante percentages and find that they are too low to support commonly observed leverage ratios. Nevertheless they still rely on the original estimates by Andrade & Kaplan (1998).

Korteweg (2010) estimates the net benefits to leverage and attempts to extend this to measure bankruptcy costs for the small subset of firms that are at or near distress. Hence his method is unable to avoid the selection bias mentioned previously. Davydenko et al. (2012) back out distress costs from market value changes upon the announcement of default. Assuming that investors do not fully anticipate default, distress costs can be estimated from the change in the market value of the firm upon announcement. They find average costs of distress of 21%, lower costs of 20.2% for highly-levered firms and higher costs for investment-grade firms (28.8%). Once again, these estimates may be biased since severely distressed firms are likely to be the ones with low bankruptcy costs.

The paper proceeds as follows. Section 2 contains the structural model. Section 3 documents the estimation procedure and describes the data. Our main results are reported in Section 4 with respect to bankruptcy costs estimates and explanatory variables. Section 5 contains the leverage regressions and test of the tradeoff theory. Section 6 concludes. Some of the technical results are contained in an appendix, as is a robustness simulation and generalizations.

2 The Structural Model

In contrast to other approaches that rely on the prices of debt securities or credit default swaps our approach relies on the use of market prices of equity and equity derivatives. This approach has several advantages. First, many debt securities are not traded at all. Second, even if they are traded, they are often illiquid and characterized by high bid-ask spreads. Also their prices depend on asset specific features, such as covenants and seniority. Third, bankruptcy may be triggered by liabilities other than debt, such as defined benefit pension plans, for which market prices do not exist. By contrast, equity is a residual claim and therefore its price is only affected by the net value once other claims are deducted, regardless of shifts in claims among these liability classes.

While equity is clearly affected by the probability of bankruptcy, it is less clear how it is affected by bankruptcy costs, since equityholders usually do not bear these costs *ex post*. However, in a dynamic model of capital structure changes over time, where firms must roll over debt, bankruptcy costs do affect equity values since they impact the price at which new debt can be issued. We therefore rely on a parsimonious dynamic capital structure model in which firms continuously refinance a constant fraction of their debt.

More specifically, we consider the debt of a firm to consist of a continuum of maturities, from zero to infinity. In any instant of time, a fraction m of the outstanding face value of total debt, B, is retired. Thus, the face value of the original debt that remains at time t is equal to $e^{-mt}B$. At any point in time, the expiring debt is replaced by a new issue with face value mB of equal seniority. This new issue consists again of a continuum of maturities, matching the original profile of the debt before refinancing. Thus, the total face value of debt, B, remains constant over time with an average maturity of $M = 1/m$ [3]. This stationary debt policy has been used in Leland (1994) and Leland (1998). Another model with alternative refinancing dynamics is Leland & Toft (1996).[4] This alternate model has fixed finite maturity debt re-issuance over time. We have applied our estimation method to simulated data obeying the assumptions of this alternative model and find that the true bankruptcy costs are recovered with negligible bias. This presentation is given in appendix B.2.

In our refinancing environment, the firm's aggregate coupon payment per unit of time is denoted by C and is assumed to be constant over time. Thus, total payments to all debt holders (debt replacement plus coupon) per unit of time, dt, are given by $(C + mB)dt$.[5]

Importantly we do not impose the requirement that equityholders control the decision to default. In general equity holders are willing to continue to pay the interest costs in return for receiving cash flows from

[3]We relax this theoretical assumption of constant face value in the empirical section, however within our data sample we find remarkably little variation in book values.

[4]Endogenous rollover is in the model of Dangl & Zechner (2016).

[5]Although we do not include issuance costs in our formal model, the model could potentially be extended in this direction. Specifically one could add a small proportional equity issuance cost in the case of negative dividends (where the equityholders are injecting capital). Also debt issuance costs could be treated as an outflow that is proportional to the face value of new debt issues.

earnings and refinancings until the unlevered value of the firm is sufficiently low. In practice, however, covenant violations might cause defaults at an earlier point. On the other hand, default at a later point could ensue due to agency costs where the decision is controlled by management rather than shareholders. We therefore model bankruptcy as the first passage time where the value of the firm strikes a constant default barrier.

The firm is assumed to generate earnings before interest and taxes, EBIT, that follows a geometric Brownian motion with drift $\hat{\mu}$ under the risk neutral measure, Q. Therefore, after-tax earnings of an all-equity firm, X_t, is given by $X_t = (1-\tau)EBIT$, with Q-dynamics given by

$$dX_t = \hat{\mu} X_t dt + \sigma X_t dW_t.$$

We define the value of unlevered assets, A_t, as the present value of future after-tax earnings:

$$\begin{aligned} A_t &\equiv E^Q\left[\int_{s=t}^{\infty} e^{-rs} X_s ds\right] \\ &= \frac{X_t}{r-\hat{\mu}} \end{aligned} \tag{1}$$

Let $\delta = \frac{X_t}{A_t} = r - \hat{\mu}$ denote the earnings yield on the unlevered asset value. Thus, the dynamics of A under the risk neutral measure satisfies

$$dA_t = (r-\delta)A_t dt + \sigma A_t dW_t.$$

We now derive the value of the levered firm, V_t. As in the standard tradeoff theory, the value of V_t is the sum of the unlevered asset value plus the present value of tax-shields minus the present value of bankruptcy costs. Let $G(t, A_t)$ be the price at time t of an Arrow-Debreu security that pays one dollar at the time of bankruptcy, T_B, when the unlevered asset value is A_B. Using risk-neutral valuation, the price of this security at time t is

$$G(t, A_t) \equiv E^Q[e^{-rT_B}] \tag{2}$$

$$= \left(\frac{A_t}{A_B}\right)^{-\eta(r)} \tag{3}$$

where

$$\begin{aligned} \eta(r) &= \frac{\mu_B + \sqrt{\mu_B^2 + 2r\sigma^2}}{\sigma^2} \\ \mu_B &= r - \delta - \frac{\sigma^2}{2} \end{aligned}$$

Therefore the levered firm value at time t is given by

$$V(A_t) = A_t + \frac{\tau C}{r}\left[1 - G(t, A_t)\right] - \alpha A_B G(t, A_t) \tag{4}$$

where the second term is the present value of the tax shield reflecting states in which the firm does not go bankrupt. The third term represents the present value of bankruptcy costs, assuming that costs are a proportion α of the value of the unlevered assets at the time of default, A_B. We do not explicitly allow for financial distress costs affecting equityholders *prior* to default. Nevertheless our model is consistent with a case in which these costs are accumulated and incurred at the time of bankruptcy. Since the present value of such costs impacts the price at which new debt can be issued, these costs therefore impact equityholders *before* bankruptcy when they refinance a proportion of the existing debt. Our model is also applicable to a situation where bankruptcy costs are *negative*. This might result from a situation where all financial claimholders are better off in bankruptcy because of the ability to extinguish a non-financial liability.

As shown by Leland (1994), if equity holders control the default decision entirely and there are no other non-financial liabilities to consider, the default boundary would be determined by the smooth pasting condition as:

$$A_B^* = \frac{\frac{C+mB}{r+m}\eta(z) - \frac{\tau C}{r}\eta(r)}{1 + (1-\alpha)\eta(z) + \alpha\eta(r)}, \tag{5}$$

where $z = r + m$. Intuitively, note that a negative bankruptcy cost, $\alpha < 0$ implies that equity holders will default later, since $\eta(z) > \eta(r)$.

2.1 Valuing Corporate Securities

We now use the above pricing equations to derive the values of corporate securities and derivatives thereof. We begin with the value of corporate debt outstanding at time t. Its value is the present value of the cash flows to debt holders before bankruptcy plus the value received by debtholders when bankruptcy occurs, i.e. the boundary A_B is reached. Because of the redemption schedule of debt, for every dollar of face value at time t, there will be $e^{-m(T_B - t)}$ dollars of the original face value outstanding at the time of bankruptcy. The time t price of an Arrow Debreu claim that pays exactly one dollar at time t if the debt claim remains outstanding at the time of bankruptcy is given by

$$G^z(t, A_t) = \left(\frac{A_t}{A_B}\right)^{-\eta(z)}.$$

Moreover the market value of existing debt at time t is given by

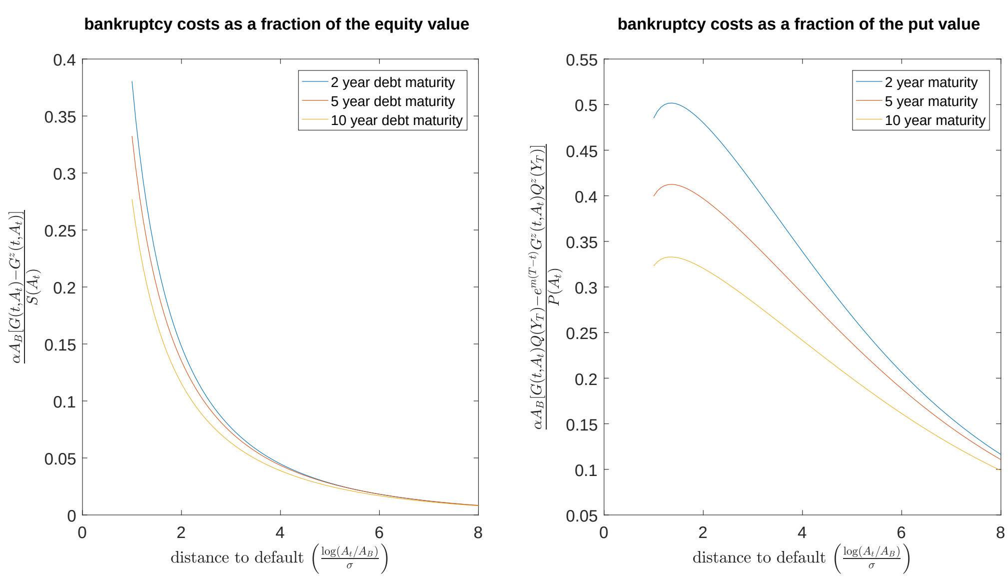
$$D(A_t) = \frac{C + mB}{z}\left[1 - G^z(t, A_t)\right] + (1-\alpha)A_B G^z(t, A_t). \tag{6}$$

Since the value of equity, $S(A_t)$, is the difference between the value of the levered firm and the value of debt, we get

$$S(A_t) = V(A_t) - D(A_t) \tag{7}$$

To see how bankruptcy costs enter the equity price, recall that αA_B are the ex-post bankruptcy costs in the event of default. The present value of these costs is given by $\alpha A_B G(t, A_t)$. Since the share of these costs borne by existing debtholders is $\alpha A_B G^z(t, A_t)$, it follows that the remaining amount, $\alpha A_B[G(t, A_t) - G^z(t, A_t)]$, is embedded in the equity price S_t. Therefore this is the crucial expression for how we identify bankruptcy costs. To illustrate, we take the theoretical model and our pricing expressions and illustrate the fraction of these costs in both equity and put prices in figure 1, as a function of different normalized distances to default. We show this for different debt maturities and varying degrees of default risk. In the case of an average maturity of 5 years, the impact of bankruptcy costs amounts to 25% of the equity value for high risk firms and 2% for firms with very low default risks. For put options the corresponding percentages are about 40% and 18%.

Figure 1: This graph shows the bankruptcy costs borne by equityholders as a fraction of total equity value and the contribution of bankruptcy costs to the put price as a fraction of the put price. The default threshold is assumed to be chosen optimally by equityholders and the unlevered asset value (A_t) is varied in such a way that the distance to default varies between 1 and 8. Calculations are based on the following parameter values: $\sigma = 0.2$, $B = 50$, $C = 2.5$, $r = 0.02$, $\delta = 0.03$, $\alpha = 0.2$, $\tau = 0.35$. The option has 90 days to maturity and its strike price is equal to 0.95 times the equity value. The three lines differ only in the maturity of debt.



In order to identify the parameters of the underlying structural model, we rely on equity as well as the price of traded derivatives, specifically put prices. We do this because the latter are even more sensitive to the possibility of bankruptcy than equity itself, and further puts–like equity–are residual claims that are not affected by the priority of various classes of debtholders. The use of put options greatly assists with the estimation. Importantly, it helps to disentangle the default boundary from the asset volatility, since both parameters tend to substitute for each other in the valuation of equity while having a complementary effect on the put valuation. Second, puts always contain a sizeable amount of the bankruptcy costs as illustrated above. Appendix E contains a detailed discussion of the benefits of put options for the identification of the model parameters.

In this framework put options are compound options, since equity itself is already a call option on the asset value. In addition a put option on a levered firm has features similar to a barrier/knock-out option because the firm can default before the option expires. To derive a put pricing formula, we split the put payoff at maturity, P_T, into a part that is paid out if the firm has not defaulted and a part paid in case the firm has defaulted[6]:

$$P_T = (K - S(A_T))^+ 1_{T_B > T} + K 1_{T_B \leq T}, \tag{8}$$

where $(K - S(A_T))^+ = \max(K - S(A_T), 0)$ and $1_{T_B > T}$ is an indicator variable taking ont he value of one whenever the event $T_B > T$ is true (and similarly for $1_{T_B \leq T}$). The put payoff (8) formula reflects the compound nature of the option since the equity value at maturity, $S(A_T)$, is itself a function of the underlying firm value. In order to derive the price of the option at time t, we first define A^* as the time-T asset value for which the option is at the money ($S(A^*) = K$). The put price can be derived as the discounted expected value of the strike price over asset paths in which the firm goes bankrupt prior to expiration plus the discounted expected value of $K - S$ in states where the firm does not go bankrupt prior to expiration and $A_T \leq A^*$. Hence the put price is equal to the following expectation under the risk neutral measure, Q.

$$P_t = e^{-r(T-t)} E^Q \left[(K - S(A_T)) 1_{A_T \leq A^* \wedge T_B > T} \right] + K e^{-r(T-t)} E^Q \left[1_{T_B \leq T} \right]$$

In the appendix, we derive the following expression for the put price by substituting the stock price into the above formula and taking expectations. We employ several changes of measure to simplify the notation. The put has a positive value at expiry either when the firm goes bankrupt or when the option expires in the money but the firm has not gone bankrupt. In the former case, the stock price is zero, so the stock price does not enter the put pricing equation. However in the latter case it does. Define the set of sample paths for which the option is in the money and the firm does not default until maturity of the option as $Y_T = \{(A_t)_{t \in [0,T]} : A_T \leq A^*, T_B > T\}$. Let 1_{Y_T} be the indicator function equal to one in the event that Y_T is true. The put pricing formula involves taking expectations, $E(1_{Y_T})$, with respect to three probability measures. The first is a pricing measure with respect to the unlevered asset process, denoted by Q^A, the second, Q^G, is the measure with respect to the claim whose price (under the risk neutral measure) is $G(t, A_t)$,

[6]To obtain an analytical solution, we assume the options are European and neglect the price difference to the American variety. For instance, Bakshi et al. (2003) find that the difference between the American option implied volatility and the European option implied volatility is within the bid-ask spread.

and the third, Q^z is the claim whose price (also under the risk neutral measure) is $G^z(t, A_t)$. The put pricing formula is derived in the appendix as

$$\begin{aligned}P_t =& e^{-r(T-t)} K \left(Q(Y_T) + Q(T_B \leq T)\right) - A_t e^{-\delta(T-t)} Q^A(Y_T) \\ & - \frac{\tau C}{r} \left(e^{-r(T-t)} Q(Y_T) - G(t, A_t) Q^G(Y_T)\right) + \alpha A_B G(t, A_t) Q^G(Y_T) \\ & + \frac{C + mB}{z} \left(e^{-r(T-t)} Q(Y_T) - e^{m(T-t)} G^z(t, A_t) Q^z(Y_T)\right) \\ & + (1-\alpha) A_B e^{m(T-t)} G^z(t, A_t) Q^z(Y_T) \end{aligned} \tag{9}$$

Equation (9) together with the equity pricing formula (7) will now be used to estimate the underlying structural parameters, including bankruptcy costs, for our sample of firms.

3 Estimation Method

We use daily pricing data on equity and put options to estimate the structural parameters of the model for every individual firm in our sample. The complicating factors are that the pricing equations are non-linear, prices are observed with error and the underlying asset value process represents an unobservable latent variable. To deal with these issues, we specify our model in state-space form and apply a nonlinear Kalman filter. The parameters of the model are estimated using maximum likelihood. Compared to a simulated methods of moments approach we are able to exploit all time-series pricing data while obviating the issue of specifying appropriate moment conditions (see Strebulaev & Whited, 2012, for a discussion of empirical approaches in structural estimation).

3.1 Estimation of Structural Parameters and the Asset Value Process

Since observed prices of stocks and put options will in general differ from the theoretical prices of our model, we follow common practice and add an error term to the pricing equations (7) and (9). The observed pricing errors may be due to various reasons such as microstructure effects or non-synchronous trading of options and stocks. We assume additive, normally distributed errors in the log-specification for stock i

$$\begin{aligned} s_{i,t} &= s(A_{i,t}; \vartheta_i) + e^S_{i,t} \\ p_{i,t} &= p(A_{i,t}; K_i, \vartheta_i) + e^P_{i,t}, \end{aligned} \tag{10}$$

such that pricing errors can be interpreted as percentage deviations. $s(A_{i,t}; \vartheta_i) = \log S(A_{i,t}; \vartheta_i)$ where $S(A_{i,t}; \vartheta_i)$ is derived from equation (7) for the stock price of firm i as a function of the asset value and the model parameter vector ϑ_i. Similarly, $p(A_{i,t}; K_i, \vartheta_i) = \log P(A_{i,t}; K_i, \vartheta_i)$ denotes price of the put option derived in equation (9) which depends on the asset value, the strike price, and the vector of model parameters ϑ_i.

Our specification requires a non-standard estimation technique, because we have both pricing errors as well as an unobservable asset value in equation (10). Hence, estimation methods, such as standard maximum likelihood as applied by Duan (1994) or Ericsson & Reneby (2005) are not applicable. Instead, a Kalman-filter is first used to infer the unobservable asset value for each date, and then model parameters and states are jointly estimated, using maximum likelihood.

For the time series regression we need to specify the dynamics of the unlevered asset value process under the physical measure. Assuming a constant market price of risk, λ, the P-dynamics are given by

$$dA_t = \mu A_t dt + \sigma A_t dw_t, \tag{11}$$

where $\mu = r - \delta + \lambda\sigma$.

Let $a_t = logA_t$. From Itō's lemma it follows that the log-asset value process can be written in discrete time as

$$a_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + a_{t-1} + \sigma\sqrt{\Delta t}\, z_t \tag{12}$$

with $z_t \overset{iid}{\sim} N(0,1)$. Since pricing errors may be autocorrelated, we follow Bates (2000) in specifiying the following process for the errors in equation (10).

$$\begin{aligned} e^S_{i,t} &= \rho_{i,S} e^S_{i,t-1} + \varepsilon^S_{i,t} \\ e^P_{i,t} &= \rho_{i,P} e^P_{i,t-1} + \varepsilon^P_{i,t}, \end{aligned} \tag{13}$$

where σ_S is the standard deviation of $\varepsilon^S_{i,t}$ and σ_P is the standard deviation of $\varepsilon^P_{i,t}$. The system to be estimated can be represented in state-space form with the asset value process (12) and the AR(1)-process (13) forming the state equations and the pricing equations (10) as the measurement equation. While the state equation is linear the measurement equation is non-linear. We use the unscented Kalman filter to deal with the non-linearity of the measurement equation.[7] The transformation, on which the unscented Kalman filter is based, enables the calculation of unbiased estimates of the mean and covariance matrix of a transformed variable. In this case the transformed variables are the stock and put prices which are both functions of the asset value. The unscented transformation captures the true mean and covariance matrix of the prices accurately to the third order, assuming as we have in our model that A_t is a geometric Brownian motion. A detailed description of the unscented Kalman filter applied to our problem is given in appendix D.

3.2 Data

We use daily equity and put prices from May 2008 to September 2010 which were obtained from Datastream. The necessary accounting data are from WorldScope. Our initial sample consists of all constituent

[7] See Wan & Van Der Merwe (2001) for a comprehensive derivation and Carr & Wu (2010) for an application to continuous-time finance-models.

firms in the S&P500 as of December 2007. Out of these relatively large firms, two firms in this sample did in fact file for chapter 11 bankruptcy protection within the estimation period: GM on June 1, 2009 and CIT Group on November 1, 2009. Both firms were included in our estimation procedure. We require the firms to have at least 50 data points with a complete set of variables (stock and put option prices, as well as accounting variables) available. For every date, we use the closing stock price plus one put option[8] The option chosen must satisfy a minimum liquidity criterion. Specifically, we require the option to fall in the 50th-percentile of the most traded options during that day. In addition the option prices must satisfy the basic intrinsic value condition and relative arbitrage bounds must hold. As a consequence, the option price series to be fitted consists of a series of different put options with changing maturities and strike prices. We thus expect the model to fit option prices less well than stock prices.

3.2.1 Parameters to be estimated

Our structural model assumes that the principal amount of debt outstanding as well as the coupon rate, the tax rate and the average debt maturity is constant. In reality, firms do change their capital structures and, in fact, several restructuring events are observed for many of the firms in our sample. Even though total liabilities do not change by much from quarter to quarter we want to take into account that markets update their information. We therefore use the most recent balance sheet value of total liabilities – which is available at quarterly frequencies – as the book value of debt outstanding.[9] With the book value of debt changing over time, we also need to allow for the coupon, the debt maturity, the default barrier and the tax shield to change over time. To account for this, we assume that the coupon and the tax shield are affine functions of the latest book value of debt. Furthermore, in this case, from equation (5), it can be shown that the default boundary, A_B, is also an affine function of the book value of debt. As mentioned above, we allow the firm to default earlier or later than *ex post* optimal for equity holders. We also use a lower bound for the estimated boundary equal to one-half of the ex post optimal boundary. Finally, the average debt maturity is inferred from the latest balance sheet data on the proportion of long and short term debt.[10] In order to derive the average maturity of total liabilities, we start by calculating a weighted average of a long-term maturity, standardized to be five years, and a short-term maturity, standardized to one year, where the weights are given by the fraction of long and short-term debt divided by total liabilities. Then, we estimate the average maturity as an affine function of this weighted average of standard maturities.

Table 1 summarizes our estimation assumptions for the capital structure variables.

[8] Since put options with different strikes behave similarly with respect to changes in the asset value and in the other model parameters, very little would be gained by using more than one option in the estimation.

[9] A similar assumption is employed in Ericsson et al. (2007), Elkamhi et al. (2012), Eom et al. (2004), Bao & Pan (2013).

[10] While a typical firm usually has several different kinds of debt outstanding our capital structure model considers only a single bond. We treat all of them as a single debt issue. Consequently, the coupon rate and the maturity of debt have to be interpreted as averages over the different forms of debt.

Table 1: Capital Structure Parameter Estimates

variable	model	estimation specification
Debt book value	B	Balance sheet value of total liabilities
Coupon	C	$\lambda_C B$
Tax shield	τC	$\lambda_\tau B$
Default barrier	A_B	$\max\left(\lambda_B B, \frac{1}{2}A_B^*\right)$
Average maturity	$\frac{1}{m}$	$\lambda_m M$ where $M = \frac{\text{longterm Debt}}{\text{total Debt}} * 5 + (1 - \frac{\text{longterm Debt}}{\text{total Debt}}) * 1$

In total there are twelve parameters to be estimated for each firm using the stock and put prices. Therefore the estimated parameter vector can be described as $\vartheta = (\mu, \delta, \sigma^2, \lambda_B, \lambda_C, \lambda_\tau, \lambda_m, \alpha, \sigma_S, \sigma_P, \rho_S, \rho_P)$.

4 Bankruptcy Cost Estimates

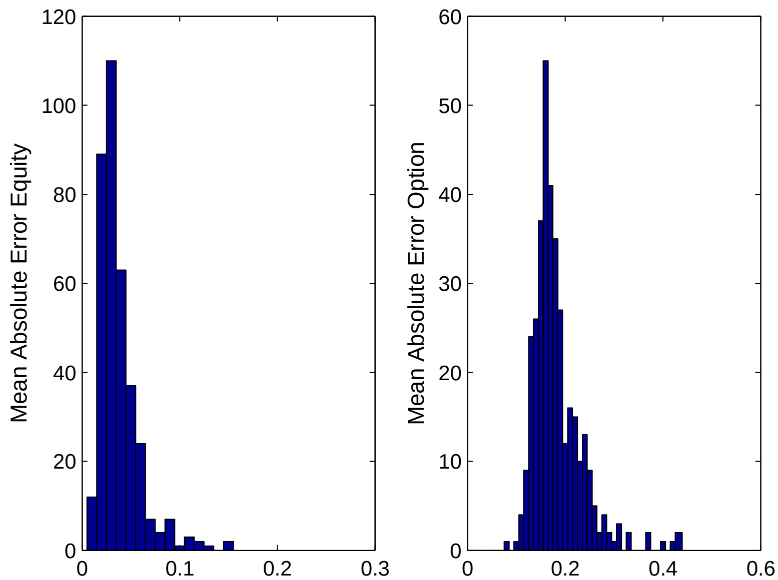
As mentioned above, we begin with the 500 constituents of the S&P 500 as of December, 2007. Out of this original population, we were unable to estimate the model for 116 firms since they lacked some relevant data (such as option prices or balance sheet liabilities). For 20 firms, the estimation procedure did not converge.[11] Therefore we were left with a remaining sample of 364 firms. For each firm we used the maximum likelihood procedure to estimate bankruptcy costs and underlying asset volatilities, along with their associated confidence bounds. In appendix section B we have performed a Monte Carlo simulation with a given bankruptcy cost and asset volatility and found that our estimation procedure results in unbiased estimates and reasonably tight confidence intervals.

To evaluate the marginal benefit of using option prices in addition to the stock prices, we attempted to estimate the parameters of the model with equity prices alone for a random subsample of the firms. The estimation did not converge in any of these cases. Therefore we conclude that the use of option prices is critical for this model specification. For our sample of 364 firms we evaluated the goodness-of-fit by computing the mean absolute value of the time series errors for the two security prices. We then aggregated the mean absolute pricing errors over all firms by computing the overall distribution of pricing errors for all firms which is indicated in Figure 2. We found that the most likely absolute error range was between 1 and 2 percent for equity prices and between 14 and 15 percent for option prices. Thus, equity prices appear to be estimated more precisely than option prices. This can be for a number of reasons. First, trading volume is lower for options than for stocks; hence microstructure effects may be more significant for the former. Also, for the options we periodically change the option series and strike price so the option is not necessarily the same over time. And of course the absolute price level of the stock price is much higher than the

[11] We did not find any systematic pattern among these firms that would indicate that they have biased our remaining sample in any significant way.

put price so it is likely that percentage deviations are much smaller.

Figure 2: Model Fit. This shows the distribution of mean absolute percentage errors of the actual and fitted stock price (left side) and the actual and fitted put option price (right side)



4.1 Industry Variation

The overall average bankruptcy cost for firms in our sample (equally weighted) is 0.20. This is substantially lower than the average obtained in Glover (2016), who observed an average of 0.45 among his sample. This can be attributed to the fact that his model imposed optimal leverage according to the tradeoff theory and such high levels of bankruptcy cost are required to prevent extreme levels of leverage from being chosen given the high benefits of the apparent tax shield. We also find a much larger variation in bankruptcy costs across industries and firms. While Glover found that bankruptcy costs varied across industries from a low of 0.35 to a high of 0.53, our estimation method produces estimates from near zero to over 0.60.

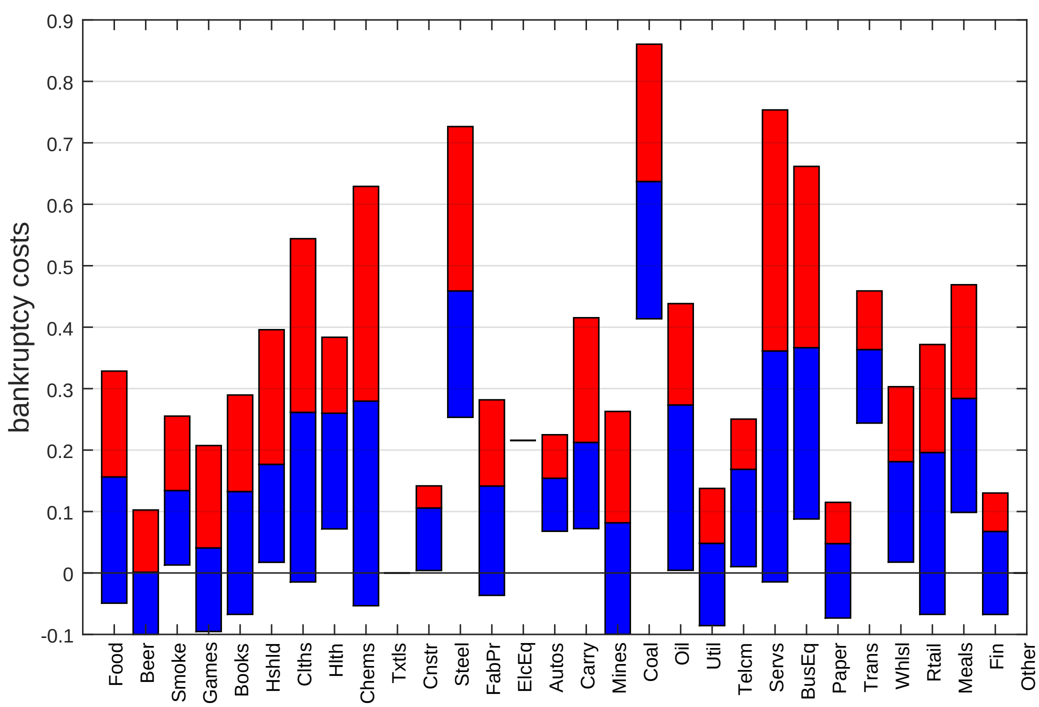
Figure 3 illustrates the differences by industry classification.[12] We display the point estimates as averages across firms in a given industry as well as the 25th percentile and 75th percentile bounds, in order to provide an idea of the intra industry spread.[13] Most of the bankruptcy cost estimates are in the range of 20-30%. Nevertheless there is huge cross-industry variation. We find that industries with high barriers to entry have

[12] We use the 30 Fama-French industry classifications available on `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_30_ind_port.html`. We have also tried other industry classifications but the results remain unaffected. Results are available from the authors upon request.

[13] When there are fewer than 4 firms in an industry the red bar is the maximum and the blue bar the minimum.

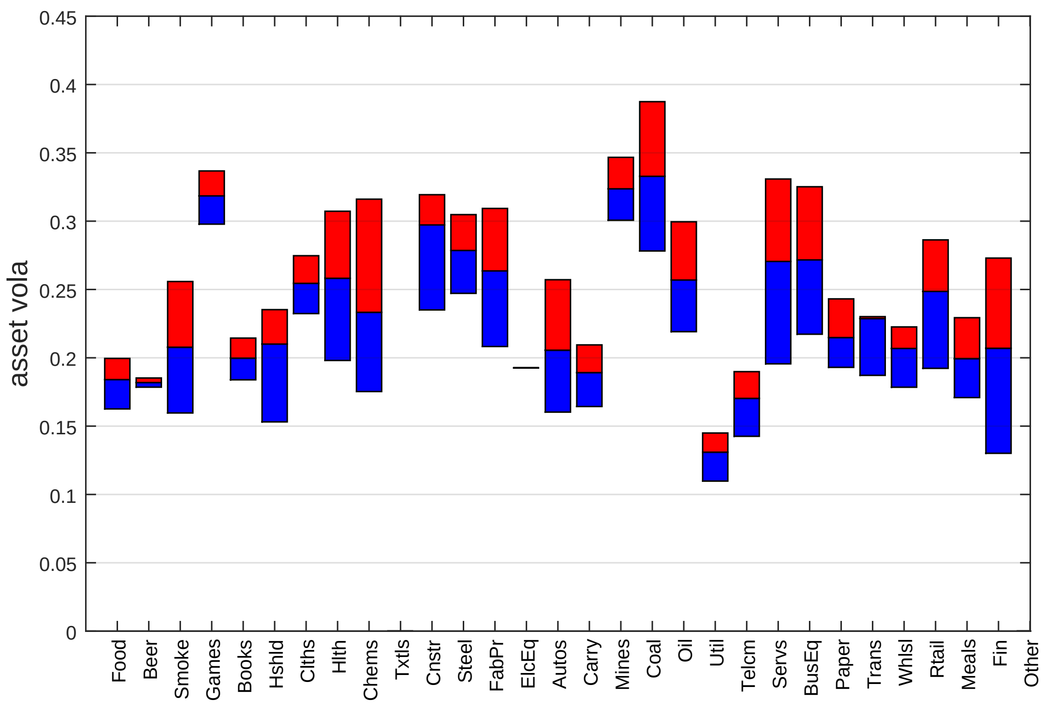
low bankruptcy costs. Food, gambling, tobacco, mining, and the financial industry are examples. This indicates that firms in such industries may continue to operate without severe adverse impacts subsequent to bankruptcy. Bankruptcy costs are higher for firms in services, business equipment and transportation. One potential reason for this finding is that they all rely on human capital and either explicit or implicit long-term contracts with customers. Such relationships may be irrevocably broken if the firm defaults. We look at these relations more specifically in the regression framework in subsection 4.2. The large within-industry variation in bankruptcy costs might be surprising at first sight. However, it confirms the result in the empirical capital structure literature that "within-industry leverage variation is twice as large as between-industry variation" (Graham & Leary, 2011). Moreover, when exploring the relationship between bankruptcy costs and firm characteristics in section 4.2, we find that bankruptcy costs are also determined by factors that are firm specific but not necessarily industry specific (e.g., corporate governance, management quality, pension plans, leasing).

Figure 3: Average Industry Distress Costs. This graph shows the percent bankruptcy costs as estimated using 30 Fama-French industry classifications. The midpoint of the bar shows the average within industry estimate; the red bar shows the variation from the mean to the 75th percentile and the blue shows the variation to the 25th percentile.



As part of our estimation procedure we derive the underlying (unlevered) asset value process, A_t. The

Figure 4: Average Industry Asset Volatility. This graph shows the average asset volatility estimates by 30 Fama-French industry classification. The midpoint of the bar graph shows the within industry average; the red bar shows the variation from the mean to the 75th percentile and the blue shows the variation to the 25th percentile.



average volatility of this process throughout our sample is displayed by industry in Figure 4. As with the previous graph, we display the point estimates for volatility as well as the 25th and 75th percentile limits. We find that point estimates of unlevered asset volatilities are around the level of 0.2, which is similar to those in papers using different methodologies.[14]

We also find some cross-industry variation. Gambling, construction, coal and oil are among the industries with the highest volatility levels. This is intuitive. Utilities have a very low asset volatility – this also accords with intuition. Within industry variation may likely depend on the breadth of the industry definition. For instance household products, chemicals, services and financials have greater variation in volatility estimates.

4.2 Determinants of Bankruptcy Costs

To better understand the size and source of bankruptcy costs, we explore whether our bankruptcy cost estimates are related to a series of explanatory variables. We investigate whether bankruptcy costs are related to firm size and cashflow risk, redeployability of assets, transferability of know-how and growth options, labor intensity, pension plans, corporate governance, and the treatment of assets in the bankruptcy procedure. In doing so we utilize a cross-sectional regression framework:

$$\alpha_i = \beta_0 + \beta_1^\top Y_i + IND_j + \varepsilon_i,$$

where Y_i represents a vector of firm characteristics for firm i, and IND_j are industry fixed effects (for industry j). The explanatory variables chosen are from the beginning of the time series estimation period (second quarter 2008) which was used to estimate the bankruptcy costs. Some of the explanatory variables derive from our estimation results. Others are calculated from other items such as balance sheet reports. The variables are defined in Table 5 in the appendix. In table 2 we report the regression results with and without industry fixed effects for two different sets of regressors. For the first set (column 1 and 2) we use the book value of total assets as a normalizing factor and for the second set (column 3 and 4) we use the unlevered firm value estimated via the Kalman filter for this purpose. The adjusted R^2 including industry fixed effects does not increase very much.[15] Hence, we conclude that most of the industry variations are already incorporated in the other right hand side variables. Overall our results are quite respectable in terms of explanatory power. Glover (2016) obtains high explanatory power for his bankruptcy cost determinant regression only when including leverage on the right hand side. Given that his bankruptcy costs were estimated by matching observed leverage ratios in the first place, his estimates are therefore not surprising.

[14] Schaefer & Strebulaev (2008) use the standard approach to unlever the equity and debt volatility and arrive at very similar values. Their 5% quantile is 0.10 and and their 95% quantile is 36% and the mean is 22%. Elkamhi et al. (2012) find values between 0.25 and 0.42.

[15] The p-value for the F-test of joint insignificance of the industry dummies is 10% for the first set regressors and 12% for the second set.

Without leverage his R^2 is around 0.20 or below. By contrast with a much smaller sample of firms we obtain R^2 at 0.38 or greater.

Our results exhibit patterns that are in accordance with what the theoretical literature has suggested. Specifically from table 2, we see that bankruptcy costs are strongly increasing in asset volatility.[16] This could be due to asymmetric information since higher asset volatility may reflect a less liquid market for the underlying assets. Moreover, asset volatility may result from larger growth options which may not be transferable in the event of bankruptcy, implying higher costs. Next, we investigate how bankruptcy costs are affected by the liquidity and transferability of a firm's assets. Tangibility relates negatively to bankruptcy costs when we use our method for estimating asset values (columns 3 and 4 in table 2). There is obviously a more liquid market for tangible assets, there are fewer informational asymmetries, and the liquidation value is close to book value, implying that there is less likelihood of a "fire sale" discount. We also find that less firm value is lost in bankruptcy if intangible assets are more fungible, which is the case for brand names and patents. The market to book ratio enters with a positive sign in terms of bankruptcy costs. This provides strong direct evidence that growth options, which are likely to be closely linked to key employees in the company, are expected to be lost in the event of bankruptcy. This finding is closely related to the concept of the inalienability of human capital in Hart & Moore (1994). While their model builds on the idea that an entrepreneur cannot pledge his human capital, we want to stress that a firm cannot credibly pledge the human capital of its key employees either. This is true especially if they have been compensated with stock and options that is now worthless in the event of bankruptcy.

Related to the last aspect, we also find a strong relationship between bankruptcy costs and labor. Bankruptcy costs in relation to human capital can arise for various reasons. Employees will start to look for other jobs and devote less of their time to fulfilling the objectives of the company, the onset of bankruptcy proceedings might distract attention and create morale problems. We use the employees to sales ratio as a proxy for labor intensity and two different measures for skill intensity: the median wage in the firm's industry and the CEO pay. The first measure should capture a firm's reliance on skilled labor and the second captures its reliance on management talent. Both measures depend on the assumption that skills and wages are positively correlated. In all four specifications in table 2, labor intensity has a highly significant positive relation to bankruptcy costs. The two skills variables are also positively related to bankruptcy costs.

Another potential determinant of bankruptcy costs is the treatment of assets in bankruptcy. Sizable costs can arise when creditors try to obtain the title to the assets of the firm in default. Costs are particularly low if the assets are exempted from the automatic stay in Chapter 11. For operating leases this can be the case. We check whether the overall fraction of the firm value lost in bankruptcy is lower if more assets are financed via operating leases. Since firms do not report the value of the operating leases but only the

[16]Our simulations in the appendix section B indicate that this relation is not the result of a spurious correlation built into our estimation procedure.

operating lease expenses, we follow the existing literature and capitalize the operational lease expenses.[17] We normalize capitalized operating leases as well as property, plant, and equipment under capital leases by total assets. The negative and significant coefficients for our capitalized operating lease variable and the insignificant coefficients for capital leases indicate that the treatment in bankruptcy matters for the costs incurred. In particular, the ability to repossess the leased assets before the bankruptcy procedure preserves value. Recent support for this perspective is given in Eisfeldt & Rampini (2009), who have developed a theoretical model for the choice between leasing and secured lending which builds on the assumption that leasing entails lower bankruptcy costs because of the lessor's ability to repossess the leased assets in the case of bankruptcy.

Clearly the skill of management and the extent of control of the board can play a role in determining bankruptcy costs since management and the board are often replaced following bankruptcy. Specifically, Gilson (1989) finds that in a sample of 69 firms filing for bankruptcy, 71% of senior managers are replaced within the period from two years prior to two years after the bankruptcy filing. Hotchkiss (1995) reports that 70% of CEOs in office two years prior to filing are replaced. We explore such governance effects on bankruptcy costs. We follow standard procedures by using the size of the board and CEO/chairman duality to construct an indicator for a weak board that previous work has found to be less efficient in monitoring and replacing management. To capture the degree by which the management can shield itself against external governance measures, we use a takeover defense variable that records the presence of a poison bill and a staggered board. Consistent with the view that bankruptcy allows a firm to replace entrenched management, both variables have a sizable negative impact on bankruptcy costs. Supermajority requirements for amendments to bylaws and endorsements of mergers are another set of corporate governance provisions that the existing literature has linked to managerial entrenchment[18]. The positive coefficient we report in table 2 suggests that supermajority provisions are an impediment to the relatively complicated decision finding process in a bankruptcy procedure. In cases where the consent of equityholders is required, such provisions most likely increase the time spent in bankruptcy, lead to suboptimal decisions, and thereby increase the value lost in bankruptcy. Finally, badly run firms should underperform their industry peers. We therefore calculate the difference between the return on assets (ROA) of a firm and the average return on assets in its industry (ind. ROA). We find that only when firms underperform their peers are bankrupcty costs reduced.

Since in our view bankruptcy involves an event that transfers control between shareholders and debtholders, the costs thereof may be impacted by other nonfinancial liabilities such as defined benefit pension plans. Frequently such plans are underfunded based on actuarial accounting. When a firm enters bankruptcy this liability due to underfunding might be expunged. The ERISA act of 1974 established the PBGC

[17]See table 5 in the appendix for the calculation details.

[18]Bebchuk et al. (2009) have constructed an entrenchment index that comprises the two takeover defense variables, the supermajority requirements, and the presence of golden parachute.

which insures the pension only up to a maximum level. We include several pension plan related explanatory variables to capture the different ways in which an existing defined benefit pension plan might affect a firm's value in bankruptcy. First of all, bankruptcy provides an opportunity to terminate the plan and avoid the future defined benefit accruals. The value gain from termination should equal the capitalized future defined benefit accruals saved. We approximate this number by assuming that pension fund contributions form a perpetuity and then relate it to the firm value in default to calculate the percentage gain from terminating a pension plan. If wage growth, interest rates, and turnover do not change much from year to year, then the accounting item "past pension service costs" will be a good proxy for the future contribution to be made. The negative coefficient we find for the pension service cost/default value variable suggests that plan termination is indeed a way to obtain benefits for the debtholders from bankruptcy. Next, we explore the relationship between underfunding and bankruptcy costs. The positive coefficient we find on amount of underfunding (pension funding gap) suggests not only that debtholders are unable to unload these underfunded liabilities in bankruptcy but also that underfunding raises bankruptcy costs. This does accord with most courts in the US that take a dim view of any attempts to deliberately use the bankruptcy process as a mechanism to transfer the liability to the PBGC. Persistent underfunding means that the sponsoring firm has lost the option to suspend pension contributions in times of financial distress and the requirement to close the gap puts a cash drain on the firm. Rauh (2006) and Bakke & Whited (2012) find that exactly such firms forgo valuable investment opportunities when they become financially distressed and our finding confirms this in the data. Interestingly we do find modest evidence that firms with a huge pension funding gap (above 30%) are expected to make offsetting gains in bankruptcy. Benmelech et al. (2012) showed that airlines with heavily underfunded plans could obtain wage concessions from their employees while in chapter 11 by threatening to terminate the plan because the PBGC covers benefits only up to a maximum amount.

In summary we have found that proportional bankruptcy costs increase with cash flow risk, while they decrease with the transferability of a firm's assets. We also found that human capital matters for two reasons. It is difficult to pledge growth options linked to a firm's key employees and labor productivity might go down because of distractions during bankruptcy procedures. Assets that can be repossessed before bankruptcy lose less value. Bankruptcy might be beneficial for firms that have bad management, use their assets less efficiently than their industry peers, or can save on future defined benefit plan accruals, but not because it allows firms to expunge their pension underfunded liability. We have found that bankruptcy costs can be sizable and are heterogeneous across industries and within industries. In the next section, we explore to what extent these bankruptcy cost estimates help to explain firms capital structure decisions.

Table 2: Regressions of bankruptcy cost, α, on firm characteristics. The regressions are performed using both the balance sheet asset value from accounting statements as well as using the estimated asset value. The balance sheet data is from Q2 2008. Regressions are also performed with and without industry fixed effects. Significance levels are indicated by *** for significance at the 1% level, ** for significance at the 5% level, and * for significance at the 10% level. We report heteroscedasticity consistent standard errors in parenthesis. The intercept is not reported.

	Balance Sheet Asset Value		Estimated Asset Value	
	(1)	(2)	(3)	(4)
Asset related				
Log sales	0.02	0.01	0.03*	0.02
	(0.02)	(0.02)	(0.02)	(0.02)
Asset volatility	1.01***	0.89***	1.37***	1.35***
	(0.28)	(0.31)	(0.25)	(0.28)
Tangibility	0.09	-0.06	-0.42***	-0.54***
	(0.15)	(0.16)	(0.12)	(0.13)
Brand and patents	-0.00*	-0.00*	-0.00***	-0.00*
	(0.00)	(0.00)	(0.00)	(0.00)
Market to book	0.08***	0.07***	0.04**	0.03*
	(0.02)	(0.02)	(0.02)	(0.02)
Labor related				
Labor intensity	15.98***	15.55***	10.80**	14.29***
	(4.38)	(5.43)	(4.38)	(5.21)
Skill intensity	0.07**	0.08*	0.07**	0.05
	(0.03)	(0.05)	(0.03)	(0.05)
CEO pay	0.11***	0.11***	0.07**	0.08**
	(0.03)	(0.03)	(0.03)	(0.03)
Bankruptcy procedure				
Operat. leases	-1.67*	-1.93*	-3.06*	-3.62**
	(0.96)	(0.99)	(1.65)	(1.71)
Capital leases	-0.20	-0.92	1.27	0.71
	(0.92)	(0.91)	(0.77)	(0.82)
Corporate governance				
Weak board	-0.05**	-0.05**	-0.05**	-0.05**
	(0.02)	(0.02)	(0.02)	(0.02)
Takover defense	-0.04*	-0.05**	-0.05**	-0.05**
	(0.02)	(0.02)	(0.02)	(0.02)
Supermajority	0.02	0.04**	0.02	0.04**
	(0.02)	(0.02)	(0.02)	(0.02)
ROA - ind. ROA (if neg.)	3.93***	5.20***	7.82**	7.43**
	(1.41)	(1.43)	(3.26)	(3.17)
Pension Plan				
Pens. service cost/	-2.43*	-2.40*	-2.27*	-1.87
def. value	(1.26)	(1.39)	(1.37)	(1.42)
Pension funding gap	0.32**	0.31**	0.29**	0.25*
	(0.14)	(0.15)	(0.13)	(0.14)
(Pension funding gap >30%)*	-0.25*	-0.19	-0.20	-0.14
Pension funding gap	(0.13)	(0.13)	(0.13)	(0.13)
R^2	0.38	0.47	0.39	0.46
adj R^2	0.34	0.37	0.34	0.37
Ind FE	N	Y	N	Y
N	264	264	267	267

5 Leverage and Bankruptcy Costs

We next analyze how bankruptcy costs affect firm leverage decisions. A key aspect of our paper is that we are able to test the "tradeoff" theory of capital structure because our bankruptcy cost estimates are not themselves a product of an estimation procedure that invokes this assumption.

Using only two determinants of leverage, bankruptcy cost and asset volatility, Figure 5 depicts this multivariate relationship. Leverage is seen to be decreasing in bankruptcy costs and asset volatility. High risk firms with high bankruptcy costs choose very low leverage ratios, whereas firms with very low bankruptcy costs and asset volatility lever up considerably. Figure 6, which plots the univariate relationship between leverage and bankruptcy costs, reveals another important aspect of a firm's capital structure decision. For each bankruptcy cost level, there exists an upper bound for leverage which firms do not exceed. Firms with high bankruptcy costs have only low leverage ratios (observations in the lower right corner) and only firms with low bankruptcy cost might choose high leverage ratios (observations in the upper left corner). However, as the observations in the lower left corner indicate that firms might choose low leverage ratios for other reasons. We explore these below in our regression framework. For instance firms with growth options subject to the debt overhang model of Myers (1977) would reduce their leverage below that implied by such a univariate relationship. Another possibility is that equityholders' incentives to decrease leverage after negative economic shocks differ from incentives to increase after positive economic shocks. This was first shown to be the case by Dangl & Zechner (2004) and was analyzed more generally by Admati et al. (2013) who term the reluctance of equityholders to reduce debt the"leverage ratchet effect". In contrast to the relationship between leverage and bankruptcy costs, the relationship to asset volatility is somewhat less pronounced.

5.1 Regression results on leverage

By virtue of our *firm specific* bankruptcy cost estimates, our model is the first to actually include bankruptcy cost directly in leverage regressions. Existing studies of leverage determinants either ignore bankruptcy costs or resort to conjectured proxies. Our explicit bankruptcy cost estimates allow us to distinguish between the traditional tradeoff theory relying on ex-post costs born by the firm's creditors in default from capital structure theories that build on leverage related costs brought about by, e.g., debt overhang, agency conflicts and corporate governance issues, or labor relations. We employ a cross-sectional regression framework for the determinants of leverage choices. Lemmon et al. (2008) and Graham & Leary (2011) report that around 60% of the variation in leverage ratios is cross-sectional variation. To clearly indicate the contribution of our specific bankruptcy cost estimates, we include many of the other variables employed in the previous section as control variables.

Our method allows us to distinguish among three leverage ratios. The first measure is defined as *market*

Figure 5: Leverage in relation to bankruptcy cost and asset volatility estimates. This shows the average leverage ratio for different groups of firms assembled according to their firms-specific asset volatility and bankruptcy cost estimates. The estimates are derived with our structural estimation procedure described in section 3. Leverage is equal to book value of debt divided by the sum of the book value of debt and the market value of equity. Bankruptcy costs are defined as the percentage of the unlevered firm value lost in the event of bankruptcy. Asset volatility represents the volatility of the unlevered asset value of a firm.

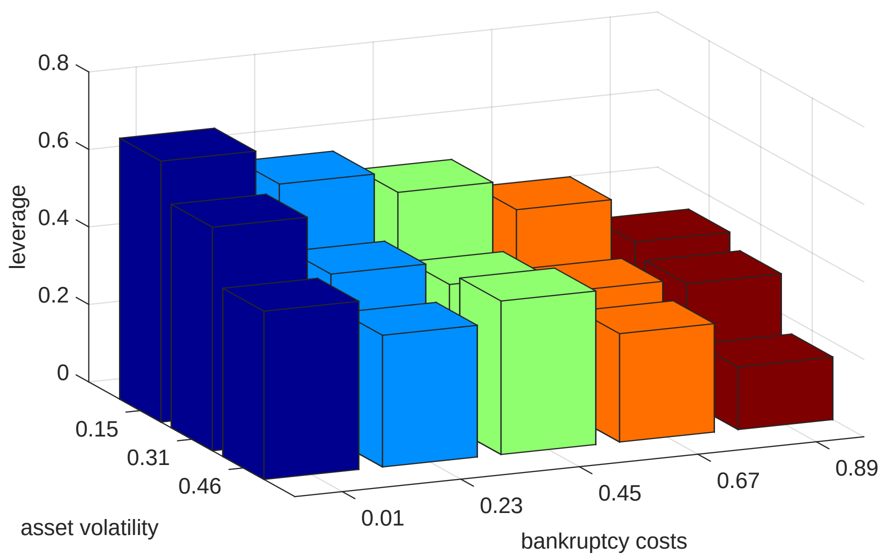
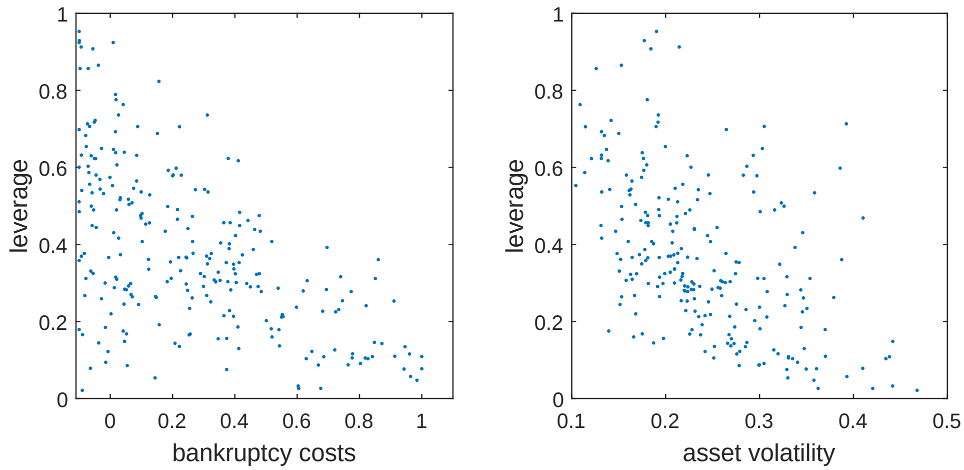


Figure 6: Bivariate relationship between leverage and bankruptcy cost as well as asset volatility estimates. Leverage is equal to book value of debt divided by the sum of the book value of debt and the market value of equity. Bankruptcy costs are defined as the percentage of the unlevered firm value lost in the event of bankruptcy. Asset volatility represents the volatility of the unlevered asset value of a firm. The estimates for bankruptcy costs and asset volatility were derived with our structural estimation procedure described in section 3.



leverage (ML), which is the ratio of the market value of debt and the market value of the levered firm using our estimation approach for both. We also employ *quasi market leverage* (QML) which is the book value of debt divided by the sum of the book value of debt plus the market value of equity. This approach therefore assumes that the book value of debt is equal to its market value. The final leverage measure is standard book leverage (BL), the ratio of book debt to total book value of assets.

The leverage estimation is given as:

$$lev_i = \beta_0 + \beta_1^\top Y_i + IND_j + \varepsilon_i,$$

where again Y_i represents a vector of firm characteristics (including bankruptcy costs, etc.) and the left hand side variable is one of the three leverage specifications (ML, QML and BL). Industry fixed effects for industry j are indicated by IND_j. Leverage ratios were calculated with market and balance sheet data from the end of the third quarter 2008 and explanatory variables are based on data from the end of the second quarter 2008.

With respect to market leverage, we obtain the regression results of Table 3. We notice most importantly that bankruptcy costs enter with a significantly negative sign in the leverage ratio regression. This is the first direct evidence that the tradeoff theory of capital structure holds with respect to bankruptcy costs. We also find very significant negative effects from asset volatility. These two characteristics are therefore separately important for leverage determinants. Most extant tests in the literature use accounting measures of asset volatility as derived for instance from earnings announcements or from the volatility of net operating profits and find a weak and mixed evidence on the impact of volatility on leverage ratios. By contrast, we use a market-based measure of unlevered asset volatility. The strong negative effect from asset volatility also supports the tradeoff theory for capital structure since the higher the volatility the higher (for a given asset asset value) is the probability of default and therefore the higher are *expected* bankruptcy costs. The bankruptcy cost and asset volatility variables by themselves explain a striking 46% of the variation in leverage ratios (see the first two columns). To formally test whether bankruptcy costs and asset volatility together improve the leverage regressions we carry out F-tests for every leverage ratio specification and every asset value definition. As expected from the large increases in R^2, the null hypotheses of no influence are strongly rejected with all p-values near zero. Leverage is also strongly positively related to tangibility. Interestingly, the ratio of property, plant, and equipment to total assets is negatively related to leverage. One possibility for this effect is that this variable is closely related to operating leverage capturing fixed costs, which would explain the negative coefficient. We also find that leverage is negatively related to profitability, especially when profitability is measured with respect to estimated asset values. Our profitability results are consistent with findings in much of the existing empirical capital structure literature.[19] We find that depreciation is negatively related to leverage, as expected. The depreciation tax-shield substitutes for the interest tax-shield.

Next, we find that firms with high growth potential, as captured by the market-to-book (MTB) ratio, have lower leverage ratios, which is consistent with the debt overhang problem in Myers (1977). In combination with our explanation for bankruptcy costs itself, these results suggest that leverage when there are high growth opportunities is affected through two separate channels. First bankruptcy costs are increased due to the inalienability of human capital. Second, it reduces the realization of future investment opportunities before bankruptcy. Without the bankruptcy cost variable, the coefficient on MTB has to capture both effects. This is indeed what we find. The coefficient is always larger in absolute terms if we leave out the bankruptcy cost variable.

We also find a negative relationship between corporate governance and the leverage ratio of firms. Firms which score high on our management entrenchment indicators (weak board and takeover defense measures) or low on the corporate governance index have higher leverage ratios. This suggests that firms with weak corporate governance use debt as a disciplining device. Management tends to pursue a less aggressive capital structure strategy if the firm's compensation policy is oriented towards long-term goals.

As with the case of growth options, the regression framework can also distinguish capital structure effects of labor prior to bankruptcy from that which occurs after bankruptcy. Berk et al. (2010) show that labor

[19]See Strebulaev (2007) for a thorough discussion on potential causes of the relationship between profitability and leverage in the cross-section.

intensive firms choose lower leverage ratios in order to decrease the likelihood of bankruptcy and, thus, the expected value of the costs imposed on employees. We capture the effect on capital structure of labor contracts before bankruptcy through a labor intensity variable. Using in particular the ratio of employment to total assets, we find strong evidence that the more important is labor in the production process the lower the level of debt. Therefore usage of labor has two reinforcing effects both lowering debt levels.

We repeat the regression analysis with leverage being measured either by quasi-market leverage (*QML*) or book leverage (*BL*) and present the results in appendix F. Table 8 shows the regression results when leverage is measured by *QML*. Most of our previous results with market leverage are preserved in this specification. Although skill intensity becomes insignificant, it retains the same negative sign. With this leverage specification, we find a significantly positive relationship between unionization and leverage. This could indicate that firms use higher leverage strategically to increase the bargaining power in wage negotiations via a higher debt burden (Hennessy & Livdan, 2009; Matsa, 2010). We find essentially the same results with respect to book leverage ratios in Table 9, with the exception of the market to book ratio. While the market to book ratio, as a measure of investment opportunities, is negatively related to market based leverage definitions it is positively related to book leverage. This dichotomy of results regarding the leverage-profitability relation when leverage is measured by market values instead of book values has also been documented in the existing literature.[20]

6 Credit Risk

Next we consider our estimates for the default boundary, A_B, and how this is related to our other estimates. In order to normalize firms for comparison purposes we actually use "distance to default". This is the measure originally employed by Moodys-KMV measuring the distance of the underlying asset value from the bankruptcy threshold in terms of standard deviations. Using the distance to default is one key ingredient into ratings on corporate debt.[21] Distance to default is defined as

$$DTD = \frac{\ln A_t - \ln A_B}{\sigma_A}. \tag{14}$$

We sort firms into quintiles, based on their average distances to default. Then we look for systematic variation in estimated bankruptcy costs, loss given default, leverage and asset volatility. Our results are presented in Table 4. For reference, the loss given default is defined as

$$LGD = 1 - \frac{(1-\alpha)A_B}{B}. \tag{15}$$

[20] See for instance Frank & Goyal (2009) and Fama & French (2002).

[21] We do not have data on the actual debt ratings of firms so we have not been able to use actual ratings in our analysis.

Table 3: This table contains the results for a regression of market leverage (ML) on various firm characteristic variables as indicated in the rows of the table. The variable definitions are in the text. The regression is performed for both firm characteristics using both balance sheet asset values and the asset values estimated from the model. The explanatory variables are from Q2 2008, the leverage ratios are calculated with Q3 2008 data. Significance levels are indicated by *** for 1%, ** for 5%, and * for 10%. Standard errors are given in parenthesis.

			Balance Sheet Asset Value				Estimated Asset Value			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	0.74***	0.85***	0.89***	0.92***	0.73***	0.70***	1.01***	1.01***	0.83***	0.74***
	(0.02)	(0.03)	(0.12)	(0.12)	(0.13)	(0.13)	(0.13)	(0.13)	(0.14)	(0.14)
Asset Volatility	-0.97***	-0.94***	-0.65***	-0.68***			-0.76***	-0.79***		
	(0.12)	(0.13)	(0.13)	(0.13)			(0.13)	(0.14)		
α	-0.29***	-0.26***	-0.12***	-0.15***			-0.17***	-0.18***		
	(0.03)	(0.03)	(0.03)	(0.03)			(0.03)	(0.03)		
PPE/Assets			-0.11*	-0.22***	-0.12*	-0.22***	-0.09	-0.29***	-0.04	-0.24**
			(0.06)	(0.06)	(0.06)	(0.06)	(0.10)	(0.10)	(0.11)	(0.11)
Tangibility			-0.09	0.03	-0.11	0.02	0.09	0.43***	0.05	0.41***
			(0.09)	(0.10)	(0.10)	(0.12)	(0.15)	(0.13)	(0.16)	(0.14)
Brand and patents			-0.00***	-0.00**	-0.00***	-0.00*	-0.00**	-0.00*	-0.00	-0.00
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Depreciation			-0.50**	-0.46**	-0.62***	-0.68***	-0.58***	-0.37	-0.75***	-0.64**
			(0.20)	(0.22)	(0.22)	(0.24)	(0.21)	(0.24)	(0.23)	(0.25)
R&D/Sales			-0.32*	-0.16	-0.52***	-0.39*	-0.17	0.06	-0.45**	-0.26
			(0.17)	(0.20)	(0.18)	(0.22)	(0.16)	(0.17)	(0.17)	(0.19)
Profit			-1.44***	-1.41***	-1.65***	-1.62***	-2.56***	-2.19***	-3.51***	-2.99***
			(0.38)	(0.39)	(0.38)	(0.39)	(0.98)	(0.81)	(1.05)	(0.85)
Market to book			-0.03***	-0.02**	-0.05***	-0.05***	-0.04***	-0.03***	-0.06***	-0.04***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Corp. govern. index			-0.15**	-0.14**	-0.16**	-0.19***	-0.17***	-0.14**	-0.18**	-0.20***
			(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.07)	(0.07)
Long-term incent.			-0.01	-0.01	-0.02**	-0.02**	-0.01	-0.02*	-0.03***	-0.04***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Weak board			0.02**	0.02*	0.03***	0.03***	0.02*	0.02**	0.04***	0.04***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Poison Pill			0.03**	0.03**	0.03**	0.04**	0.03**	0.03**	0.03	0.04**
			(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)
Unionized			-0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Skill intensity			-0.04**	-0.04*	-0.05**	-0.06**	-0.01	-0.01	-0.02	-0.03
			(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
L over K			-0.02***	-0.03***	-0.02***	-0.03***	-0.01	-0.01	-0.01	-0.01
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Log sales			0.00	-0.00	0.01	0.01	-0.00	0.00	0.01	0.01*
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.46	0.59	0.66	0.71	0.59	0.64	0.60	0.67	0.48	0.56
adj R^2	0.46	0.55	0.64	0.66	0.57	0.58	0.58	0.62	0.45	0.50
Ind FE	N	Y	N	Y	N	Y	N	Y	N	Y
N	300	300	300	300	300	300	303	303	303	303

Table 4: Firms are sorted into 5 quintiles representing distance to default. The resulting average bankruptcy costs, LGD, leverage, and asset volatility are displayed. In addition the distance to default is related to the ratio of estimated default threshold to ”optimal” default thresholds, A_B/A_B^* and the relationship between estimated default threshold to book values of debt, A_B/B.

Distance to default	2.73	4.20	5.29	6.40	8.64
Bankruptcy costs	0.03	0.18	0.38	0.28	0.15
LGD	0.11	0.18	0.43	0.40	0.23
Leverage	0.73	0.62	0.56	0.54	0.55
Asset volatility	0.22	0.22	0.25	0.25	0.21
A_B/A_B^*	1.42	1.42	0.81	0.72	0.53
A_B/B	0.93	1.03	1.06	0.97	0.97

We find very plausibly that bankruptcy costs increase with firms’ distances to default, at least up to a value of five standard deviations away from the default boundary. However, at the upper range, bankruptcy costs are decreasing somewhat. We find similar patterns for the LGD: there is a strong increase of estimated LGD with DTD over the range where firms have measurable default risks. Firms with the lowest distance to default tend to have high levels of leverage. Interestingly, asset volatilities do not vary much at all with respect to distance to default. This result contrasts strongly with Glover (2016), who finds that highly rated firms have low asset volatilities, and Elkamhi et al. (2012). Nevertheless our results are in accord with other studies such as Schaefer & Strebulaev (2008) who find that the different rating classes from AAA to B have nearly the same average asset volatility. The reason that asset volatility estimates are monotonic in Glover (2016) and Elkamhi et al. (2012) as compared to our paper could conceivably come from their assumption that default occurs exactly when optimal for equity holders. If in fact equity holders on average default sooner than according to this “optimal” level, implied asset volatilities are biased upward, with the bias more severe for firms closer to default.

Indeed Table 4 offers further support for the fact that equity holders do not default exactly when it appears to be optimal only from a cash flow standpoint. We find that for firms closest to default, the estimated default threshold is 42 percent higher than the optimal default threshold. This makes sense in the case where such firms have “precommitted” to default earlier through tough covenants and are thus forced into bankruptcy. However, we also find that many firms far away from bankruptcy have estimated default boundaries that are significantly below the equity maximizing levels. At the extreme, firms more than eight standard deviations away from bankruptcy have default boundaries only 50 percent of the optimal ones. These cases may represent situations where equityholders desire to continue to put in capital beyond where they can expect a financial return commensurate with their outside opportunities. These may be situations where some large shareholders may enjoy additional benefits of ownership, or situations where self-interested managers are able to persuade equity holders to continue. Another explanation for this finding could be that debtholders find it in their interest to engage in partial debt forgiveness, interest reductions or maturity extensions, etc. since this may reduce the expected bankruptcy costs borne by them.

7 Hidden Debt

For our empirical analysis to this point we have taken the book debt level from the balance sheet of the firms and estimated, among other things, the default threshold implied by observed market prices. We did not require that the estimated default threshold be equal to the one that would be optimal for equity-holders in the theoretical model, i.e. the one where the smooth-pasting condition is satisfied. In fact, we found considerable deviations from this “optimal” default threshold. As discussed above, covenants and agency considerations may play a role in this discrepancy. Another possibility, however, is, that the true set of liabilities faced by equityholders is not fully reflected in the accounting statements of the firm. For example, since our sample consists mostly of large US corporations, health care obligations can be an important liability omitted from the balance sheet. To investigate this we explore the presence of such *hidden debts*. In order to implement this, we now assume that the actual default threshold equals the optimal threshold for equityholders *which also reflects these hidden debts.*. Therefore we solve equation (5) for the B which equates the theoretical with the estimated default barrier. We denote this implicit face value of total liabilities by B^H. Therefore B^H satisfies

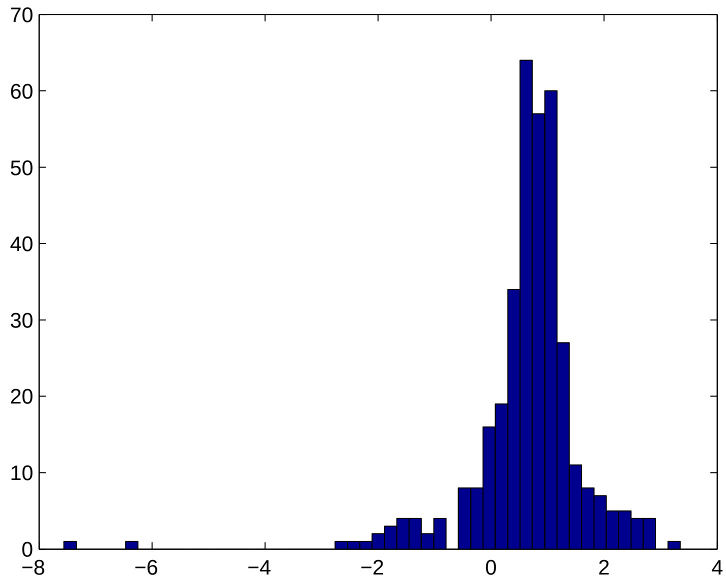
$$B^H = \frac{1}{m}\left(\left[(1+(1-\alpha)\eta(z)+\alpha\eta(r))\,A^*{}_B + \frac{\tau C}{r}\eta(r)\right]\frac{r+m}{\eta(z)} - C\right).$$

For most of the firms in our sample, B^H is greater than B, consistent with the existence of positive hidden debts. This is true whenever the estimated default threshold is higher than the optimal default threshold, using balance sheet liabilities. However, sometimes B^H is less than B, and in some cases B^H is even negative (for 49 firms this is indeed the case).

One hypothesis for the existence of negative hidden debts is that firms in financial distress may be able to recontract with parties, such as their employees, under more favorable terms. Indeed this seems to have been the case for many of the airline bankruptcies that have occurred in recent years, e.g. American Airlines.

Figure 7 displays the distribution of the ratios of implied to balance sheet liabilities, B^H/B. There is a concentration around one indicating that the most likely situation is that the average firm does not exhibit hidden debts as a major consideration. We conjecture that in particular firms with large legacy costs due to retirees as well as other former employees would be candidates to have negative hidden debts. Also, firms with relatively high labor costs within an industry would be candidates to have negative hidden debts, since financial distress allows these firms to recontract.

Figure 7: Hidden Debt. This illustrates a histogram of the ratios of the total estimated debt levels (including hidden debt) divided by the balance sheet value of debt.



8 Conclusions

As part of the literature on capital structure, the issue of the magnitude of bankruptcy costs has been recognized as having fundamental importance. In order to reconcile observed debt levels, if there is any relevance to the tradeoff theory of capital structure, bankruptcy costs should be economically significant. However, measuring these costs has been fraught with considerable difficulty. First, in the cross-section of firms, only a small proportion actually goes bankrupt. Second, observing total bankruptcy costs is not easy and often omits indirect and opportunity costs. Finally, there is a well-known selection bias in extending ex post observations to ex ante expectations.

This paper has taken a novel approach to this critical subject. We have utilized a broad based sample of S&P 500 firms in 2007 and applied a new method for inferring bankruptcy costs from equity and equity-linked put option prices during 2008 to 2010. Unlike previous approaches, our sample does not suffer by only considering highly levered firms or ones that have gone bankrupt. Moreover, our approach does not assume that firms continuously optimize their capital structures by trading off bankruptcy costs against tax advantages of interest deductibility. As a result of this structural modeling approach we are therefore able to look at the key determinants of bankruptcy costs *without* relying directly on the leverage ratio. And further, we are able to test the veracity of the (optimal) tradeoff theory itself.

We illustrate the efficacy of our method by utilizing data from the financial crisis period, which was characterized by wild swings in stock markets. Applying this estimation procedure using Kalman filtering techniques gives specific estimates that are reasonable and significant in magnitude – averaging 20% of unlevered asset values. These are somewhat lower than other recent approaches. We further find large differences within and across industries, which is reasonable given the heterogeneity of firms in the economy. Some firms even have negative bankruptcy cost estimates which is consistent with the idea that default may be necessary in order to achieve efficiencies.

In our cross sectional analysis we found that asset volatility and growth options as measured by market to book ratios, labor and skill intensity have a significant positive impact, while tangibility and size have significant negative impact. Furthermore, pension deficits have a negative and labor intensity has a positive effect on bankruptcy costs, but these relations are less significant. We also provide evidence that bankruptcy can be profitable for firms that have weak corporate governance, an entrenched management, employ their assets less efficiently than their industry peers, or have defined benefit pension plans in place. Finally, bankruptcy costs are lower for assets that can be repossessed more easily. While we identify important and significant variables that determine the size of bankruptcy costs, they can only explain less than half of the cross-sectional variation of bankruptcy cost estimates. This implies that our new estimation method has the potential to improve tests for capital structure theories.

We augment capital structure tests that regress leverage on firm characteristics by including firm-specific bankruptcy cost estimates and show that this improves explanatory power significantly. Our approach also enables us to estimate market values for debt securities which allows us to analyze market leverage ratios instead of book leverage ratios. In addition we recover underlying unlevered asset values from a nonlinear Kalman filter. Our bankruptcy cost variable significantly negatively impacts leverage ratios. This negative impact is over and above that of other firm characteristics such as asset intangibility and asset volatility. We also find a negative leverage profitability relationship using market leverage values, consistent with earlier literature. In sum, we find strong support for the tradeoff theory of capital structure.

In a final application of our method, we infer hidden debts that cannot be otherwise inferred from balance sheets. These are debts which could conceivably be expunged in bankruptcy thus possibly reducing bankruptcy costs. The best examples are long term legacy contracts that may not reflect current labor market conditions. While there is more room for work in this area, we believe that our study supports the view that hidden debts can be another significant factor in explaining likelihoods and consequences of bankruptcy.

Using this method of estimating bankruptcy costs there are a number of potential extensions. For instance one could look at the variation of bankruptcy costs over time. It would be interesting also to see how these costs are related to the competitiveness of industries. And finally what happens to bankruptcy costs after corporate events like mergers and divestitures. These are only a few of the many research extensions that arise from having precise firm-level bankruptcy costs.

A Variable Definitions

Table 5: This table contains the description of the variables used in the estimation of the firm-specific parameters and the regressions of bankruptcy costs and leverage ratios.

Variable	**Description**
α	The estimate of the firm-specific bankruptcy costs expressed as a percentage of the unlevered asset value in the case of default.
Asset volatility	The firm-specific estimate for the volatility of the unlevered asset value.
Brand and patents	Value of a firm's brand and patents divided by the number of employees.
Capital leases	Property, plant, and equipment leased divided by total assets
CEO = chair	Dummy variable equal to one, if the CEO of a firm is also the chair of the board of directors
Corp. govern. Index	Corporate governance index from the ASSET4 database in Datastream. Low values correspond to weak corporate governance.
Depreciation	Depreciation is the ratio of the accounting item depreciation to property, plant, and equipment taken from the balance sheet.
Equity price	End-of-day price of a firm's stock obtained from Datastream.
CEO pay	Highest remuneration package within the company in US dollars.
Industry fixed effects	We use the 30 Fama-French industry classifications available on `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_30_ind_port.html`.
Labor Intensity	Number of employees divided by sales
Long term incentives	The maximum time horizon of targets to reach full senior executives' compensation, taken from the ASSET4 database in Datastream.
L over K	Number of employees divided by total assets
Market to book	Market-to-book ratio is defined in the numerator by the market value of equity + short-term debt + long term debt + preferred liquidation value - deferred taxes and investment tax credits. In the denominator the book value of total assets is used.

Operating leases	Following Lim et al. (2003) and Miller & Upton (1976), the present value of operating leases is equal to the average of the current year rental expenses and the next year's minimum operating lease payments, discounted as a perpetuity. This value is divided by total assets.
Pension funding gap	Following Rauh (2009) we construct a measure of the pension gap as the ratio of pension assets minus pension liabilities to pension liabilities. Pension assets correspond to the fair value of plan assets and pension liabilities to the projected benefit obligation.
Pension funding gap >10%	A dummy variable that takes the value one if the pension funding gap of a firm exceeds 30%
Pension service costs	Represents the expense caused by the increase in pension benefits payable (the projected benefit obligation) to employees due to services rendered during the current year.
Poison pill	Dummy variable equal to one, if the company has a poison pill in place.
PPE	Property, plant and equipment;
Profitability	Profitability equals after-tax operating income before depreciation divided by total assets taken either from the balance sheet or from the estimation results.
Put Price	End-of-day price of a put option written on a firm's stock obtained from Datastream.
R&D/Assets	R&D expenses over sales
ROA minus Ind. ROA	Return on assets of a firm minus the average return on assets in the firm's industry. We split this variable up into the negative and the positive realizations.
Skill Intensity	Logarithm of the average weekly wage of a firm's industry in 2012 obtained from the Bureau of Labor Statistics
Staggered board	Dummy variable equal to one if the company has a staggered board structure.
Supermajority	Supermajority is the sum of two dummy variables related to supermajority requirements. The first dummy variable equal to one, if the company has a supermajority vote requirement for amendments of charters and bylaws and the second dummy variable is equal to one, if the company has a supermajority vote requirement in the case of significant company transactions such as M&As.

Takeover defense	Sum of the poison pill and staggered board dummy variables.
Tangibility	Tangibility is quantified by the measure from Berger et al. (1996) which was also used in Almeida & Campello (2007). The measure is defined as $Tangibility = 0.715 \times Receivables + 0.547 \times Inventory + 0.535 \times Capital$, where Capital equals property, plant and equipment. Cash holdings are added to this value and the sum is scaled by total assets
Total assets	We either use the balance sheet value of total assets or our estimate of the unlevered asset value.
Total liabilities	represent all short and long term obligations expected to be satisfied by the company;
Weak board	Sum of the CEO=chair dummy and a large-boad dummy equal to one if the board size is larger than the median.

B Robustness

B.1 Applying the estimation method to simulated data

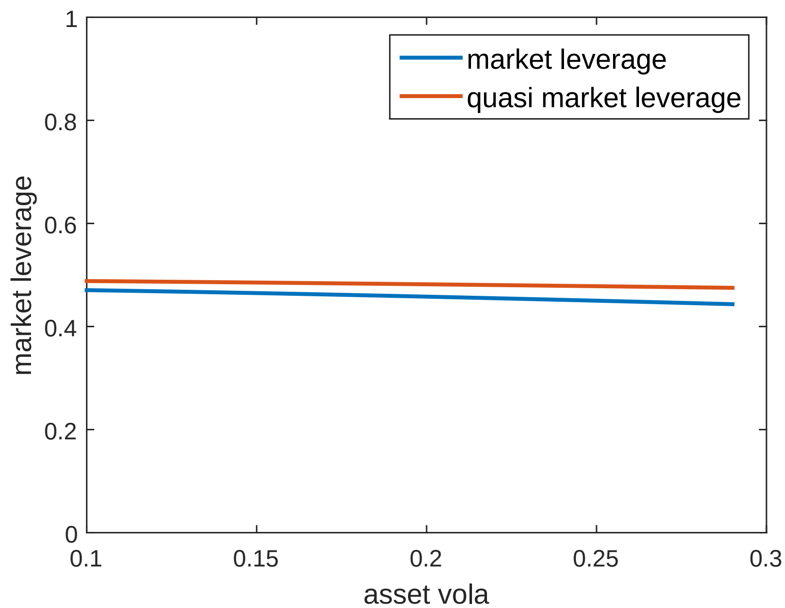
We would like to use our structural model to understand how the empirical findings, in particular the negative relationship between leverage and asset volatility and between leverage and bankruptcy costs, are related to firms' capital structure decisions. We note that our estimation method does not impose on firms that they make either static or dynamically optimal capital structure decisions.

First, we would like to explore to what extent the strong negative relation between asset volatility and leverage could be showing up even if firms are not optimally choosing their leverage ratios at the beginning of the sample period. To this end, we fix the book leverage and then derive market and quasi market leverage ratios from our theoretical pricing model for a representative firm with different unlevered asset volatilities. Figure 8 depicts the effect of asset volatility on market and quasi market-leverage, produced by the impact of asset volatility on theoretical equity and debt values via the default threshold and probability of default. Note that, although the slope is slightly negative, it is essentially zero for both market and quasi-market leverage ratios compared to the significantly negative empirical estimates.

We have performed a similar exercise with respect to bankruptcy costs. Here, for fixed nominal debt levels, the theoretical relationship is actually positive[22], whereas the empirical evidence is strongly negative.

[22] Higher bankruptcy costs increase a firm's refinancing costs and decrease dividends. Consequently, the equity price drops and market leverage goes up.

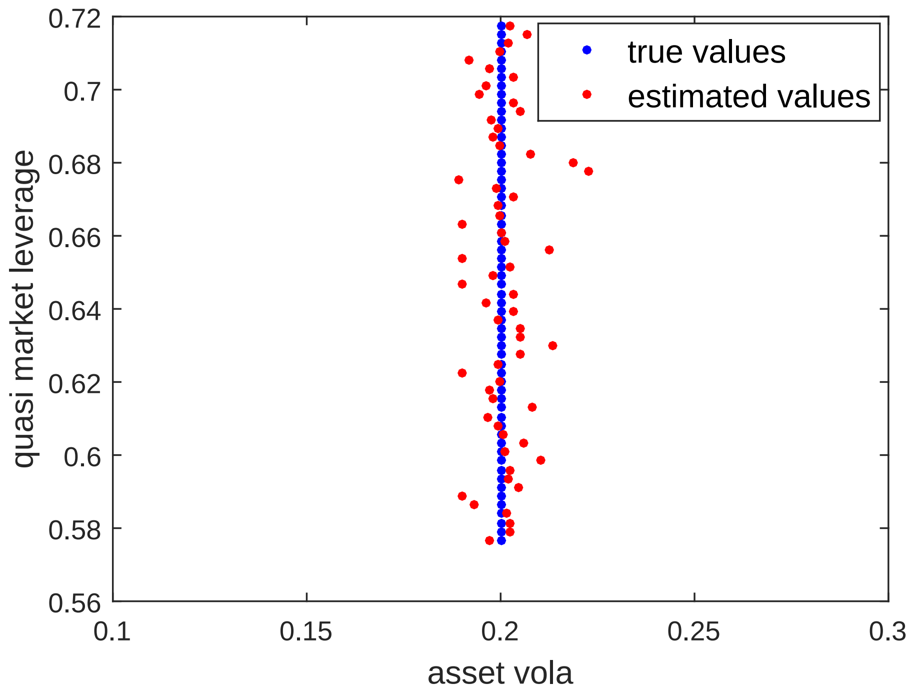
Figure 8: Leverage vs. Asset Volatility. This graph illustrates the theoretical relationship between asset volatility on market leverage (blue, lower line) and on quasi market leverage (red upper line). The relationship is generated by the equity and debt pricing models in the paper.



Second, we want to ensure that the observed negative relation between leverage and asset volatility and leverage and bankruptcy costs is not purely an artifact of our estimation procedure. To check whether the pronounced negative relation between leverage and asset volatility or bankruptcy costs is generated artificially we test our estimation method on simulated data. We construct a sample of firms which, by assumption, does not exhibit a negative correlation between leverage and asset volatility or bankruptcy costs. For all firms, the asset volatility and the bankruptcy costs are the same but the book value of debt varies. Given these parameters we simulate sample paths of equity and option prices for 60 firms. Then we estimate the structural parameters of the firms in the same fashion as we did for the actual data.

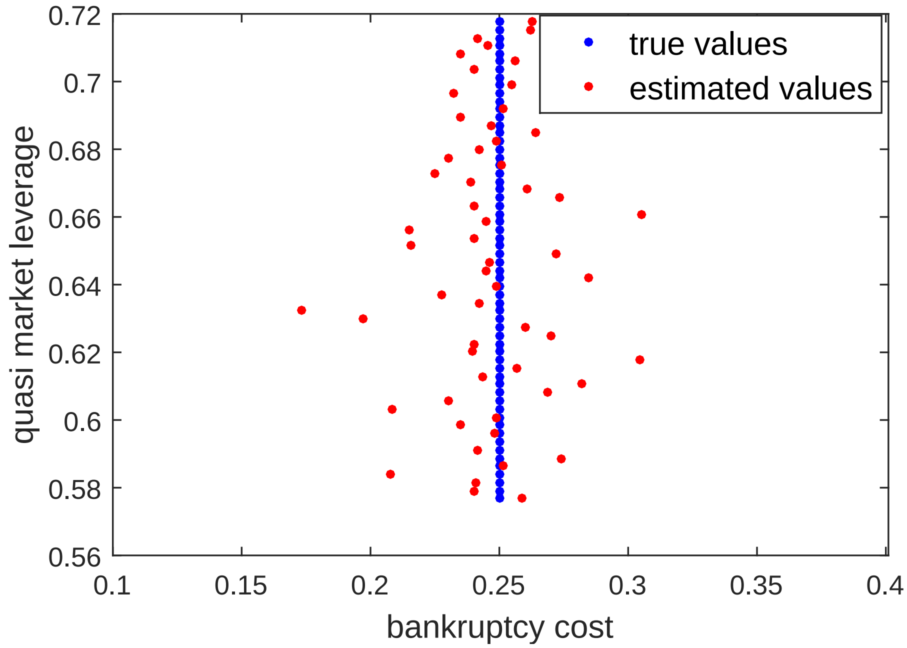
Figure 9 depicts the outcome of the simulation with respect to estimated volatilities. The blue points represent the true quasi market leverage ratios of the firms in the simulation. The volatility was fixed at $\sigma = 0.2$ while the market leverage ratio varied between $lev = 0.58$ and $lev = 0.72$ for the simulated firms. The red points depict the corresponding estimated values of asset volatility. The correlation between estimated asset volatilities and estimated market leverage is close to zero (0.03), indicating that the estimation procedure does not impose the documented negative correlation between these two variables. Similarly Figure 10 illustrates the estimates of bankruptcy cost obtained by simulating all 60 firms in the study. The true bankruptcy cost value is fixed at $\alpha = 0.25$. The blue points represent the true values, while the red points indicate the estimated bankruptcy cost values. While there is more estimation error in determining the bankruptcy cost than with respect to unlevered asset volatilities, there is no noticeable bias in the esti-

Figure 9: Leverage vs. Asset Volatility. This illustrates the results of a simulation study in which sixty artificial firms were simulated with different leverage ratios but the same asset volatility. The linear blue dots indicate the true values and the red random dots indicate the results from the simulation.



mates. The correlation between bankruptcy costs and leverage is somewhat higher than that for volatilities (-0.07) but also insignificant. Table 6 summarizes the results of our simulation study. Since the mean of

Figure 10: Leverage vs. Bankruptcy Costs. This illustrates the results of a simulation study in which sixty artificial firms were simulated with different leverage ratios but the same bankruptcy cost. The linear blue dots indicate the true values and the red random dots indicate the results from the simulation.



the estimates equals exactly the true values for volatility and bankruptcy cost, there is no bias in either. The mean squared errors for volatility are lower than for the bankruptcy costs. Nevertheless, the square root of the MSE for bankruptcy cost is a small fraction of the average estimate. This table also reports the correlations with leverage and the cross-correlation and shows that they are all insignificantly different from zero using a *t*-test.

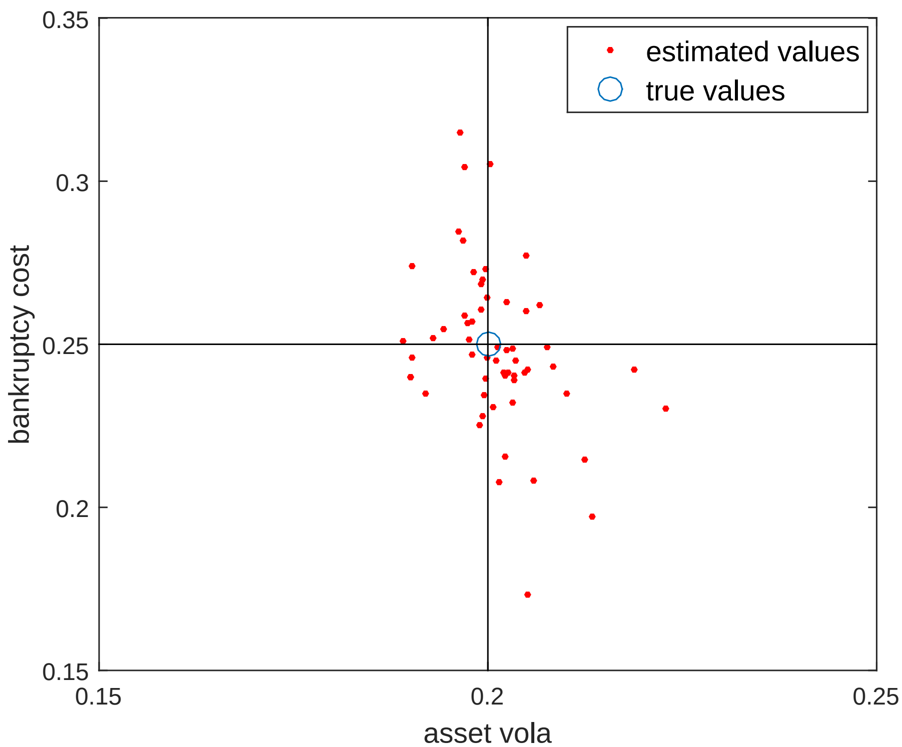
Table 6: Simulation Results: This table reports the results of a simulation study in which sixty artificial firms were simulated with different leverage ratios but the same asset volatility and bankruptcy cost.

	Estimation			Correlation with			
	true value	mean	$\sqrt{\text{MSE}}$	leverage	t-stat	asset volatility	t-stat
asset volatility	0.20	0.20	0.01	0.03	0.21		
bankruptcy cost	0.25	0.25	0.04	-0.07	-0.53	-0.16	-1.19

We have also computed the confidence bounds for both bankruptcy cost and volatility in the simulation. We find that 95% of the time, the bankruptcy cost is between 0.20 and 0.31 while the true value is 0.25. For asset volatilities, the 95% confidence band is between 0.19 and 0.22 for a true value of 0.20.

Third, we want to test whether the positive relationship between a firm's bankruptcy costs and its asset volatility documented in table 2 is a spurious result of the estimation procedure. The correlation between the true bankruptcy costs and asset volatilities is zero in our simulated sample of 60 firms, because both values are fixed as constants. As table 6 reports, the slightly negative correlation of the estimated parameters is not significantly different from zero. This is also illustrated in figure 11.

Figure 11: Bankruptcy Costs vs Asset Volatility: This illustrates the results of a simulation study in which sixty artificial firms were simulated with different leverage ratios but the same bankruptcy cost and asset volatility. The true values for asset volatility and bankruptcy cost are 0.2 and 0.25 and are represented by the crossing point of the straight lines. The red random dots indicate the results from the estimation.



B.2 Robustness to alternative refinancing strategies

We have employed the Leland (1998) model in our theoretical derivation, however to better understand the results we also have applied this against simulated data coming from another well-known refinancing model, namely that of Leland & Toft (1996). Recall that our model assumes a continuum of maturities extending to infinity with refinancing rate of m, and average maturity equal to a constant $1/m$. By contrast in the model of Leland & Toft (1996) the firm starts off with finite maturity debt and then refinances a constant fraction by reissuing new debt with the same fixed finite maturity. Therefore in this model the original debt remaining declines along a linear path and is completely extinguished at the original maturity.

We have derived the equivalent equity and put pricing formulas under this refinancing policy.[23] We perform a daily simulation over a two year period for 100 sample paths for a representative firm. We assume that the puts have 180 days to maturity and every 30 days we pick another strike price which is equal to 0.90 times the current equity value. In addition we assume the following true parameters in our model: $\sigma = 0.2$, $B = 50$, $C = 1.5$, $r = 0.02$, $\delta = 0.03$, $\alpha = 0.25$, $\tau = 0.35$. The finite maturity of originally issued debt is chosen to be equal to 4 years; which means that the average maturity is also equal to 4 years, since newly issued debt replaces exactly the amount that is retired.

Table 7 shows the results of applying our estimation procedure under the 'mis-specified' refinancing model. Remarkably we recover almost exactly the true values for asset volatility and bankruptcy costs. In the table the mean squared error from our estimates is only a small percentage of the estimated values and there appears to be almost no bias in the estimates.

Table 7: Simulation Results: This table reports the results of a simulation study in which 100 artificial firms were simulated according to the model in Leland & Toft (1996). The following parameter values were used: $\sigma = 0.2$, $B = 50$, $C = 1.5$, $r = 0.02$, $\delta = 0.03$, $\alpha = 0.25$, $\tau = 0.35$, and the maturity of debt was set to 4 years. Put options on these simulated firms have 180 days to maturity. Every 30 days we pick another strike price which is set equal to 0.90 times the current equity value. We use our finite maturity model to estimate the parameters from the simulated equity and put option prices of these Leland & Toft (1996) type firms.

	asset vola	bankruptcy costs
true value	0.2000	0.2500
mean estimate	0.2016	0.2570
$\sqrt{\text{MSE}}$	0.0032	0.0290

This simulation exercise shows that even though the maturity structure of the firms are modeled differently, the asset volatility and bankruptcy costs can be recovered, with the equity volatility parameter more precisely estimated then the bankruptcy cost parameter.

C Derivation of the Put Pricing Formula

Let $(\Omega, F, \mathcal{F}, P)$ be a filtered probability space with the filtration $\mathcal{F} = \{F_t : t \geq 0\}$ generated by the Brownian motion W_t^P, and let $Q \sim P$ be the martingale measure with the risk-free bank account as the

[23]These are available from the authors upon request.

numeraire. The Q-dynamics of the unlevered asset value process A_t are given by

$$dA_t = \left(\mu_B + \frac{\sigma^2}{2}\right) A_t dt + \sigma A_t dW_t \tag{16}$$

where W_t is a Q-Wiener process and $\mu_B = r - \delta - \frac{\sigma^2}{2}$ is the drift of $\ln A_t$.

The payoff of a put option, (17), depends on whether the underlying firm has defaulted or not:

$$P_T = (K - S(A_T))^+ 1_{T_B > T} + K 1_{T_B \leq T} \tag{17}$$

In order to derive the price of the option at time 0, we first define A^* as the time-T unlevered asset value such that the option is at the money ($S(T, A^*) = K$). If markets are arbitrage free, the put price can be written as the discounted expected value of the payoff, with the risk-free rate serving as the discount rate under the risk-neutral measure Q:

$$P_0 = e^{-rT} E_0^Q \left[(K - S(A_T)) 1_{A_T \leq A^* \wedge T_B > T}\right] + K e^{-rT} E^Q \left[1_{T_B \leq T}\right] \tag{18}$$

with the stock price given by

$$\begin{aligned} S(A_T) = & A_T + \frac{\tau C}{r}\left[1 - G(T, A_T)\right] - \alpha A_B G(T, A_T) \\ & - \frac{C + mB}{z}\left[1 - G^z(T, A_T)\right] - (1 - \alpha) A_B G^z(T, A_T) \end{aligned}$$

The pricing formula (18) includes the stochastic variable A_T, as well as $G(T, A_T)$ and $G^z(T, A_T)$ which are non-linear functions of A_T together with the indicator function 1_{Y_T} where $Y_T = \{A_T \leq A^* \wedge T_B > T\}$ is the event that the option is in the money and the firm has not defaulted prior to maturity of the option. As the put formula can be expressed as terms involving the payoffs $A_T 1_{Y_T}$, $G(T, A_T) 1_{Y_T}$, and $G^z(T, A_T) 1_{Y_T}$ we will derive their time-0 values explicitly in the next three lemmas. To facilitate calculations we will change the probability measure by choosing convenient likelihood processes (see Ericsson & Reneby, 1998, 2003, for a discussion of this approach). We make sure that the likelihood processes are chosen in such a way as to guarantee that the new measures are also probability measures. In addition, the new measures will be martingale measures with A_T, $G(T, A_T)$, and $G^z(T, A_T)$ as the respective numeraires. Finally, Girsanov's theorem (see Duffie, 2001, app D) will tell us the drift rate of A_t under the new measures.

The first term involves the time-T value of the unlevered asset price. For this transformation we use the unlevered asset value as 'numeraire'.

Lemma C.1 *The price of the time-T payoff $A_T 1_{Y_T}$ at time 0 is given by*

$$E_0^Q \left[e^{-rT} A_T 1_{Y_T}\right] = A_0 e^{-\delta T} Q^A(Y_T) \tag{19}$$

with the likelihood process

$$L_Q^A(t) = \frac{dQ^A}{dQ}, \quad on\ F_t, \quad 0 \leq t \leq T$$

given by

$$L_Q^A(t) = \frac{A_t e^{\delta t}}{B_t A(0)}$$

The Girsanov kernel for the transition from Q to Q^A is equal to σ which changes the drift of A under Q^A to

$$\mu_A = \mu_B + \sigma^2$$

Proof The Likelihood process $L_Q^A(t) = \frac{A_t e^{\delta t}}{B_t A(0)}$ is a Q-martingale and $E_0^Q\left[L_Q^A(T)\right] = 1$. The pricing formula (19) follows from $E_0^Q\left[e^{-rT}A_T 1_{Y_T}\right] = E_0^A\left[L_A^Q(T)e^{-rT}A_T 1_{Y_T}\right]$ where $L_A^Q(t) = \frac{1}{L_Q^A(t)}$.

$$\begin{aligned} E_0^Q\left[e^{-rT}A_T 1_{Y_T}\right] =& E_0^A\left[\frac{B_T A_0 e^{-\delta T}}{B_0 A_T} e^{-rT} A_T 1_{Y_T}\right] \\ =& A_0 e^{-\delta T} E_0^A\left[1_{Y_T}\right] \\ =& A_0 e^{-\delta T} Q^A(Y_T) \end{aligned} \tag{20}$$

To derive the price of the future \$1 in-default claim, we use this claim itself to factor it out of the expectation.

Lemma C.2 *The price of the time-T payoff $G(T, A_T)1_{Y_T}$ at time 0 is given by*

$$E_0^Q\left[e^{-rT}G(T, A_T)1_{Y_T}\right] = G(0, A_0)Q^G(Y_T) \tag{21}$$

In this case, the likelihood process is given by

$$L_Q^G(t) = \frac{G(t, A_t)}{B_t G(0, A_0)}$$

The Girsanov kernel for the transition from Q to Q^G is equal to $-\eta(r)\sigma$ which changes the drift of A under Q^G to

$$\mu_G = \mu_B - \eta(r)\sigma^2$$

Proof The steps of the proof are the same as for lemma C.1.

The final term involves $G^z(T, V_T)$ which is a claim to $e^{-m(T_B - T)}$ dollars if the firm defaults at T_B.

Lemma C.3 *The price of the time-T payoff $G^z(T, A_T)1_{Y_T}$ at time 0 is given by*

$$E_0^Q \left[e^{-rT} G^z(T, A_T) 1_{Y_T} \right] = e^{mT} G^z(0, A_0) Q^z(Y_T) \tag{22}$$

with the likelihood process given by

$$L_Q^z(t) = \frac{G^z(t, A_t) e^{-mt}}{B_t G^z(0, A_0)}$$

The Girsanov kernel for the transition from Q to Q^z is equal to $-\eta(z)\sigma$ which changes the drift of A under Q^z to

$$\mu_z = \mu_B - \eta(z)\sigma^2$$

Proof The steps of the proof are the same as for lemma C.1.

The put pricing formula contains the probability of the event Y_t evaluated under different martingale measures with respect to different numeraires, namely, A_t, $G(t, A_t)$, and $G^z(t, A_t)$. The probabilities $Q(A_T)$, $Q^A(A_T)$, $Q^G(A_T)$, and $Q^z(A_T)$ can be easily derived from the density of an absorbed Brownian motion with the respective drift rates $\hat{\mu}$, μ_A, μ_G, and μ_z (e.g. Bjoerk, 2004, ch 18).

Using the previous results, the price of the put option is stated in the following proposition:

Proposition C.4 *Given the Q-dynamics of A_t in (16), the price of the put option with time-T payoff defined in (17) is*

$$\begin{aligned} P_t =& e^{-r(T-t)} K \left(Q(Y_T) + Q(T_B < T) \right) - A_t e^{-\delta(T-t)} Q^A(Y_T) \\ & - \frac{\tau C}{r} \left(e^{-r(T-t)} Q(Y_T) - G(t, A_t) Q^G(Y_T) \right) + \alpha A_B G(t, A_t) Q^G(Y_T) \\ & + \frac{C + mB}{z} \left(e^{-r(T-t)} Q(Y_T) - e^{m(T-t)} G^z(t, A_t) Q^z(Y_T) \right) \\ & + (1 - \alpha) A_B e^{m(T-t)} G^z(t, A_t) Q^z(Y_T) \end{aligned} \tag{23}$$

D The Unscented Kalman Filter

Our model has the following state space representation:

$$x_t = A + F x_{t-1} + \varepsilon_t \tag{24}$$

$$y_t = g(x_t) \tag{25}$$

As explained in section 3.1, the state equation comprises the process for the unlevered asset value and the AR(1) specification for the pricing errors, i.e. $x_t = (a_t, e_t^S, e_t^P)'$, where a_t is the log asset-value and e_t^S, e_t^P are the pricing errors for the stock and the put price. Therefore, the covariance matrix of the state equation errors (Q) contains the asset volatility and the variance of the noise terms in the pricing error processes.

$$\begin{aligned} Q &= E\left[\varepsilon\varepsilon'\right] \\ &= \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma_S^2 & 0 \\ 0 & 0 & \sigma_P^2 \end{pmatrix} \end{aligned} \tag{26}$$

The measurement equation (25), which summarizes equation (10) in vector form, contains the two observable security prices, the stock price and the put option price ($y_t = (s_{i,t}, p_{i,t})'$). The non linear pricing functions g can be further simplified to $g_k(x_t, \vartheta) = \bar{g}_k(a_t, \vartheta) + e_t^k$, $k \in \{S, P\}$ with only a_t entering the non linear part.

As the state equation (24) is linear, the state propagation is the same as in the linear Kalman filter. Therefore, the update of the state variable and its mean squared error matrix (MSE), $P_{t|t-1}$, is given by:

$$\begin{aligned} \hat{x}_{t|t-1} &= A + B\hat{x}_{t-1|t-1} \\ P_{t|t-1} &= FP_{t-1|t-1}F' + Q \end{aligned}$$

The measurement update, however, differs, since the state variables enter in a non linear way in the measurement equation (25). To approximate the distribution of y_t, which is a non linear transformation of the distribution of x_t, we rely on the unscented Kalman filter (see Wan & Van Der Merwe, 2001, for a detailed description) to give us an approximation for the mean and the covariance matrix. The unscented transformation captures the true mean and covariance matrix of the prices accurately to the third order (if A_t where not Gaussian, then to the second order). Figure 12 depicts the gain in accuracy obtained by the use of the unscented transformation.

We construct $2L + 1$ sigma vectors, χ_i, where $L = 2$ is the number of state variables. The sigma vectors are chosen in such a way that the mean and the covariance matrix of y_t is approximated accurately up to the third order. Each sigma vector comes with corresponding weights, W_i^m and W_i^c, to calculate the mean and the covariance matrix is the weighted average of the sigma points. The sigma vectors and weights are given by

$$\begin{array}{lll} \chi_0 = \hat{x}_{t|t-1} & W_0^m = \frac{\lambda}{\lambda+L} & W_0^c = \frac{\lambda}{\lambda+L} + 1 - \alpha^2 + \beta \\ \chi_i = \hat{x}_{t|t-1} + \sqrt{(L+\lambda)(P_{t|t-1})_i} & W_i^m = W_i^c = \frac{1}{2(\lambda+L)} & i = 1, \ldots, L \\ \chi_i = \hat{x}_{t|t-1} - \sqrt{(L+\lambda)(P_{t|t-1})_{i-L}} & W_i^m = W_i^c = \frac{1}{2(\lambda+L)} & i = L+1, \ldots, 2L \end{array} \tag{27}$$

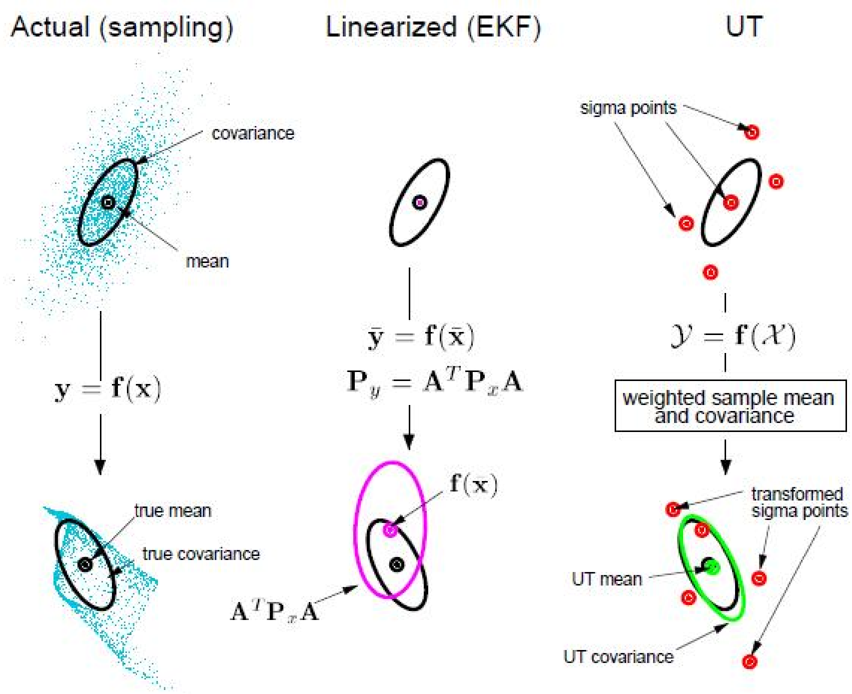


Figure 12: Example for the unscented transformation for mean and covariance propagation comparing actual moments to moments derived under first-order linearization (extended Kalman filter), and unscented Kalman filter. Source: Wan & Van Der Merwe (2001).

where $\lambda = \alpha^2(L+\kappa) - L$ is a scaling parameter. We follow the general recommendations (e.g., see Wan & Van Der Merwe, 2001, section 7.2) and set $\alpha = 1e-3$, $\kappa = 0$, and $\beta = 2$. The non-linear function g is applied to the sigma vectors to generate $y_i = g(\chi_i)$, $i = 0, \ldots, 2L$. The measurement update is then given by

$$\begin{aligned}
\hat{y}_{t|t-1} &= \sum_{i=0}^{2L} W_i^m y_i \\
\Psi_t &= \sum_{i=0}^{2L} W_i^c (y_i - \hat{y}_{t|t-1})(y_i - \hat{y}_{t|t-1})' \\
P_{t|t-1}^{xy} &= \sum_{i=0}^{2L} W_i^c (\chi_i - \hat{x}_{t|t-1})(y_i - \hat{y}_{t|t-1})' \\
K_t &= P_{t|t-1}^{xy} \Psi^{-1} \\
\hat{x}_{t|t} &= \hat{x}_{t|t-1} + K_t (y_t - \hat{y}_{t|t-1}) \\
P_{t|t} &= P_{t|t-1} + K_t \Psi_t K_t'
\end{aligned}$$

Finally, the log-likelihood function is given by

$$l_t(\hat{\vartheta}) = -\frac{1}{2}\log|\Psi_t| - \frac{1}{2}(y_t - \hat{y}_{t|t-1})\Psi_t^{-1}(y_t - \hat{y}_{t|t-1})' \tag{28}$$

E Importance of Put Options

The additional use of put option prices improves the estimation for two reasons. First, the unlevered asset value is filtered more precisely because two different price series are available to back out this unobservable time series. Figure 13 shows the precision improvement. The mean absolute percentage deviation of the filtered unlevered asset value is 71 basis points if equity prices alone are used. This number drops to 22 basis points if both equity prices and put option prices are used. Second and more importantly, some parameters affect put option prices differently than equity prices. Put options are particularly helpful in identifying the asset volatility and the default threshold, which are hard to identify from equity prices alone. The difficulty comes from the fact that both a higher asset volatility and a lower default threshold increase the price of equity and different combinations of asset volatility and bankruptcy threshold values lead to similar equity prices. This problem is highlighted in the right panel of figure 14, which shows the log-likelihood function based on equity prices alone assuming that the unlevered asset value is observable. Put option prices increase with a higher asset volatility but contrary to equity prices they also increase with a higher bankruptcy threshold. The difference in the effect of the bankruptcy threshold allows us to identify both parameters, as can be seen from the right panel of figure 14. In the final estimation, which also includes the filtering of the unlevered asset value process, the asset volatility parameter is estimated from the pricing functions through the time-series behavior of the unlevered asset value. This helps to identify the parameter but without put prices the log-likelihood function is rather flat (left panel of figure 15). With put prices, the objective function becomes much steeper (see the right panel of figure 15).

Figure 13: This figure shows the deviation of the filtered asset value from the true asset value. We simulated 500 observations of the unlevered asset value and calculated equity and put prices. Then we used either equity prices alone or both the equity and put prices to back out the unlevered asset value.

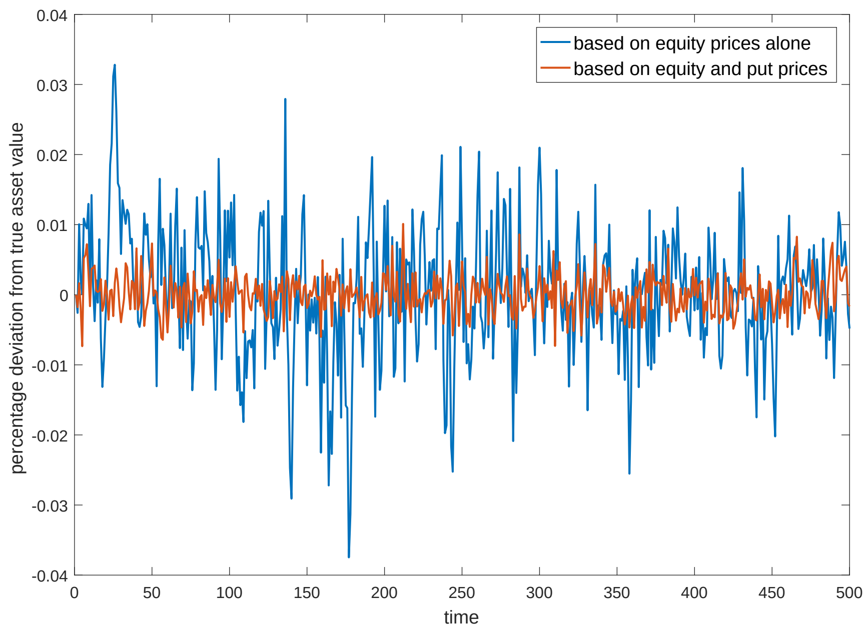


Figure 14: This figure shows log likelihood function for different asset volatility and bankruptcy threshold combinations assuming that the unlevered asset value is known. The true asset volatility is 0.2 and the true bankruptcy threshold is 0.95. The left panel shows the log likelihood function based on equity prices alone and the right panel the log likelihood function using equity and put prices.

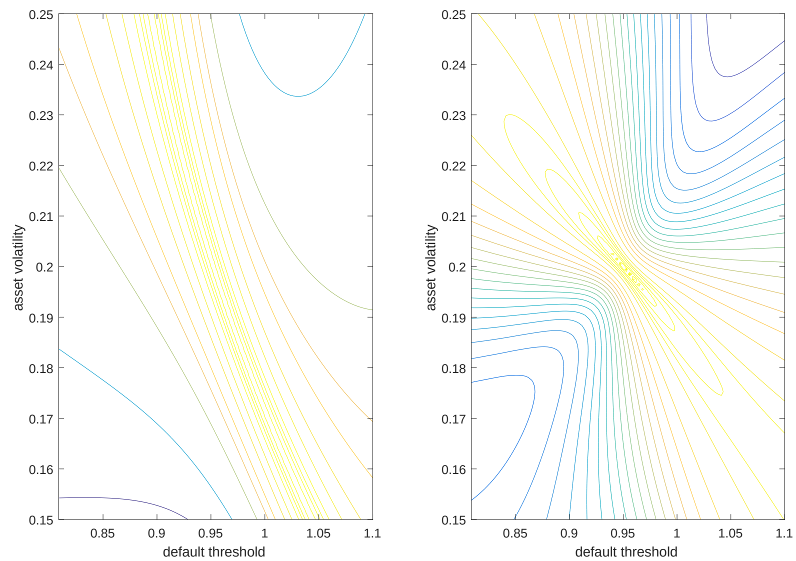
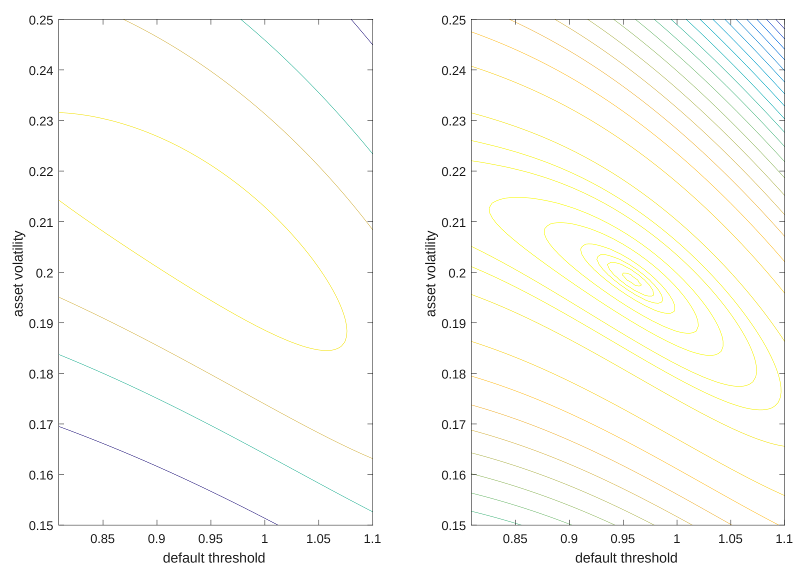


Figure 15: This figure shows log likelihood function for different asset volatility and bankruptcy threshold combinations without assuming that the unlevered asset value is known. The true asset volatility is 0.2 and the true bankruptcy threshold is 0.95. The left panel shows the log likelihood function based on equity prices alone and the right panel the log likelihood function using equity and put prices.



F Additional leverage regressions

Table 8: This table contains the results for a regression of quasi market leverage (QML) on various firm characteristic variables as indicated in the rows of the table. The variable definitions are in the text. The regression is performed for both firm characteristics using both balance sheet asset values and the asset values estimated from the model. The explanatory variables are from Q2 2008, the leverage ratios are calculated with Q3 2008 data. Significance levels are indicated by *** for 1%, ** for 5% and * for 10%. Standard errors are given in parenthesis.

			Balance Sheet Asset Value				Estimated Asset Value			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	0.70***	0.82***	0.64***	0.62***	0.58***	0.55***	0.88***	0.83***	0.77***	0.66***
	(0.03)	(0.04)	(0.13)	(0.14)	(0.13)	(0.13)	(0.14)	(0.14)	(0.13)	(0.13)
Asset volatility	-0.95***	-0.84***	-0.32**	-0.30**			-0.48***	-0.48***		
	(0.15)	(0.15)	(0.14)	(0.14)			(0.15)	(0.15)		
α	-0.32***	-0.28***	-0.09**	-0.11***			-0.12***	-0.11***		
	(0.04)	(0.04)	(0.04)	(0.03)			(0.03)	(0.03)		
PPE/Assets			-0.15**	-0.26***	-0.16***	-0.27***	-0.14	-0.36***	-0.11	-0.34***
			(0.06)	(0.07)	(0.06)	(0.07)	(0.10)	(0.10)	(0.11)	(0.11)
Tangibility			-0.07	0.06	-0.07	0.07	0.18	0.60***	0.16	0.59***
			(0.09)	(0.11)	(0.09)	(0.12)	(0.15)	(0.13)	(0.15)	(0.14)
Brand and patents			-0.00***	-0.00	-0.00***	-0.00	-0.00	-0.00	-0.00	-0.00
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Depreciation			-0.67***	-0.67***	-0.74***	-0.77***	-0.71***	-0.46*	-0.83***	-0.63**
			(0.23)	(0.25)	(0.24)	(0.27)	(0.23)	(0.26)	(0.24)	(0.26)
R&D/Sales			-0.52**	-0.38	-0.63***	-0.49*	-0.28	-0.03	-0.46**	-0.23
			(0.20)	(0.25)	(0.21)	(0.25)	(0.19)	(0.22)	(0.19)	(0.21)
Profit			-1.99***	-1.78***	-2.16***	-1.95***	-2.26**	-1.51**	-2.90***	-2.01***
			(0.44)	(0.46)	(0.43)	(0.44)	(0.91)	(0.71)	(0.94)	(0.71)
Market to book			-0.05***	-0.05***	-0.06***	-0.06***	-0.09***	-0.07***	-0.10***	-0.08***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Corp. govern. index			-0.13**	-0.13**	-0.14**	-0.17**	-0.17**	-0.15**	-0.18**	-0.18**
			(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
Long-term incent.			-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02*	-0.03**
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Weak board			0.02	0.02	0.02*	0.02**	0.02*	0.03**	0.03***	0.04***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Poison Pill			0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03
			(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
unionized			0.01***	0.01**	0.01***	0.01**	0.01	0.01	0.01	0.01
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Skill intensity			-0.02	-0.04	-0.02	-0.05*	0.01	0.00	0.01	-0.01
			(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
L over K			-0.03***	-0.03***	-0.03***	-0.03***	-0.01	-0.01	-0.01	-0.01
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Log sales			0.01*	0.01*	0.02**	0.02**	0.00	0.01	0.01	0.02**
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.40	0.55	0.69	0.73	0.67	0.71	0.67	0.74	0.63	0.71
adj R^2	0.39	0.51	0.67	0.69	0.65	0.67	0.65	0.70	0.61	0.66
Ind FE	N	Y	N	Y	N	Y	N	Y	N	Y
N	300	300	300	300	300	300	303	303	303	303

Table 9: This table contains the results for a regression of book leverage (BL) on various firm characteristic variables as indicated in the rows of the table. The variable definitions are in the text. The regression is performed for both firm characteristics using both balance sheet asset values and the asset values estimated from the model. The explanatory variables are from Q2 2008, the leverage ratios are calculated with Q3 2008 data. Significance levels are indicated by *** for 1%, ** for 5% and * for 10%. Standard errors are given in parenthesis.

			Balance Sheet Asset Value				Estimated Asset Value			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	0.89***	0.94***	0.99***	0.84***	0.69***	0.46***	1.13***	0.97***	0.84***	0.62***
	(0.03)	(0.04)	(0.17)	(0.18)	(0.16)	(0.17)	(0.18)	(0.18)	(0.16)	(0.15)
Asset volatility	-1.14***	-1.08***	-1.06***	-1.03***			-0.94***	-0.88***		
	(0.16)	(0.17)	(0.21)	(0.22)			(0.21)	(0.22)		
α	-0.11**	-0.09**	-0.09*	-0.10**			-0.07	-0.07		
	(0.05)	(0.05)	(0.05)	(0.05)			(0.05)	(0.05)		
PPE/Assets			-0.11	-0.16*	-0.10	-0.15	-0.15	-0.23*	-0.08	-0.18
			(0.08)	(0.09)	(0.08)	(0.10)	(0.11)	(0.13)	(0.12)	(0.14)
Tangibility			-0.09	0.08	-0.16	0.04	0.06	0.28	-0.05	0.20
			(0.12)	(0.15)	(0.13)	(0.17)	(0.15)	(0.18)	(0.15)	(0.18)
Brand and patents			-0.00*	-0.00**	-0.00*	-0.00*	-0.00	-0.00	-0.00	-0.00
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Depreciation			-0.32	-0.55	-0.49	-0.87**	-0.33	-0.46	-0.49	-0.75*
			(0.35)	(0.34)	(0.39)	(0.38)	(0.36)	(0.36)	(0.39)	(0.40)
R&D/Sales			-0.76***	-0.76**	-1.04***	-1.07***	-0.53*	-0.50	-0.82***	-0.82**
			(0.29)	(0.35)	(0.30)	(0.37)	(0.28)	(0.33)	(0.27)	(0.32)
Profit			-1.42**	-1.25*	-1.55**	-1.34*	-1.85**	-1.25	-2.63***	-1.86**
			(0.67)	(0.74)	(0.67)	(0.73)	(0.88)	(0.85)	(0.86)	(0.78)
Market to book			0.06***	0.06***	0.03	0.03	-0.00	0.00	-0.02	-0.01
			(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Corp. govern. index			-0.02	-0.06	-0.01	-0.10	-0.08	-0.11	-0.07	-0.14
			(0.09)	(0.10)	(0.11)	(0.11)	(0.10)	(0.10)	(0.11)	(0.12)
Long-term incent.			-0.01	-0.01	-0.02*	-0.02	-0.01	-0.01	-0.03**	-0.02*
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Weak board			0.01	0.02	0.03*	0.03**	0.01	0.02	0.03	0.04**
			(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Poison Pill			-0.03	-0.03	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02
			(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Unionized			0.01**	0.01	0.01***	0.01	0.01*	0.00	0.01**	0.01
			(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)
Skill intensity			-0.04	-0.03	-0.05*	-0.06	-0.02	-0.01	-0.02	-0.03
			(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)
L over K			-0.02	-0.04**	-0.02	-0.04**	-0.01	-0.02	-0.00	-0.02
			(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)
Log sales			-0.00	0.00	0.01	0.02	-0.01	0.00	0.00	0.02*
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.26	0.36	0.38	0.45	0.26	0.36	0.33	0.41	0.23	0.35
adj R^2	0.25	0.30	0.34	0.36	0.22	0.27	0.29	0.32	0.19	0.25
Ind FE	N	Y	N	Y	N	Y	N	Y	N	Y
N	300	300	300	300	300	300	303	303	303	303

References

Admati, A. R., DeMarzo, P. M., Hellwig, M. F., & Pfleiderer, P. (2013). *The Leverage Ratchet Effect*. Working paper series of the max planck institute for research on collective goods, Max Planck Institute for Research on Collective Goods.

Almeida, H. & Campello, M. (2007). Financial constraints, asset tangibility, and corporate investment. *Review of Financial Studies*, 20(5), pp. 1429–1460.

Almeida, H. & Philippon, T. (2007). The risk-adjusted cost of financial distress. *Journal of Finance*, 62(6), 2557–2586.

Altman, E. I. (1984). A further empirical investigation of the bankruptcy cost question. *Journal of Finance*, 39(4), pp. 1067–1089.

Andrade, G. & Kaplan, S. N. (1998). How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed. *Journal of Finance*, 53(5), 1443–1493.

Ang, J. S., Chua, J. H., & McConnell, J. J. (1982). The administrative costs of corporate bankruptcy: A note. *Journal of Finance*, 37(1), pp. 219–226.

Bakke, T.-E. & Whited, T. M. (2012). Threshold events and identification: A study of cash shortfalls. *The Journal of Finance*, 67(3), 1083–1111.

Bakshi, G., Kapadia, N., & Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies*, 16(1), 101–143.

Bao, J. & Pan, J. (2013). Bond illiquidity and excess volatility. *Review of Financial Studies*.

Bates, D. S. (2000). Post-'87 crash fears in the s&p 500 futures option market. *Journal of Econometrics*, 94(1-2), 181–238.

Bebchuk, L., Cohen, A., & Ferrell, A. (2009). What matters in corporate governance? *Review of Financial Studies*, 22(2), 783–827.

Benmelech, E., Bergman, N. K., & Enriquez, R. J. (2012). Negotiating with labor under financial distress. *Review of Corporate Finance Studies*, 1(1), 28–67.

Berger, P. G., Ofek, E., & Swary, I. (1996). Investor valuation of the abandonment option. *Journal of Financial Economics*, 42(2), 257 – 287.

Berk, J. B., Stanton, R., & Zechner, J. (2010). Human capital, bankruptcy, and capital structure. *Journal of Finance*, 65(3), 891–926.

Bjoerk, T. (2004). *Arbitrage Theory in Continuous Time*. Oxford: Oxford University Press.

Bris, A., Welch, I., & Zhu, N. (2006). The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *Journal of Finance*, 61(3), 1253–1303.

Carr, P. & Wu, L. (2010). Stock options and credit default swaps: A joint framework for valuation and estimation. *Journal of Financial Econometrics*, 8(4), 409–449.

Dangl, T. & Zechner, J. (2004). Credit risk and dynamic capital structure choice. *Journal of Financial Intermediation*, 13(2), 183–204.

Dangl, T. & Zechner, J. (2016). *Debt Maturity and the Dynamics of Leverage*. SSRN Working Paper Series 89022, WU Wien.

Davydenko, S. A., Strebulaev, I. A., & Zhao, X. (2012). A market-based study of the cost of default. *Review of Financial Studies*, 25(10), 2959–2999.

Duan, J.-C. (1994). Maximum likelihood estimation using price data of the derivative contract. *Mathematical Finance*, 4(2), 155–167.

Duffie, D. (2001). *Dynamic Asset Pricing Theory*. Princeton University Press, 3rd edition.

Eisfeldt, A. L. & Rampini, A. A. (2009). Leasing, ability to repossess, and debt capacity. *Review of Financial Studies*, 22(4), 1621–1657.

Elkamhi, R., Ericsson, J., & Parsons, C. A. (2012). The cost and timing of financial distress. *Journal of Financial Economics*, 105(1), 62 – 81.

Eom, Y. H., Helwege, J., & Huang, J.-Z. (2004). Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies*, 17(2), 499–544.

Ericsson, J. & Reneby, J. (1998). A framework for valuing corporate securities. *Applied Mathematical Finance*, 5(3-4), 143–163.

Ericsson, J. & Reneby, J. (2003). Stock options as barrier contingent claims. *Applied Mathematical Finance*, 10(2), 121–147.

Ericsson, J. & Reneby, J. (2005). Estimating structural bond pricing models. *Journal of Business*, 78(2), 707–706.

Ericsson, J., Reneby, J., & Wang, H. (2007). Can structural models price default risk? new evidence from bond and credit derivative markets. *working paper*.

Fama, E. F. & French, K. R. (2002). Testing trade?off and pecking order predictions about dividends and debt. *Review of Financial Studies*, 15(1), 1–33.

Frank, M. Z. & Goyal, V. K. (2009). Capital structure decisions: Which factors are reliably important? *Financial Management*, 38(1), 1–37.

Gilson, S. C. (1989). Management turnover and financial distress. *Journal of Financial Economics*, 25(2), 241–262.

Glover, B. (2016). The expected cost of default. *Journal of Financial Economics*, 119(2), 284 – 299.

Graham, J. R. & Leary, M. T. (2011). A Review of Empirical Capital Structure Research and Directions for the Future. *Annual Review of Financial Economics*, 3(1), 309–345.

Hart, O. & Moore, J. (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics*, 109(4), 841–79.

Hennessy, C. A. & Livdan, D. (2009). Debt, bargaining, and credibility in firm-supplier relationships. *Journal of Financial Economics*, 93(3), 382–399.

Hotchkiss, E. S. (1995). Postbankruptcy performance and management turnover. *The Journal of Finance*, 50(1), 3–21.

Korteweg, A. (2010). The net benefits to leverage. *Journal of Finance*, 65(6), 2137–2170.

Leland, H. (1994). *Bond Prices, Yield Spreads, and Optimal Capital Structure with Default Risk*. Finance working paper 240, Haas School of Business, U.C. Berkeley.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. *Journal of Finance*, 53(4), pp. 1213–1243.

Leland, H. E. & Toft, K. B. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, 51(3), 987–1019.

Lemmon, M. L., Roberts, M. R., & Zender, J. F. (2008). Back to the beginning: Persistence and the cross-section of corporate capital structure. *Journal of Finance*, 63(4), 1575–1608.

Lim, S., Mann, S., & V.Mihov (2003). *Market evalution of off-balance sheet financing: You can run but you can't hide*. Working paper, Texas Christian University.

Matsa, D. A. (2010). Capital structure as a strategic variable: Evidence from collective bargaining. *The Journal of Finance*, 65(3), 1197–1232.

Miller, M. H. & Upton, C. W. (1976). Leasing, buying, and the cost of capital services. *The Journal of Finance*, 31(3), pp. 761–786.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2), 147–175.

Rauh, J. D. (2006). Investment and financing constraints: Evidence from the funding of corporate pension plans. *The Journal of Finance*, 61(1), 33–71.

Schaefer, S. M. & Strebulaev, I. A. (2008). Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *Journal of Financial Economics*, 90(1), 1 – 19.

Strebulaev, I. A. (2007). Do tests of capital structure theory mean what they say? *The Journal of Finance*, 62(4), 1747–1787.

Strebulaev, I. A. & Whited, T. M. (2012). Dynamic Models and Structural Estimation in Corporate Finance. *Foundations and Trends(R) in Finance*, 6(1�2), 1–163.

Wan, E. & Van Der Merwe, R. (2001). The Unscented Kalman Filter. In *Kalman Filtering and Neural Networks* chapter 7, (pp. 221–280). John Wiley & Sons, Inc.

Weiss, L. A. (1990). Bankruptcy resolution: Direct costs and violation of priority of claims. *Journal of Financial Economics*, 27(2), 285 – 314.