

Dissecting Characteristics Nonparametrically*

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Abstract

We propose a nonparametric method to study which characteristics provide incremental information for the cross section of expected returns. We use the adaptive group LASSO to select characteristics and to estimate how they affect expected returns nonparametrically. Our method can handle a large number of characteristics, allows for a flexible functional form, and is insensitive to outliers. Many of the previously identified return predictors do not provide incremental information for expected returns, and nonlinearities are important. Our proposed method has higher out-of-sample explanatory power compared to linear panel regressions.

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I Introduction

In his presidential address, Cochrane (2011) argues the cross section of the expected return “is once again descending into chaos.” Harvey et al. (2016) identify “hundreds of papers and factors” that have predictive power for the cross section of expected returns.¹ Many economic models, such as the consumption CAPM of Lucas (1978), Breeden (1979), and Rubinstein (1976), instead predict that only a small number of state variables suffice to summarize cross-sectional variation in expected returns.

Researchers typically employ two methods to identify return predictors: (i) (conditional) portfolio sorts based on one or multiple characteristics, such as size or book-to-market, and (ii) linear regression in the spirit of Fama and MacBeth (1973). Both methods have many important applications, but they fall short in what Cochrane (2011) calls the multidimensional challenge: “[W]hich characteristics really provide *independent* information about average returns? Which are subsumed by others?” Portfolio sorts are subject to the curse of dimensionality when the number of characteristics is large, and linear regressions make strong functional-form assumptions and are sensitive to outliers.² In addition, in many empirical settings, most of the variation in characteristic values and returns are in the extremes of the characteristic distribution and the association between characteristics and returns appears nonlinear (see Fama and French (2008)). Cochrane (2011) speculates, “To address these questions in the zoo of new variables, I suspect we will have to use different methods.”

We propose a nonparametric method to determine which firm characteristics provide incremental information for the cross section of expected returns without making strong functional-form assumptions. Specifically, we use a group LASSO (least absolute shrinkage and selection operator) procedure developed by Huang, Horowitz, and Wei (2010) for model selection and nonparametric estimation. Model selection deals with the question of which characteristics have incremental predictive power for expected returns,

¹Figure 2 documents the number of discovered factors over time.

²We discuss these, and related concerns in Section II and compare current methods with our proposed framework in Section III.

given the other characteristics. Nonparametric estimation deals with estimating the effect of important characteristics on expected returns without imposing a strong functional form.

We show three applications of our proposed framework. First, we study which characteristics provide incremental information for the cross section of expected returns. We estimate our model on 62 characteristics including size, book-to-market, beta, and other prominent variables and anomalies on a sample period from July 1965 to June 2014. Only 13 variables, including size, total volatility, and past return-based predictors, have incremental explanatory power for expected returns for the full sample period and all stocks. A hedge portfolio going long the stocks with the 50% highest predicted returns and shorting the 50% of stocks with the lowest predicted returns has an in-sample Sharpe ratio of more than 3. Only 11 characteristics have predictive power for returns in the first half of our sample. In the second half, instead, we find 14 characteristics are associated with cross-sectional return premia. For stocks whose market capitalization is above the 20% NYSE size percentile, only nine characteristics, including changes in shares outstanding, past returns, and standardized unexplained volume, remain incremental return predictors. The in-sample Sharpe ratio is still 2.37 for large stocks.

Second, we compare the out-of-sample performance of the nonparametric model with a linear model. Estimating flexible functional forms raises the concern of in-sample overfitting. We estimate both the linear and the nonparametric model over a period until 1990 and select return predictors. We then use 10 years of data to estimate the models on the selected characteristics. In the first month after the end of our estimation period, we take the selected characteristics, predict one-month-ahead returns, and construct a hedge portfolio similar to our in-sample exercise. We roll the estimation and prediction period forward by one month and repeat the procedure until the end of the sample.

Specifically, we perform model selection once until December 1990 for both the linear model and the nonparametric model. Our first estimation period is from December of 1981 until November of 1990, and the first out-of-sample prediction is for January 1991

using characteristics from December 1990.³ We then move the estimation and prediction period forward by one month. The nonparametric model generates a Sharpe ratio of 2.83 compared to 0.75 for the linear model.⁴ The characteristics we study are not a random sample, but have been associated with cross sectional return premia in the past. Therefore, we focus mainly on the comparison across models rather than emphasizing the overall magnitude of the Sharpe ratios.

The linear model selects 30 characteristics in-sample compared to only eleven for the nonparametric model, but performs worse out-of-sample and nonlinearities are important. We find an increase in out-of-sample Sharpe ratios relative to the Sharpe ratio of the linear model when we employ the nonparametric model for prediction but use the 30 characteristics the linear model selects. The linear model appears to overfit the data in-sample. We find an identical Sharpe ratio for the linear model when we use the 11 characteristics selected by the nonparametric model, as we do with the 30 characteristics selected by the linear model. This latter result underscores once more the importance of nonlinearities. With the same set of 11 characteristics the nonlinear model selects, we find the nonparametric model has a Sharpe ratio that is larger by a factor of three relative to the Sharpe ratio of the linear model using the same set of characteristics.

Third, we study whether the predictive power of characteristics for expected returns varies over time. We estimate the model using 120 months of data on all characteristics we select in our baseline analysis, and then estimate rolling one-month-ahead return forecasts. We find substantial time variation in the predictive power of characteristics for expected returns. As an example, momentum returns conditional on other return predictors vary substantially over time, and we find a momentum crash similar to Daniel and Moskowitz (2016) as past losers appreciated during the recent financial crisis. Size conditional on the other selected return predictors, instead, has a significant predictive power for expected returns throughout our sample period similar to the findings in Asness, Frazzini, Israel,

³We merge balance-sheet variables to returns following the Fama and French (1993) convention of requiring a lag of at least six months, and our results are therefore indeed out-of-sample.

⁴The linear model we estimate and the results for the linear model are similar to Lewellen (2015).

Moskowitz, and Pedersen (2017).

The paper provides a new method in empirical asset pricing to understand which of the previously published firm characteristics provide information for expected returns conditional on other characteristics. We see this exercise as a natural first step in the “multidimensional challenge.” Once we understand which characteristics indeed provide incremental information, we can aim to relate characteristics to factor exposures, estimate factors and stochastic discount factors directly, or relate characteristics and factors to economic models.

A Related Literature

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) predicts that an asset’s beta with respect to the market portfolio is a sufficient statistic for the cross section of expected returns. Fama and MacBeth (1973) provide empirical support for the CAPM. Subsequently, researchers identified many variables, such as size (Banz (1981)), the book-to-market ratio (Rosenberg et al. (1985)), leverage (Bhandari (1988)), earnings-to-price ratios (Basu (1983)), or past returns (Jegadeesh and Titman (1993)) that contain additional independent information for expected returns. Sorting stocks into portfolios based on these characteristics often led to rejecting the CAPM because the spread in CAPM betas could not explain the spread in returns. Fama and French (1992) synthesize these findings, and Fama and French (1993) show that a three-factor model with the market return, a size factor, and a value factor can explain cross sections of stocks sorted on characteristics that appeared anomalous relative to the CAPM. In this sense, Fama and French (1993) and Fama and French (1996) achieve a significant dimension reduction: researchers who want to explain the cross section of stock returns only have to explain the size and value factors.

Daniel and Titman (1997), on the contrary, argue that characteristics have higher explanatory power for the cross section of expected returns than loadings on pervasive risk factors. Chordia, Goyal, and Shanken (2015) develop a method to estimate

bias-corrected return premia from cross-sectional data for individual stocks. They find firm characteristics explain more of the cross-sectional variation in expected returns compared to factor loadings. Kozak, Nagel, and Santosh (2017a) show horse races between factor loadings and characteristics do not allow researchers to disentangle rational from behavioral explanations for return spreads. We study which characteristics provide incremental information for expected returns but do not aim to investigate whether rational models or behavioral explanations drive our findings.

In the 20 years following the publication of Fama and French (1992), many researchers joined a “fishing expedition” to identify characteristics and factor exposures that the three-factor model cannot explain. Harvey, Liu, and Zhu (2016) provide an overview of this literature and list over 300 published papers that study the cross section of expected returns. They propose a t -statistic of 3 for new factors to account for multiple testing on a common data set. Figure 3 shows the suggested adjustment over time. However, even employing the higher threshold for the t -statistic still leaves approximately 150 characteristics as useful predictors for the cross section of expected returns. Chordia, Goyal, and Saretto (2017) build on this line of work and propose a combination of statistical criteria and economic considerations to choose return predictors. Fama and French (2015) take a different route and augment the three-factor model of Fama and French (1993) with an investment and profitability factor (Haugen and Baker (1996) and Novy-Marx (2013)). Fama and French (2016) test the five-factor model on a small set of anomalies and find substantial improvements relative to a three-factor model, but also substantial unexplained return variation across portfolios. Hou et al. (2015) test a q -factor model consisting of four factors on 35 anomalies that are univariately associated with cross-sectional return premia, and find their model can reduce monthly alphas to an average of 0.20%. Barillas and Shanken (2016) develop a new method to directly compare competing factor models.

The large number of significant predictors is not a shortcoming of Harvey et al. (2016), who address the issue of multiple testing. Instead, authors in this literature

usually consider their proposed return predictor in isolation without conditioning on previously discovered return predictors. Haugen and Baker (1996) and Lewellen (2015) are notable exceptions. They employ Fama and MacBeth (1973) regressions to combine the information in multiple characteristics. Lewellen (2015) jointly studies the predictive power of 15 characteristics and finds that only a few are significant predictors for the cross section of expected returns. Green, Hand, and Zhang (2016) adjust Fama-MacBeth regressions to avoid overweighting microcaps and adjust for data snooping bias and find for a sample starting in 1980 that many return predictors do not provide independent information. Although Fama-MacBeth regressions carry a lot of intuition, they do not offer a formal method of model selection. We build on Lewellen (2015) and provide a framework that allows for nonlinear association between characteristics and returns, provide a formal framework to disentangle important from unimportant return predictors, and study many more characteristics.

Light, Maslov, and Rytchkov (2016) use partial least squares (PLS) to summarize the predictive power of firm characteristics for expected returns. PLS summarizes the predictive power of all characteristics and therefore does not directly disentangle important from unimportant characteristics and does not reduce the number of characteristics for return prediction. Brandt, Santa-Clara, and Valkanov (2009) parameterize portfolio weights as a function of stock characteristics to sidestep the task of modeling the joint distribution of expected returns and characteristics. DeMiguel, Martin-Utrera, Nogales, and Uppal (2016) extend the parametric portfolio approach of Brandt et al. (2009) to study which characteristics provide incremental information for the cross section of returns. Specifically, DeMiguel et al. (2016) add long-short characteristic-sorted portfolios to benchmark portfolios, such as the value-weighted market portfolio, and ask which portfolios increase investor utility. Moritz and Zimmermann (2016) use tree-based conditional portfolio sorts to study which past-return based predictors provide independent information for future returns.

We also contribute to a small literature estimating nonlinear asset-pricing models

using semi- and nonparametric methods. Bansal and Viswanathan (1993) extend the arbitrage pricing theory (APT) of Ross (1976) and estimate the stochastic discount factor semiparametrically using neural nets. They allow for payoffs to be nonlinear in risk factors and find their APT is better able to explain the returns of size-sorted portfolios. Chapman (1997) explains the size effect with a consumption-based model in which he approximates the stochastic discount factor with orthonormal polynomials in a low number of state variables. Connor, Hagmann, and Linton (2012) propose a nonparametric regression method relating firm characteristics to factor loadings in a nonlinear way. They find momentum and stock volatility have explanatory power similar to size and value for the cross section of expected returns.

We build on a large literature in economics and statistics using penalized regressions. Horowitz (2016) gives a general overview of model selection in high-dimensional models, and Huang, Horowitz, and Wei (2010) discuss variable selection in a nonparametric additive model similar to the one we implement empirically. Recent applications of LASSO methods in finance are Huang and Shi (2016), who use an adaptive group LASSO in a linear framework and construct macro factors to test for determinants of bond risk premia. Chinco, Clark-Joseph, and Ye (2015) assume the irrepresentable condition of Meinshausen and Bühlmann (2006) to achieve model-selection consistency in a single-step LASSO. They use a linear model for high-frequency return predictability using past returns of related stocks, and find their method increases predictability relative to OLS. Goto and Xu (2015) use a LASSO to obtain a sparse estimator of the inverse covariance matrix for mean variance portfolio optimization.

Bryzgalova (2016) highlights that weak identification in linear factor models could result in an overstatement of significant cross-sectional risk factors.⁵ She proposes a shrinkage-based estimator to detect possible rank deficiency in the design matrix and to identify strong asset-pricing factors. Gagliardini, Ossola, and Scaillet (2016) develop a weighted two-pass cross-sectional regression method to estimate risk premia from an

⁵See also Jagannathan and Wang (1998), Kan and Zhang (1999), Kleibergen (2009), Gospodinov, Kan, and Robotti (2014), Kleibergen and Zhan (2015), and Burnside (2016).

unbalanced panel of individual stocks. Giglio and Xiu (2016) instead propose a three-pass regression method that combines principal component analysis and a two-stage regression framework to estimate consistent factor risk premia in the presence of omitted factors when the cross section of test assets is large. Kelly, Pruitt, and Su (2017) generalize standard PCA to allow for time-varying loadings and extract common factors from the universe of individual stocks. Kozak, Nagel, and Santosh (2017b) exploit economic restrictions relating expected returns to covariances to construct stochastic discount factors. We, instead, are mainly concerned with formal model selection, that is, which characteristics provide incremental information in the presence of other characteristics.

II Current Methodology

A Expected Returns and the Curse of Dimensionality

One aim of the empirical asset-pricing literature is to identify characteristics that predict expected returns, that is, find a characteristic C in period $t-1$ that predicts excess returns of firm i in the following period, R_{it} . Formally, we try to describe the conditional mean function,

$$E[R_{it} \mid C_{1,it-1}, \dots, C_{S,it-1}]. \quad (1)$$

We often use portfolio sorts to approximate equation (1) for a single characteristic. We typically sort stocks into 10 portfolios and compare mean returns across portfolios. Portfolio sorts are simple, straightforward, and intuitive, but they also suffer from several shortcomings. First, we can only use portfolio sorts to analyze a small set of characteristics. Imagine sorting stocks jointly into five portfolios based on CAPM beta, size, book-to-market, profitability, and investment. We would end up with $5^5 = 3125$ portfolios, which is larger than the number of stocks at the beginning of our sample.⁶

⁶The curse of dimensionality is a well-understood shortcoming of portfolio sorts. See Fama and French (2015) for a recent discussion in the context of the factor construction for their five-factor model. They also argue not-well-diversified portfolios have little power in asset-pricing tests.

Second, portfolio sorts offer little formal guidance to discriminate between characteristics. Consider the case of sorting stocks into five portfolios based on size, and within these, into five portfolios based on the book-to-market ratio. If we now find the book-to-market ratio only leads to a spread in returns for the smallest stocks, do we conclude it does not matter for expected returns? Fama and French (2008) call this second shortcoming “awkward.” Third, we implicitly assume expected returns are constant over a part of the characteristic distribution, such as the smallest 10% of stocks, when we use portfolio sorts as an estimator of the conditional mean function. Fama and French (2008) call this third shortcoming “clumsy.”⁷ Nonetheless, portfolio sorts are by far the most commonly used technique to analyze which characteristics have predictive power for expected returns.

Instead of (conditional) double sorts, we could sort stocks into portfolios and perform spanning tests, that is, we regress long-short portfolios on a set of risk factors. Take 10 portfolios sorted on profitability and regress the hedge return on the three Fama and French (1993) factors. A significant time-series intercept would correspond to an increase in Sharpe ratios for a mean-variance investor relative to the investment set the three Fama and French (1993) factors span (see Gibbons, Ross, and Shanken (1989)). The order in which we test characteristics matters, and spanning tests cannot solve the selection problem of which characteristics provide incremental information for the cross section of expected returns.

An alternative to portfolio sorts and spanning tests is to *assume* linearity of equation (1) and run linear panel regressions of excess returns on S characteristics, namely,

$$R_{it} = \alpha + \sum_{s=1}^S \beta_s C_{s,it-1} + \varepsilon_{it}. \quad (2)$$

Linear regressions allow us to study the predictive power for expected returns of many characteristics jointly, but they also have potential pitfalls. First, no a priori reason

⁷Portfolio sorts are a restricted form of nonparametric regression. We will use the similarities of portfolio sorts and nonparametric regressions to develop intuition for our proposed framework below.

exists why the conditional mean function should be linear.⁸ Fama and French (2008) estimate linear regressions as in equation (2) to dissect anomalies, but raise concerns over potential nonlinearities. They make ad hoc adjustments and use, for example, the log book-to-market ratio as a predictive variable. Second, linear regressions are sensitive to outliers and extreme observations of the characteristics might drive point estimates. Researchers often use ad hoc techniques to mitigate these concerns, such as winsorizing observations and estimating linear regressions separately for small and large stocks (see Lewellen (2015) for a recent example).

Cochrane (2011) synthesizes many of the challenges that portfolio sorts and linear regressions face in the context of many return predictors, and suspects “we will have to use different methods.”

B Equivalence between Portfolio Sorts and Regressions

Cochrane (2011) conjectures in his presidential address, “[P]ortfolio sorts are really the same thing as nonparametric cross-sectional regressions, using nonoverlapping histogram weights.” Additional assumptions are necessary to show a formal equivalence, but his conjecture contains valuable intuition to model the conditional mean function formally. We first show a formal equivalence between portfolio sorts and regressions and then use the equivalence to motivate the use of nonparametric methods.⁹

Suppose we observe excess returns R_{it} and a single characteristic C_{it-1} for stocks $i = 1, \dots, N_t$ and time periods $t = 1, \dots, T$. We sort stocks into L portfolios depending on the value of the lagged characteristic, C_{it-1} .¹⁰ Specifically, stock i is in portfolio l at time t if $C_{it-1} \in I_{tl}$, where I_{tl} indicates an interval of the distribution for a given firm characteristic. For example, take a firm with lagged market cap in the 45th percentile

⁸Fama and MacBeth (1973) regressions also assume a linear relationship between expected returns and characteristics. Fama-MacBeth point estimates are numerically equivalent to estimates from equation (2) when characteristics are constant over time.

⁹Cattaneo et al. (2016) develop inference methods for a portfolio-sorting estimator and also show the equivalence between portfolio sorting and nonparametric estimation.

¹⁰We only consider univariate portfolio sorts in this example to gain intuition.

of the firm size distribution. We would sort that stock in the 5th out of 10 portfolios in period t . For each time period t , let N_{tl} be the number of stocks in portfolio l , $N_{tl} = \sum_{i=1}^{N_t} \mathbf{1}(C_{it-1} \in I_{tl})$. The excess return of portfolio l at time t , P_{tl} , is then

$$P_{tl} = \frac{1}{N_{tl}} \sum_{i=1}^N R_{it} \mathbf{1}(C_{it-1} \in I_{tl}).$$

Alternatively, we can run a pooled time-series cross-sectional regression of excess returns on dummy variables, which equal 1 if firm i is in portfolio l in period t . We denote the dummy variables by $\mathbf{1}(C_{it-1} \in I_{tl})$ and write,

$$R_{it} = \sum_{l=1}^L \beta_l \mathbf{1}(C_{it-1} \in I_{tl}) + \varepsilon_{it}.$$

Let \mathcal{R} be the $NT \times 1$ vector of excess returns and let X be the $NT \times L$ matrix of dummy variables, $\mathbf{1}(C_{it-1} \in I_{tl})$. Let $\hat{\beta}$ be an OLS estimate, $\hat{\beta} = (X'X)^{-1}X'\mathcal{R}$. It is easy to show that

$$\hat{\beta}_l = \frac{1}{T} \sum_{t=1}^T \frac{N_{tl}}{\frac{1}{T} \sum_{t=1}^T N_{tl}} P_{tl}.$$

Now suppose we have the same number of stocks in each portfolio l for each time period t , that is, $N_{tl} = \bar{N}_l$ for all t . Then

$$\hat{\beta}_l = \frac{1}{T} \sum_{t=1}^T P_{tl}$$

and

$$\hat{\beta}_l - \hat{\beta}_{l'} = \frac{1}{T} \sum_{t=1}^T (P_{tl} - P_{tl'}).$$

Hence, the slope coefficients in pooled time-series cross-sectional regressions are equivalent to average portfolio returns, and the difference between two slope coefficients is the excess return between two portfolios.

If the number of stocks in the portfolios changes over time, then portfolio sorts and

regressions typically differ. We can restore equivalence in two ways. First, we could take the different number of stocks in portfolio l over time into account when we calculate averages, and define excess return as

$$\frac{1}{\sum_{t=1}^T N_{tl}} \sum_{t=1}^T N_{tl} P_{tl} - \frac{1}{\sum_{t=1}^T N_{tl'}} \sum_{t=1}^T N_{tl'} P_{tl'},$$

which equals $\hat{\beta}_l - \hat{\beta}_{l'}$.

Second, we could use the weighted least squares estimator, $\tilde{\beta} = (X'WX)^{-1}X'W\mathcal{R}$, where the $NT \times NT$ weight matrix W is a diagonal matrix with the inverse number of stocks on the diagonal, $\text{diag}(1/N_{tl})$. With this estimator, we again get

$$\tilde{\beta}_l - \tilde{\beta}_{l'} = \frac{1}{T} \sum_{t=1}^T (P_{tl} - P_{tl'}).$$

III Nonparametric Estimation

We now use the relationship between portfolio sorts and regressions to develop intuition for our nonparametric estimator, and show how we can interpret portfolio sorts as a special case of nonparametric estimation. We then show how to select characteristics with incremental information for expected returns within that framework.

Suppose we knew the conditional mean function $m_t(c) \equiv E[R_{it} | C_{it-1} = c]$.¹¹ Then,

$$E[R_{it} | C_{it-1} \in I_{tl}] = \int_{I_{tl}} m_t(c) f_{C_{it-1}|C_{it-1} \in I_{tl}}(c) dc,$$

where $f_{C_{it-1}|C_{it-1} \in I_{tl}}$ is the density function of the characteristic in period $t-1$, conditional on $C_{it-1} \in I_{tl}$. Hence, to obtain the expected return of portfolio l , we can simply integrate the conditional mean function over the appropriate interval of the characteristic distribution. Therefore, the conditional mean function contains all information for portfolio returns. However, knowing $m_t(c)$ provides additional information about

¹¹We take the expected excess return for a fixed time period t .

nonlinearities in the relationship between expected returns and characteristics, and the functional form more generally.

To estimate the conditional mean function, m_t , consider again regressing excess returns, R_{it} , on L dummy variables, $\mathbf{1}(C_{it-1} \in I_{tl})$,

$$R_{it} = \sum_{l=1}^L \beta_l \mathbf{1}(C_{it-1} \in I_{tl}) + \varepsilon_{it}.$$

In nonparametric estimation, we call indicator functions of the form $\mathbf{1}(C_{it-1} \in I_{tl})$ constant splines. Estimating the conditional mean function, m_t , with constant splines, means we approximate it by a step function. In this sense, portfolio sorting is a special case of nonparametric regression. A step function is nonsmooth and therefore has undesirable theoretical properties as a nonparametric estimator, but we build on this intuition to estimate m_t nonparametrically.¹²

Figures 4–6 illustrate the intuition behind the relationship between portfolio sorts and nonparametric regressions. These figures show returns on the y-axis and book-to-market ratios on the x-axis, as well as portfolio returns and the nonparametric estimator we propose below for simulated data.

We see in Figure 4 that most of the dispersion in book-to-market ratios and returns is in the extreme portfolios. Little variation in returns occurs across portfolios 2-4 in line with empirical settings (see Fama and French (2008)). Portfolio means offer a good approximation of the conditional mean function for intermediate portfolios. We also see, however, that portfolios 1 and 5 have difficulty capturing the nonlinearities we see in the data.

Figure 5 documents that a nonparametric estimator of the conditional mean function provides a good approximation for the relationship between book-to-market ratios and returns for intermediate values of the characteristic, but also in the extremes of the distribution.

¹²We formally define our estimator in Section III. *D* below.

Finally, we see in Figure 6 that portfolio means provide a better fit in the tails of the distribution once we allow for more portfolios. Portfolio mean returns become more comparable to the predictions from the nonparametric estimator the larger the number of portfolios.

A Multiple Regression & Additive Conditional Mean Function

Both portfolio sorts and regressions theoretically allow us to look at several characteristics simultaneously. Consider small (S) and big (B) firms and value (V) and growth (G) firms. We could now study four portfolios: (SV) , (SG) , (BV) , and (BG) . However, portfolio sorts quickly become infeasible as the number of characteristics increases. For example, if we have four characteristics and partition each characteristic into five portfolios, we end up with $5^4 = 625$ portfolios. Analyzing 625 portfolio returns would, of course, be impractical, but would also result in poorly diversified portfolios.

In nonparametric regressions, an analogous problem arises. Estimating the conditional mean function, m_t , fully nonparametrically with many regressors results in a slow rate of convergence and imprecise estimates in practice.¹³ Specifically, with S characteristics and N_t observations, assuming technical regularity conditions, the optimal rate of convergence in mean square is $N_t^{-4/(4+S)}$, which is always smaller than the rate of convergence for the parametric estimator of N_t^{-1} . Notice the rate of convergence decreases as S increases.¹⁴ Consequently, we get an estimator with poor finite sample properties if the number of characteristics is large.

As an illustration, suppose we observe one characteristic, in which case, the rate of convergence is $N_t^{-4/5}$. Now suppose instead we have 11 characteristics, and let N_t^* be the number of observations necessary to get the same rate of convergence as in the case with

¹³The literature refers to this phenomenon as the “curse of dimensionality” (see Stone (1982) for a formal treatment).

¹⁴We assume the conditional mean function, m_t , is twice continuously differentiable.

one characteristic. We get,

$$(N_t^*)^{-4/15} = N_t^{-4/5} \Rightarrow N_t^* = N_t^3.$$

Hence, in the case with 11 characteristics, we have to raise the sample size to the power of 3 to obtain the same rate of convergence and comparable finite sample properties as in the case with only one characteristic. Consider a sample size, N_t , of 1,000. Then, we would need 1 billion return observations to obtain similar finite sample properties of an estimated conditional mean function with 11 characteristics.

Conversely, suppose $S = 11$ and we have $N_t^* = 1,000$ observations. This combination yields similar properties as an estimation with one characteristic and a sample size $N_t = (N_t^*)^{1/3}$ of 10.

Nevertheless, if we are interested in which characteristics provide incremental information for expected returns given other characteristics, we cannot look at each characteristic in isolation. A natural solution in the nonparametric regression framework is to assume an additive model,

$$m_t(c_1, \dots, c_S) = \sum_{s=1}^S m_{ts}(c_s),$$

where $m_{ts}(\cdot)$ are unknown functions. The main theoretical advantage of the additive specification is that the rate of convergence is always $N_t^{-4/5}$, which does not depend on the number of characteristics S (see Stone (1985), Stone (1986), and Horowitz et al. (2006)).

An important restriction of the additive model is

$$\frac{\partial^2 m_t(c_1, \dots, c_S)}{\partial c_s \partial c_{s'}} = 0$$

for all $s \neq s'$; therefore, the additive model does not allow for cross dependencies between characteristics. For example, the predictive power of the book-to-market ratio

for expected returns does not vary with firm size (conditional on size). One way around this shortcoming is to add certain interactions as additional regressors. For instance, we could interact every characteristic with size to see if small firms are really different. An alternative solution is to estimate the model separately for small and large stocks. Brandt et al. (2009) make a similar assumption, but also stress that we can always interpret characteristics c as the cross product of a more basic set of characteristics. In our empirical application, we show results for all stocks and all-but micro caps, but also show results when we interact each characteristic with size.

Although the assumption of an additive model is somewhat restrictive, it provides desirable econometric advantages. In addition, we always make this assumption when we estimate multivariate regressions and in our context this assumption is far less restrictive than assuming linearity right away, as we do in Fama-MacBeth regressions. Another major advantage of an additive model is that we can jointly estimate the model for a large number of characteristics, select important characteristics, and estimate the summands of the conditional mean function, m_t , simultaneously, as we explain in Section *D*.

***B* Comparison of Linear & Nonparametric Models**

We now want to compare portfolio sorts and a linear model with nonparametric models in some specific numerical examples. The comparison helps us understand the potential pitfalls from assuming a linear relationship between characteristics and returns, and gain some intuition for why we might select different characteristics in a linear model in our empirical tests in Section *IV*.

Suppose we observe excess returns, R_{it} , and a single characteristic, C_{it-1} , distributed according to $C_{it-1} \sim U[0, 1]$ for $i = 1, \dots, N$ and $t = 1, \dots, T$ with the data-generating process,

$$R_{it} = m_t(C_{it-1}) + \varepsilon_{it},$$

where $E[\varepsilon_{it} | C_{it-1}] = 0$.

Without knowing the conditional mean function m_t , we could sort stocks into portfolios according to the distribution of the characteristic. C_{it-1} predicts returns if mean returns differ significantly across portfolios. For example, we could construct 10 portfolios based on the percentiles of the distribution and test if the first portfolio has a significantly different return than the 10th portfolio.

If we knew the conditional mean function m_t , we could conclude that C_{it-1} predicts returns if m_t is not constant on $[0, 1]$. Moreover, knowing the conditional mean function allows us to construct portfolios with a large spread in returns. Instead of sorting stocks based on the values of the characteristic C_{it-1} , we could sort stocks directly based on the conditional mean function $m_t(C_{it-1})$. For example, let $q_t(\alpha)$ be the α -quantile of $m_t(C_{it-1})$ and let stock i be in portfolio l at time t if $m_t(C_{it-1}) \in [q_t((l-1)/10), q_t(l/10)]$. That is, we construct 10 portfolios based on return predictions. Portfolio 1 contains the 10% of stocks with the lowest predicted returns, and portfolio 10 contains the 10% of stocks with the highest predicted returns.

If m_t is *monotone* and we only study a single characteristic, both sorting based on the value of the characteristic and sorting based on predicted returns $m_t(C_{it-1})$ results in the same portfolios. However, if m_t is not monotone, the “10-1 portfolio” return is higher when we sort based on $m_t(C_{it-1})$. As a simple example, suppose $m_t(c) = (c - 0.5)^2$. Then the expected “10-1 portfolio” return when sorting based on characteristic, C_{it-1} , is 0.

We now consider two characteristics, $C_{1,it-1} \sim U[0, 1]$ and $C_{2,it-1} \sim U[0, 1]$, and assume the following data-generating process:

$$R_{it} = m_{t1}(C_{1,it-1}) + m_{t2}(C_{2,it-1}) + \varepsilon_{it},$$

where $E[\varepsilon_{it} \mid C_{1,it-1}, C_{2,it-1}] = 0$. Again, we can construct portfolios with a large spread in predicted returns based on the value of the conditional mean function, m_t . The idea is similar to constructing trading strategies based on the predicted values of a linear model,

$$R_{it} = \beta_0 + \beta_1 C_{1,it-1} + \beta_2 C_{2,it-1} + \varepsilon_{it}.$$

We will now, however, illustrate the potential pitfalls of the linear model and how a nonparametric model can alleviate them.

Assume the following return-generating process:

$$R_{it} = -0.2 + 0.3\sqrt{C_{1,it-1}} + 0.25C_{2,it-1}^2 + \varepsilon_{it}.$$

In this example, a regression of returns R_{it} on the characteristics $C_{1,it-1}$ and $C_{2,it-1}$ yields slope coefficients of around 0.25 in large samples. Therefore, for predictions, the linear model treats $C_{1,it-1}$ and $C_{2,it-1}$ almost identically, although they affect returns very differently.

We now compare the performance of the linear and nonparametric models for the “10-1” hedge portfolio. The table below shows monthly returns, standard deviations, and Sharpe ratios from a simulation for 2,000 stocks and 240 periods for both models.¹⁵

Predicted returns for the nonparametric model are slightly higher compared to the linear model, with almost identical standard deviations resulting in larger Sharpe ratios with the nonparametric method. Nevertheless, the linear model is a good approximation in this example, and the nonparametric method improves only marginally on the linear model.

	Linear	Nonparametric
Return	0.1704	0.1734
Std	0.2055	0.2054
Sharpe Ratio	0.8329	0.8480

Instead, now we study the following data-generating process:

$$R_{it} = -0.3 + 0.3\Phi((C_{1,it-1} - 0.1)/0.1) + 0.3\Phi((C_{2,it-1} - 0.9)/0.1) + \varepsilon_{it},$$

¹⁵The numbers in the table are averages of portfolio means, standard deviations, and Sharpe ratios of 1,000 simulated data sets. We use the first 120 periods to estimate the conditional mean functions as we explain below, and form portfolios for each remaining period based on the estimates. Therefore, the portfolio means in the table are based on 120 time periods.

where Φ denotes the standard normal cdf. Figure 1 plots the two functions, along with a parametric and a nonparametric estimate for a representative data set.

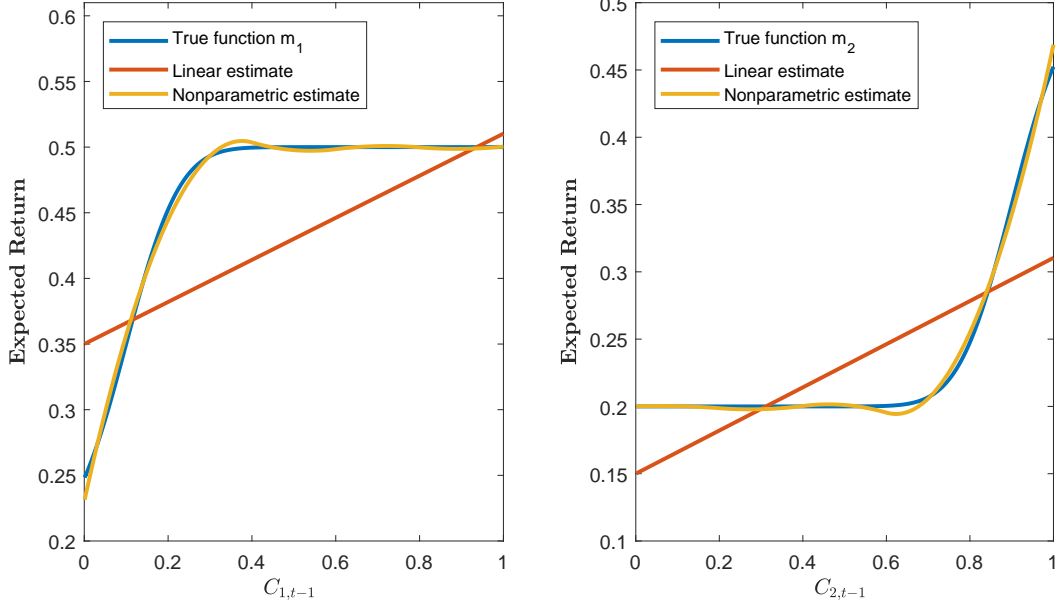


Figure 1: Regression functions and estimates

In this example, a regression of R_{it} on $C_{1,it-1}$ and $C_{2,it-1}$ yields two slope coefficients of around 0.15. Hence, as in the previous example, the linear model treats $C_{1,it-1}$ and $C_{2,it-1}$ identically for predictions.

	Linear	Nonparametric
Return	0.1154	0.1863
Std	0.1576	0.1576
Sharpe Ratio	0.7352	1.1876

The portfolio returns using the nonparametric model are now substantially higher compared to the linear model, with almost identical standard deviations, resulting in much larger Sharpe ratios. In this example, the linear model is a poor approximation of the true data-generating process even though returns increase monotonically in both characteristics.¹⁶

¹⁶We thank Seth Pruitt for raising this point.

Note that we do not know the true data-generating process, and the linear model may provide a good or poor approximation. Therefore, nonparametric methods are the natural choice.

In a last example, we want to discuss how the linear and nonparametric models treat nonlinear transformations of variables. This example helps us understand why a linear model might select more variables in empirical settings. Consider the following data-generating process:

$$R_{it} = C_{1,it-1} + C_{2,it-1} + \varepsilon_{it},$$

with $C_{2,it-1} = C_{1,it-1}^2$; that is, the second characteristic is just the square of the first characteristic. In the linear model, both characteristics are important to describe the conditional mean function, whereas in the nonparametric model, m_t is a function of $C_{1,it-1}$ only (or alternatively, $C_{2,it-1}$ only). In Section *D*, we consider model selection next to estimation, and these differences between the linear and the nonparametric model will play an important role.

C Normalization of Characteristics

We now describe a suitable normalization of the characteristics, which will allow us to map our nonparametric estimator directly to portfolio sorts. As before, define the conditional mean function m_t for S characteristics as

$$m_t(C_{1,it-1}, \dots, C_{S,it-1}) = E[R_{it} \mid C_{1,it-1}, \dots, C_{S,it-1}].$$

For each characteristic s , let $F_{s,t}(\cdot)$ be a known strictly monotone function and denote its inverse by $F_{s,t}^{-1}(\cdot)$. Define $\tilde{C}_{s,it-1} = F_{s,t}(C_{s,it-1})$ and

$$\tilde{m}_t(C_1, \dots, C_S) = m_t(F_{1,t}^{-1}(C_1), \dots, F_{S,t}^{-1}(C_S)).$$

Then,

$$m_t(C_{1,it-1}, \dots, C_{S,it-1}) = \tilde{m}_t(\tilde{C}_{1,it-1}, \dots, \tilde{C}_{S,it-1}).$$

Knowledge of the conditional mean function m_t is equivalent to knowing the transformed conditional mean function \tilde{m}_t . Moreover, using a transformation does not impose any additional restrictions and is therefore without loss of generality.

Instead of estimating m_t , we will estimate \tilde{m}_t for a rank transformation that has desirable properties and nicely maps to portfolio sorting. When we sort stocks into portfolios, we are typically not interested in the value of a characteristic in isolation, but rather in the rank of the characteristic in the cross section. Consider firm size. Size grows over time, and a firm with a market capitalization of USD 1 billion in the 1960s was considered a large firm, but today it is not. Our normalization considers the relative size in the cross section rather than the absolute size, similar to portfolio sorting.

Hence, we choose the rank transformation of $C_{s,it-1}$ such that the cross-sectional distribution of a given characteristic lies in the unit interval; that is, $C_{s,it-1} \in [0, 1]$. Specifically, let

$$F_{s,t}(C_{s,it-1}) = \frac{\text{rank}(C_{s,it-1})}{N_t + 1}.$$

Here, $\text{rank}(\min_{i=1, \dots, N_t} C_{s,it-1}) = 1$ and $\text{rank}(\max_{i=1, \dots, N_t} C_{s,it-1}) = N_t$. Therefore, the α quantile of $\tilde{C}_{s,it-1}$ is α . We use this particular transformation because portfolio sorting maps into our estimator as a special case.¹⁷

Although knowing m_t is equivalent to knowing \tilde{m}_t , in finite samples, the estimates of the two typically differ; that is,

$$\hat{m}_t(c_1, \dots, c_S) \neq \hat{\tilde{m}}_t(F_{1,t}^{-1}(c_1), \dots, F_{S,t}^{-1}(c_S)).$$

In simulations and in the empirical application, we found \tilde{m}_t yields better out-of-sample predictions than m_t . The transformed estimator appears to be less sensitive to outliers

¹⁷The general econometric theory we discuss in Section *D* (model selection, consistency, etc.) also applies to any other monotonic transformation or the non-transformed conditional mean function.

thanks to the rank transformation, which could be one reason for the superior out-of-sample performance.

In summary, the transformation does not impose any additional assumptions, directly relates to portfolio sorting, and works well in finite samples because it appears more robust to outliers.¹⁸

D Adaptive Group LASSO

We use a group LASSO procedure developed by Huang et al. (2010) for estimation and to select those characteristics that provide incremental information for expected returns, that is, for model selection. To recap, we are interested in modeling excess returns as a function of characteristics; that is,

$$R_{it} = \sum_{s=1}^S \tilde{m}_{ts}(\tilde{C}_{s,it-1}) + \varepsilon_{it}, \quad (3)$$

where $\tilde{m}_s(\cdot)$ are unknown functions and $\tilde{C}_{s,it-1}$ denotes the rank-transformed characteristic.

The idea of the group LASSO is to estimate the functions \tilde{m}_{ts} nonparametrically, while setting functions for a given characteristic to 0 if the characteristic does not help predict returns. Therefore, the procedure achieves model selection; that is, it discriminates between the functions \tilde{m}_{ts} , which are constant, and the functions that are not constant.¹⁹

In portfolio sorts, we approximate \tilde{m}_{ts} by a constant within each portfolio. We instead propose to estimate quadratic functions over parts of the normalized characteristic distribution. Let $0 = t_0 < t_1 < \dots < t_{L-1} < t_L = 1$ be a sequence of increasing numbers between 0 and 1 similar to portfolio breakpoints, and let \tilde{I}_l for $l = 1, \dots, L$ be a partition of the unit interval, that is, $\tilde{I}_l = [t_{l-1}, t_l)$ for $l = 1, \dots, L - 1$ and $\tilde{I}_L = [t_{L-1}, t_L]$. We

¹⁸Cochrane (2011) stresses the sensitivity of regressions to outliers. Our transformation is insensitive to outliers and nicely addresses his concern.

¹⁹The “adaptive” part indicates a two-step procedure, because the LASSO selects too many characteristics in the first step and is therefore not model-selection consistent unless restrictive conditions on the design matrix are satisfied (see Meinshausen and Bühlmann (2006) and Zou (2006) for an in-depth treatment of the LASSO in the linear model).

refer to t_0, \dots, t_{L-1} as knots and choose $t_l = l/L$ for all $l = 0, \dots, L - 1$ in our empirical application. Because we apply the rank transformation to the characteristics, the knots correspond to quantiles of the characteristic distribution and we can think of \tilde{I}_l as the l^{th} portfolio.

To estimate \tilde{m}_t , we use *quadratic* splines; that is, we approximate \tilde{m}_t as a quadratic function on each interval \tilde{I}_l . We choose these functions so that the endpoints are connected and \tilde{m}_t is differentiable on $[0, 1]$. We can approximate each \tilde{m}_{ts} by a series expansion with these properties, i.e.,

$$\tilde{m}_{ts}(\tilde{c}) \approx \sum_{k=1}^{L+2} \beta_{tsk} p_k(\tilde{c}), \quad (4)$$

where $p_k(c)$ are known basis functions.²⁰

The number of intervals L is a user-specified smoothing parameter, similar to the number of portfolios. As L increases, the precision of the approximation increases, but so does the number of parameters we have to estimate and hence the variance. Recall that portfolio sorts can be interpreted as approximating the conditional mean function as a constant function over L intervals. Our estimator is a smooth and more flexible estimator, but follows a similar idea (see again Figures 4 – 6).

We now discuss the two steps of the adaptive group LASSO. In the first step, we obtain estimates of the coefficients as

$$\tilde{\beta}_t = \arg \min_{b_{sk}: s=1, \dots, S; k=1, \dots, L+2} \sum_{i=1}^N \left(R_{it} - \sum_{s=1}^S \sum_{k=1}^{L+2} b_{sk} p_k(\tilde{C}_{s,it-1}) \right)^2 + \lambda_1 \sum_{s=1}^S \left(\sum_{k=1}^{L+2} b_{sk}^2 \right)^{\frac{1}{2}}, \quad (5)$$

where $\tilde{\beta}_t$ is an $(L + 2) \times S$ vector of estimates and λ_1 is a penalty parameter.

The first part of equation (5) is just the sum of the squared residuals as in ordinary least squares regressions; the second part is the LASSO group penalty function. Rather than penalizing individual coefficients, b_{sk} , the LASSO penalizes all coefficients associated with a given characteristic. Thus, we can set the point estimates of an entire expansion

²⁰In particular, $p_1(c) = 1$, $p_2(c) = c$, $p_3(c) = c^2$, and $p_k(c) = \max\{c - t_{k-3}, 0\}^2$ for $k = 4, \dots, L + 2$. See Chen (2007) for an overview of series estimation.

of \tilde{m}_t to 0 when a given characteristic does not provide incremental information for expected returns. Due to the penalty, the LASSO is applicable even when the number of characteristics is larger than the sample size. Yuan and Lin (2006) propose to choose λ_1 in a data-dependent way to minimize Bayesian Information Criterion (BIC) which we follow in our application.

However, as in a linear model, the first step of the LASSO selects too many characteristics unless restrictive conditions on the design matrix hold. Informally speaking, the LASSO selects all characteristics that predict returns, but also selects some characteristics that have no predictive power. A second step addresses this problem.

We first define the following weights:

$$w_{ts} = \begin{cases} \left(\sum_{k=1}^{L+2} \tilde{\beta}_{sk}^2 \right)^{-\frac{1}{2}} & \text{if } \sum_{k=1}^{L+2} \tilde{\beta}_{sk}^2 \neq 0 \\ \infty & \text{if } \sum_{k=1}^{L+2} \tilde{\beta}_{sk}^2 = 0. \end{cases} \quad (6)$$

Intuitively, these weights guarantee we do not select any characteristic in the second step that we did not select in the first step.

In the second step of the adaptive group LASSO, we solve

$$\check{\beta}_t = \arg \min_{b_{sk}: s=1, \dots, S; k=1, \dots, L+2} \sum_{i=1}^N \left(R_{it} - \sum_{s=1}^S \sum_{k=1}^{L+2} b_{sk} p_k(\tilde{C}_{s,it-1}) \right)^2 + \lambda_2 \sum_{s=1}^S \left(w_{ts} \sum_{k=1}^{L+2} b_{sk}^2 \right)^{\frac{1}{2}}. \quad (7)$$

We again follow Yuan and Lin (2006) and choose λ_2 to minimize BIC.

Huang et al. (2010) provide conditions under which $\check{\beta}_t$ is model-selection consistent; that is, it correctly selects the non-constant functions with probability approaching 1 as the sample size grows large. However, the estimators have unfavorable statistical properties because they are not oracle efficient. We re-estimate the model for the selected characteristics with OLS to address this problem.

Denote the estimated coefficients for characteristic s by $\hat{\beta}_{ts}$. The estimator of the

function \tilde{m}_{ts} is then

$$\hat{\tilde{m}}_{ts}(\tilde{c}) = \sum_{k=1}^{L+2} \hat{\beta}_{tsk} p_k(\tilde{c}).$$

If the cross section is sufficiently large, model selection and estimation could be performed period by period. Hence, the method allows for the importance of characteristics and the shape of the conditional mean function to vary over time. For example, some characteristics might lose their predictive power for expected returns over time. McLean and Pontiff (2016) show that for 97 return predictors, predictability decreases by 58% post publication. However, if the conditional mean function was time-invariant, pooling the data across time would lead to more precise estimates of the function and therefore more reliable predictions. In our empirical application in Section IV, we estimate our model over subsamples and also estimate rolling specifications to investigate the variation in the conditional mean function over time.

***E* Interpretation of the Conditional Mean Function**

In a nonparametric additive model, the locations of the functions are not identified. Consider the following example. Let α_s be S constants such that $\sum_{s=1}^S \alpha_s = 0$. Then,

$$\tilde{m}_t(\tilde{c}_1, \dots, \tilde{c}_S) = \sum_{s=1}^S \tilde{m}_{ts}(\tilde{c}_s) = \sum_{s=1}^S (\tilde{m}_{ts}(\tilde{c}_s) + \alpha_s).$$

Therefore, the summands of the transformed conditional mean function, \tilde{m}_s , are only identified up to a constant. The model-selection procedure, expected returns, and the portfolios we construct do not depend on these constants. However, the constants matter when we plot an estimate of the conditional mean function for one characteristic.

We report estimates of the functions using the common normalization that the functions integrate to 0, which is identified.

F Confidence Bands

We also report uniform confidence bands for the estimated functions in the plots later to gain some intuition for estimation uncertainty. Note that the set of characteristics the LASSO selects does not rely on these confidence bands. As explained above, we assume that

$$R_{it} = \sum_{s=1}^S \tilde{m}_{ts}(\tilde{C}_{s,it-1}) + \varepsilon_{it}.$$

In a linear model, we could report confidence intervals for the individual slope coefficients. Analogously, because we are mainly interested in the slopes of the functions \tilde{m}_{ts} and because the levels of the functions are not separately identified, we report estimates and confidence bands for the functions $\tilde{m}_{ts}(\tilde{c}_s) - \int \tilde{m}_{ts}(\tilde{c}_s)d\tilde{c}_s$. That is, we normalize the functions such that they are 0 on average. By inspecting the confidence bands we can then test hypotheses that do not depend on the levels of the functions, such as whether a constant function or a linear function is consistent with the data. However, the bands are not informative about the levels of the estimated functions similar to confidence intervals for slope coefficients in a linear model.

Recall that we approximate $\tilde{m}_{ts}(\tilde{c}_s)$ by $\sum_{k=1}^{L+2} \beta_{tsk} p_k(\tilde{c}_s)$ and estimate it by $\sum_{k=1}^{L+2} \hat{\beta}_{tsk} p_k(\tilde{c}_s)$. Let $\tilde{p}_k(\tilde{c}_s) = p_k(\tilde{c}_s) - \int p_k(\tilde{c}_s)d\tilde{c}_s$ be the normalized basis functions and let $\tilde{p}(\tilde{c}_s) = (\tilde{p}_1(\tilde{c}_s), \dots, \tilde{p}_{L+2}(\tilde{c}_s))'$ be the corresponding vector of basis functions. Next let Σ_{ts} be the $L+2 \times L+2$ covariance matrix of $\sqrt{n}(\hat{\beta}_{ts} - \beta_{ts})$. We define $\hat{\Sigma}_{ts}$ as the heteroscedasticity-consistent estimator of Σ_{ts} and define $\hat{\sigma}_{ts}(\tilde{c}_s) = \sqrt{\tilde{p}(\tilde{c}_s)' \hat{\Sigma}_{ts} \tilde{p}(\tilde{c}_s)}$, which is the estimated standard error of $\sum_{k=1}^{L+2} \hat{\beta}_{tsk} \tilde{p}_k(\tilde{c}_s)$. Just as in the linear model, $\hat{\sigma}_{ts}(\tilde{c})$ depends on which other characteristics are included in the model. For example, if two characteristics are highly correlated, the standard deviations of the estimated functions are typically high.

The uniform confidence band for $\tilde{m}_{ts}(\tilde{c}_s) - \int \tilde{m}_{ts}(\tilde{c}_s)d\tilde{c}_s$ is of the form

$$\left[\sum_{k=1}^{L+2} \hat{\beta}_{tsk} \tilde{p}_k(\tilde{c}_s) - d_{ts} \hat{\sigma}_{ts}(\tilde{c}_s), \sum_{k=1}^{L+2} \hat{\beta}_{tsk} \tilde{p}_k(\tilde{c}_s) + d_{ts} \hat{\sigma}_{ts}(\tilde{c}_s) \right],$$

where d_{ts} is a constant. Thus, the width of the confidence band is proportional to the standard deviation of the estimated function. To choose the constant, let $Z \sim N(0, \hat{\Sigma}_{ts})$ and let \hat{d}_{ts} be such that

$$P \left(\sup_{\tilde{c}_s \in [0,1]} \left| \frac{Z' \tilde{p}(\tilde{c}_s)}{\hat{\sigma}_{ts}(\tilde{c}_s)} \right| \leq \hat{d}_{ts} \mid \hat{\Sigma}_{ts} \right) = 1 - \alpha.$$

We can calculate the probability on the left-hand side using simulations.

Given consistent model selection and under the conditions in Belloni, Chernozhukov, Chetverikov, and Kato (2015), it follows that

$$P \left(\tilde{m}_{ts}(\tilde{c}_s) - \int \tilde{m}_{ts}(\tilde{c}_s) d\tilde{c}_s \in \left[\sum_{k=1}^{L+2} \hat{\beta}_{tsk} \tilde{p}_k(\tilde{c}_s) - \hat{d}_{ts} \hat{\sigma}_{ts}(\tilde{c}_s), \sum_{k=1}^{L+2} \hat{\beta}_{tsk} \tilde{p}_k(\tilde{c}_s) + \hat{d}_{ts} \hat{\sigma}_{ts}(\tilde{c}_s) \right] \forall \tilde{c}_s \in [0, 1] \right)$$

converges to $1 - \alpha$ as the sample size increases.

To better understand why these bands are useful, suppose that no linear function fits in the confidence band. Then, we can reject the null hypothesis that $\tilde{m}_{ts}(\tilde{c}_s) - \int \tilde{m}_{ts}(\tilde{c}_s) d\tilde{c}_s$ is linear at a significance level of $1 - \alpha$. But since $\tilde{m}_{ts}(\tilde{c}_s) - \int \tilde{m}_{ts}(\tilde{c}_s) d\tilde{c}_s$ is linear if and only if $\tilde{m}_{ts}(\tilde{c}_s)$ is linear, we can then also reject the null hypothesis that $\tilde{m}_{ts}(\tilde{c}_s)$ is linear. Similar, by inspecting the band we can test if $\tilde{m}_{ts}(\tilde{c}_s)$ is constant.

We want to stress that the selection of characteristics in the LASSO does not rely on these confidence bands and we report the confidence bands only to provide intuition and to summarize sampling uncertainty.

G Knot Selection

Theory tells us the number of interpolation points should grow as the sample size grows. Empirically, this statement is not too helpful in guiding our choices. We therefore document that the number and identify of characteristics is stable for reasonable variations in the number of knots (see Figure 11 which we discuss below).

IV Empirical Application

We now discuss the universe of characteristics we use in our empirical application and study which of the 62 characteristics provide incremental information for expected returns, using the adaptive group LASSO for selection and estimation.

A Data

Stock return data come from the Center for Research in Security Prices (CRSP) monthly stock file. We follow standard conventions and restrict the analysis to common stocks of firms incorporated in the United States trading on NYSE, Amex, or Nasdaq.

Balance-sheet data are from the Standard and Poor's Compustat database. We use balance-sheet data from the fiscal year ending in calendar year $t-1$ for estimation starting in June of year t until May of year $t+1$ predicting returns from July of year t until June of year $t+1$.

Table 1 provides an overview of the 62 characteristics we apply our method to. We group them into six categories: past return based predictors such as momentum (\mathbf{r}_{12-2}) and short-term reversal (\mathbf{r}_{2-1}), investment-related characteristics such as the annual percentage change in total assets (**Investment**) or the change in inventory over total assets (**IVC**), profitability-related characteristics such as gross profitability over the book-value of equity (**Prof**) or return on operating assets (**ROA**), intangibles such as operating accruals (**OA**) and tangibility (**Tan**), value-related characteristics such as the book-to-market ratio (**BEME**) and earnings-to-price (**E2P**), and trading frictions such as the average daily bid-ask spread (**Spread**) and standard unexplained volume (**SUV**). We follow Hou, Xue, and Zhang (2015) in the classification of characteristics.

To alleviate a potential survivorship bias due to backfilling, we require that a firm has at least two years of Compustat data. Our sample period is July 1965 until June 2014. Table 2 reports summary statistics for various firm characteristics and return predictors. We calculate all statistics annually and then average over time. On average we have 1.6

million observations in our analysis.

Section A.1 in the online appendix contains a detailed description of the characteristics, the construction, and the relevant references.

B Selected Characteristics and Their Influence

The purpose of this section is to show different applications of the adaptive group LASSO. We do not aim to exhaust all possible combinations of characteristics, sample periods, and firm sizes, or all possible applications but rather aim to provide some insights into the flexibility of the method.

Table 3 reports average annualized returns with standard errors in parentheses of 10 equally-weighted portfolios sorted on the characteristics we study. Most of the 62 characteristics have individually predictive power for expected returns in our sample period and result in large and statistically significant hedge portfolio returns and alphas relative to the Fama and French three-factor model (Table 4). Thirty-one sorts have annualized hedge returns of more than 5%, and 13 characteristics are even associated with excess returns of more than 10%. Thirty-six characteristics have a t-statistic above 2. Correcting for exposure to the Fama-French three-factor model has little impact on these findings. The vast majority of economic models, that is, the ICAPM (Merton (1973)) or consumption-based models, as surveyed in Cochrane (2007), suggest a low number of state variables can explain the cross section of returns. Therefore, all characteristics are unlikely to provide incremental information for expected returns.

To tackle the “multidimensional challenge,” we now estimate the adaptive group LASSO with 10, 15, 20, and 25 knots. The number of knots corresponds to the smoothing parameter we discuss in Section III. Ten knots corresponds to 11 portfolios in sorts.

We first show in a series of figures a few characteristics which provide large cross sectional return premia univariately. However, some of the characteristics do not provide incremental predictive power once we condition on other firm characteristics.

Figure 7 and Figure 8 plot estimates of the function $\tilde{m}(\tilde{C}_{it-1})$ for adjusted turnover

(**DTO**), idiosyncratic volatility (**Idio vol**), the change in inventories (**IVC**), and net operating assets (**NOA**). The left panels report the unconditional mean functions, whereas the right panels plot the associations between the characteristics and expected returns conditional on all selected characteristics.²¹

Stocks with low change in inventories, low net operating assets but high turnover and high idiosyncratic volatility have higher expected returns than stocks with high change in inventories, net operating assets, and low turnover or idiosyncratic volatility unconditionally. These results are consistent with our findings for portfolio sorts in Table 3. Portfolio sorts result in average annualized hedge portfolio returns of around 13%, 3%, 8%, and 9% for sorts on turnover, idiosyncratic volatility, change in inventories, and net operating profits, respectively. Change in inventories, net operating assets, and turnover have t-stats relative to the Fama-French three-factor model substantially larger than the threshold Harvey et al. (2016) suggest (see Table 4).

These characteristics, however, are correlated with other firm characteristics. We now want to understand whether they have marginal predictive power for expected returns conditional on other firm characteristics. We see in the right panels that the association of these characteristics with expected returns vanishes once we condition on other stock characteristics. The estimated conditional mean functions are now close to constant and do not vary a lot with the value of the characteristic. The constant conditional mean functions imply turnover, idiosyncratic volatility, the change in inventories, and net operating assets have no marginal predictive power for expected returns once we condition on other firm characteristics.

The examples of turnover, idiosyncratic volatility, the change in inventories, and net operating assets show the importance of conditioning on other characteristics to infer the predictive power of characteristics for expected returns. We now study this question systematically for 62 firm characteristics using the adaptive group LASSO.

Table 5 reports the selected characteristics of the nonparametric model for different

²¹We estimate the plots over the full sample and all firms using 20 interpolations points, see column (1) of Table 5.

numbers of knots, sets of firms, and sample periods. We see in column (1) that the baseline estimation for all stocks over the full sample period using 20 knots selects 13 out of the universe of 62 firm characteristics. The change in shares outstanding, investment, size, share turnover, the adjusted profit margin, short-term reversal, momentum, intermediate momentum, closeness to the 52 weeks high, the return on cash, standard unexplained volume, and total volatility all provide incremental information conditional on all other selected firm characteristics.

When we allow for a wider grid in column (2) with only 15 knots, we also select the book to market ratio, net operating assets, and long-term reversal. We instead select the same characteristics when we impose a finer grid and estimate the group LASSO with 25 interpolation points (see column (3)).

Figure 11 shows how the number of characteristics we select varies with the number of interpolation points. We see the number of selected characteristics is stable around 20 interpolation points and varies between 16 when we use only 10 knots and 12 when we use 30 interpolation points.

We estimate the nonparametric model only on large stocks above the 10%- and 20%-size quantile of NYSE stocks in columns (4) to (6), reducing the sample size from more than 1.6 million observations to around 760,000.

The change in shares outstanding, investment, short-term reversal, momentum, intermediate momentum, the return on cash, standard unexplained volume, and total volatility are significant return predictors both for a sample of firms above the 10%-size threshold and the sample of all stocks in column (1), whereas the sales to price ratio becomes a significant return predictor. For firms above the 20%-size threshold of NYSE firms, we also see momentum losing predictive power, but returns over the last six months becoming a significant return predictor. When we impose a coarser grid with only 10 knots for a sample of firms above the 20%-size threshold of NYSE firms in column (6), we see closeness to the 52 weeks high and long-term reversal regaining predictive power, whereas

standard momentum driving out intermediate momentum.²²

Columns (7) and (8) split our sample in half and re-estimate our benchmark nonparametric model in both sub-samples separately to see whether the importance of characteristics for predicted returns varies over time. Only 11 characteristics have predictive power for expected returns in the sample until 1990, whereas 14 characteristics provide incremental predictive power in the second half of the sample until 2014.

The change in shares outstanding, short-term reversal, momentum, the closeness to the previous 52-week high, the return on cash, standardized unexplained volume, and total volatility are the most consistent return predictors across different sample periods, number of interpolation points, and sets of firms.

Figure 9 and Figure 10 plot the conditional and unconditional mean functions for short-term reversal, the closeness to the previous 52-week high, size, and standard unexplained volume. We see in Figure 9 both for reversal and closeness to the 52 weeks high a monotonic association between the characteristic distribution and expected returns both unconditionally and once we condition on other characteristics in the right panel. Size matters for returns for all firms in the right panel of Figure 10 and the conditional association is more pronounced than the unconditional relationship in the left panel. This finding is reminiscent of Asness, Frazzini, Israel, Moskowitz, and Pedersen (2017), who argue “size matters, if you control your junk.” We see in the lower panels, standardized unexplained volume is both unconditionally and conditionally positively associated with expected returns.

This section shows that many of the univariately significant return predictors do not provide incremental predictive power for expected returns once we condition on other stock characteristics.

²²The number of knots increases with the sample size. The penalty function instead increases in the number of knots, which is why we select fewer characteristics with more knots.

C Interactions of Firm Characteristics and Selection in the Linear Model

We discuss in Section III the impact of estimating our model fully nonparametrically on the rate of convergence of the estimator, the so-called curse of dimensionality, and that imposing an additive structure on the conditional mean function offers a solution. The additive structure implies the effect of one characteristic on returns is independent of other characteristics once we condition on them, just as in any multivariate regression. We argued above that creating pseudo characteristics, which are themselves interactions of firm characteristics, offers a possible solution to the additive structure and we now show a simple application. Specifically, we interact each of the 61 firm characteristics other than firm size with firm size for a total of 123 firm characteristics. For example, one of the new characteristics is $LME \times BEME$, firm size interacted with the book-to-market ratio.

Table 6 tabulates the results. Instead of selecting 13 characteristics as in the baseline (see column (1) of Table 5), we now select a total of 25 out of the 123 firm characteristics. The model selects 10 of the 13 characteristics it already selected in the baseline. Interestingly, return on cash, which is one of the most consistent return predictors in our baseline table across specifications, is no longer a significant return predictor once we allow for interactions with firm size. Contrary to our baseline, we also no longer select firm size in levels in the model with interactions. Among the 25 characteristics we select in the new model with interactions, almost half are interactions with firm size.

We see in columns (2) to (4) of Table 6 that interactions with firm size are mainly important among small stocks. Once we focus on stocks above the 10%- and 20%-size quantile of NYSE stocks only short-term reversal, momentum, and return over the previous six months interact with firms size and provide incremental information for expected returns.

The previous results are reassuring for previous research which relied on multivariate

regressions to dissect anomalies, especially for papers which tested models on different parts of the firm-size distribution.

Table 7 estimates a linear model with the adaptive LASSO to gain some intuition for the importance of nonlinearities. Specifically, we endow the linear model with the same machinery we use for our nonparametric model and report how many and which characteristics the linear model selects in-sample.

When we compare column (1) in Table 5 for the nonparametric model with column (1) in Table 7 for the linear model, we see the linear model selects nine more characteristics in-sample for a total of 24. Interestingly, the linear model selects eight of the 13 characteristics the nonparametric model selects but, e.g., also selects the book-to-market ratio, the earnings-to-price ratio, or the average bid-ask spread over the previous month.

So far, we used raw characteristics for the linear model, whereas we applied the rank transformation to characteristics we discuss above in the nonparametric model. We now estimate a linear model with the adaptive LASSO to see whether the use of raw characteristics might explain the larger number of characteristics we select in the linear model. We see in column (2) of Table 7 that estimating a linear model on rank-transformed characteristics results in an even larger number of characteristics which seem to provide incremental information for expected returns.

Table 7 shows nonlinearities between characteristics and returns might result in a larger number of selected characteristics in a linear model. The selection of more characteristics for the linear model is something which we will see again below when we compare the out-of-sample performance of our nonparametric model with the linear model.

***D* Time Variation in Return Predictors**

McLean and Pontiff (2016) document substantial variation over time in the predictive power of many characteristics for expected returns. Figure 12 to Figure 15 show the conditional mean function for a subset of characteristics for our baseline nonparametric

model for all stocks and ten knots over time. We perform model selection on the first 10 years of data. We then fix the selected characteristics and estimate the nonparametric model on a rolling basis using 10 years of data.

We see in the top panel of Figure 12 that the conditional mean function is non-constant throughout the sample period for lagged market cap. Small firms have higher expected returns compared to large firms, conditional on all other selected return predictors. Interestingly, the size effect seems largest during the end of our sample period, contrary to conventional wisdom (see Asness et al. (2017) for a related finding). The bottom panel shows that firms with higher profit margin relative to other firms within the same industry have higher expected returns conditional on other firm characteristics, contrary to the unconditional association (see Table 3).

We see in the top panel of Figure 13 that intermediate momentum has a significant conditional association with expected returns throughout the sample period. Interestingly, we do not observe a crash for intermediate momentum, because intermediate losers have always lower returns compared to intermediate winners. In the bottom panel, we see momentum conditional on other firm characteristics was a particular strong return predictor in the middle sample but lost part of the predictive power for expected returns in the more recent period because of high returns of past losers, consistent with findings in Daniel and Moskowitz (2016).

Figure 14 shows the effect of short-term reversal on expected returns has been strongest in the early sample period because recent losers used to appreciate more than they currently do. The bottom panel shows the association of the change in shares outstanding and returns has been almost flat until the early 1990s and only afterwards did stocks with the highest level of issuances earn substantially higher returns than all other stocks conditional on other firm characteristics.

Figure 15 plots the conditional mean function for turnover and standard unexplained volume over time. Both high unexplained volume and turnover are associated with high returns but whereas the effect of unexplained volume conditional on other characteristics

appears stronger early on, the predictive power of turnover seems stronger in the second part of the sample.

We see those figures as one application of our proposed method for the cross section of stock returns and do not want to put too much weight on the *eyeball econometrics* we performed in the previous section. Ultimately, we cannot tell causal stories and the results might change when we condition on additional firm characteristics.

***E* Out-of-Sample Performance and Model Comparison**

We argued in Section II that the nonparametric method we propose overcomes potential shortcomings of more traditional methods, and show potential advantages of the adaptive group LASSO in simulations.

We now want to compare the performance of the nonparametric model with the linear model out-of-sample. The out-of-sample context ensures that in-sample overfit does not explain a potentially superior performance of the nonparametric model.

We estimate the nonparametric model for a period from 1965 to 1990 and carry out model selection with the adaptive group LASSO with ten knots, but also use the adaptive LASSO for model selection in the linear model over the same sample period, that is, we give both the nonparametric model and the linear model the same machinery and, hence, equal footing. We then use 10 years of data to estimate the model on the selected characteristics. In the next month, we take the selected characteristics and predict one-month-ahead returns and construct a hedge portfolio going long stocks with the highest predicted returns and shorting stocks with the lowest predicted returns. We then roll the estimation and prediction period forward by one month and repeat the procedure until the end of the sample.

Specifically, in our first out-of-sample predictions, we use return data from January 1981 until December 1990 and characteristics data from December 1980 until November 1990 to get estimates $\hat{\beta}$.²³ We then take the estimated coefficients and characteristics

²³To be more precise, for returns until June 1981, many of the balance-sheet variables will be from the fiscal year ending in 1979.

data of December 1990 to predict returns for January 1991 and form two portfolios for each method. We buy the stocks with the highest predicted returns and sell the stocks with the lowest predicted returns. We then move our estimation sample forward by one month from February 1981 until January 1991, get new estimates $\hat{\beta}$, and predict returns for February 1991.

Table 8 reports the out-of-sample Sharpe ratios for both the nonparametric and linear models for different sample periods and firms when we go long the 50% of firms with highest predicted returns and short the 50% of firms with lowest predicted return. For a sample from 1991 to 2014 and ten knots, the nonparametric model generates an out-of-sample Sharpe ratio for an equally-weighted hedge portfolio of 2.83 compared to 0.75 for the linear model (compare columns (1) and (2)). The linear model selects 30 characteristics in-sample compared to only 11 for the nonparametric model, but performs worse out-of-sample.²⁴

Nonlinearities are important. We find a substantial increase in out-of-sample Sharpe ratios relative to the Sharpe ratio of the linear model when we employ the nonparametric model for prediction on the 30 characteristics the linear model selects (see column (3)).

The linear model appears to overfit the data in-sample. When we use the 11 characteristics we select with the nonparametric model, we find the Sharpe ratio for the linear model is identical to the one we find when we use the 30 characteristics the linear model selects (see column (4)). But even with the same set of 11 characteristics, we find the Sharpe ratio for the linear model is still substantially smaller compared to the Sharpe ratio of the nonparametric model. In line with our findings above, it appears the linear model selects many characteristics in-sample that do not provide incremental information for return prediction, but also that nonlinearities are important.

Columns (5) and (6) focus on a longer out-of-sample period starting in 1973 to be comparable to results in the literature.²⁵ Results are very similar to when we split the

²⁴The linear model might be misspecified and therefore select more variables (see discussion and simulation results in Section III).

²⁵The linear model we estimate and the results are similar to Lewellen (2015). Specifically, he finds for a linear model out-of-sample Sharpe ratios of 1.65.

sample in half.

We see in columns (7) to (10) that Sharpe ratios drop substantially for both models when we exclude firms below the 10th or 20th percentile of NYSE stocks. Lewellen (2015) also finds Sharpe ratios for an equally-weighted hedge portfolio that are lower by 50% when he excludes “all but tiny stocks.” The Sharpe ratios are still close to 1 for the nonparametric model for both sets of stocks, whereas Sharpe ratios are only around 0.10 for the linear model.

So far, we invested in all stocks. Table A.1 in the online appendix reports qualitatively and quantitatively very similar results when we only go long in the 10% of stocks with highest predicted returns and short the 10% of stocks with lowest predicted returns.

Results are also similar when we perform rolling selection. So far, we performed model selection once, fixed the selected characteristics for the nonparametric and linear model, and performed rolling model estimation and return prediction. In a robustness check, we performed annual model selection on a constant sample size and an increasing sample size in addition to rolling estimation and prediction, but found results which are very similar to our baseline results discussed above.

V Conclusion

We propose a nonparametric method to tackle the challenge posed by Cochrane (2011) in his presidential address, namely, which firm characteristics provide incremental information for expected returns. We use the adaptive group LASSO to select important return predictors and to estimate the model.

We document the properties of our framework in three applications: (i) Which characteristics have incremental forecasting power for expected returns? (ii) Does the predictive power of characteristics vary over time? (iii) How does the nonparametric model compare to a linear model out-of-sample?

Our results are as follows: (i) Out of 62 characteristics, only nine to 16 provide

incremental information depending on the number of interpolation points (similar to the number of portfolios in portfolio sorts), sample period, and universe of stocks (large versus small stocks). (ii) Substantial time variation is present in the predictive power of characteristics. (iii) The nonparametric model selects fewer characteristics than the linear model in-sample and has a Sharpe ratio that is larger by a factor of 3 out-of-sample.

We see our paper as a starting point only and pose the following questions for future research. Are the characteristics we identify related to factor exposures? How many factors are important? Can we achieve a dimension reduction and identify K factors that can summarize the N independent dimensions of expected returns with $K \ll N$ similar to Fama and French (1993) and Fama and French (1996)?

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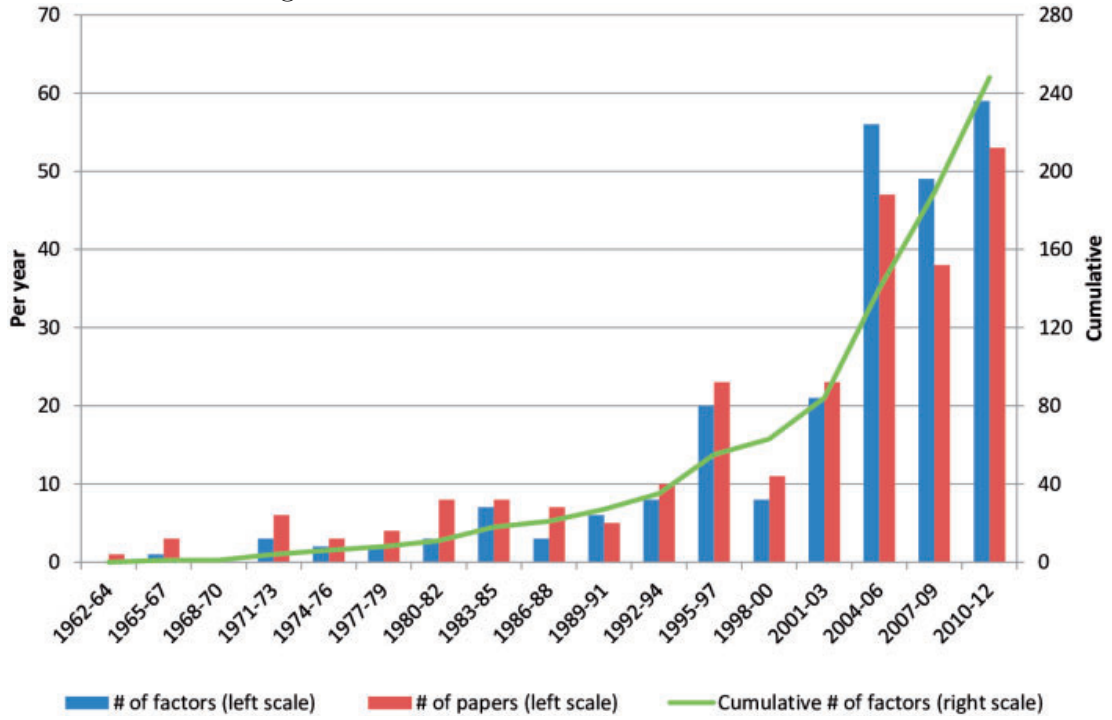
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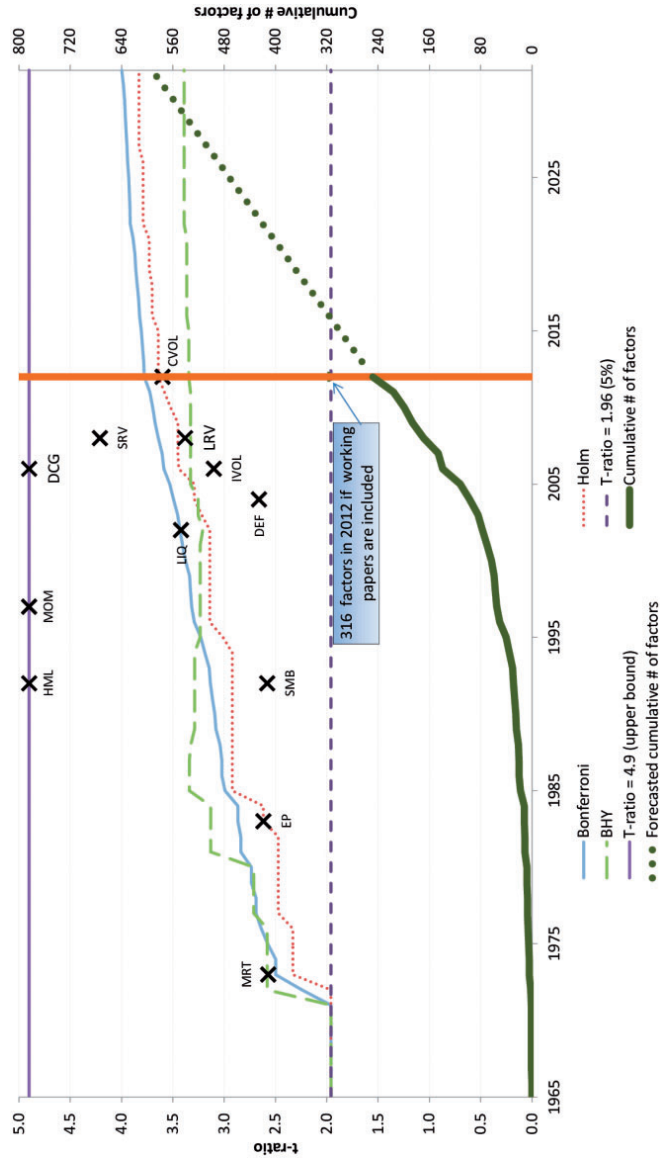
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Figure 2: Numbers of Published Factors



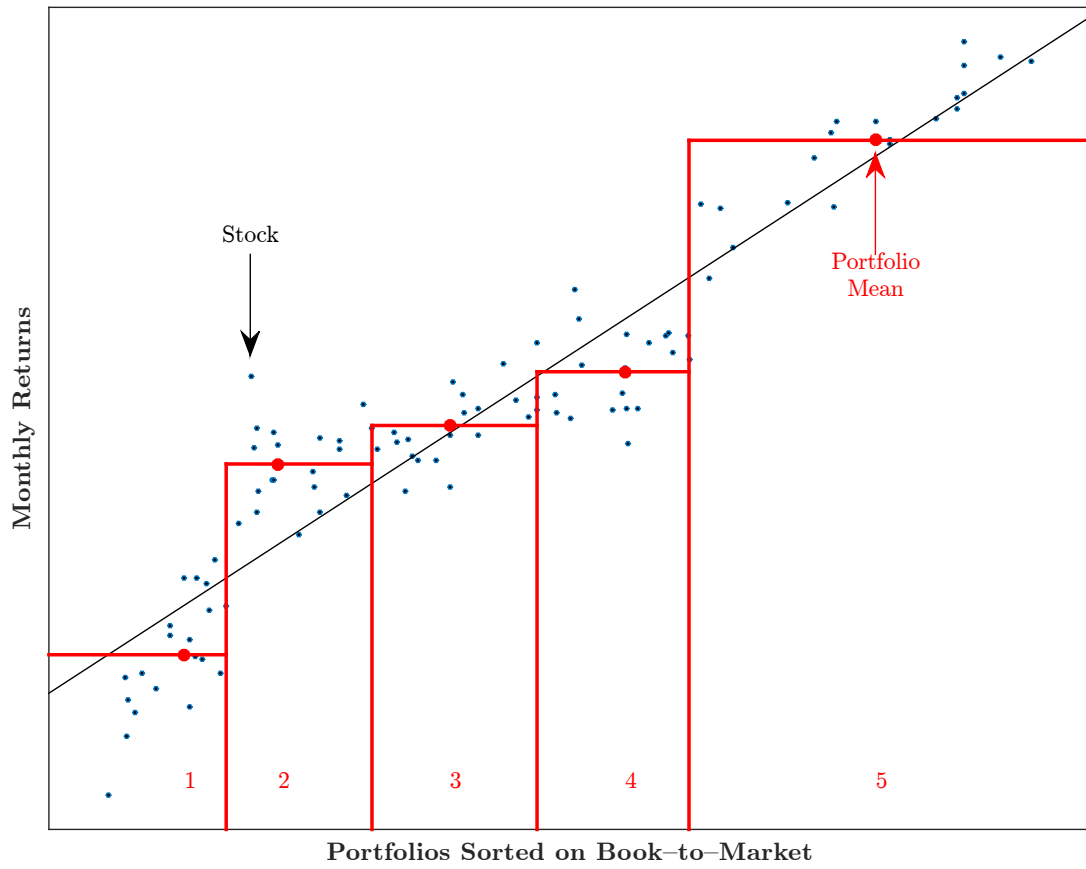
Number of discovered factors over time. Source: Figure 2 of Harvey, Liu, and Zhu (2016).

Figure 3: Suggested t-stats Adjustment



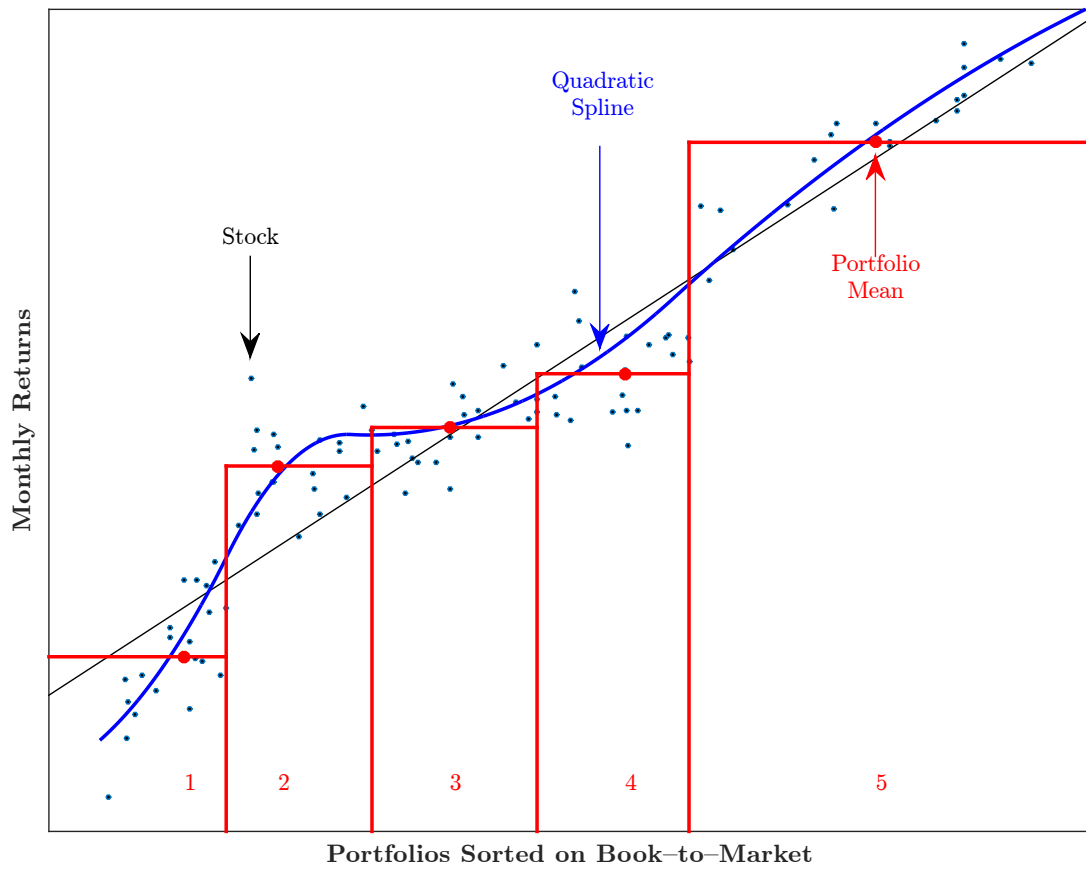
Recommendation to adjust t-statistics for multiple testing problem. Source: Figure 3 of Harvey, Liu, and Zhu (2016).

Figure 4: 5 Portfolios Sorted on Book-to-Market



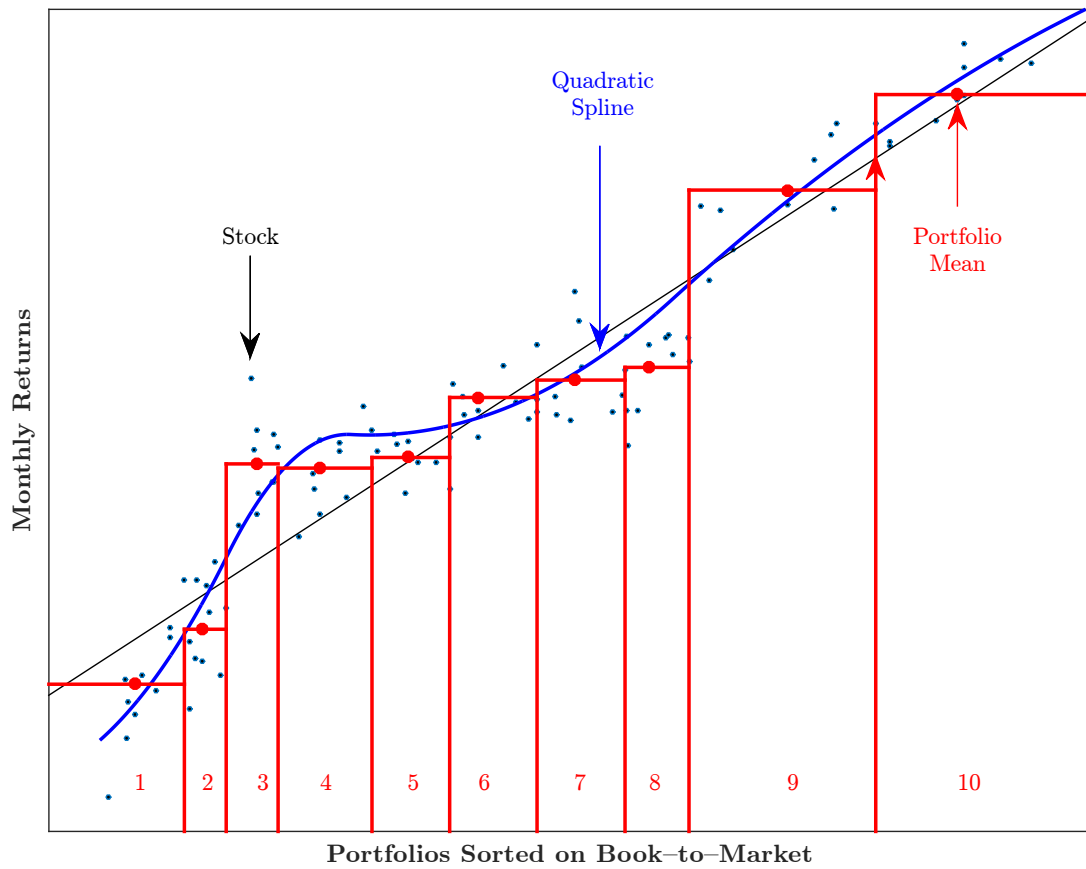
This figure plots returns on the y-axis against the book-to-market ratio on the x-axis as well as portfolio mean returns for simulated data.

Figure 5: 5 Portfolios Sorted on Book-to-Market and Nonparametric Estimator



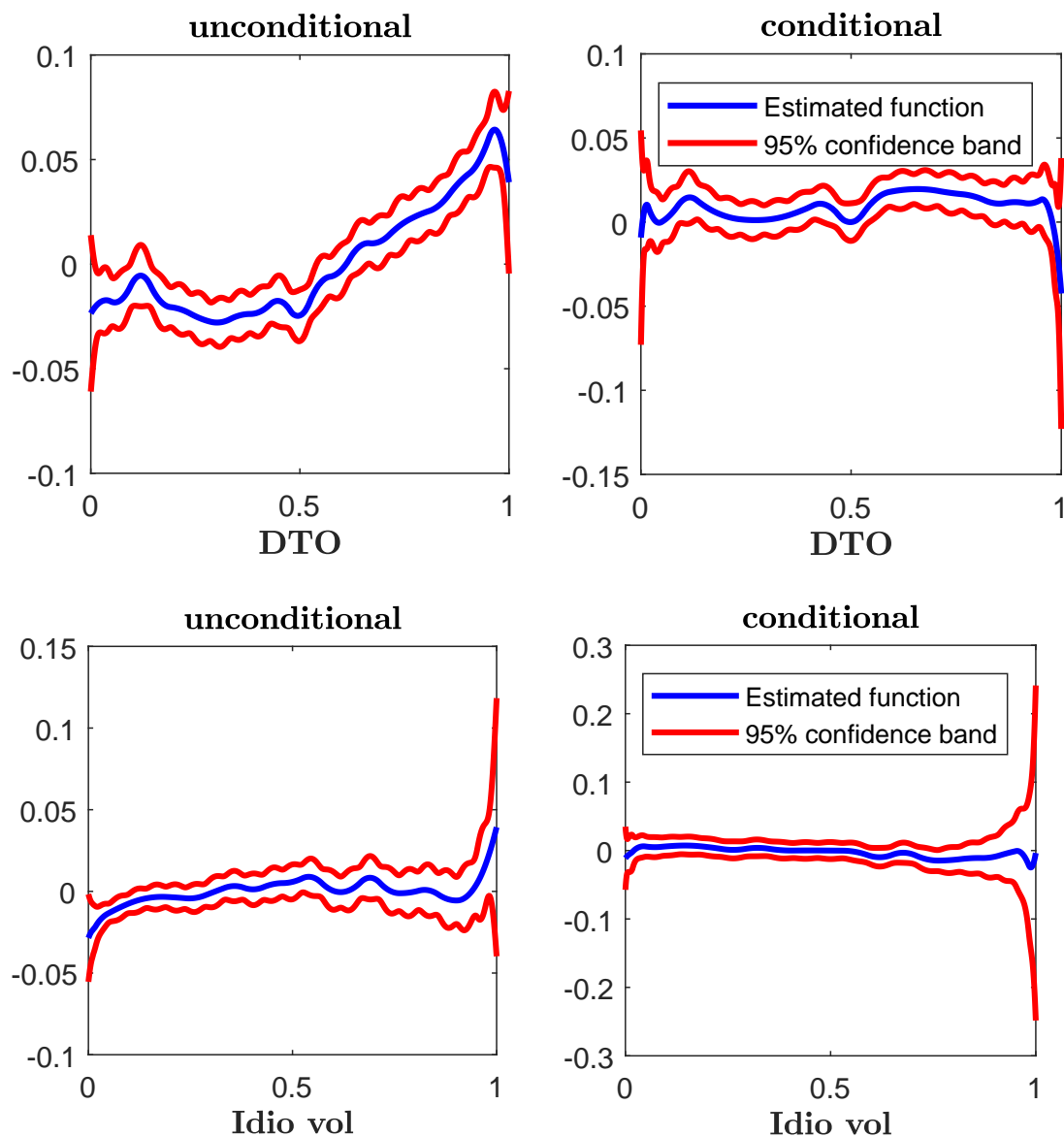
This figure plots returns on the y-axis against the book-to-market ratio on the x-axis as well as portfolio mean returns and a nonparametric conditional mean function for simulated data.

Figure 6: 10 Portfolios sorted on Book-to-Market and Nonparametric Estimator



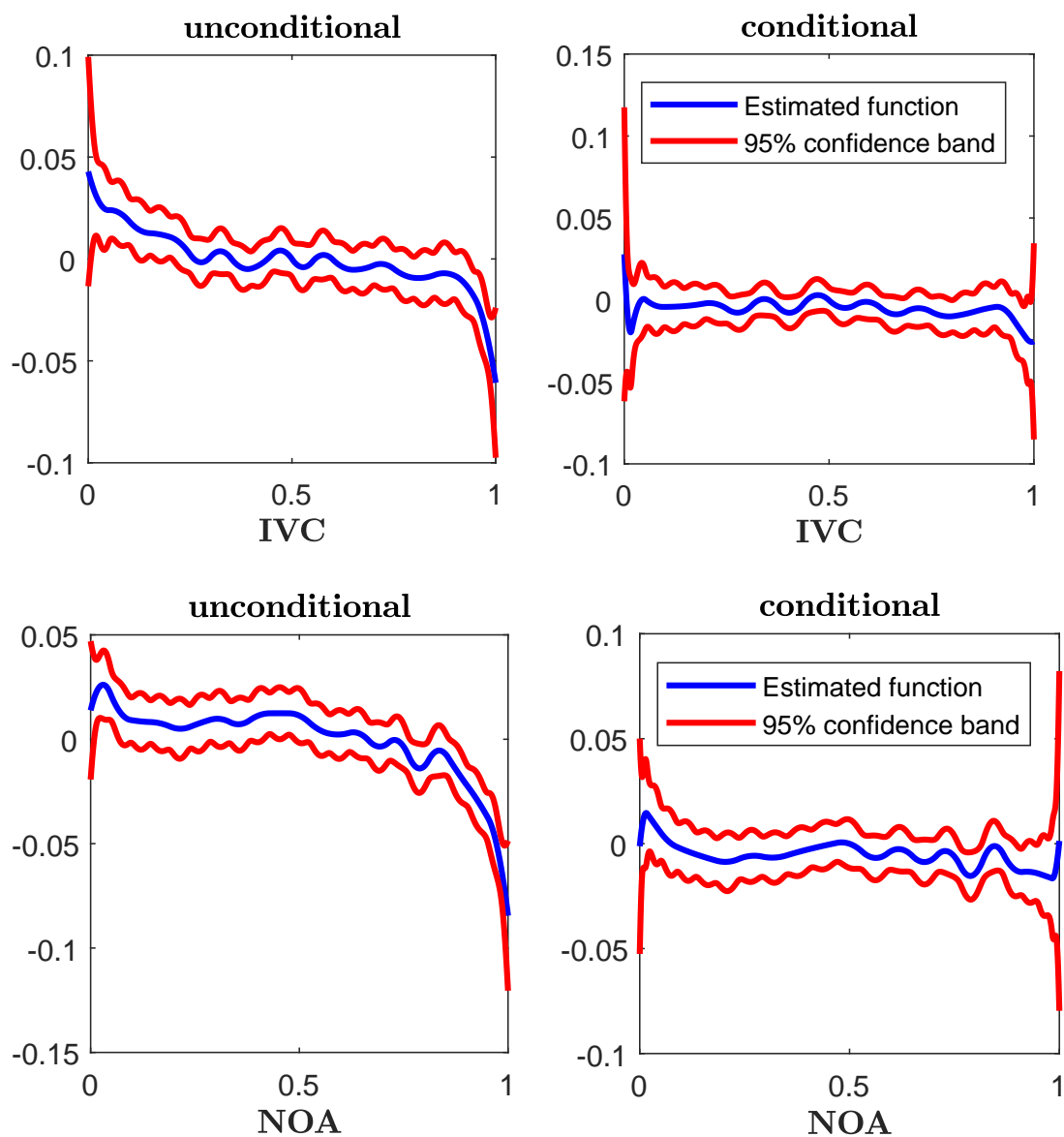
This figure plots returns on the y-axis against the book-to-market ratio on the x-axis as well as portfolio mean returns and a nonparametric conditional mean function for simulated data.

Figure 7: Unconditional and Conditional Mean Function: Adjusted Turnover (DTO) and Idiosyncratic Volatility (Idio vol)



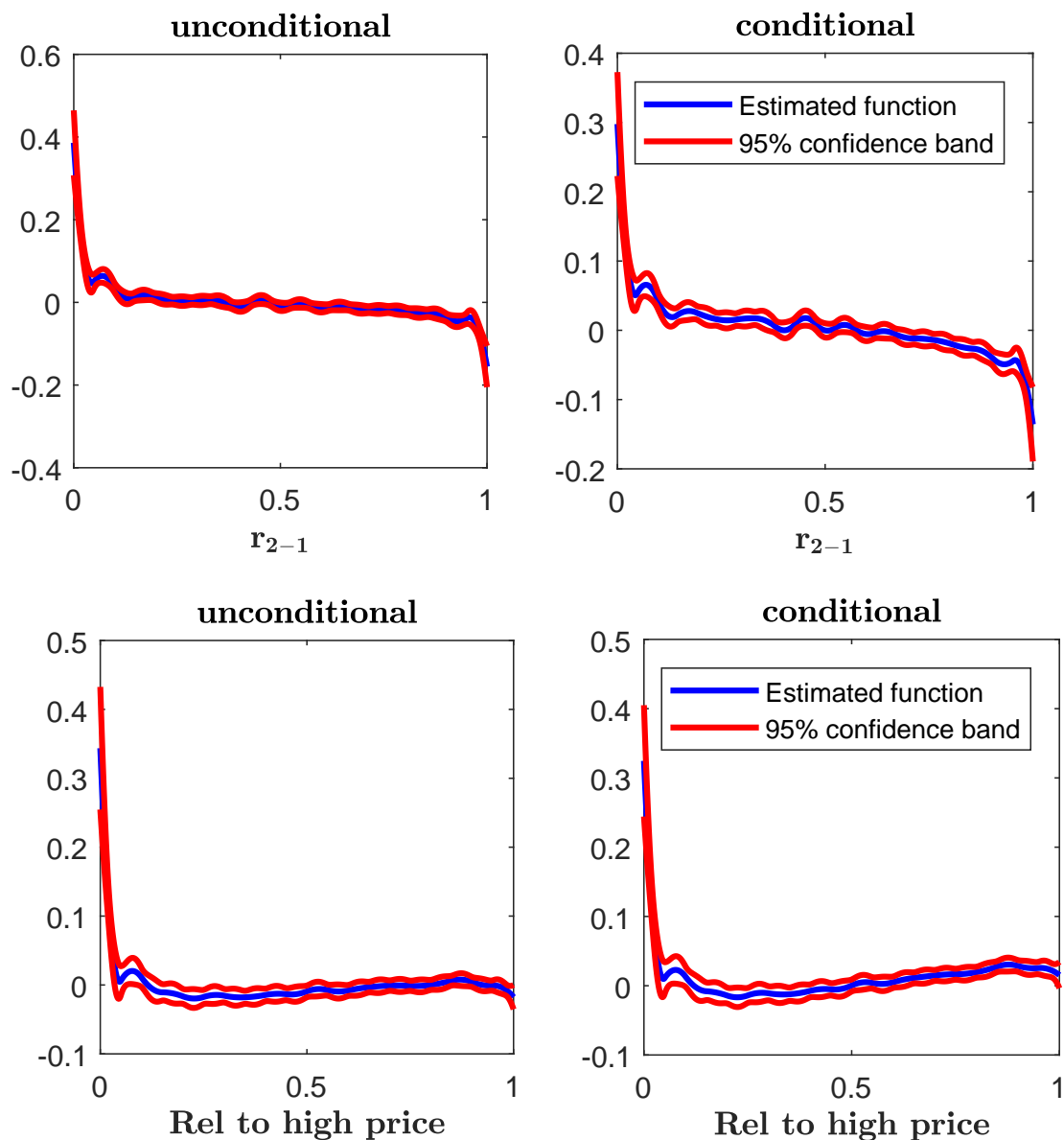
Effect of normalized adjusted turnover (DTO) and idiosyncratic volatility (Idio vol) on average returns (see equation (3)). The left panels report unconditional associations between a characteristic and returns, and the right panels report associations conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 8: Unconditional and Conditional Mean Function: Change in Inventories (IVC) and Net Operating Assets (NOA)



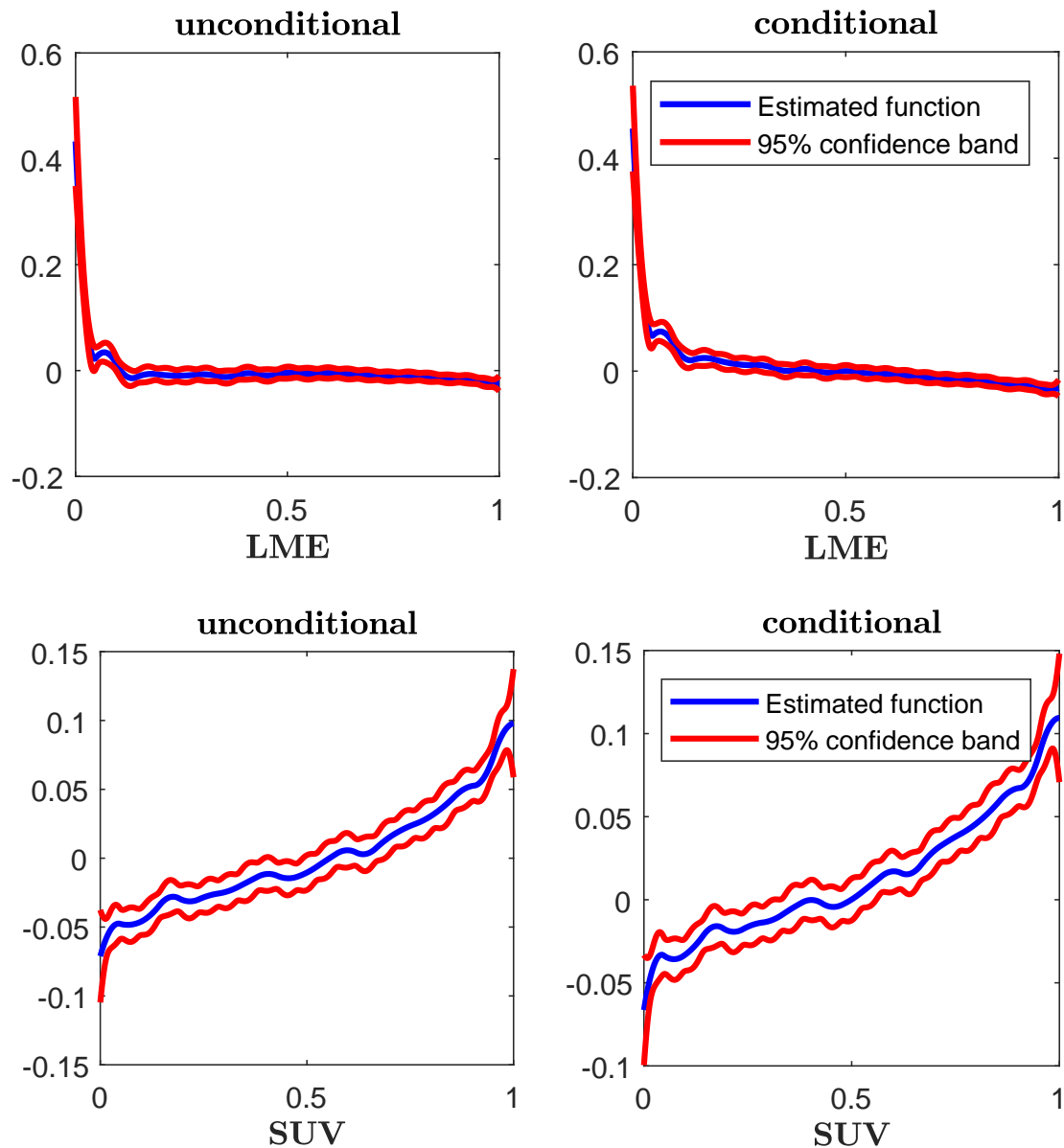
Effect of normalized change in inventories (IVC) and net operating assets (NOA) on average returns (see equation (3)). The left panels report unconditional associations between a characteristic and returns and the right panels report associations conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 9: Unconditional and Conditional Mean Function: Short-Term Reversal (r_{2-1}) and Closeness to 52 week's High (Rel to high price)



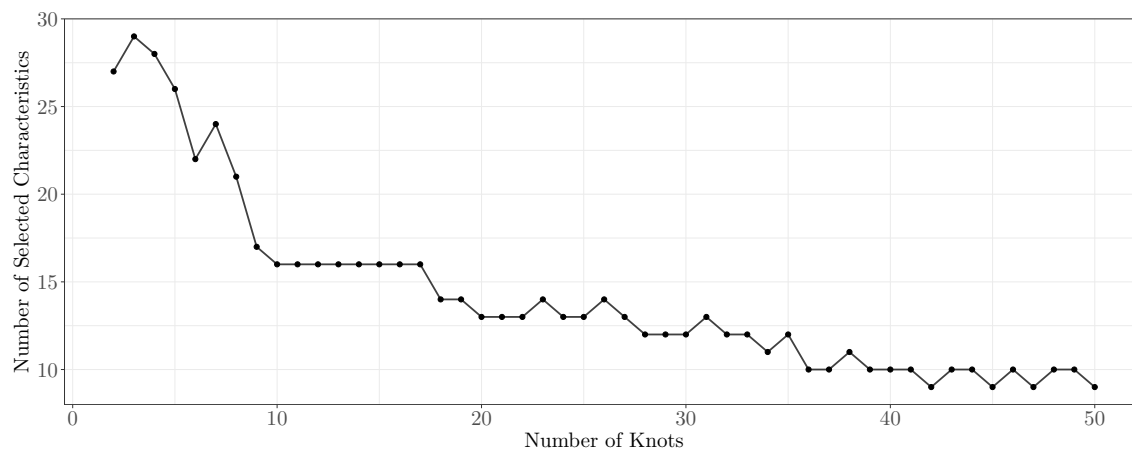
*Effect of normalized short-term reversal (r_{2-1}) and closeness to 52 week's high (**Rel to high price**) on average returns (see equation (3)). The left panels report unconditional associations between a characteristic and returns, and the right panels report associations conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.*

Figure 10: Unconditional and Conditional Mean Function: Size (LME) and Standard Unexplained Volume (SUV)



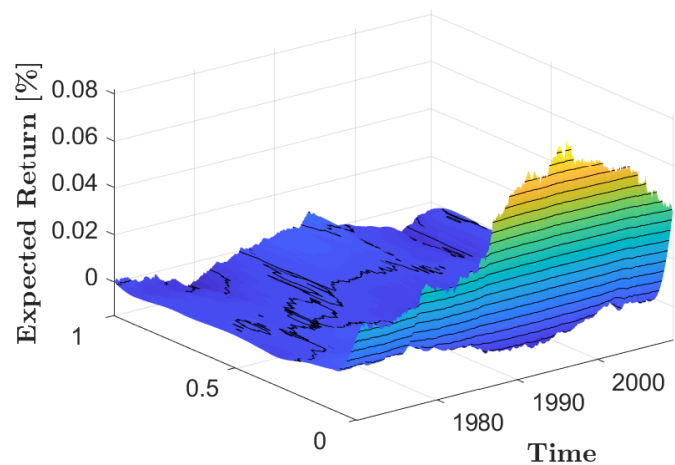
Effect of normalized size (LME) and standard unexplained volume (SUV) on average returns (see equation (3)). The left panels report unconditional associations between a characteristic and returns, and the right panels report associations conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 11: Number of Selected Characteristics versus Number of Interpolation Points

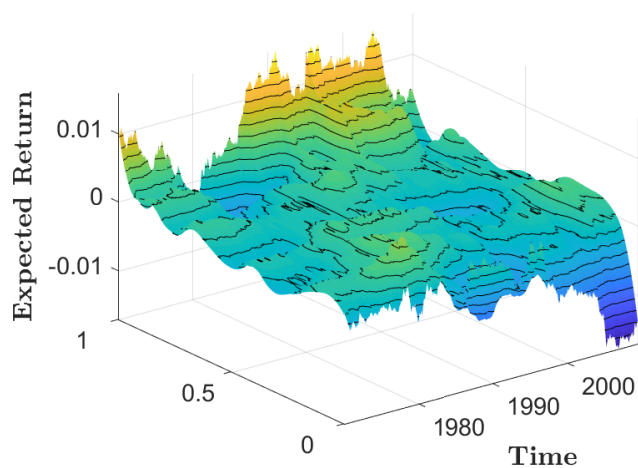


This figure plots the number of firms characteristics we select against the number of interpolation points in our baseline analysis. We use the adaptive group LASSO to select significant return predictors out of a universe of 63 characteristics during a sample period from 1965 to 2014. We detail the method in Section III.

Figure 12: Time-varying Conditional Mean Function: Size (LME) and adjusted Profit Margin (PM_adj)



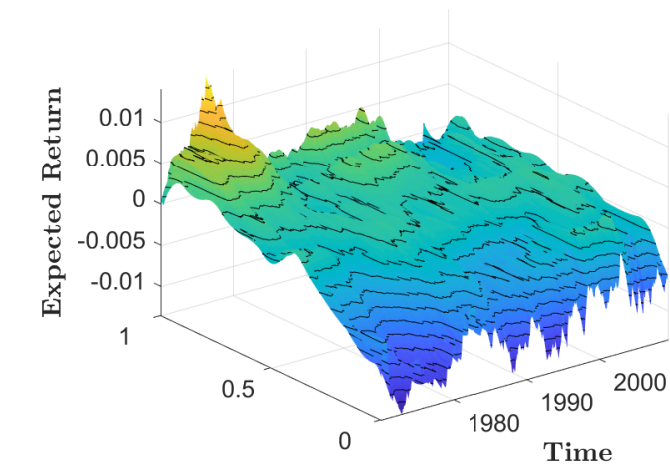
Market Cap (normalized)



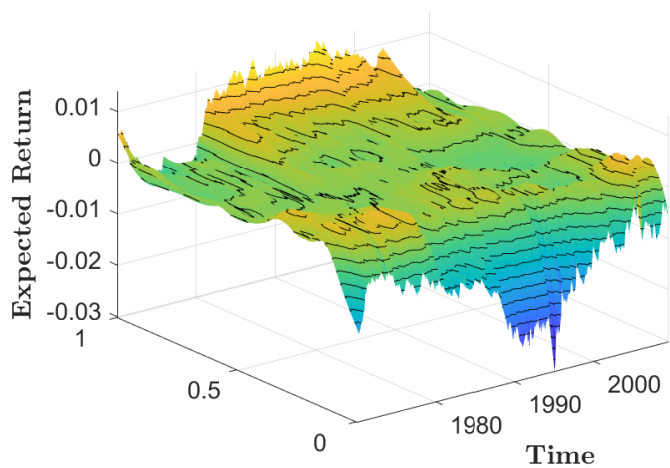
Adjusted Profit Margin (normalized)

Effect of normalized size (LME) and adjusted profit margin (PM_adj) on average returns over time (see equation (3)) conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 13: Time-varying Conditional Mean Function: Intermediate Momentum (r_{12-7}) and Standard Momentum (r_{12-2})



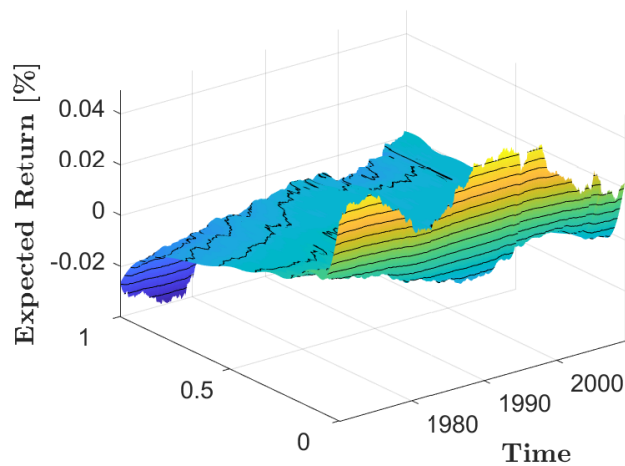
Intermediate Momentum (normalized)



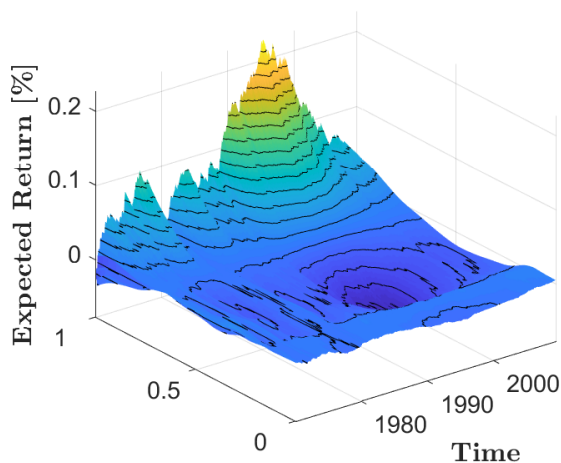
Momentum (normalized)

Effect of normalized intermediate momentum (r_{12-7}) and standard momentum (r_{12-2}) on average returns over time (see equation (3)) conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 14: **Time-varying Conditional Mean Function: Short-Term Reversal (r_{2-1}) and Change in Shares Outstanding (ΔSh_{rout})**



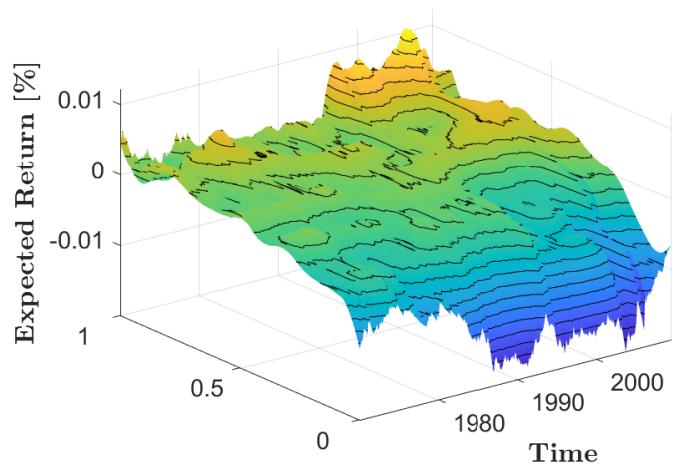
One month lagged return (normalized)



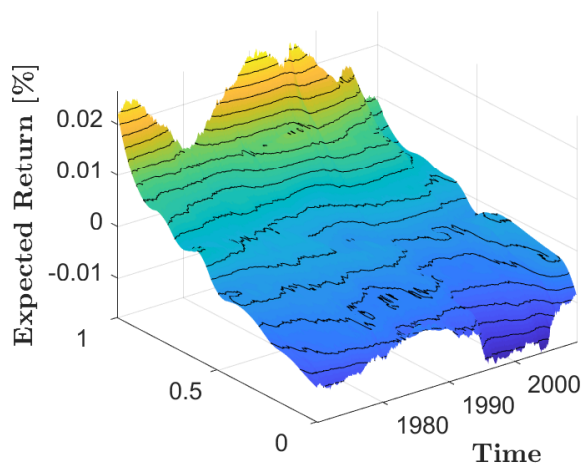
Change in Shares Outstanding (normalized)

Effect of normalized short-term reversal (r_{36-13}) and the percentage change in shares outstanding (ΔSh_{rout}) on average returns over time (see equation (3)) conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Figure 15: Time-varying Conditional Mean Function: Turnover ($Lturnover$) and Standard Unexplained Volume (SUV)



Turnover (normalized)



Standard Unexplained Volume (normalized)

Effect of normalized turnover ($Lturnover$) and standard unexplained volume (SUV) on average returns over time (see equation (3)) conditional on all other selected characteristics. The sample period is January 1965 to June 2014. See Section A.1 in the online appendix for variable definitions.

Table 2: Descriptive Statistics for Firm Characteristics

This table reports average returns, medians, and time series standard deviations for the firm characteristics discussed in Section A.1 of the online appendix. Frequency is the frequency at which the firm characteristics varies. *m* is monthly and *y* is yearly. The sample period is January 1965 to June 2014.

	Mean	Median	Std	Freq		Mean	Median	Std	Freq
Past-returns:					Value:				
r_{2-1}	0.01	0.00	(0.13)	m	A2ME	3.04	1.62	(5.75)	y
r_{6-2}	0.06	0.03	(0.31)	m	BEME	0.94	0.77	(0.80)	y
r_{12-2}	0.14	0.07	(0.51)	m	BEME _{adj}	0.01	-0.13	(0.77)	m
r_{12-7}	0.08	0.04	(0.34)	m	C	0.13	0.07	(0.15)	y
r_{36-13}	0.35	0.17	(0.96)	m	C2D	0.17	0.17	(1.26)	y
Investment:					Δ SO	0.03	0.00	(0.12)	y
Investment	0.14	0.08	(0.44)	y	Debt2P	0.86	0.34	(2.37)	y
Δ CEQ	0.18	0.06	(2.00)	y	E2P	0.01	0.07	(0.36)	y
Δ PI2A	0.09	0.06	(0.22)	y	Free CF	-0.23	0.05	(9.70)	y
Δ Shrout	0.01	0.00	(0.10)	m	LDP	0.02	0.01	(0.05)	m
IVC	0.02	0.01	(0.06)	y	NOP	0.01	0.01	(0.12)	y
NOA	0.67	0.67	(0.38)	y	O2P	0.03	0.02	(0.13)	y
Profitability:					Q	1.63	1.20	(1.47)	y
ATO	2.52	1.94	(21.51)	y	S2P	2.75	1.60	(4.38)	y
CTO	1.35	1.18	(1.11)	y	Sales_g	0.37	0.09	(9.81)	y
$\Delta(\Delta$ G _M - Δ Sales)	-0.29	0.00	(17.42)	y	Trading frictions:				
EPS	1.76	1.19	(21.66)	y	AT	2,906.94	243.22	(19,820.90)	y
IPM	-1.01	0.07	(35.76)	m	Beta	1.05	0.99	(0.55)	m
PCM	-0.60	0.32	(34.01)	y	Beta daily	0.89	0.81	(1.52)	m
PM	-0.99	0.08	(35.90)	y	DTO	0.00	0.00	(0.01)	m
PM _{adj}	0.39	0.09	(35.79)	m	Idio vol	0.03	0.02	(0.02)	m
Prof	1.01	0.64	(11.50)	y	LME	1,562.03	166.44	(7,046.08)	m
RNA	0.21	0.14	(6.79)	y	LME _{adj}	287.02	-683.49	(6,947.60)	m
ROA	0.03	0.04	(0.15)	y	Lturnover	0.08	0.05	(0.12)	m
ROC	-6.86	-1.44	(332.86)	m	Rel _{to_high_price}	0.75	0.79	(0.18)	m
ROE	0.06	0.10	(1.42)	y	Ret max	0.07	0.05	(0.07)	m
ROIC	0.06	0.07	(0.12)	y	Spread	0.03	0.02	(0.04)	m
S2C	84.77	15.32	(970.18)	y	Std turnover	0.31	0.16	(0.68)	m
SAT	1.21	1.08	(0.93)	y	Std volume	162.84	33.51	(583.97)	m
SAT _{adj}	0.02	-0.06	(0.74)	m	SUV	0.22	-0.15	(2.39)	m
Intangibles:					Total vol	0.03	0.02	(0.02)	m
AOA	5.23	0.07	(285.41)	y					
OL	1.10	0.95	(0.91)	y					
Tan	0.54	0.55	(0.12)	y					
OA	-0.47	-0.03	(78.52)	y					

Table 3: Returns of 10 Portfolios Sorted on Characteristics

This table reports equally-weighted returns with standard errors in parentheses for ten portfolios sorted on firm characteristics discussed in Section A.1 of the online appendix. The sample period is July 1963 to June 2014.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1
<u>Past-return based:</u>											
r_{2-1}	34.36 (3.22)	20.48 (4.38)	17.55 (2.62)	16.76 (2.80)	15.92 (2.52)	15.07 (3.36)	13.21 (2.71)	12.21 (2.56)	10.34 (2.97)	4.34 (2.44)	-13.43 (2.97)
r_{6-2}	15.13 (3.33)	14.70 (4.46)	14.62 (2.61)	14.99 (3.46)	15.74 (2.56)	15.54 (2.96)	15.41 (2.79)	16.31 (2.73)	17.47 (2.51)	20.47 (2.47)	5.34 (3.33)
r_{12-2}	13.90 (3.45)	12.57 (4.49)	13.02 (2.55)	14.02 (3.04)	14.28 (3.44)	15.14 (2.51)	16.49 (2.57)	18.38 (2.69)	19.73 (2.48)	22.82 (2.83)	8.92 (3.46)
r_{12-7}	12.37 (3.44)	12.22 (3.96)	13.58 (2.57)	14.80 (2.57)	15.96 (2.66)	15.86 (2.94)	16.72 (2.53)	17.97 (2.86)	19.67 (2.69)	21.28 (3.20)	8.92 (2.43)
r_{36-13}	23.45 (3.37)	19.32 (4.26)	17.56 (2.54)	15.91 (2.99)	15.26 (2.70)	14.97 (3.40)	15.23 (2.56)	13.69 (2.48)	13.54 (2.49)	11.47 (2.81)	-11.99 (2.86)
<u>Investment:</u>											
Investment	22.50 (3.45)	20.08 (3.88)	18.59 (2.48)	17.38 (2.78)	15.74 (3.07)	15.49 (2.49)	15.09 (3.08)	14.20 (2.72)	12.85 (2.62)	8.64 (2.49)	-13.87 (1.88)
Δ CEQ	19.98 (3.46)	19.73 (3.88)	17.47 (2.44)	16.34 (2.62)	17.62 (3.07)	15.49 (2.80)	15.26 (2.64)	15.38 (2.52)	13.76 (3.14)	9.55 (2.51)	-10.43 (1.89)
Δ PI2A	20.93 (3.30)	18.65 (3.46)	17.95 (2.68)	16.50 (2.73)	16.31 (2.71)	16.18 (3.02)	15.37 (2.94)	14.99 (2.66)	13.37 (2.83)	10.28 (2.61)	-10.65 (1.60)
Δ Shrout	16.27 (2.71)	15.86 (3.01)	15.54 (2.85)	15.70 (2.83)	14.78 (2.90)	15.51 (2.89)	15.15 (2.86)	15.04 (2.89)	16.76 (2.86)	19.78 (2.89)	3.51 (1.22)
IVC	20.84 (3.33)	18.43 (3.44)	16.24 (2.69)	16.69 (2.69)	15.18 (3.10)	15.76 (2.84)	15.36 (2.70)	15.33 (2.91)	14.04 (2.55)	12.59 (2.61)	-8.25 (1.35)
NOA	18.36 (3.16)	17.88 (2.94)	17.62 (2.83)	17.31 (2.85)	18.30 (2.95)	17.09 (2.73)	16.04 (2.94)	14.64 (2.73)	13.94 (2.76)	9.47 (2.95)	-8.89 (1.49)

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Table 3: Continued from Previous Page

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1
Profitability:											
ATO	14.78 (3.05)	15.14 (2.46)	15.06 (2.85)	16.48 (2.73)	17.14 (3.02)	16.93 (3.15)	17.04 (2.99)	16.44 (2.95)	15.90 (2.89)	15.66 (2.89)	0.88 (1.70)
CTO	14.72 (3.00)	14.10 (2.57)	15.99 (2.95)	16.89 (2.90)	16.44 (3.06)	16.65 (2.69)	17.58 (3.01)	16.36 (2.98)	16.16 (3.00)	15.69 (2.93)	0.97 (1.65)
$\Delta(\Delta\text{Gm}-\Delta\text{Sales})$	14.31 (3.26)	16.04 (3.45)	16.21 (2.59)	16.58 (2.71)	15.82 (2.62)	16.47 (2.79)	15.78 (2.64)	16.79 (2.83)	15.98 (2.89)	16.58 (3.03)	2.28 (1.17)
EPS	18.60	17.77	19.27	17.03	15.04	14.60	14.02	14.85	14.72	14.57	-4.03
IPM	(2.27)	(4.13)	(2.85)	(2.39)	(2.56)	(3.57)	(3.09)	(2.71)	(2.29)	(3.94)	(2.99)
PCM	17.72	18.78	18.39	17.84	16.75	15.15	15.00	14.02	13.57	13.32	-4.40
PM	(2.36)	(4.34)	(2.79)	(2.73)	(2.63)	(3.19)	(3.68)	(2.97)	(2.49)	(2.36)	(3.08)
PM_adj	17.88	15.88	15.68	15.46	16.27	15.85	15.81	15.52	15.52	16.53	-1.35
Prof	(2.87)	(3.38)	(2.90)	(2.80)	(2.84)	(2.85)	(2.71)	(2.94)	(2.92)	(2.71)	(1.52)
RNA	17.24	19.35	18.03	18.04	16.30	15.69	14.88	13.79	13.81	13.44	-3.79
ROA	(2.27)	(4.29)	(2.84)	(2.72)	(2.57)	(3.18)	(2.77)	(3.65)	(2.37)	(2.97)	(3.27)
ROC	16.98	17.27	17.06	15.64	15.51	14.17	15.89	15.28	16.37	16.31	-0.67
ROE	(3.09)	(3.49)	(2.64)	(2.67)	(2.91)	(2.77)	(3.16)	(2.70)	(3.16)	(3.00)	(1.91)
ROIC	14.82	13.64	15.03	15.31	15.43	16.44	16.39	16.93	18.35	18.14	3.32
S2C	(3.21)	(3.18)	(2.75)	(2.73)	(2.89)	(2.49)	(2.94)	(2.66)	(3.03)	(3.09)	(1.67)
SAT	17.31	18.47	16.60	16.43	17.08	16.47	15.53	15.20	13.70	13.73	-3.58
SAT_adj	(2.93)	(3.87)	(2.67)	(2.92)	(2.69)	(2.74)	(2.68)	(2.94)	(3.14)	(2.65)	(2.03)
	18.26	19.82	17.42	15.11	15.83	15.61	16.85	15.10	13.91	12.64	-5.62
	(2.87)	(4.42)	(2.51)	(2.52)	(2.54)	(2.67)	(3.68)	(2.65)	(2.97)	(2.71)	(2.72)
	16.90	16.11	18.15	19.70	19.18	18.40	17.15	13.85	11.65	9.58	-7.32
	(2.85)	(2.69)	(2.93)	(3.20)	(2.71)	(3.10)	(2.99)	(2.84)	(3.11)	(2.95)	(1.71)
	17.96	19.21	17.29	16.92	16.10	15.19	15.07	15.27	14.42	13.10	-4.86
	(3.03)	(4.42)	(2.45)	(2.67)	(2.44)	(2.95)	(2.47)	(2.63)	(3.59)	(2.77)	(2.69)
	18.59	17.06	16.88	15.27	16.09	16.15	16.00	15.58	14.49	14.35	-4.24
	(4.24)	(2.63)	(2.68)	(2.91)	(2.65)	(2.66)	(2.71)	(2.74)	(3.46)	(2.75)	(2.81)
	14.96	16.06	15.39	16.15	17.13	15.96	16.05	15.82	16.48	16.51	1.55
	(2.78)	(3.10)	(2.92)	(2.86)	(2.84)	(2.93)	(2.89)	(2.86)	(2.91)	(2.86)	(1.68)
	13.68	13.44	14.16	15.92	15.67	16.78	16.94	17.52	17.64	18.76	5.07
	(2.92)	(2.56)	(2.99)	(2.85)	(3.04)	(2.71)	(3.02)	(3.00)	(3.03)	(3.01)	(1.60)
	13.84	15.02	15.20	14.49	16.08	15.21	15.75	17.48	18.61	18.82	4.98
	(2.94)	(3.15)	(2.60)	(2.76)	(2.93)	(2.97)	(2.79)	(3.15)	(2.94)	(2.67)	(1.09)

Table 3: Continued from Previous Page

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1
Intangibles:											
AOA	15.73 (2.74)	15.38 (3.55)	16.42 (2.70)	16.03 (2.61)	17.42 (2.99)	16.85 (2.59)	17.24 (2.83)	16.42 (2.66)	16.24 (2.74)	12.89 (3.29)	-2.84 (1.42)
OL	14.05 (3.01)	13.33 (2.42)	13.81 (3.08)	15.13 (2.82)	15.74 (2.62)	16.00 (3.15)	17.53 (3.03)	17.75 (3.09)	18.06 (2.97)	19.07 (3.10)	5.02 (1.90)
Tan	16.05 (3.38)	14.63 (3.03)	15.27 (2.73)	15.17 (2.82)	16.36 (2.93)	16.21 (2.68)	16.56 (2.87)	16.85 (2.81)	16.19 (2.64)	17.16 (3.12)	1.11 (1.91)
OA	17.36 (3.27)	17.69 (3.46)	17.66 (2.57)	17.24 (2.63)	16.67 (2.96)	16.46 (2.73)	15.90 (2.83)	16.09 (2.76)	14.65 (2.63)	10.95 (2.90)	-6.41 (1.31)
Value:											
A2ME	9.22 (3.31)	13.04 (3.22)	14.19 (2.80)	15.57 (2.80)	16.80 (2.92)	17.34 (2.78)	18.24 (2.87)	18.48 (2.92)	18.72 (3.06)	19.06 (2.85)	9.84 (2.69)
BEME	9.20 (3.30)	12.47 (3.32)	13.32 (2.72)	14.51 (2.95)	15.66 (2.70)	16.97 (2.74)	16.77 (2.71)	18.18 (2.91)	20.21 (3.10)	23.24 (2.85)	14.04 (2.27)
BEME_adj	11.19 (2.86)	12.22 (3.37)	13.42 (2.75)	13.57 (2.76)	14.84 (2.94)	16.12 (2.84)	16.67 (2.75)	18.00 (2.85)	20.99 (2.98)	23.42 (2.89)	12.22 (1.81)
C	15.28 (3.40)	14.45 (2.61)	15.37 (2.82)	15.21 (2.78)	16.60 (2.82)	16.72 (3.22)	16.65 (2.74)	16.72 (2.95)	16.29 (3.03)	17.20 (2.89)	1.92 (2.13)
C2D	18.12 (2.63)	17.78 (4.41)	14.59 (2.70)	15.06 (2.76)	16.15 (2.69)	16.10 (2.75)	16.79 (3.50)	16.12 (2.72)	15.28 (2.72)	14.49 (2.66)	-3.63 (2.64)
ΔSO	19.56 (3.39)	17.68 (2.64)	17.37 (2.77)	17.17 (3.06)	16.77 (3.00)	16.59 (3.22)	16.38 (2.62)	15.98 (2.85)	13.07 (2.76)	10.02 (2.70)	-9.53 (1.74)
Debt2P	16.17 (2.93)	14.39 (3.35)	13.50 (2.78)	14.71 (2.74)	16.45 (2.80)	16.57 (3.18)	17.04 (2.78)	15.97 (2.90)	16.71 (3.00)	18.86 (2.85)	2.68 (2.01)
E2P	20.17 (2.91)	13.80 (4.25)	12.39 (2.68)	14.17 (2.95)	14.44 (3.20)	15.60 (2.59)	15.30 (3.47)	16.09 (2.57)	17.98 (2.49)	20.24 (2.49)	0.06 (2.47)
Free CF	14.96 (2.80)	15.64 (4.04)	16.44 (2.72)	15.62 (2.78)	16.29 (2.63)	16.05 (2.58)	15.62 (3.38)	16.58 (2.96)	16.61 (2.62)	16.70 (2.57)	1.74 (2.25)
LDP	18.76 (2.18)	17.92 (3.68)	16.13 (3.09)	14.32 (2.28)	15.06 (3.02)	15.74 (3.54)	14.56 (2.62)	14.67 (2.40)	16.15 (3.21)	17.06 (3.70)	-1.70 (2.48)
NOP	13.11 (2.49)	15.84 (3.71)	16.80 (2.86)	15.44 (2.36)	16.51 (2.29)	15.89 (3.18)	15.51 (2.65)	16.31 (2.46)	16.30 (3.84)	18.80 (3.56)	5.69 (2.13)
O2P	15.82 (2.58)	18.75 (3.51)	15.44 (2.87)	14.64 (2.39)	14.43 (2.26)	14.92 (3.26)	15.51 (2.63)	16.35 (2.46)	16.32 (3.78)	18.29 (3.49)	2.47 (1.71)
Q	22.62 (3.32)	19.76 (3.08)	18.04 (2.75)	17.29 (2.77)	16.36 (2.83)	15.91 (2.98)	14.63 (2.73)	14.26 (3.08)	12.49 (3.00)	9.15 (2.93)	-13.47 (2.14)
S2P	10.10 (3.42)	11.35 (3.43)	12.86 (2.66)	14.44 (2.61)	15.85 (2.65)	16.36 (3.22)	17.92 (2.97)	19.19 (2.92)	20.14 (2.76)	22.31 (2.76)	12.21 (2.45)
Sales.g	19.80 (3.69)	18.47 (3.44)	17.09 (2.47)	16.91 (2.54)	16.35 (2.87)	16.04 (2.54)	16.56 (2.70)	15.25 (3.13)	13.77 (2.92)	10.34 (2.67)	-9.47 (1.56)

Table 3: Continued from Previous Page

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1
Trading frictions:											
AT	21.00 (2.31)	19.48 (3.79)	17.44 (2.95)	15.59 (2.71)	16.33 (3.31)	15.52 (2.76)	14.89 (2.54)	14.05 (2.85)	13.50 (3.57)	12.62 (3.14)	-8.38 (3.13)
Beta	15.85 (1.91)	16.45 (4.77)	17.03 (2.58)	16.82 (3.43)	16.58 (2.85)	16.42 (3.11)	15.99 (2.46)	15.56 (2.10)	15.13 (3.91)	14.74 (2.29)	-1.11 (3.66)
Beta daily	18.26 (3.17)	15.99 (4.25)	15.68 (2.49)	15.61 (2.47)	15.43 (3.13)	15.80 (2.84)	16.60 (2.34)	16.64 (2.39)	16.12 (3.51)	14.37 (2.64)	-3.89 (2.34)
DTO	12.47 (3.58)	13.49 (3.37)	11.50 (2.48)	11.50 (2.53)	12.16 (3.03)	13.66 (2.78)	17.59 (2.87)	19.83 (2.73)	22.33 (3.16)	25.80 (2.60)	13.33 (1.53)
Idio vol	12.54 (4.48)	14.76 (1.77)	15.35 (2.76)	16.67 (3.00)	18.19 (2.12)	17.34 (2.35)	17.04 (3.54)	16.69 (3.26)	16.49 (3.97)	15.59 (2.54)	3.06 (3.67)
LME	31.91 (2.21)	16.09 (4.05)	14.97 (3.08)	14.77 (2.70)	14.50 (3.44)	15.36 (2.48)	14.83 (2.88)	13.81 (3.21)	12.66 (3.01)	11.18 (3.22)	-20.73 (3.48)
LME_adj	19.83 (2.24)	16.76 (3.30)	16.56 (3.31)	18.08 (2.98)	17.49 (2.64)	16.91 (3.03)	16.01 (3.20)	14.77 (3.17)	12.68 (2.84)	11.42 (3.11)	-8.41 (2.28)
Lturnover	12.04 (2.11)	14.31 (4.10)	15.72 (2.79)	16.17 (3.28)	17.04 (2.56)	17.74 (2.88)	17.53 (3.67)	17.37 (3.04)	17.62 (2.64)	15.16 (2.40)	3.13 (3.08)
Rel_to_high_price	26.19 (2.11)	15.27 (4.85)	13.47 (2.85)	13.78 (2.50)	14.78 (3.78)	15.26 (3.06)	15.74 (2.66)	15.30 (3.37)	16.21 (2.26)	14.19 (2.39)	-12.00 (4.01)
Ret max	15.10 (4.23)	16.45 (1.93)	17.01 (2.74)	16.66 (3.21)	17.74 (2.37)	17.04 (3.52)	17.28 (2.96)	16.93 (2.18)	15.02 (3.82)	11.50 (2.58)	-3.60 (3.25)
Spread	13.32 (2.40)	14.61 (3.97)	15.52 (2.85)	15.54 (3.32)	16.42 (2.68)	16.30 (3.16)	15.96 (3.54)	15.40 (2.52)	16.01 (2.42)	21.33 (3.04)	8.01 (2.99)
Std turnover	10.69 (2.01)	13.10 (3.86)	14.85 (2.80)	16.06 (3.16)	16.88 (2.33)	17.56 (3.34)	18.14 (3.00)	17.46 (3.59)	18.34 (2.51)	17.60 (2.65)	6.91 (2.76)
Std volume	15.88 (2.35)	17.20 (3.05)	17.06 (3.04)	17.75 (3.04)	16.74 (3.02)	16.46 (2.72)	16.64 (3.06)	14.94 (3.14)	15.07 (3.11)	12.89 (2.91)	-2.99 (2.26)
SUV	6.55 (3.22)	9.26 (2.84)	10.94 (2.72)	12.87 (2.78)	13.47 (2.83)	15.99 (2.83)	17.69 (2.90)	20.27 (2.78)	23.73 (3.11)	29.62 (2.97)	23.07 (1.87)
Total vol	12.85 (4.57)	14.45 (1.71)	15.60 (2.73)	16.72 (2.06)	18.03 (2.50)	17.77 (3.64)	16.98 (3.28)	17.52 (2.28)	15.43 (2.98)	15.31 (4.04)	2.46 (3.75)

Table 4: **Fama & French Three-Factor alphas for Characteristic-sorted Portfolios**

This table reports Fama&French three-factor alphas of long-short portfolios sorted on the characteristics we describe in Section A.1 of the online appendix with standard errors in parentheses and t-statistics. The sample period is July 1963 to June 2015.

	α_{FF3}	SE	t-stat		α_{FF3}	SE	t-stat
<u>Past-return based:</u>				<u>Value:</u>			
r_{2-1}	-26.48	(2.94)	-8.99	A2ME	2.75	(1.93)	1.42
r_{6-2}	9.13	(3.34)	2.73	BEME	7.80	(1.56)	5.00
r_{12-2}	12.73	(3.48)	3.66	BEME_adj	8.60	(1.56)	5.50
r_{12-7}	10.79	(2.46)	4.39	C	4.48	(1.69)	2.65
r_{36-13}	-6.68	(2.59)	-2.58	C2D	0.23	(2.23)	0.10
				ΔSO	-9.14	(1.54)	-5.95
Investment:				Debt2P	-3.21	(1.63)	-1.97
Investment	-11.85	(1.76)	-6.74	E2P	0.56	(2.19)	0.25
ΔCEQ	-8.02	(1.78)	-4.50	Free CF	3.29	(2.04)	1.61
$\Delta PI2A$	-9.32	(1.53)	-6.07	LDP	0.99	(1.68)	0.59
$\Delta ShROUT$	3.57	(1.17)	3.04	NOP	5.74	(1.61)	3.56
IVC	-7.30	(1.34)	-5.45	O2P	2.74	(1.34)	2.04
NOA	-9.56	(1.50)	-6.36	Q	-8.09	(1.50)	-5.40
				S2P	5.44	(1.91)	2.85
				Sales_g	-7.52	(1.50)	-5.02
<u>Profitability:</u>				<u>Trading frictions:</u>			
ATO	0.85	(1.45)	0.59	AT	-7.01	(2.27)	-3.09
CTO	0.57	(1.52)	0.37	Beta	-7.83	(2.38)	-3.29
$\Delta(\Delta Gm - \Delta Sales)$	3.15	(1.18)	2.66	Beta daily	-6.39	(1.97)	-3.24
EPS	1.01	(2.14)	0.47	DTO	13.08	(1.56)	8.37
IPM	-0.41	(2.47)	-0.17	Idio vol	-2.92	(2.74)	-1.06
PCM	1.87	(1.36)	1.37	LME	-15.30	(2.76)	-5.53
PM	-0.88	(2.59)	-0.34	LME_adj	-4.76	(1.51)	-3.14
PM_adj	3.96	(1.70)	2.32	Lturnover	0.44	(2.03)	0.22
Prof	1.73	(1.69)	1.03	Rel_to_high_price	-5.46	(3.54)	-1.54
RNA	-0.37	(1.81)	-0.21	Ret max	-8.41	(2.40)	-3.51
ROA	-1.70	(2.36)	-0.72	Spread	3.06	(2.74)	1.12
ROC	-4.07	(1.37)	-2.96	Std turnover	4.03	(1.79)	2.26
ROE	-1.89	(2.39)	-0.79	Std volume	-3.55	(1.85)	-1.92
ROIC	-1.75	(2.49)	-0.70	SUV	21.88	(1.89)	11.59
S2C	-0.45	(1.56)	-0.29	Total vol	-3.94	(2.74)	-1.43
SAT	4.43	(1.52)	2.91				
SAT_adj	5.36	(1.10)	4.88				
<u>Intangibles:</u>							
AOA	-4.34	(1.24)	-3.48				
OL	4.01	(1.67)	2.39				
Tan	4.29	(1.67)	2.58				
OA	-5.92	(1.34)	-4.41				

Table 5: Selected Characteristics in Nonparametric Model

This table reports the selected characteristics from the universe of 62 firm characteristics we discuss in Section A.1 of the online appendix for different numbers of knots, the sample size, and in-sample Sharpe ratios of an equally-weighted hedge portfolio going long stocks with the highest predicted returns and shorting stocks with lowest predicted returns. q indicates the size percentile of NYSE firms. The sample period is January 1965 to June 2014 unless otherwise specified.

Firms	All	All	All	Size > q_{10}	Size > q_{20}	Size > q_{20}	All	All	
Sample	Full	Full	Full	Full	Full	Full	1965-1990	1991-2014	
Knots	20	15	15	15	15	10	15	15	
Sample Size	1,629,155	1,629,155	1,629,155	959,757	763,850	763,850	603,658	1,025,497	
# Selected	13	16	13	10	9	11	11	14	
Sharpe Ratio	3.23	3.19	3.26	2.55	2.37	3.19	3.79	2.78	
Characteristics	# Selected	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
BEME	2		BEME				BEME		
Δ Shrout	8	Δ Shrout	Δ Shrout	Δ Shrout	Δ Shrout	Δ Shrout	Δ Shrout	Δ Shrout	
Δ SO	7	Δ SO	Δ SO	Δ SO	Δ SO	Δ SO	Δ SO	Δ SO	
Investment	5	Investment	Investment	Investment	Investment	Investment	Investment	Investment	
LDP	1						LDP		
LME	5	LME	LME	LME	LME	LME	LME	LME	
Lturnover	4	Lturnover	Lturnover	Lturnover	Lturnover	Lturnover	Lturnover	Lturnover	
NOA	2		NOA					NOA	
NOP	1						NOP		
PM_adj	4	PM_adj	PM_adj	PM_adj	PM_adj	PM_adj	PM_adj	PM_adj	
t_{2-1}	8	t_{2-1}	t_{2-1}	t_{2-1}	t_{2-1}	t_{2-1}	t_{2-1}	t_{2-1}	
t_{6-2}	2						t_{6-2}		
t_{12-2}	6	t_{12-2}	t_{12-2}	t_{12-2}	t_{12-2}	t_{12-2}	t_{12-2}	t_{12-2}	
t_{12-7}	7	t_{12-7}	t_{12-7}	t_{12-7}	t_{12-7}	t_{12-7}	t_{12-7}	t_{12-7}	
t_{36-13}	3		t_{36-13}				t_{36-13}	t_{36-13}	
Rel.to.high-price	6	Rel.to.high	Rel.to.high	Rel.to.high	Rel.to.high	Rel.to.high	Rel.to.high	Rel.to_high	
Ret max	1						Ret max		
ROC	7	ROC	ROC	ROC	ROC	ROC	ROC	ROC	
S2P	3						S2P		
SUV	8	SUV	SUV	SUV	SUV	SUV	SUV	SUV	
Total vol	7	Total vol	Total vol	Total vol	Total vol	Total vol	Total vol	Total vol	

Never selected: A2ME, AOA, AT, ATO, BEME_adj, Beta, Beta daily, C, C2D, CTO, Δ CEQ, $\Delta(\Delta$ Gm- Δ Sales), Δ PI2A, Debt2P, DTO, E2P, EPS, Free CF, Idio vol, IPM, IVC, LME_adj, O2P, OL, OA, PCM, PM, Prof, Q, RNA, ROA, ROE, ROIC, S2C, Sales.g, SAT, SAT_adj, Spread, Std turnover, Std volume, Tan

Table 6: Selected Characteristics in Nonparametric Model: Size Interactions

This table reports the selected characteristics from the universe of 62 firm characteristics we discuss in Section A.1 of the online appendix for different numbers of knots, the sample size, and in-sample Sharpe ratios of an equally-weighted hedge portfolio going long stocks with the highest predicted returns and shorting stocks with lowest predicted returns. We interact each firm characteristic with the previous month's market capitalization. q indicates the size percentile of NYSE firms. The sample period is January 1965 to June 2014.

Firms		All	$Size > q_{10}$	$Size > q_{20}$	$Size > q_{20}$
Sample		Full	Full	Full	Full
Knots		20	15	15	10
Sample Size		1,629,155	959,757	763,850	763,850
# Selected		25	15	9	13
Sharpe Ratio		3.41	2.98	2.42	2.69
Characteristics	# Selected	(1)	(2)	(3)	(4)
BEME	1	BEME			
Δ Shrout	4	Δ Shrout	Δ Shrout	Δ Shrout	Δ Shrout
Δ SO	4	Δ SO	Δ SO	Δ SO	Δ SO
DTO	1	DTO			
Investment	1	Investment			
Lturnover	2	Lturnover	Lturnover		
NOA	1		NOA		
PM_adj	1	PM_adj			
r_{2-1}	1	r_{2-1}			
r_{6-2}	2		r_{6-2}		r_{6-2}
r_{12-2}	1		r_{12-2}		
r_{12-7}	4	r_{12-7}	r_{12-7}	r_{12-7}	r_{12-7}
r_{36-13}	3	r_{36-13}	r_{36-13}		r_{36-13}
Rel.to_high_price	2	Rel.to_high_price	Rel.to_high_price		Rel.to_high_price
S2P	3		S2P	S2P	S2P
SUV	4	SUV	SUV	SUV	SUV
Total vol	4	Total vol	Total vol	Total vol	Total vol
Characteristics \times Size					
A2ME	1	A2ME			
BEME_adj	1	BEME_adj			
DTO	1	DTO			
EPS	1	EPS			
NOA	1	NOA			
r_{2-1}	4	r_{2-1}	r_{2-1}	r_{2-1}	r_{2-1}
r_{6-2}	4	r_{6-2}	r_{6-2}	r_{6-2}	r_{6-2}
r_{12-2}	4	r_{12-2}	r_{12-2}	r_{12-2}	r_{12-2}
Rel.to_high_price	1	Rel.to_high_price			
Ret max	1	Ret max			
ROC	1				ROC
ROE	1	ROE			
SUV	1	SUV			

Table 7: Selected Characteristics in Linear Model

This table reports the selected characteristics from the universe of 62 firm characteristics we discuss in Section A.1 of the online appendix for a linear model and raw characteristics in column (1) and a linear model and ranked-transformed characteristics in column (2) and in-sample Sharpe ratios of an equally-weighted hedge portfolio going long stocks with the highest predicted returns and shorting stocks with lowest predicted returns. The sample period is January 1965 to June 2014.

Firms		All	All
Model		Linear Model	Linear Model: rank normalized
Sample		Full	Full
Sample Size		1,629,155	1,629,155
# Selected		24	35
Sharpe Ratio		1.26	2.44
Characteristics	# Selected	(1)	(2)
AOA	1		AOA
AT	1		AT
BEME	2	BEME	BEME
BEME.adj	1	BEME.adj	
Beta	1	Beta	
C	2	C	C
CTO	1		CTO
Δ CEQ	1		Δ CEQ
Δ Shrout	2	Δ Shrout	Δ Shrout
Δ SO	1	Δ SO	
Debt2P	1		Debt2P
DTO	2	DTO	DTO
E2P	2	E2P	E2P
EPS	1		EPS
Idio vol	2	Idio vol	Idio vol
Investment	2	Investment	Investment
LDP	2	LDP	LDP
LME	1		LME
Lturnvoer	2	Lturnover	Lturnover
OA	1		OA
OL	1		OL
PCM	1		PCM
PM	1		PM
PM.adj	1		PM.adj
Prof	1		Prof
Q	1	Q	
r_{2-1}	2	r_{2-1}	r_{2-1}
r_{6-2}	1	r_{6-2}	
r_{12-2}	1		r_{12-2}
r_{12-7}	2	r_{12-7}	r_{12-7}
r_{36-13}	2	r_{36-13}	r_{36-13}
Rel.to.high_price	2	Rel.to.high_price	Rel.to.high_price
Ret max	1	Ret max	
ROA	1	ROA	
ROE	1		ROE
ROIC	2	ROIC	ROIC
S2C	1		S2C
S2P	1		S2P
SAT.adj	2	SAT.adj	SAT.adj
Spread	2	Spread	Spread
Std volume	1		Std volume
SUV	2	SUV	SUV

Table 8: **Out-of-Sample Sharpe Ratios**

This table reports out-of-sample (oos) Sharpe ratios of hedge portfolios going long the 50% of stocks with highest predicted returns and shorting the 50% of stocks with lowest predicted return for different sets of firms, out-of-sample periods, number of interpolation points, for the nonparametric and linear models. q indicates the size percentile of NYSE firms. We perform model selection from January 1965 until the months before start of the out-of-sample prediction.

	(1)	(2)	(3)	(4)	(5)	(6)
Firms	All	All	All	All	All	All
oos period	1991-2014	1991-2014	1991-2014	1991-2014	1973-2014	1973-2014
Knots	10		10		10	
Sample Size	1,025,497	1,025,497	1,025,497	1,025,497	1,541,922	1,541,922
Model	NP	Linear	NP	Linear	NP	Linear
# Selected	11	30	30	11	12	30
Model for Selection	NP	Linear	Linear	NP	NP	Linear
Sharpe Ratio	2.83	0.75	2.32	0.78	3.08	1.14
	(7)	(8)	(9)	(10)		
Firms	$Size > q_{10}$	$Size > q_{10}$	$Size > q_{20}$	$Size > q_{20}$		
oos period	1991-2014	1991-2014	1991-2014	1991-2014		
Knots	10		10			
Sample Size	959,757	959,757	763,850	763,850		
Model	NP	Linear	NP	Linear		
# Selected	9	24	9	24		
Model for Selection	NP	Linear	NP	Linear		
Sharpe Ratio	1.12	0.15	0.86	0.05		

Online Appendix: Dissecting Characteristics Nonparametrically

Joachim Freyberger, Andreas Neuhierl, and Michael Weber

Not for Publication

A.1 Data

This section details the construction of variables we use in the main body of the paper with CRSP and Compustat variable names in parentheses and the relevant references. Unless otherwise specified, we use balance-sheet data from the fiscal year ending in year $t - 1$ for returns from July of year t to June of year $t + 1$ following the Fama and French (1993) timing convention.

A2ME: We follow Bhandari (1988) and define assets-to-market cap as total assets (AT) over market capitalization as of December t-1. Market capitalization is the product of shares outstanding (SHROUT) and price (PRC).

AOA: We follow Bandyopadhyay et al. (2010) and define AOA as absolute value of operation accruals (OA) which we define below.

AT Total assets (AT) as in Gandhi and Lustig (2015).

ATO: Net sales over lagged net operating assets as in Soliman (2008). Net operating assets are the difference between operating assets and operating liabilities. Operating assets are total assets (AT) minus cash and short-term investments (CHE), minus investment and other advances (IVAO). Operating liabilities are total assets (AT), minus debt in current liabilities (DLC), minus long-term debt (DLTT), minus minority interest (MIB), minus preferred stock (PSTK), minus common equity (CEQ).

BEME: Ratio of book value of equity to market value of equity. Book equity is shareholder equity (SH) plus deferred taxes and investment tax credit (TXDITC), minus preferred stock (PS). SH is shareholders' equity (SEQ). If missing, SH is the sum of common equity (CEQ) and preferred stock (PS). If missing, SH is the difference between total assets (AT) and total liabilities (LT). Depending on availability, we use the redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for PS. The market value of equity is as of December t-1. The market value of equity is the product of shares outstanding (SHROUT) and price (PRC). See Rosenberg, Reid, and Lanstein (1985) and Davis, Fama, and French (2000).

BEME_adj: Ratio of book value of equity to market value of equity minus the average industry ratio of book value of equity to market value of equity at the Fama-French 48 industry level as in Asness et al. (2000). Book equity is shareholder

equity (SH) plus deferred taxes and investment tax credit (TXDITC), minus preferred stock (PS). SH is shareholders' equity (SEQ). If missing, SH is the sum of common equity (CEQ) and preferred stock (PS). If missing, SH is the difference between total assets (AT) and total liabilities (LT). Depending on availability, we use the redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for PS. The market value of equity is as of December t-1. The market value of equity is the product of shares outstanding (SHROUT) and price (PRC).

Beta: We follow Frazzini and Pedersen (2014) and define the CAPM beta as product of correlations between the excess return of stock i and the market excess return and the ratio of volatilities. We calculate volatilities from the standard deviations of daily log excess returns over a one-year horizon requiring at least 120 observations. We estimate correlations using overlapping three-day log excess returns over a five-year period requiring at least 750 non-missing observations.

Beta_daily: Beta_daily is the sum of the regression coefficients of daily excess returns on the market excess return and one lag of the market excess return as in Lewellen and Nagel (2006).

C: Ratio of cash and short-term investments (CHE) to total assets (AT) as in Palazzo (2012).

C2D: Cash flow to price is the ratio of income and extraordinary items (IB) and depreciation and amortization (dp) to total liabilities (LT).

CTO: We follow Haugen and Baker (1996) and define capital turnover as ratio of net sales (SALE) to lagged total assets (AT).

Debt2P: Debt to price is the ratio of long-term debt (DLTT) and debt in current liabilities (DLC) to the market capitalization as of December t-1 as in Litzenberger and Ramaswamy (1979). Market capitalization is the product of shares outstanding (SHROUT) and price (PRC).

Δ ceq: We follow Richardson et al. (2005) in the definition of the percentage change in the book value of equity (CEQ).

$\Delta(\Delta$ Gm- Δ Sales): We follow Abarbanell and Bushee (1997) in the definition of the difference in the percentage change in gross margin and the percentage change in sales (SALE). We define gross margin as the difference in sales (SALE) and costs of goods sold (COGS).

Δ So: Log change in the split adjusted shares outstanding as in Fama and French (2008). Split adjusted shares outstanding are the product of Compustat shares outstanding (CSHO) and the adjustment factor (AJEX).

Δ shrout: We follow Pontiff and Woodgate (2008) in the definition of the percentage change in shares outstanding (SHROUT).

Δ PI2A: We define the change in property, plants, and equipment following Lyandres, Sun, and Zhang (2008) as changes in property, plants, and equipment (PPEGT) and inventory (INVT) over lagged total assets (TA).

DTO: We follow Garfinkel (2009) and define turnover as ratio of daily volume (VOL) to shares outstanding (SHROUT) minus the daily market turnover and de-trend it by its 180 trading day median. We follow Anderson and Dyl (2005) and scale down the volume of NASDAQ securities by 38% after 1997 and by 50% before that to address the issue of double-counting of volume for NASDAQ securities.

E2P: We follow Basu (1983) and define earnings to price as the ratio of income before extraordinary items (IB) to the market capitalization as of December $t-1$. Market capitalization is the product of shares outstanding (SHROUT) and price (PRC).

EPS: We follow Basu (1977) and define earnings per share as the ratio of income before extraordinary items (IB) to shares outstanding (SHROUT) as of December $t-1$.

Free CF: Cash flow to book value of equity is the ratio of net income (NI), depreciation and amortization (DP), less change in working capital (WCAPCH), and capital expenditure (CAPX) over the book-value of equity defined as in the construction of BEME (see Hou et al. (2011)).

Idio vol: Idiosyncratic volatility is the standard deviation of the residuals from a regression of excess returns on the Fama and French (1993) three-factor model as in Ang, Hodrick, Xing, and Zhang (2006). We use one month of daily data and require at least fifteen non-missing observations.

Investment: We define investment as the percentage year-on-year growth rate in total assets (AT) following Cooper, Gulen, and Schill (2008).

IPM: We define pre-tax profit margin as ratio of pre-tax income (PI) to sales (SALE).

IVC: We define IVC as change in inventories (INVT) between $t - 2$ and $t - 1$ over the average total assets (AT) of years $t - 2$ and $t - 1$ following Thomas and Zhang (2002).

Lev: Leverage is the ratio of long-term debt (DLTT) and debt in current liabilities (DLC) to the sum of long-term debt, debt in current liabilities, and stockholders' equity (SEQ) following Lewellen (2015).

LDP: We follow Litzenberger and Ramaswamy (1979) and define the dividend-price ratio as annual dividends over last months price (PRC). We measure annual dividends as the sum of monthly dividends over the last 12 months. Monthly dividends are the scaled

difference between returns including dividends (RET) and returns excluding dividends (RETX).

LME: Size is the total market capitalization of the previous month defined as price (PRC) times shares outstanding (SHROUT) as in Fama and French (1992).

LME_adj: Industry-adjusted-size is the total market capitalization of the previous month defined as price (PRC) times shares outstanding (SHROUT) minus the average industry market capitalization at the Fama-French 48 industry level as in Asness et al. (2000).

LTurnover: Turnover is last month's volume (VOL) over shares outstanding (SHROUT) (Datar, Naik, and Radcliffe (1998)).

NOA: Net operating assets are the difference between operating assets minus operating liabilities scaled by lagged total assets as in Hirshleifer, Hou, Teoh, and Zhang (2004). Operating assets are total assets (AT) minus cash and short-term investments (CHE), minus investment and other advances (IVAO). Operating liabilities are total assets (AT), minus debt in current liabilities (DLC), minus long-term debt (DLTT), minus minority interest (MIB), minus preferred stock (PSTK), minus common equity (CEQ).

NOP: Net payout ratio is common dividends (DVC) plus purchase of common and preferred stock (PRSTKC) minus the sale of common and preferred stock (SSTK) over the market capitalization as of December as in Boudoukh, Michaely, Richardson, and Roberts (2007).

O2P: payout ratio is common dividends (DVC) plus purchase of common and preferred stock (PRSTKC) minus the change in value of the net number of preferred stocks outstanding (PSTKRV) over the market capitalization as of December as in Boudoukh, Michaely, Richardson, and Roberts (2007).

OA: We follow Sloan (1996) and define operating accruals as changes in non-cash working capital minus depreciation (DP) scaled by lagged total assets (TA). Non-cash working capital is the difference between non-cash current assets and current liabilities (LCT), debt in current liabilities (DLC) and income taxes payable (TXP). Non-cash current assets are current assets (ACT) minus cash and short-term investments (CHE).

OL: Operating leverage is the sum of cost of goods sold (COGS) and selling, general, and administrative expenses (XSGA) over total assets as in Novy-Marx (2011).

PCM: The price-to-cost margin is the difference between net sales (SALE) and costs of goods sold (COGS) divided by net sales (SALE) as in Gorodnichenko and Weber (2016) and D'Acunto, Liu, Pflueger, and Weber (2017).

PM: The profit margin is operating income after depreciation (OIADP) over sales (SALE) as in Soliman (2008).

PM_adj: The adjusted profit margin is operating income after depreciation (OIADP) over net sales (SALE) minus the average profit margin at the Fama-French 48 industry level as in Soliman (2008).

Prof: We follow Ball, Gerakos, Linnainmaa, and Nikolaev (2015) and define profitability as gross profitability (GP) divided by the book value of equity as defined above.

Q: Tobin's Q is total assets (AT), the market value of equity (SHROUT times PRC) minus cash and short-term investments (CEQ), minus deferred taxes (TXDB) scaled by total assets (AT).

Rel to High: Closeness to 52-week high is the ratio of stock price (PRC) at the end of the previous calendar month and the previous 52 week high price defined as in George and Hwang (2004).

Ret_max: Maximum daily return in the previous month following Bali, Cakici, and Whitelaw (2011).

RNA: The return on net operating assets is the ratio of operating income after depreciation to lagged net operating assets (Soliman (2008)). Net operating assets are the difference between operating assets minus operating liabilities. Operating assets are total assets (AT) minus cash and short-term investments (CHE), minus investment and other advances (IVAO). Operating liabilities are total assets (AT), minus debt in current liabilities (DLC), minus long-term debt (DLTT), minus minority interest (MIB), minus preferred stock (PSTK), minus common equity (CEQ).

ROA: Return-on-assets is income before extraordinary items (IB) to lagged total assets (AT) following Balakrishnan, Bartov, and Faurel (2010).

ROC: ROC is the ratio of market value of equity (ME) plus long-term debt (DLTT) minus total assets to Cash and Short-Term Investments (CHE) as in Chandrashekar and Rao (2009).

ROE: Return-on-equity is income before extraordinary items (IB) to lagged book-value of equity as in Haugen and Baker (1996).

ROIC: Return on invested capital is the ratio of earnings before interest and taxes (EBIT) less nonoperating income (NOPI) to the sum of common equity (CEQ), total liabilities (LT), and Cash and Short-Term Investments (CHE) as in Brown and Rowe (2007).

r₁₂₋₂ : We define momentum as cumulative return from 12 months before the return prediction to two months before as in Fama and French (1996).

r₁₂₋₇ : We define intermediate momentum as cumulative return from 12 months before the return prediction to seven months before as in Novy-Marx (2012).

r₆₋₂ : We define r_{6-2} as cumulative return from 6 months before the return prediction to two months before as in Jegadeesh and Titman (1993).

r₂₋₁ : We define short-term reversal as lagged one-month return as in Jegadeesh (1990).

r₃₆₋₁₃ : Long-term reversal is the cumulative return from 36 months before the return prediction to 13 months before as in De Bondt and Thaler (1985).

S2C: Sales-to-cash is the ratio of net sales (SALE) to Cash and Short-Term Investments (CHE) following Ou and Penman (1989).

S2P: Sales-to-price is the ratio of net sales (SALE) to the market capitalization as of December following Lewellen (2015).

Sales_g: Sales growth is the percentage growth rate in annual sales (SALE) following Lakonishok, Shleifer, and Vishny (1994).

SAT: We follow Soliman (2008) and define asset turnover as the ratio of sales (SALE) to total assets (AT).

SAT_adj: We follow Soliman (2008) and define adjusted asset turnover as the ratio of sales (SALE) to total assets (AT) minus the average asset turnover at the Fama-French 48 industry level.

SGA2S: SG&A to sales is the ratio of selling, general and administrative expenses (XSGA) to net sales (SALE).

Spread: The bid-ask spread is the average daily bid-ask spread in the previous months as in Chung and Zhang (2014).

Std_turnover: Std_turnover is the standard deviation of the residuals from a regression of daily turnover on a constant as in Chordia, Subrahmanyam, and Anshuman (2001). Turnover is the ratio of volume (VOL) times shares outstanding (SHROUT) We use one month of daily data and require at least fifteen non-missing observations.

Std_volume: Std_volume is the standard deviation of the residuals from a regression of daily volume on a constant as in Chordia, Subrahmanyam, and Anshuman (2001). We use one month of daily data and require at least fifteen non-missing observations.

SUV: Standard unexplained volume is difference between actual volume and predicted volume in the previous month. Predicted volume comes from a regression of daily volume on a constant and the absolute values of positive and negative returns. Unexplained volume is standardized by the standard deviation of the residuals from the regression as in Garfinkel (2009).

Tan: We follow Hahn and Lee (2009) and define tangibility as $(0.715 \times \text{total receivables (RECT)} + 0.547 \times \text{inventories (INVT)} + 0.535 \times \text{property, plant and equipment (PPENT)} + \text{cash and short-term investments (CHE)}) / \text{total assets (AT)}$.

Total_vol: Total volatility is the standard deviation of the residuals from a regression of excess returns on a constant as in Ang, Hodrick, Xing, and Zhang (2006). We use one month of daily data and require at least fifteen non-missing observations.

Table A.1: **Out-of-Sample Sharpe Ratios (90–10)**

This table reports out-of-sample (oos) Sharpe ratios of hedge portfolios going long the 10% of stocks with highest predicted returns and shorting the 10% of stocks with lowest predicted return for different sets of firms, out-of-sample periods, number of interpolation points, for the nonparametric and linear models. q indicates the size percentile of NYSE firms. We perform model selection from January 1965 until the months before start of the out-of-sample prediction.

	(1)	(2)	(3)	(4)	(5)	(6)
Firms	All	All	All	All	All	All
oos period	1991-2014	1991-2014	1991-2014	1991-2014	1973-2014	1973-2014
Knots	10		10		10	
Sample Size	1,025,497	1,025,497	1,025,497	1,025,497	1,541,922	1,541,922
Model	NP	Linear	NP	Linear	NP	Linear
# Selected	11	30	30	11	12	30
Model for Selection	NP	Linear	Linear	NP	NP	Linear
Sharpe Ratio	2.75	1.06	2.61	1.09	3.11	1.41

	(7)	(8)	(9)	(10)
Firms	<i>Size > q_{10}</i>	<i>Size > q_{10}</i>	<i>Size > q_{20}</i>	<i>Size > q_{20}</i>
oos period	1991-2014	1991-2014	1991-2014	1991-2014
Knots	10		10	
Sample Size	959,757	959,757	763,850	763,850
Model	NP	Linear	NP	Linear
# Selected	9	24	9	24
Model for Selection	NP	Linear	NP	Linear
Sharpe Ratio	1.22	0.13	0.89	0.06