The Fiscal Multiplier*

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Abstract

We measure the size of the fiscal multiplier using a heterogeneous-agent model with incomplete markets, capital and rigid prices and wages. The environment encompasses the essential elements necessary for a quantitative analysis of fiscal policy. First, output is partially demand-determined due to pricing frictions in product and labor markets, so that a fiscal stimulus increases aggregate demand. Second, incomplete markets deliver a realistic distribution of dynamic consumption and investment responses to stimulus policies across the population. These elements give rise to the standard textbook Keynesian-cross logic which, and unlike conventional wisdom would suggest, is significantly reinforced in our dynamic forward looking model.

We find that market incompleteness is key to determining the size of the fiscal multiplier, which is uniquely determined in our model for any combination of fiscal and monetary policies of interest. The multiplier is 1.34 if deficit-financed and 0.61 if contemporaneously tax-financed for a pegged nominal interest rate, with similar values in a liquidity trap. If monetary policy follows a Taylor rule, the numbers drop to 0.66 and 0.54, respectively. We elucidate the importance of market incompleteness for our results and contrast them to models featuring complete markets or hand-to-mouth consumers.

Keywords: Fiscal Multiplier, Incomplete Markets, Sticky Prices, Liquidity Trap

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1 Introduction

In an attempt to stabilize the economy during the Great Recession, monetary authorities lowered nominal interest rates to nearly zero and fixed them at that level for several years. Having reached the limit of traditional monetary policy, U.S. legislators stepped in with the largest fiscal stimulus since the 1930s. Almost a trillion dollars was to be spent by the government, much of it early on, but also with significant spending budgeted to future years. Although attempts to stabilize the economy through spending occur in virtually every recession, the questions of how much and through which channels an increase in government spending affects output, employment and investment are far from being answered.

The traditional logic describing the effects of these policies is well known. A government spending stimulus increases aggregate demand which leads to higher labor demand and thus higher employment and wages. Higher labor income then stimulates consumption, in particular of poor households, which leads to even higher aggregate demand, and thus higher employment, higher income, more consumption and so on. The equilibrium impact of an initial government spending of $1 on output - the fiscal multiplier - is then the sum of the impacts of the initial increase in government spending and of the induced private consumption response.

This simple argument is based on two essential elements. The first element is that output is demand determined that ensures that the increase in government spending stimulates aggregate demand in the first place. The typical underlying assumption is that prices are rigid so that firms adjust quantities and not only prices in response to increased government demand. Firms increase production to satisfy this demand by raising employment, capital and wages, which leads to higher household income. This direct effect of an increase in spending differs from the full equilibrium effect because it keeps prices and taxes unchanged and does not take into account indirect multiplier effects that arise from higher private consumption and investment. The second essential element of the argument is a significant deviation from the permanent income hypothesis, such that households have a high marginal propensity to consume (MPC) out of the transitory increase in income induced by the stimulus, generating a nontrivial indirect effect. Higher private consumption directly induced by government spending then leads to further increase in labor demand, income, consumption and so on.

A quantitative assessment of a stimulus policy and of the size of the direct and indirect effects requires both elements to be disciplined by the empirical behavior of households and firms. This requires a model that, first, features the right amount of nominal rigidities, so that the aggregate demand channel is as in the data. Second, it requires incorporating observed marginal propensities to consume that imply a substantial deviation from the permanent income hypothesis. However, this is not the path traditionally followed in the literature. Instead, one strand of the literature (e.g., Baxter and King, 1993; Heathcote, 2005; Brinca et al., 2016; Ferriere and Navarro, 2018) assumes flexible prices. This framework is limited in its ability to provide a full assessment as only the supply but not the demand channel is operative—that is, the first essential element is not present. Another strand of the literature uses Representative Agent New Keynesian (RANK) models with sticky prices and wages e.g., Christiano et al. (2011). Nominal rigidities provide a role for the demand channel but now the second essential element is missing because in these models households are assumed to be representative agents who behave exactly like permanent-income ones and respond little to a temporary income shock. This stands in the face of the findings of a large empirical literature that has documented substantial MPC heterogeneity and large consumption responses to transitory income and transfer payments.

In this paper we measure the size of the fiscal multiplier in a dynamic equilibrium model featuring these two elements disciplined by the observed behavior in micro data. Specifically, we extend the standard Bewley-Imrohoroglu-Huggett-Aiyagari model to include New-Keynesian style nominal price and wage rigidities.[1] Introducing incomplete markets allows the model to match the rich joint distribution of income, earnings and wealth. Such heterogeneity is crucial in generating a realistic distribution of MPCs and, more generally, for assessing the effects of policies that induce redistribution. The nominal rigidities allow for the model to have a meaningful demand channel operating. In addition, the government budget constraint in the model is specified, as in the data, in nominal terms. This lets us exploit the result in Hagedorn (2016, 2018) that incomplete market models with (partially) nominal fiscal policy imply price level

[1]See Kaplan and Violante (2018) for a recent review of this emerging literature combining incomplete-markets models with nominal rigidities. Additional references include, among others, Oh and Reis (2012), Guerrieri and Lorenzoni (2017), Gornemann et al. (2012), Kaplan et al. (2018), Auclert (2016), Lütticke (2015), McKay et al. (2016), Bayer et al. (2019), Ravn and Sterk (2017), Den Haan et al. (2017), Bhandari et al. (2017a), Auclert and Rognlie (2017), Hagedorn et al. (2019) and Hagedorn et al. (2018b). Dupor et al. (2018) build on our insights to quantify the size of the fiscal multiplier in the Great Recession in the US exploiting cross-regional evindence and a multi-region HANK model.

determinacy. This allows us to measure the size of fiscal multiplier for arbitrary combinations of monetary and fiscal policies of practical interest, including a constant nominal interest rate, without having to impose restrictions on policy rules to ensure determinacy.

Introducing these features into the model allows us to move beyond the analysis based on the theoretically elegant Two Agent New Keynesian (TANK) model where one fraction of households is hand-to-mouth and the other fraction behaves according to the permanent income hypothesis (Bilbiie, 2008, 2017). Models in this class can feature a high MPC and the demand channel, but they exhibit the same determinacy problem as the RANK model and lack propagation and anticipation effects that we will show are crucial in determining the impact of a fiscal stimulus. In particular, a TANK model delivers a much smaller multiplier even for the same MPC - relating current income to current consumption - than our model. The reason for this stark difference is the different consumption response to future income increases in the two models. Hand-to-mouth consumers spend the full increase in current income but do not respond to increases in future income. In contrast, households in our model respond to both current and future income changes, making the logic underlying the size of the multiplier dynamic. An increase in fiscal spending leads to higher current income, part of which is saved to be spent in the future, which raises private consumption demand not only today but also in all subsequent periods. This higher path of demand leads to a higher income path, relaxing precautionary savings motives and allowing households to consume even more today. Indeed, we find that future income increases (as opposed to increases in current income) drive the contemporaneous consumption response, while only the latter effect is featured by the TANK model. Finally, agents in our model anticipate and act on their expectations of future policies, for example, how and when the stimulus will be repaid enabling us to evaluate the entire policy path of interest.

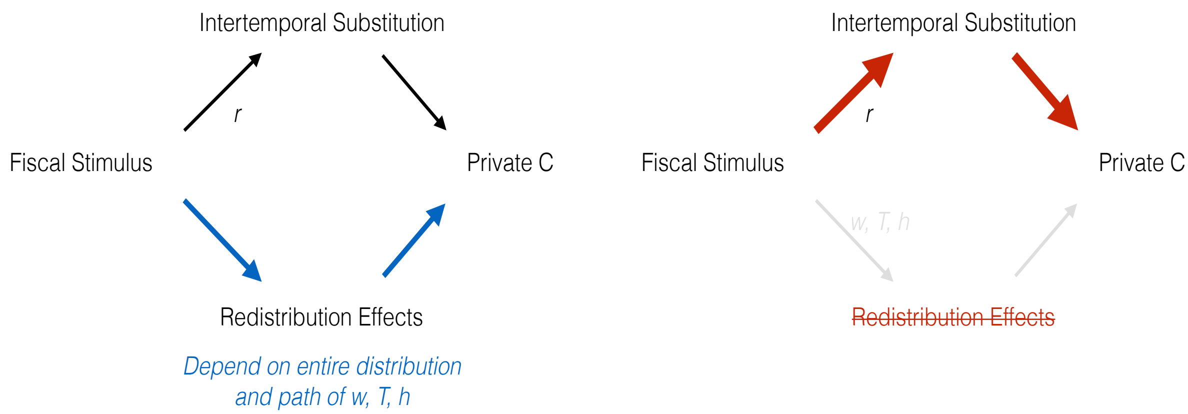
To assess the effects of fiscal policies, we also need to include capital accumulation into the model. A fiscal stimulus affects demand for both consumption and investment, and the two channels interact significantly in equilibrium. Moreover, if the financing of fiscal spending involves changes in government debt, it may imply a crowding out of investment, the consequences of which need to be taken into account in policy evaluation.

Clearly, "the fiscal multiplier" is not a single number – its size crucially depends on how it is financed (debt, distortionary taxation, reduction of transfers), how persistent fiscal policy is, what households and firms expect about future policy changes, and whether spending is increased or transfers are directed to low-income households. These important details can be incorporated in the model but are difficult to control for in empirical studies. Perhaps it is due to these difficulties that, despite the importance of this research question, no consensus on the size of the multiplier has been reached and findings come with substantial uncertainty (see Ramey, 2011, for a survey).[2] Of course, we are not the first to attempt to sidestep these difficulties faced in empirical work by relying on a more theoretical approach. Instead, our contribution is to provide a quantitative laboratory that simultaneously features a demand channel and realistic consumption and investment responses to changes in current and future incomes, and that allows researchers to assess the fiscal multiplier for any configuration of realistic monetary and fiscal policies. We also put the laboratory to use by quantifying the size of the multiplier in several policy scenarios that are of independent interest and allow to isolate various channels through which fiscal policy operates and to compare them to the implications of key alternative models in the literature.[3]

In our model, fiscal stimulus affects both private consumption and investment demand and the incompleteness of markets affects how both demand components operate. For private consumption, the fiscal multiplier operates through two channels — intertemporal substitution and redistribution — with interesting interactions. The intertemporal substitution channel describes how government spending affects real interest rates and how this changes private consumption. The strength of this channels depends on the magnitude of the response of real interest rates and on the responsiveness of private consumption to changes in real interest rates. The redistribution channel describes the distributional consequences of changes in prices,

[2]Most of the empirical studies use aggregate data to measure the strength of the fiscal multiplier, which range from around 0.6 to 1.8, although "reasonable people can argue, however, that the data do not reject 0.5 or 2.0" (see Ramey, 2011). Another more recent branch of the literature looks at cross-state evidence and typically finds larger multipliers. However, as Ramey (2011) and Farhi and Werning (2016) have pointed out, the size of the local multipliers found in those studies may not be very informative about the magnitude of aggregate multipliers. For example, the local multiplier could be 1.5 whereas the aggregate multiplier is 0.

[3]In contemporaneous and complementary work, Auclert et al. (2018) also study the size of the multiplier in a heterogenous-agent model with nominal rigidities. They characterize theoretically the constant-real-rate multiplier in a version of the model with no redistribution across households. Further, they develop a general equilibrium sufficient-statistics approach based on a matrix of "iMPCs" - intertemporal marginal propensities to consume. In contrast to our paper, their approach requires more restrictive assumptions on monetary and fiscal policy, and focuses on first-order deviations from steady-state, whereas we study a broader class of monetary and fiscal policies and investigate the importance of state dependence in a liquidity trap. However, our broad conclusions regarding the effect of different forms on financing on the size of the multiplier and the differences between heterogenous-agent and two-agent models are congruous.



(a) Incomplete Markets (b) Complete Markets

Figure 1: Channels of Fiscal Stimulus: The response of private consumption (C) to changes in real rates (r), real wages (w), hours (h) and taxes (T).

income, taxes, etc induced by government spending. The strength of this channel depends on the magnitude of the redistribution induced by spending, and on how it affects private consumption. Here it is important that the response of labor earnings is in line with the data because (1) for asset poor workers income moves basically one-to-one with earnings and (2) the profits of firms move roughly inversely with wages in response to a demand stimulus. Introducing wage rigidities—to match the empirical properties of wages—bounds the volatility of profits in the model, which is crucial for policy evaluation as the distributional effects arising from the distribution of profits have first order implications. This is why we extend previous work on Heterogeneous Agent New Keynesian (HANK) models to allow wages to be as rigid as observed in the data. In addition, these two channels do not operate independently of each other in general equilibrium, but may reinforce or attenuate each other as changes in the real interest rate have distributional effects, and redistribution itself affects the equilibrium real interest rate. Panel (a) of Figure 1 illustrates the two channels in incomplete markets. In contrast, Panel (b) shows the mechanism in complete markets. In this special but standard case only the intertemporal channel is operative.

Our paper is the first to quantify the size of those channels in a model where both are present in a meaningful way and to compare the results to the standard model with complete markets. Even qualitatively the comparison is ex-ante unclear. On the one hand, the theoretical findings in Hagedorn (2016, 2018) suggest that the response of real interest rate to changes in government spending is smaller in incomplete markets models. Kaplan et al. (2018) show that a given change in real interest rate has smaller effects in incomplete markets models. Both arguments together - a smaller response and a smaller impact of real interest rates - imply that the intertemporal channel is weaker here than if markets were complete. On the other hand, the redistributional channel is larger in incomplete markets models (as it is absent in complete markets), suggesting that the multiplier could be larger here than in complete market models. In terms of investment, the incompleteness of markets implies that Ricardian equivalence does not hold such that a deficit-financed government spending increase crowds-out private investment. At the same time higher aggregate demand leads to higher investment and we use our model to inform us which of the two effects dominates.

Our quantitative analysis combines all channels and their interaction in equilibrium. To isolate and measure the effects of fiscal policy as opposed to a convolution of changes in both monetary and fiscal policies, our benchmark experiments assume that monetary policy does not respond to the fiscal stimulus and keeps the nominal interest fixed. We find that the impact multiplier of an increase in government spending is equal to 0.61 if spending is financed by a contemporaneous increase in taxes and 1.34 if it is deficit-financed. The multiplier is high on impact but dies out quite quickly so that the cumulative multiplier, which is the discounted average multiplier over time, falls to 0.43 and 0.55 for tax and deficit-financed spending, respectively. We then assess the size of the fiscal multiplier in a liquidity trap, a question that has received renewed interest in the aftermath of the Great Recession. We therefore engineer a liquidity trap where the natural real interest rate falls below zero and is consistent with salient aggregate dynamics during the Great Recession. The results from the benchmark analysis are relatively little changed. For example, the impact multiplier is now about 0.73 for tax financed and 1.39 for deficit financed spending, implying that the multiplier is only mildly state-dependent in our non-linear model.

Two stark differences to the complete markets case require an explanation: the size of the multiplier depends on how it is financed, and is much smaller in the liquidity trap than in complete markets models (while being within a reasonable range of empirical estimates Ramey (2011)). The dependence on the type of financing, specifically deficit-financed spending being more effective in stimulating the economy than tax-financed, is not surprising in models where Ricardian equivalence is violated. Increasing spending and taxes at the same time first stimulates demand but then offsets it through raising taxes that particularly affects high-MPC households. In contrast, with deficit financing, the newly issued debt is mainly bought by low-MPC households whereas high-MPC households consume a large fraction of their additional income. Deficit financing thus implicitly redistributes from asset-rich households with low MPCs who finance their consumption more from asset income to low-asset households with high MPCs who rely more on labor income so that the aggregate MPC increases. The effect is partially offset, however, because of the crowding out of investment by the expansion in government debt.

The reason why we find smaller, when compared to complete markets, multipliers in a liquidity trap is that the response of the real interest rate is much smaller in our model than in complete markets models.[4] As a result, the magnitude of the intertemporal channel is smaller in our model and the sizes of both intertemporal substitution and redistribution channels are similar in and outside of the liquidity trap. Our decomposition of the strength of the two channels shows that the intertemporal channel contributes 0.97 (1.1) and the redistributional channel contributes 0.37 (-0.5) to the multiplier of 1.34 (0.61) if deficit (tax) financed with a similar decomposition in a liquidity trap.

Our quantitative analysis shows that the multiplier puzzles that have been documented for RANK models do not carry over to our model.[5] In RANK models, the multiplier increases if prices become more flexible and is unbounded when price rigidities vanish implying a discontinuity at fully flexible prices where the multiplier is smaller than one. The reason is that the inflation response is larger when prices are more flexible and that the private consumption response is one-to-one related to the inflation rate since only the intertemporal substitution channel is operating in RANK models. In contrast, we find that the multiplier in a liquidity trap becomes smaller as prices become more flexible and that the discontinuity at fully flexible prices disappears. The muted intertemporal substitution channel in combination with different responses of inflation and real interest rates explains this finding. In particular, we do not find large deflations as observed in liquidity traps in RANK models.

[4] Standard RANK models are linear and do not feature state dependence of the size of the multiplier. Instead, the large differences in the size of the multipliers in and outside a liquidity trap arise only because of different monetary policies.

[5] Thus, our model overcomes a Catch-22 of HANK models with fiscal policy fully specified in real terms. As Bilbiie (2018) shows, some calibrations of these models can resolve the puzzles but then the multiplier is small. Other calibrations can generate large multipliers but then the puzzles are even aggravated, a Catch-22. In contrast, government bonds are nominal in our HANK model, which allows us to resolve the puzzles and at the same time multipliers can be, depending on the distributional channel and the type of financing, large.

We also show that promises to increase government spending in the future are less effective than immediate spending increases and that the multiplier decreases in the size of spending. If we deviate from the benchmark assumption of monetary policy not responding to fiscal stimulus and assume instead that monetary policy is described by an interest rate feedback rule, the fiscal stimulus becomes considerably less effective, especially if it is deficit-financed. We also study the effectiveness and welfare implications of stimulative policies that take the form of transfers rather that an increase in spending. Although these are commonly used policies in practice, they cannot be evaluated using the traditional RANK models.

The paper is organized as follows. Section 2 presents our incomplete markets model with price and wage rigidities. In Section 3 we quantitatively study the size and properties of the fiscal multiplier. Section 4 concludes.

2 Model

The model is a standard New Keynesian model with capital with one important modification: Markets are incomplete as in Huggett (1993); İmrohoroğlu (1989, 1992); Aiyagari (1994, 1995). Price setting is constrained by costly prices adjustments as in Rotemberg (1982) leading to price rigidities. As is standard in the New Keynesian literature, final output is produced in several intermediate steps. Final good producers combine the intermediate goods to produce and sell their output in a competitive goods market. Intermediate goods producers are monopolistically competitive. They set the price they charge to the final good producer to maximize profits taking into account the price adjustment costs they face. The intermediate goods producer rent inputs, capital and a composite of differentiated labor, in competitive factor markets. We also allow for sticky wages and assume that differentiated labor is monopolistically supplied as well.

2.1 Households

The economy consists of a continuum of agents normalized to measure 1 who are ex-ante heterogenous with respect to their subjective discount factors, have CRRA preferences over consumption and additively separable preferences for leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where

$$u(c,h) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} - v(h) & \text{if } \sigma \neq 1 \\ \log(c) - v(h) & \text{if } \sigma = 1, \end{cases}$$

$\beta \in (0,1)$ is the household-specific subjective discount factor and $v(h)$ is the disutility of labor.

Agents' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic and is characterized by an N-state Markov chain that can take on values $s_t \in \mathcal{S} = \{s_1, \cdots, s_N\}$ with transition probability characterized by $\gamma(s_{t+1}|s_t)$ and $\int s = 1$. Agents rent their labor services, $h_t s_t$, to firms for a real wage w_t and their nominal assets a_t to the asset market for a nominal rent i_t^a and a real return $(1 + r_t^a) = \frac{1+i_t^a}{1+\pi_t}$, where $1 + \pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate (P_t is the price of the final good). The nominal return on bonds is i_t with a real return $(1 + r_t) = \frac{1+i_t}{1+\pi_t}$. There are two classes of assets, bonds and capital with potentially different returns, but households can invest in one asset, A, that a mutual fund (described below) collects and allocates to bonds and capital.

To allow for sticky wages in our heterogeneous worker economy we need to extend the literature which models wage rigidities in representative agent models (Erceg et al., 2000). We follow the representative agent literature and assume that each household i provides differentiated labor services which are transformed by a representative, competitive labor recruiting firm into an aggregate effective labor input, H_t, using the following technology:

$$H_t = \left(\int_0^1 s_{it} (h_{it})^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{1}$$

where ϵ_w is the elasticity of substitution across differentiated labor.

A middleman firm (e.g. a union) sells households labor services $s_{it} h_{it}$ at the wage W_{it} to the labor recruiter, which, given aggregate labor demand H_t by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{it} s_{it} h_{it} di, \tag{2}$$

implying a demand for the labor services of household i:

$$h_{it} = h(W_{it}; W_t, H_t) = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon_w} H_t, \tag{3}$$

where W_t is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left(\int_0^1 s_{it} W_{it}^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}.$$

The union sets a nominal wage $\hat{W}_t$ for an effective unit of labor (so that $W_{it} = \hat{W}_t$) to maximize profits subject to wage adjustment costs modeled similarly to the price adjustment costs in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity s_{it}, are measured in units of aggregate output, and are given by a quadratic function of the change in wages above and beyond steady state wage inflation $\overline{\Pi}^w$,

$$\Theta\left(s_{it}, W_{it} = \hat{W}_t, W_{it-1} = \hat{W}_{t-1}; H_t\right) = s_{it}\frac{\theta_w}{2}\left(\frac{W_{it}}{W_{it-1}} - \overline{\Pi}^w\right)^2 H_t = s_{it}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t.$$

Our extension to an heterogeneous agent economy requires to make assumptions on the aggregation of heterogeneous workers. There is not a unique way to do so and we choose one that is tractable and delivers the representative agent outcome when all heterogeneity is shut down. The union's wage setting problem is assumed to maximize[6]

$$\begin{aligned} V_t^w\left(\hat{W}_{t-1}\right) \equiv & \max_{\hat{W}_t} \int \Big(\frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{v(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\Big) di \\ - & \int s_{it}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t di + \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right), \end{aligned} \tag{4}$$

where C_t is aggregate consumption and τ_t is a proportional tax on labor income.[7]

Some algebra (see Appendix II) yields, using $h_{it} = H_t$ and $\hat{W}_t = W_t$ and defining the real

[6]Equivalently one can think of a continuum of unions each setting the wage for a representative part of the population with $\int s = 1$ at all times.

[7]Any decision problem in a heterogeneous group requires assumptions on the aggregation of individual needs and this wage setting problem is no exception. Fortunately, our choices here have virtually no effect on our findings. We divide $v(h(\hat{W}_t; W_t, H_t))$ by $u'(C_t)$ but using individual consumption and dividing by $u'(C_{it})$ instead has virtually no effect. We discount with $\frac{1}{1+r_t}$ but discounting using $\frac{1}{1+r_t^a}$ or β instead also has negligible effect our findings.

wage $w_t = \frac{W_t}{P_t}$, the wage inflation equation

$$\theta_w \left(\pi_t^w - \overline{\Pi}^w\right) \pi_t^w = (1-\tau_t)(1-\epsilon_w)w_t + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} + \frac{1}{1+r_t}\theta_w \left(\pi_{t+1}^w - \overline{\Pi}^w\right) \pi_{t+1}^w \frac{H_{t+1}}{H_t}. \tag{5}$$

Thus, at time t an agent faces the following budget constraint:

$$P_t c_t + a_{t+1} = (1+i_t^a)a_t + (1-\tau_t)P_t w_t h_t s_t + T_t,$$

where T_t is a nominal lump sum transfer. Households take prices as well as wages and hours from the middleman's wage setting problem as given. Thus, we can rewrite the agent's problem recursively as follows:

$$V(a, s, \beta; \Omega) = \max_{c \geq 0, a' \geq 0} u(c, h) + \beta \sum_{s \in \mathcal{S}} \gamma(s'|s) V(a', s', \beta; \Omega') \tag{6}$$

$$\text{subj. to} \quad Pc + a' = (1+i^a)a + P(1-\tau)whs + T$$

$$\Omega' = \Upsilon(\Omega),$$

where $\Omega(a, s, \beta)$ is the distribution on the space $X = \mathcal{A} \times \mathcal{S} \times \mathcal{B}$ of agents' asset holdings $a \in \mathcal{A}$, labor productivity $s \in \mathcal{S}$ and discount factor $\beta \in \mathcal{B}$, across the population, which will, together with the policy variables, determine the equilibrium prices. Let $\mathbb{B}(X) = \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{S}) \times \mathcal{P}(\mathcal{B})$ be the σ-algebra over X, defined as the cartesian product over the Borel σ-algebra on $\mathcal{A}$ and the power sets of $\mathcal{S}$ and $\mathcal{B}$. Define our space $M = (X, \mathbb{B}(X))$, and let $\mathcal{M}$ be the set of probability measures over M. Υ is an equilibrium object that specifies the evolution of the distribution Ω.

2.2 Production

2.2.1 Final Good Producer

A competitive representative final goods producer aggregates a continuum of intermediate goods y_{jt} indexed by $j \in [0, 1]$ and with prices p_{jt}:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon - 1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon - 1}}.$$

where ϵ is the elasticity of substitution across goods. Given a level of aggregate demand Y_t, cost minimization for the final goods producer implies that the demand for the intermediate good j is given by

$$y_{jt} = y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t, \tag{7}$$

where P is the (equilibrium) price of the final good which can be expressed as

$$P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}.$$

2.2.2 Intermediate-Goods Firms

Each intermediate good j is produced by a monopolistically competitive producer using the technology:

$$Y_{jt} = \begin{cases} K_{jt}^{\alpha} H_{jt}^{1-\alpha} - F & \text{if} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

where $0 < \alpha < 1$, K_{jt} is capital services rented, H_{jt} is labor services rented and the fixed cost of production is denoted $F > 0$.

Intermediate-goods firms rent capital and labor in perfectly competitive factor markets. A firm's real marginal cost is $mc_{jt} = \partial \Xi_t(Y_{jt})/\partial Y_{jt}$, where

$$\Xi_t(Y_{jt}) = \min_{K_{jt}, H_{jt}} r_t^k K_{jt} + w_t H_{jt}, \text{ and } Y_{jt} \text{ is given by (8).} \tag{9}$$

Given our functional forms, we have

$$mc_t = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (r_t^k)^{\alpha} (w_t)^{1-\alpha} \tag{10}$$

and

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k}. \tag{11}$$

Prices are sticky as intermediate-goods firms face Rotemberg (1982) price adjustment costs. Given last period's individual price p_{jt-1} and the aggregate state $(P_t, Y_t, Z_t, w_t, r_t)$, the firm chooses this period's price p_{jt} to maximize the present discounted value of future profits, satisfying all demand. The intermediate-goods firm's pricing problem is

$$V_t^{IGF}(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - \Xi(y(p_{jt}; P_t, Y_t)) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t - F + \frac{1}{1+r_t} V_{t+1}^{IGF}(p_{jt}).$$

Some algebra (see Appendix II) yields the New Keynesian Phillips Curve

$$(1-\epsilon)+\epsilon mc_t-\theta\left(\pi_t-\overline{\Pi}\right)\pi_t+\frac{1}{1+r_t}\theta\left(\pi_{t+1}-\overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}=0.$$

The equilibrium real profit of each intermediate goods firm, fully taxed by the government, is

$$d_t=Y_t-F-\Xi(Y_t).$$

2.3 Mutual Fund

The mutual fund collects households savings $A_{t+1}/P_{t+1}\equiv\int a_{it}/P_{t+1}\,di$, pays a real return $\tilde{r}^a_t$, and invests them in real bonds B_{t+1}/P_{t+1} and capital K_{t+1}. It maximizes

$$V^{MF}(K_t)\equiv\max_{K_{t+1},\frac{B_{t+1}}{P_{t+1}}}(1+r^k_{t+1}-\delta)K_{t+1}+(1+r_{t+1})B_{t+1}/P_{t+1}-(1+\tilde{r}^a_{t+1})(A_{t+1}/P_{t+1})+\frac{V^{MF}(K_{t+1})}{(1+\tilde{r}^a_{t+2})}$$

$$\text{subj. to}\quad A_{t+1}/P_{t+1}=K_{t+1}+B_{t+1}/P_{t+1}+\Phi(K_{t+1},K_t)$$

for adjustment costs $\Phi(K_{t+1},K_t)$ and taking K_t and K_{t+2} as given. Note that we drop expectation operators for simplicity since we consider perfect foresight experiments only. The equilibrium first-order conditions are

$$r_{t+1}=\tilde{r}^a_{t+1}, \tag{12}$$

$$1+r^k_{t+1}-\delta=(1+\tilde{r}^a_{t+1})(1+\Phi_1(K_{t+1},K_t))+\Phi_2(K_{t+2},K_{t+1}), \tag{13}$$

$$A_{t+1}/P_{t+1}=K_{t+1}+B_{t+1}/P_{t+1}+\Phi(K_{t+1},K_t). \tag{14}$$

The total dividends of the fund are

$$D^{MF}_{t+1}=(1+r^k_{t+1}-\delta)K_{t+1}+(1+r_{t+1})B_{t+1}/P_{t+1}-(1+\tilde{r}^a_{t+1})(A_{t+1}/P_{t+1}),$$

and after-tax per unit of investment they are $d^{MF}_{t+1}=(1-\tau_k)D^{MF}_{t+1}/(A_{t+1}/P_{t+1})$. Households therefore receive (or have to pay) $d^{MF}_{t+1}A_{t+1}/P_{t+1}$ in period $t+1$ per unit invested such that households' real return equals

$$1+r^a_{t+1}=1+\tilde{r}^a_{t+1}+d^{MF}_{t+1}$$

and corresponding nominal return is

$$1+i^a_{t+1}=(1+r^a_{t+1})\frac{P_{t+1}}{P_t}.$$

2.4 Government

The government obtains revenue from taxing labor income, profits and dividends and issuing bonds. Household labor income is taxed progressively with a nominal lump-sum transfer T and a proportional tax τ:

$$\tilde{T}(wsh,T)=-T+\tau Pwsh.$$

The government issues nominal bonds denoted by B^g, with negative values denoting government asset holdings and fully taxes away profits of intermediate goods firms, obtaining nominal revenue Pd. The government also taxes dividend income at the rate τ_k. The government uses the revenue to finance exogenous nominal government expenditures, G_t, interest payments on bonds and transfers to households. The government budget constraint is therefore given by:

$$B^g_{t+1}=(1+i_t)B^g_t+G_t-P_t d_t-\tau_k P_t D^{MF}_t-\int\tilde{T}(w_t s_t h_t,T_t)d\Omega. \tag{15}$$

2.5 Equilibrium

Market clearing requires that the labor demanded by firms is equal to the aggregate labor supplied by households, that the demand for bonds issued by the government and for capital equal their supplies and that the amount of assets provided by households equals the demand for them by the mutual fund:

$$H_t=\int H_{jt}dj=H_{jt}=\int h_{it}di=h_{it} \tag{16}$$

$$B_t=B^g_t \tag{17}$$

$$K_t=\int K_{jt}dj \tag{18}$$

$$K_{t+1}+B_{t+1}/P_{t+1}+\Phi(K_{t+1},K_t)=A_{t+1}/P_{t+1}=\int\frac{a_{t+1}(a_t,s_t,\beta)}{P_{t+1}}d\Omega_t, \tag{19}$$

where $a_{t+1}(a_t,s_t,\beta)$ is the asset choice of an agent with asset level a_t, labor productivity s_t and discount factor β.

The aggregate resource constraint is given by

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} = \int c_t(a_t, s_t, \beta) d\Omega_t + \frac{G_t}{P_t} + F + K_{t+1} - (1-\delta)K_t + \Phi(K_{t+1}, K_t) \tag{20}$$

. It does not include price and wage adjustment costs because we interpret those costs as being virtual—they affect optimal choices but do not cause real resources to be expended. We make this assumption to avoid significant movements of these adjustment costs and to avoid them becoming a non-trivial fraction of output in e.g., the liquidity trap. In a liquidity trap prices fall, leading to large price adjustment costs, which, if they were actual resource costs, would lead to a boom in the price adjustment cost industry and may imply an increase in aggregate output (see Hagedorn et al., 2018a, for details). Assuming that price adjustment costs are as-if avoids these outcomes and moves our model closer to one with price setting à la Calvo.

Definition: A monetary competitive equilibrium is a sequence of tax rates τ_t and τ_k, nominal transfers T_t, nominal government spending G_t, supply of government bonds B_t^g, value functions $V_t \colon X \times \mathcal{M} \to \mathcal{R}$ with policy functions $a_t \colon X \times \mathcal{M} \to \mathcal{R}_+$ and $c_t \colon X \times \mathcal{M} \to \mathcal{R}_+$, hours choices $H_t, H_{jt}, h_{it} \colon \mathcal{M} \to \mathcal{R}_+$, capital decisions $K_t, K_{jt} \colon \mathcal{M} \to \mathcal{R}_+$, bond choices $B_t \colon \mathcal{M} \to \mathcal{R}_+$, price levels $P_t \colon \mathcal{M} \to \mathcal{R}_+$, pricing functions $r_t, r_t^a, r_t^k, \tilde{r}_t^a \colon \mathcal{M} \to \mathcal{R}$ and $w_t \colon \mathcal{M} \to \mathcal{R}_+$, and a law of motion $\Upsilon \colon \mathcal{M} \to \mathcal{M}$, such that:

1. V_t satisfies the Bellman equation with corresponding policy functions a_t and c_t given price sequences r_t^a, w_t and hours h_t.
2. Firms maximize profits taking prices P_t, r_t^k, w_t as given.
3. Wages are set optimally by middlemen.
4. The mutual fund maximizes profits taking prices as given.
5. For all $\Omega \in \mathcal{M}$ market clearing conditions (16) - (19) and the resource constraint (20) are satisfied.
6. Aggregate law of motion Υ generated by a' and γ.

3 The Fiscal Multiplier

In this section we calculate the fiscal multiplier in our model with incomplete markets, conducting the following experiment. Assume that the economy is in steady state with nominal bonds B_{ss}, government spending G_{ss}, transfers T_{ss}, labor tax rate τ_{ss}, dividend tax $\tau_{k,ss}$, and where the price level is P_{ss}. The real value of bonds is then B_{ss}/P_{ss}, the real value of government expenditure is G_{ss}/P_{ss} and so on. We then consider an M.I.T. (unexpected and never-again-occurring) shock to government expenditures and compute the impulse response to this persistent innovation in G. Agents are assumed to have perfect foresight after the shock. Eventually the economy will reach the new steady state characterized by government spending $G_{ss}^{new} = G_{ss}$, government bonds $B_{ss}^{new} = B_{ss}$, transfers $T_{ss}^{new} = T_{ss}$, tax rates $\tau_{ss}^{new} = \tau_{ss}$ and $\tau_{k,ss}^{new} = \tau_{k,ss}$, and the price level P_{ss}^{new}.

3.1 The Fiscal Multiplier in Incomplete Market Models

In our model the fiscal multiplier affects both aggregate consumption and investment. In terms of consumption, the stimulus operates through two interdependent channels — intertemporal substitution and redistribution. The intertemporal substitution channel describes how government spending changes real interest rates and how this changes private consumption. The distributional channel describes how government spending changes prices, income, taxes, etc., the redistribution induced by these changes and the resulting impact on private consumption. In terms of investment, the multiplier operates through changes in real interest rates, through crowding-out of capital when deficit financing is used and through stimulating demand which makes firms hire not only more labor but also more capital. In addition to capturing the relevant transmission mechanisms for fiscal policy, our HANK framework also delivers price level determinacy (Hagedorn, 2016, 2018). This allows us to study arbitrary combinations of monetary and fiscal policies of interest, in particular a constant nominal interest rate as prevails at the zero lower bound (ZLB). We do not face the indeterminacy issues with the representative-agent New Keynesian model at the ZLB raised by Cochrane (2017) and have a uniquely determined fiscal multiplier at the ZLB, one of the cases where we are most interested in knowing its size.

We now explain the role of the two consumption channels in shaping the fiscal multiplier, how they interact with price level determinacy, what determines their strengths, and explain the differences to complete markets. After that we turn to the investment channel. Finally, we formally define the fiscal multiplier in our environment as well as the corresponding multipliers that would arise in RANK and TANK models faced with same dynamics of prices as arising in our model. This will be helpful in illustrating the role of various features of our model that lead to quite different conclusions from the ones implied by those prominent alternative frameworks.

3.1.1 Intertemporal Substitution Channel

To understand the workings of the intertemporal substitution channel in our model it is instructive to start with the complete markets case where this is the only channel operating. We then move to incomplete markets to elucidate the differences.

With complete markets, the size of the multiplier m is determined by the response of the real interest rates only. The consumption Euler equation for our utility function, $\frac{C^{1-\sigma}}{1-\sigma}+...$, is

$$C_t^{-\sigma} = \beta(1+r_{t+1})C_{t+1}^{-\sigma}. \tag{21}$$

Iterating this equation and assuming that consumption is back to the steady-state level at time T, $C_T = C_{ss}$, we obtain for consumption at time $t=1$ when spending is increased,

$$C_1^{-\sigma} = \Big(\prod_{t=1}^{T-1} \beta(1+r_{t+1})\Big) C_T^{-\sigma}, \tag{22}$$

so that the initial percentage increase in consumption equals

$$\frac{C_1}{C_{ss}} = \Big(\prod_{t=1}^{T-1} \beta(1+r_{t+1})\Big)^{\frac{-1}{\sigma}} = \Big(\prod_{t=1}^{T-1} \frac{1+r_{t+1}}{1+r_{ss}}\Big)^{\frac{-1}{\sigma}}, \tag{23}$$

where we have used that $\beta(1+r_{ss}) = 1$ in a complete markets steady state. The fiscal multiplier m - the dollar change in output for each dollar increase in g - is one-to-one related to the percentage change in private consumption

$$m = 1 + \Big(\frac{C_1}{C_{ss}} - 1\Big)\frac{C_{ss}}{\Delta g} \tag{24}$$

and is thus one-to-one related to the accumulated response of the real interest rate which is induced by the fiscal stimulus,

$$m = 1 + \frac{C_{ss}}{\Delta g}\left(\Big(\prod_{t=1}^{T-1}\frac{1+r_{t+1}}{1+r_{ss}}\Big)^{\frac{-1}{\sigma}} - 1\right). \tag{25}$$

The multiplier is then proportional to the consumption response

$$\log(\frac{C_1}{C_{ss}}) = \underbrace{\frac{1}{\sigma}}_{\text{Intertemporal Substitution}} \sum_{t=1}^{T-1}\underbrace{(\log(1+r_{ss}) - \log(1+r_{t+1}))}_{\text{Change of real interest rates}}, \tag{26}$$

which can be decomposed in the change in the real interest rate, $\approx r_t - r_{ss}$, and the effect of this change on consumption, whose strength is governed by the IES, $\frac{1}{\sigma}$.

Both components of the intertemporal substitution channel tend to be weaker in incomplete markets models. The effect of the real interest rate on consumption is smaller since some households are credit constrained and thus not on their Euler equation, breaking the tight link between consumption and real interest rates. In addition, the change in real interest rates is smaller. To understand the difference, assume for simplicity that the nominal interest rate is fixed at i_{ss} and that the steady state is reached after T periods so that

$$\prod_{t=1}^{T-1}(1+r_{t+1}) = \prod_{t=1}^{T-1}(\frac{1+i_{ss}}{1+\pi_{t+1}}) = \prod_{t=1}^{T-1}(1+i_{ss})\frac{P_T}{P_1} = \prod_{t=1}^{T-1}(1+i_{ss})\frac{P_{ss}^{new}}{P_1}, \tag{27}$$

i.e., the response of $\prod_{t=1}^{T-1}(1+r_{t+1})$ is one-to-one related to the response of P_{ss}^{new}/P_1. This response is quite large in complete market models (Christiano et al., 2011) but small here.

This is a consequence of the result that incomplete markets combined with fiscal policy specified partially in nominal terms delivers a globally determined price level independently of how monetary policy is specified. We refer the reader to Hagedorn (2016, 2018) for details and only provide the intuition for the key result that $P_{ss}^{new} = P_{ss}$. Define households' real steady-state asset demand as S. The asset demand S, the real stock of capital, K_{ss}, and the amount of nominal bonds is the same, $B_{ss}^{new} = B_{ss}$, in both steady states and therefore both price levels solve the same asset market clearing condition[8]

$$S(1+r_{ss},...) = K_{ss} + \frac{B_{ss}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}^{new}}. \tag{28}$$

Together these arguments imply that the intertemporal substitution channel is weaker in our incomplete markets model than in the corresponding complete markets model, where $P_{ss}^{new} > P_{ss}$. Note that since P_1 typically increases in response to a stimulus, $P_{ss}^{new}/P_1 < 1$, and thus $\prod_{t=1}^{T-1}(1+r_{t+1}) < \prod_{t=1}^{T-1}(1+i_{ss})$ i.e., the intertemporal substitution channel by itself implies a multiplier smaller than one, $m < 1$. The multiplier here is smaller than one whereas it is equal to one in Woodford (2011) since the real interest rate is constant in the small-open economy experiment considered in Woodford (2011) but endogenous and time-varying here.[9]

3.1.2 Distributional Consequences of a Stimulus

An increase in spending, the necessary adjustments in taxes and transfers, and the resulting responses of prices and hours induce redistribution across economic agents. For example, changes in the tax code naturally deliver winners and losers. An increase in the price level and of labor income leads to a redistribution from households who finance their consumption more from asset income to households who rely more on labor income. Changes in interest rates redistribute between debtors (the government) and lenders (Erosa and Ventura, 2002; Doepke and Schneider, 2006).

These distributional effects matter due to the endogenous heterogeneity in the MPCs in the data that is replicated in the model. This heterogeneity together with the properties of the redistribution determine the aggregate consumption response, and, since output is demand determined due to price rigidities, also affect output. Individual household's consumption c_t at time t depends on the sequence of transfers T, tax rates τ, labor income wh, prices P and

[8]Price level determinacy is a consequence of only two empirically well-grounded assumptions, incomplete markets and nominal government bonds, and in particular does not require any further selection criteria, such as a price or inflation targeting rule.

[9]We also overcome an indeterminacy problem in Woodford (2011). He assumes that after a fiscal stimulus consumption converges back to its pre-stimulus level, ruling out other belief driven steady states. Since the real interest rate is constant, this implies that consumption is equal to its steady-state level in all periods, implying a multiplier equal to one. If instead households believe income to change permanently by $x\%$, then consumption demand increases by $x\%$ as well, confirming the initial belief as an equilibrium outcome. There is a steady state for each $x \neq 0$, assumed away in Woodford (2011) who focuses instead on the $x = 0$ equilibrium. Such assumptions to ensure determinacy are not needed here.

nominal interest rates i^a, so that aggregate private consumption satisfies

$$C_t\big(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0}\big) = \int c_t(a, s, \beta; \{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0}) d\Omega_t. \tag{29}$$

In our model hours are a household choice variable but they are demand determined as well. Of course, consumption and hours worked are jointly determined in equilibrium but to understand the demand response of the fiscal stimulus it turns out to be useful to consider wh as exogenous for consumption decisions here. In particular it allows us to distinguish between the initial impact,"first round" demand impulse due to the policy change and "second, third, ... round" due to equilibrium responses. Those arise in our model since an initial policy-induced demand stimulus leads to more employment by firms, and thus higher labor income which in turn implies more consumption demand, which again leads to more employment and so on until an equilibrium is reached where all variables are mutually consistent. Denoting pre-stimulus variables by a bar, we can now decompose the aggregate consumption response,

$$(\Delta C)_t = C_t\big(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l\geq 0}\big), \tag{30}$$

into its different channels:

$$\begin{aligned}
(\Delta C)_t \quad = \quad & \underbrace{C_t\big(\{T_l, \tau_l, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Impact of Transfers and Taxes}} & (31)\\
+ \quad & \underbrace{C_t\big(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{T_l, \tau_l, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Equilibrium Effect: Labor Income}} & (32)\\
+ \quad & \underbrace{C_t\big(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0}\big) - C_t\big(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l\geq 0}\big)}_{\text{Price and Interest Rate Adjustment}}. & (33)
\end{aligned}$$

Total demand is the sum of private consumption demand C and real government consumption $g = G/P$, which both determine output. The private consumption response does not directly depend on G/P but it does indirectly. First, transfers T and taxes τ have to adjust to balance the intertemporal government budget constraint. Second, increases in G/P translate one-for-one into increases in demand. On impact an increase by Δg increases demand by Δg and thus hours worked from $\bar{h}$ to $\bar{h} + \Delta h^g$, where Δh^g is the amount of hours needed to produce Δg while keeping the capital stock unchanged. As before, this increase in labor income stimulates private demand which in turn leads to higher employment, then again higher consumption and so on until convergence. We therefore decompose the total demand effect ΔD of an increase in government spending by Δg as

$$
\begin{aligned}
(\Delta D)_t &= \underbrace{(\Delta g)_t}_{\text{Direct Govt' Spending Response}} + \underbrace{(\Delta C)_t}_{\text{Private Consumption Response}} && (34)\\
&= (\Delta g)_t + \underbrace{C_t\big(\{\bar{T},\bar{\tau},\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{\bar{T},\bar{\tau},\bar{w}\bar{h},\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Direct Impact on Private Consumption}} && (35)\\
&+ \underbrace{C_t\big(\{T_l,\tau_l,w_l h_l,P_l,i^a_l\}_{l\geq 0}\big) - C_t\big(\{\bar{T},\bar{\tau},\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Private Consumption Response}}. && (36)
\end{aligned}
$$

A fiscal stimulus, in addition to the immediate impact on government demand, also leads to higher employment and labor income and thus stimulates private consumption, the *Direct Impact on Private Consumption*. The remainder of the private consumption is as above the sum of the direct impact of transfers and taxes, the indirect equilibrium effects of labor income and price and interest rate adjustment, such that the full decomposition of the total demand effect $(\Delta D)_t$ is

$$
\begin{aligned}
(\Delta D)_t &= \underbrace{(\Delta g)_t}_{\text{Direct G Impact}} && (37)\\
&+ \underbrace{C_t\big(\{\bar{T},\bar{\tau},\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{\bar{T},\bar{\tau},\bar{w}\bar{h},\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Direct G Impact on C}} && (38)\\
&+ \underbrace{C_t\big(\{T_l,\tau_l,\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{\bar{T},\bar{\tau},\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Tax/Transfer Impact}} && (39)\\
&+ \underbrace{C_t\big(\{T_l,\tau_l,w_l h_l,\bar{P},\bar{i}^a\}_{l\geq 0}\big) - C_t\big(\{T_l,\tau_l,\bar{w}(\bar{h}+\Delta h^g),\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Labor Income Impact}} && (40)\\
&+ \underbrace{C_t\big(\{T_l,\tau_l,w_l h_l,P_l,i^a_l\}_{l\geq 0}\big) - C_t\big(\{T_l,\tau_l,w_l h_l,\bar{P},\bar{i}^a\}_{l\geq 0}\big)}_{\text{Indirect Price and Interest Impact}}. && (41)
\end{aligned}
$$

3.1.3 Investment Channel

Investment demand is another component of aggregate demand and the strength of this channel depends both on the cost of investment – the real interest rate r^k_t – and the demand for intermediate goods. Intermediate goods firms set prices subject to Rotemberg adjustment costs and have to satisfy the resulting demand Y_{jt} for their product through hiring labor H_{jt} or buying capital goods K_{jt}, which leads to the cost minimization condition (11):

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k}.$$

Given prices w_t and r_t^k, a higher demand Y_{jt} leads to an increase both in capital and employment, the demand channel of investment. Higher capital costs r_t^k dampen investment demand but only if they increase more than wages. Since firm have to satisfy demand, the relative costs of the factor inputs matter. As an example, suppose that r_t^k increases by 1% but that wages increase by 2%, so that $\frac{\alpha w_t}{(1-\alpha) r_t^k}$ increases. In this case, firms would substitute from labor to capital although capital costs have increased just because the costs of the other input factor, wages, has increased even more.[10] Two features of our calibration below make this scenario less likely. High capital adjustment costs imply a strong response of adjustment costs and thus of r_t^k to capital changes and wage rigidities dampen the movement of wages in response to demand fluctuations. In addition to these partial equilibrium considerations, general equilibrium requires asset market clearing, that is the real interest rate received by households has to be such that they are willing to absorb all assets supplied, bonds and capital,

$$S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0}\big) = \int \frac{a_{t+1}(a, s; \{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l\geq 0})}{P_{t+1}} d\Omega_t \tag{42}$$

$$= K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t), \tag{43}$$

such that the change in aggregate savings equals the sum of the change in capital, real bonds and adjustment costs,

$$(\Delta S)_{t+1} = (\Delta K_{t+1}) + (\Delta \frac{B_{t+1}}{P_{t+1}}) + (\Delta \Phi(K_{t+1}, K_t)). \tag{44}$$

This asset market clearing condition implies that, if the stimulus is financed by increasing government debt, capital could be partially crowded out since the increase in savings is partially absorbed by higher government debt. To assess the magnitude of this channel we compute the sequence of real interest rates $r^{a,Crowding}$ and capital stocks K^{Crowd} which would clear the

[10] The findings of Rupert and Šustek (2019), derived in a complete markets New Keyneian model with capital, flexible wages and low adjustment costs of capital, suggest that this is not only a theoretical possibility.

asset market if bonds were fixed at their steady-state level B_{ss},

$$S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, r_l^{a,Crowding}\}_{l\geq 0}\big) = K_{t+1}^{Crowd} + B_{ss}/P_{t+1} + \Phi(K_{t+1}^{Crowd}, K_t^{Crowd}), \tag{45}$$

such that the difference in capital stocks,

$$K_{t+1} - K_{t+1}^{Crowd}, \tag{46}$$

is the effect of crowding out.

The asset market clearing condition also has a supply side. Households have to be willing to increase their savings. We decompose the change in aggregate savings into two channels. The same redistributive forces that affect consumption behavior in turn affect the savings behavior of households, the redistributive channel of savings. The second channel describes the effect of higher real interest rates on savings. The decomposition of aggregate savings is

$$\begin{aligned}
(\Delta S)_{t+1} &= S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, r_l^a\}_{l\geq 0}\big) - S_{t+1}\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{r}^a\}_{l\geq 0}\big) && (47)\\
&= \underbrace{S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, \bar{r}^a\}_{l\geq 0}\big) - S_{t+1}\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{r}^a\}_{l\geq 0}\big)}_{\text{Distributional Impact}} && (48)\\
&+ \underbrace{S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, r_l^a\}_{l\geq 0}\big) - S_{t+1}\big(\{T_l, \tau_l, w_l h_l, P_l, \bar{r}^a\}_{l\geq 0}\big)}_{\text{Interest Rate Impact}}. && (49)
\end{aligned}$$

3.1.4 Multiplier: Definition

We now follow Farhi and Werning (2016) in computing the response of the economy to a fiscal stimulus. Concretely, we compute the response of the economy to an unexpected increase in the path of nominal government spending to $G_0, G_1, G_2, \ldots, G_t, \ldots, G_{ss}$, where G_{ss} is the steady-state nominal spending level and $G_t \geq G_{ss}$.

We summarize the effects of spending on output in several ways. The government spending elasticity of date t real output equals

$$\frac{\frac{Y_t - Y_{ss}}{Y_{ss}}}{\frac{\frac{G_0}{P_0} - \frac{G_{ss}}{P_{ss}}}{\frac{G_{ss}}{P_{ss}}}}, \tag{50}$$

where P_{ss}, G_{ss}, Y_{ss} are the steady state price level, nominal spending and real output respectively and $\frac{G_0}{P_0}$ is real government spending at date $t = 0$.

We multiply this elasticity by $\frac{Y_{ss}}{G_{ss}/P_{ss}}$ to convert it to dollar equivalents, which yields the path of the incomplete markets multipliers as the sequence of

$$m_t^{IM} = \frac{\frac{Y_t}{Y_{ss}} - 1}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}}. \tag{51}$$

We also compute the present value multipliers for a discount factor $\tilde{\beta}$ as

$$m_t^{PV} = \frac{\sum_{k=0}^{t} \tilde{\beta}^k (\frac{Y_k}{Y_{ss}} - 1)}{\sum_{k=0}^{t} \tilde{\beta}^k (\frac{G_k P_{ss}}{P_k G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \tag{52}$$

where the two statistics coincide at $t = 0$ and represent the impact multiplier. A useful statistic is then the cumulative multiplier, which represents the discounted percentage change in real output relative to the discounted percentage change in real government spending for any path of government spending:

$$\overline{M} = m_\infty^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t (\frac{Y_t}{Y_{ss}} - 1)}{\sum_{t=0}^{\infty} \tilde{\beta}^t (\frac{G_t P_{ss}}{P_t G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}. \tag{53}$$

For comparison with the complete markets case, we also compute the as-if dynamic complete markets multiplier, m_t^{CM}. The objective is not to compute the equilibrium of the complete markets model, but to use the paths of r^a, w_t and r_t obtained from our incomplete markets model to derive the full consumption and investment paths that would have arisen in a complete-markets model given those prices. That allows us to isolate the intertemporal substitution effect on the multiplier as the distribution effect is not present in the complete markets model. Computing the as-if consumption path is straightforward since intertemporal substitution is the only channel operating as we discussed in Section 3.1.1 and we use the incomplete markets path of real interest rates. Following the line of arguments in that section and iterating the consumption Euler equation using the real return $1 + r_{t+1}^a$ on households' assets yields the as-if percentage response of aggregate consumption,

$$\frac{C_t^{CM}}{C_{ss}} - 1 = \prod_{s=t}^{\infty} \frac{1 + r_{ss}^a}{1 + r_{t+1}^a} - 1.$$

To compute the as-if investment path we use the as-if path of consumption, wages w_t and the real return on bonds $1+r_{t+1}$ as inputs to solve for the paths of K_{t+1} as well as of $1+r_t^k$ and H_t.

To computese the paths requires solving a system of three equations in each time period: the cost-minimization condition (11), the mutual fund first-order condition (13) and the resource constraint (20). Jointly solving this system of equations delivers a path of investment, I_t^{CM}, hours, H_t^{CM} and the return on capital r_t^k.

Since the multiplier is in terms of units of consumption and not in percentages, adjusting for the magnitudes of steady state consumption, output and government spending,

$$\begin{aligned} m_t^{CM} &= \frac{\frac{C_t^{CM}-C_{ss}}{C_{ss}}\frac{C_{ss}}{Y_{ss}} + \frac{I_t^{CM}-I_{ss}}{I_{ss}}\frac{I_{ss}}{Y_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{Y_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}} \\ &= \frac{\frac{C_t^{CM}-C_{ss}}{C_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{C_{ss}}{G_{ss}/P_{ss}} + \frac{\frac{I_t^{CM}-I_{ss}}{I_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{I_{ss}}{G_{ss}/P_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{G_0/P_0 - G_{ss}/P_{ss}}. \end{aligned}$$

The as-if complete markets multiplier also allows us to decompose the incomplete markets multiplier. The incomplete markets multiplier combines three channels – the investment, the intertemporal substitution and the distributional one – while the as-if complete markets multiplier captures only the first two channels but shuts down the distributional one. This allows us to interpret the difference between these two multipliers as the contribution of the distributional channel,

$$m_t^{Distribution} = m_t^{IM} - m_t^{CM}, \tag{54}$$

and the as-if multiplier as the contribution of the intertemporal substitution and the investment channel.

We proceed similarly for a comparison with a Two Agent New Keynesian (TANK) model, that combines a permanent income household with a hand-to-mouth household. The permanent income household behaves like a representative agent so that the full consumption path can be computed as above using the consumption Euler equation. The consumption of hand-to-mouth households equals $(1-\tau)w_t h_t + T_t$, where we import w_t and T_t from our incomplete markets model. We assume that the demand for hours h_t is the same for all households. The paths of investment and hours are computed as described above in the representative agent economy. Combining consumption of both groups yields an aggregate consumption path which we use to compute the as-if TANK multiplier path, m_t^{TANK}.

3.2 Calibration

To quantitatively assess the size of the fiscal multiplier, we now calibrate the model.

Preferences Households have separable preferences over labor and constant relative risk aversion preferences for consumption. We set the risk-aversion parameter, σ, equal to 1. Following Krueger et al. (2016), we assume permanent discount factor heterogeneity across agents. We allow for two values of the discount factor, which we choose along with the relative proportions to match the Gini of net worth net of home equity, the ratio of median and 30th percentile of net worth net of home equity in the 2013 SCF, and the ratio of aggregate savings net of home equity to quarterly GDP of 11.46.[11] This allows us to capture the overall level of wealth in the economy and important distributional moments. We assume the functional form for the distulity of labor v:

$$v(h) = \psi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \tag{55}$$

We set the Frisch elasticity, $\varphi = 0.5$, following micro estimates. We choose $\psi = 0.6$ so that in steady state $h = 1/3$.

Productivity Process Krueger et al. (2016) use data from the Panel Study of Income Dynamics to estimate a stochastic process for labor productivity. They find that log income consists of a persistent and a transitory component. They estimate that the persistent shock has an annual persistence of 0.9695 and a variance of innovations of 0.0384. The transitory shock is estimated to have variance 0.0522. We follow Krueger et al. (2016) in converting these annual estimates into a quarterly process. We discretize the persistent shock into a seven state Markov chain using the Rouwenhorst method and integrate over the transitory shock using Gauss-Hermite quadrature with three nodes.

Production Technology We set the capital share $\alpha = 0.36$. We choose the quarterly depreciation rate $\delta = 0.032$ to generate the same real return on capital net of depreciation

[11] We calibrate to a capital to quarterly output ratio of 10.26, and government debt to quarterly GDP ratio of 1.2.

and on bonds when the capital output ratio is 10.26. We assume the functional form for Φ:

$$\Phi(K', K) = \frac{\phi_k}{2}\left(\frac{K' - K}{K}\right)^2 K, \tag{56}$$

and set $\phi_k = 17$ to match estimates of the elasticity of investment to Tobin's q from Eberly et al. (2008). We choose the elasticity of substitution between intermediate goods, $\epsilon = 10$, to match an average markup of 10%. The adjustment cost parameter on prices and wages is set to $\theta = \theta_w = 300$ to match the slope of the NK Philips curve, $\epsilon/\theta = 0.03$, in Christiano et al. (2011). We set the firm operating cost F equal to the steady state markup such that steady state profits equal zero (Basu and Fernald, 1997). Profits, which are nonzero only off steady state, are fully taxed and are distributed to households as lump-sum transfers in the benchmark.

Government We set the proportional labor income tax τ equal to 25%, and the dividend tax τ_k equal to 36% (Trabandt and Uhlig, 2011). We set nominal government spending G in steady state equal to 6% of output (Brinca et al., 2016). The value of of lump-sum transfers T is set to 8.55% of output such that roughly 40% of households receive a net transfer from the government (Kaplan et al., 2018). This generates a government debt to quarterly GDP ratio of 1.2 in steady state.

Monetary Policy As discussed above, for the benchmark specification we assume that the monetary authority operates a constant interest rate peg of $i = 0$. For purposes of comparison, we will also consider a specification where we assume that the monetary authority follows a Taylor rule, which sets the nominal interest rate according to:

$$i_{t+1} = \max(X_{t+1}, 0), \tag{57}$$

where

$$X_{t+1} = \left(\frac{1}{\zeta}\right)\left(\frac{P_t}{P_{ss}}\right)^{\phi_1(1-\rho_R)}\left(\frac{Y_t}{Y_{ss}}\right)^{\phi_2(1-\rho_R)}[\zeta(1+i_t)]^{\rho_R} - 1.$$

We follow the literature in setting $\rho_R = 0.8$, $\phi_1 = 0.5$, $\phi_2 = 0.25$ and $\zeta = 1/(1 + r_{ss})$.

Steady State Model Fit In the calibrated model 3% of agents have 0 wealth, and 10% of agents have less than $\$1,000$. The wealth to annual income ratio is 4.9. The annual MPC out of transitory income equals 0.4, which is in the middle range of empirical estimates 0.2-0.6 (e.g., Johnson et al., 2006).[12]

3.3 Households' Demand and Marginal Propensity to Consume

Before moving to the full general equilibrium analysis, we provide several partial equilibrium consumption results to build intuition for our general equilibrium findings and to show that our incomplete markets model is closer to the empirical findings in the micro consumption literature than standard frameworks. In all of the following experiments, we consider standard household consumption/savings problems for prices fixed at their steady-state values. To illustrate the properties of the MPC in our model, the experiments differ in the timing and the amount of transfers households receive.

Each of the four panels in Figure 2 plots four separate experiments, where each line corresponds to the aggregate consumption path in response to finding out at date 0 that all households will receive a transfer either at date 0, or at date 4, or at date 8, or at date 12 without an obligation to ever repay. The four panels differ in the size of the transfer received, either \$10, \$100, \$1,000 or \$10,000.

Consider first the experiment of giving a household \$10 today. A permanent income household would save basically all of the money and consume a small fraction. In our model households face idiosyncratic income risk, inducing a desire to smooth income but also credit constraints, preventing perfect smoothing. A borrowing constrained household would consume the full \$10. Unconstrained, but low asset households will also consume a large fraction of the transfer because it relaxes precautionary savings motives. These arguments together imply an initial MPC significantly larger than in complete markets models. The fraction of the transfers not spent in the initial period is spent in the following periods at a decaying rate. If households receive larger transfers, the initial MPC falls, mainly because larger transfers are more likely to relax the credit constraints. For example, a \$10,000 transfer is sufficient to relax all households' credit constraints, so that even the borrowing constrained household will

[12] We compute the annual MPC using the quarterly MPC via the formula: $MPC_a = 1 - (1 - MPC_q)^4$. The quarterly MPC in the model is 0.12.

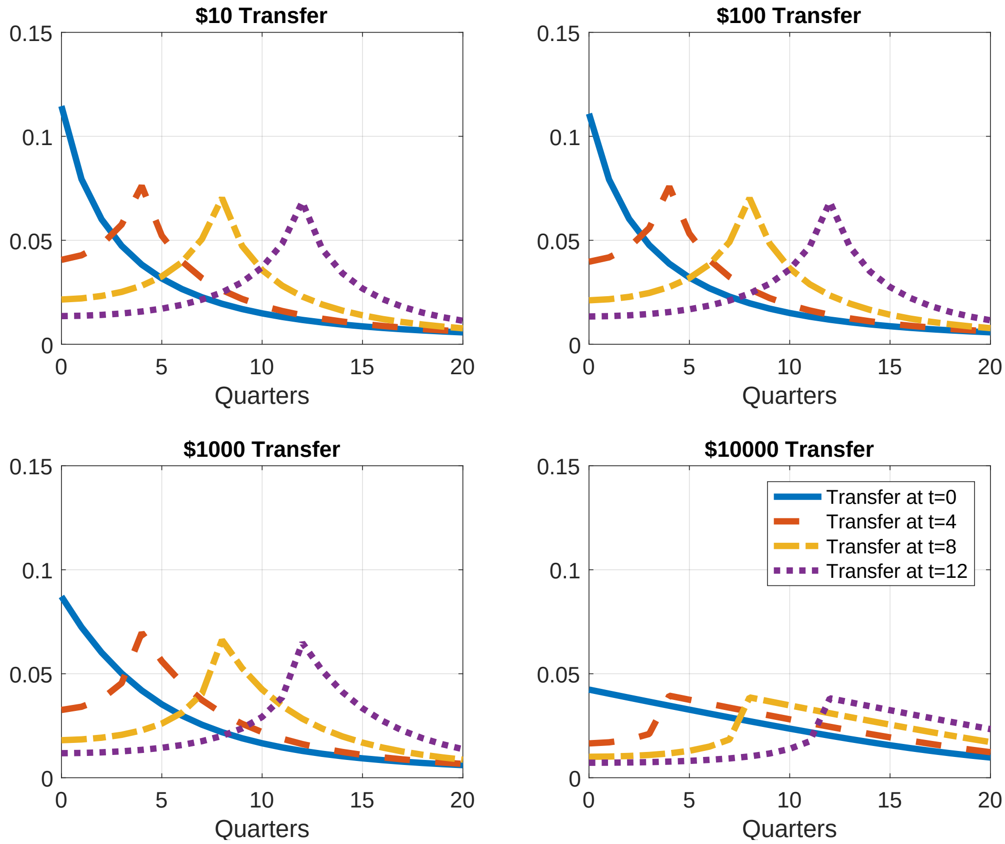


Figure 2: Propensity to Consume for Transfers of Size $10, $100 $1,000 and $10,000.

not consume the full transfer. Now, suppose households do not receive the transfer today but only learn today that they will receive a transfer in a future period, at date 4, 8, or 12. The credit constrained households cannot respond until the transfer arrives. The unconstrained households are able to smooth consumption, but their MPC is lower, so the initial rise in consumption is smaller than before.

These model properties are consistent with the data but inconsistent with a complete markets RANK model.[13] They are also inconsistent with a TANK model which deviates from the RANK model by introducing two types of agents – a permanent income household and a hand-to-mouth household. The TANK model can match high MPCs and is theoretically tractable. However, as shown in Figure 3, when we replicate the same experiment in a TANK model, instead of tent-shaped impulse responses we obtain in our HANK model, the TANK model delivers spiky responses. The TANK model also misses out on the sensitivity to the size of the transfer and all of the dynamic anticipation and propagation effects because the response

[13] For a discussion on the empirical evidence on intertemporal MPCs see Auclert et al. (2018).

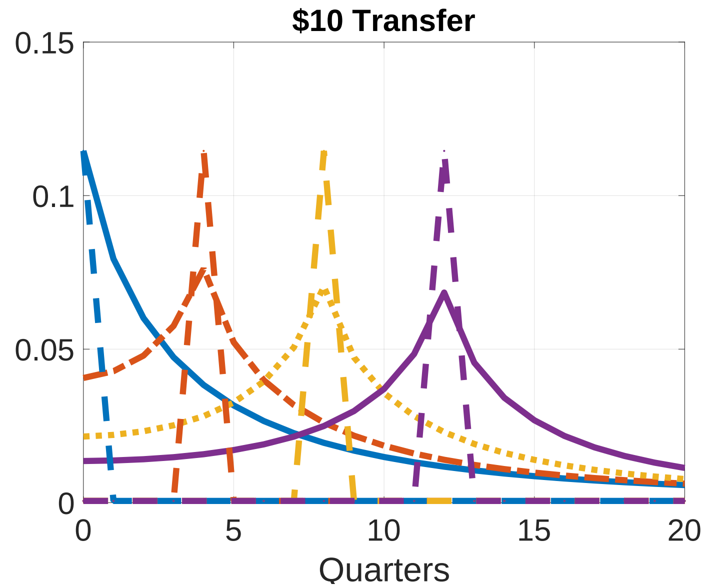


Figure 3: Propensity to Consume for Transfers of Size 10$, HANK vs TANK.

of permanent income households is minimal and consumption of hand-to-mouth households responds to current income only. To understand why this is important we need to think about how the general equilibrium Keynesian cross multiplier logic works.

As we saw, if a household receives a transfer in the first period, it is spent over all future periods. In general equilibrium, that would mean an increase in aggregate demand and also in income not only today but also in all future periods. Consumption today increases not only because of the increase in income today but also because of the increases in future income by relaxing precautionary savings motives. To illustrate this point we now combine the four previous experiments, so that households learn in period 0 that they will receive a transfer at *all* dates 0, 4, 8 and 12. Figure 4 shows that now the impact response of consumption is nearly twice as high, even though in period 0 the same transfer is received. This is a partial equilibrium example, but in general equilibrium the demand and income increases at different times reinforce each other, generating what Auclert et al. (2018) have coined as an intertemporal keynesian cross. The TANK economy fails to reproduce the dynamic multiplier path because the anticipation and propagation effects in that model are basically zero.

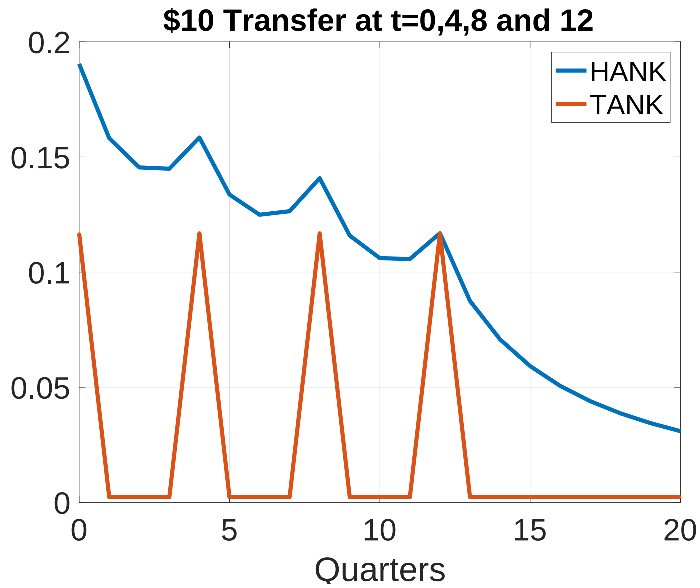


Figure 4: Propensity to Consume for Transfers of Size 10$ that are Received in periods 0, 4, 8 and 12, HANK vs TANK.

3.4 Results

We can now compute the equilibrium response of prices, employment, output and consumption to a persistent increase in nominal government spending for 20 quarters. Specifically, we assume that spending increases by one percent in period t and after that the amount of additional spending decays at rate $1-\rho_g$ per quarter for the subsequent 19 quarters. We set $\rho_g = 0.9$ in the baseline experiments (implying a half-life of 7 quarters) and vary this parameter later. Balancing the government budget when government spending is increased requires to adjust taxes or debt or both. As Ricardian equivalence does not hold in our model, different assumptions on the path of taxes and debt will have different implications for the path of aggregate consumption and therefore prices and the change in output. We consider two benchmark scenarios:

1. Transfer are adjusted period by period to keep nominal debt constant.
2. Deficit financing and delayed reduction of transfers to pay back debt after 40 quarters.

We will also consider a third scenario,

3. Deficit financing and delayed cuts in government expenditure after 40 quarters.

For each of these scenarios, we report the dynamic response of hours, consumption, output, prices, tax revenue and debt as well as the paths of incomplete markets multiplier m_t^{IM}, of

Table I: Main Results Consumption and Multipliers

Experiment:	Benchmark		Taylor Rule		Forward	G-Financed	Transfer
Financing:	Tax	Deficit	Tax	Deficit	Deficit	Deficit	Deficit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Impact Mult.	0.61	1.34	0.54	0.66	0.80	1.30	0.66
Cumul Mult.	0.43	0.55	0.40	0.29	0.17	-0.15	0.1
$100 \times \Delta C_0$	-2.7	1.4	-2.9	-0.4	0.2	1.5	5.2
$100 \times \Delta I_0$	0.3	0.6	-0.02	-1.6	-1.4	0.3	0.4
Decomposition of Consumption ($100 \times \Delta$)							
Direct G on C	1.2	1.2	1.2	1.2	0.9	1.1	0.0
Tax/Transfers	-3.1	0.5	-3.1	-0.2	0.4	0.7	4.6
Indirect Income	-0.7	0.2	-0.8	-0.7	-0.2	0.2	1.1
Prices	-0.1	-0.5	-0.2	-0.8	-0.9	-0.6	-0.5

Note - The table contains the impact and the cumulative multiplier $\overline{M}$ (using definition (58) for the last column and (59) otherwise) as well as the initial consumption and investment responses, ΔC_0 and ΔI_0 (as a % of output). The last four rows show the decomposition of the initial aggregate consumption response (also multiplied by 100) into the direct G impact on C (Eq. 38), the effect of taxes/transfers (Eq. 39), indirect income effects (Eq. 40) and the price and interest rate effects (Eq. 41).

the as-if complete markets multiplier m_t^{CM} and of the as-if TANK multiplier m_t^{TANK}. The cumulative multiplier $\overline{M}$ and various additional statistics across the experiments we conduct are collected in Table I.

3.4.1 Tax Financing: Constant Nominal Debt

Under the first financing scheme, we assume that the government adjusts lump-sum transfers period by period to keep the level of nominal debt constant. The four panels of Figure 5 show the results for the private consumption and output response, the different multipliers, the decomposition of private consumption, and the evolution of government bonds.

The bottom right panel plotting the efolution of fiscal policy illustrates that the level of nominal government bonds is unchanged since the stimulus is tax-financed. The top left panel shows that on impact G increases by 1% (0.06% of output) and consumption decreases by 0.027% of output. As illustrated in the top right panel, this leads to an impact multiplier of

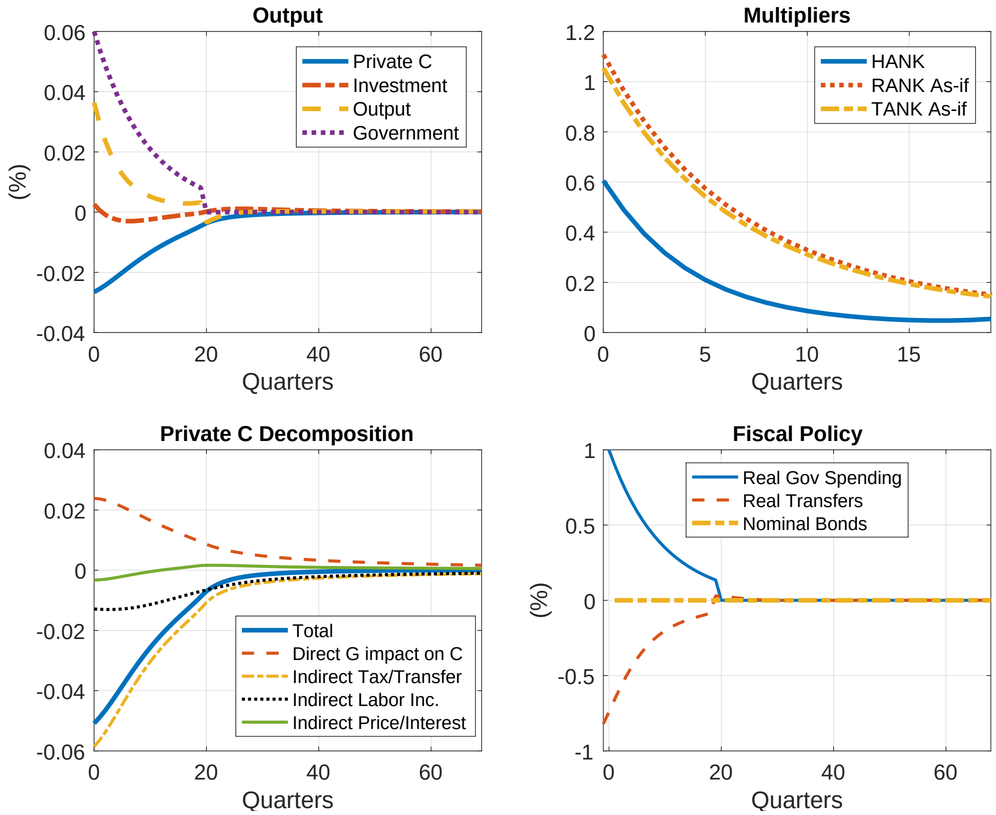


Figure 5: Fiscal Multiplier and Aggregate Consumption: Tax Financing

0.61. The incomplete markets multiplier converges to zero since the consumption response, although negative, slowly dies out and becomes small relative to the initial government spending increase. The decomposition of the consumption response in the bottom left panel reveals the quantitative importance of the direct, the indirect and the price effects. The stimulus of 0.06% directly increases households' labor supply by the same amount, leading to a aggregate consumption increase of 0.012% of output (Eq. 38). The contemporaneous cut in transfers lowers aggregate consumption by -0.031% on impact (Eq. 39), implying a total initial negative effect of -0.018%. This effect is negative since the government spending increases household's income proportional to their productivity and thus benefits high income households more whereas the transfer cut is uniform across all income groups and thus negatively affects high MPC households. This decrease leads to lower consumption demand, which in turn leads to lower labor demand, lower labor income and again lower consumption demand until an equilibrium is reached. These indirect income effects sum up to -0.007% (Eq. 40) further lowering the consumption response. Finally, the decomposition shows that the price increase and (the unchanged) interest rate effects are small (Eq. 41). The investment channel is also contributing very little.

Whereas the incomplete markets multiplier is significantly below one, $m_0^{IM} = 0.61$, the as-if multiplier in the complete markets model is, $m_0^{CM} = 1.11$, so that the contribution of the distributional channel (Eq. 54), $m_0^{Distribution} = -0.5$. The results is unsurprising given the foregoing discussion, as the permanent income representative household responds little to the cut in transfers. We find similar results for the TANK economy with $m_0^{TANK} = 1.05$ and a negative distributional channel of -0.44. The as-if multiplier decreases slightly, due to the higher average MPC in the TANK economy relative to RANK. However, while the TANK and our HANK economies feature the same quarterly MPC, the large difference between them is due to dynamic anticipation effects arising in the HANK model that are absent in TANK. The hand-to-mouth agents in TANK respond only to the contemporaneous decrease in transfers, but not to the anticipated cut in future transfers (as occurs in our HANK economy), leading to a much smaller drop in private consumption demand. The larger drop in aggregate household consumption demand in the incomplete markets model then translates through the Keynesian cross logic into a smaller general equilibrium multiplier of 0.61 relative to 1.05.

The impulse response of the remaining variables to a 1% innovation in government spending are plotted in Figure A-1 in the appendix. The cumulative multiplier, in Table I, is 0.43.

3.4.2 Deficit Financing

Under this scenario we assume that real transfers are kept constant during the first 40 quarters after the innovation to government spending. Then, the government is assumed to adjust transfers linearly over eight quarters, keep them constant for eight quarters, and then allow transfers to revert back to the real steady state level with an autocorrelation parameter of 0.8. Thus, under this timing scheme, the government chooses only the level of adjustment to transfers to guarantee that nominal government debt returns to its original level. The four panels of Figure 6 summarize the evolution of the key variables of interest.

Deficit instead of tax financing increases the initial multiplier m_0^{CM} from 0.61 to 1.34 and the initial aggregate consumption response from -0.027% to 0.014%. The decomposition of the consumption response makes clear why. The direct impact of the spending stimulus is identical (0.012%) but now there is no initial offsetting effect from contemporaneously higher taxes/lower transfers. The indirect income effects now accumulate to 0.002% and the

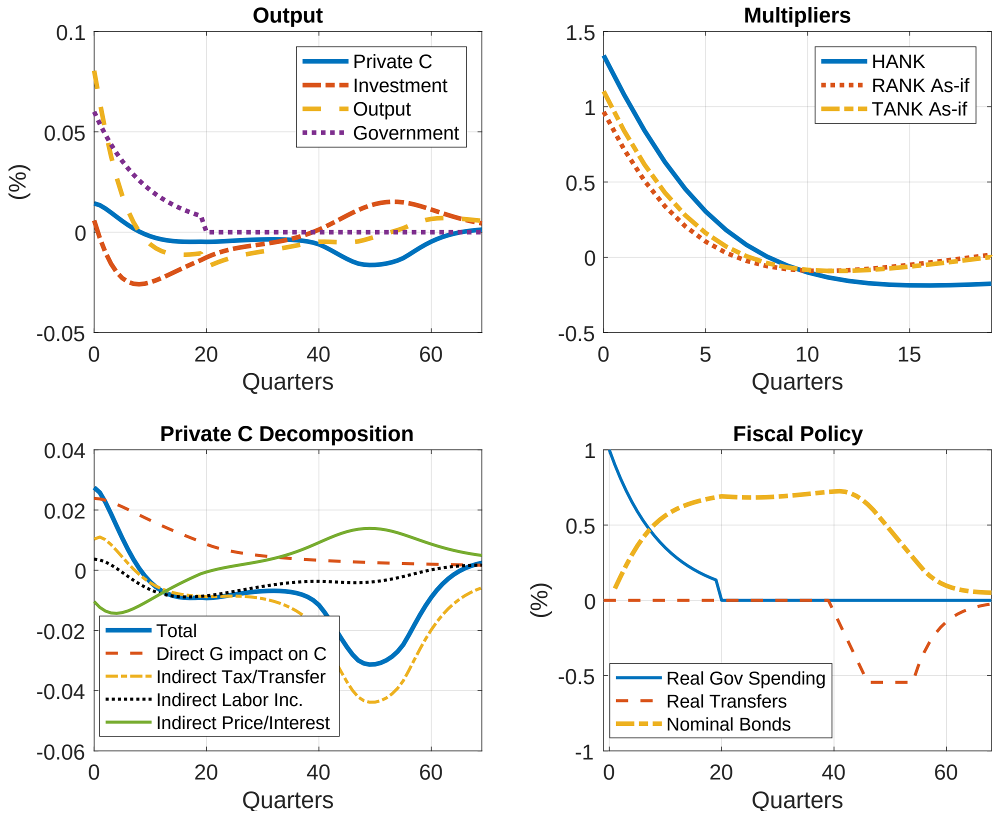


Figure 6: Fiscal Multiplier and Aggregate Consumption: Deficit Financing

deficit financing leads to an increase in government bonds and the consumption response becomes negative only from period 9 onwards. However, the increase in government spending is ultimately financed through a future reduction in transfers, which results in a contraction in future output. Thus, the cumulative multiplier, reported in Table I is only 0.55, significantly smaller than the impact multiplier. The impulse responses of the remaining variables are plotted in Figure A-2 in the appendix.

Since the as-if multipliers $m_0^{CM} = 0.97$ and $m_0^{TANK} = 1.10$ are still close to one, the contribution of the distributional channel, is now positive $m_0^{Distribution} = 0.37$ for complete markets and 0.24 for the TANK model. The fiscal expansion increases employment and thus labor income but because of deficit financing there is no offsetting effect through higher taxes. This benefits high MPC households, who primarily rely on labor income and not asset income, and allows them to increase consumption demand. The additional government debt on the other hand is bought by low MPC households which reduce their consumption. In total, this redistribution towards high MPC households increases aggregate consumption demand, implying $m_0^{Distribution} > 0$.

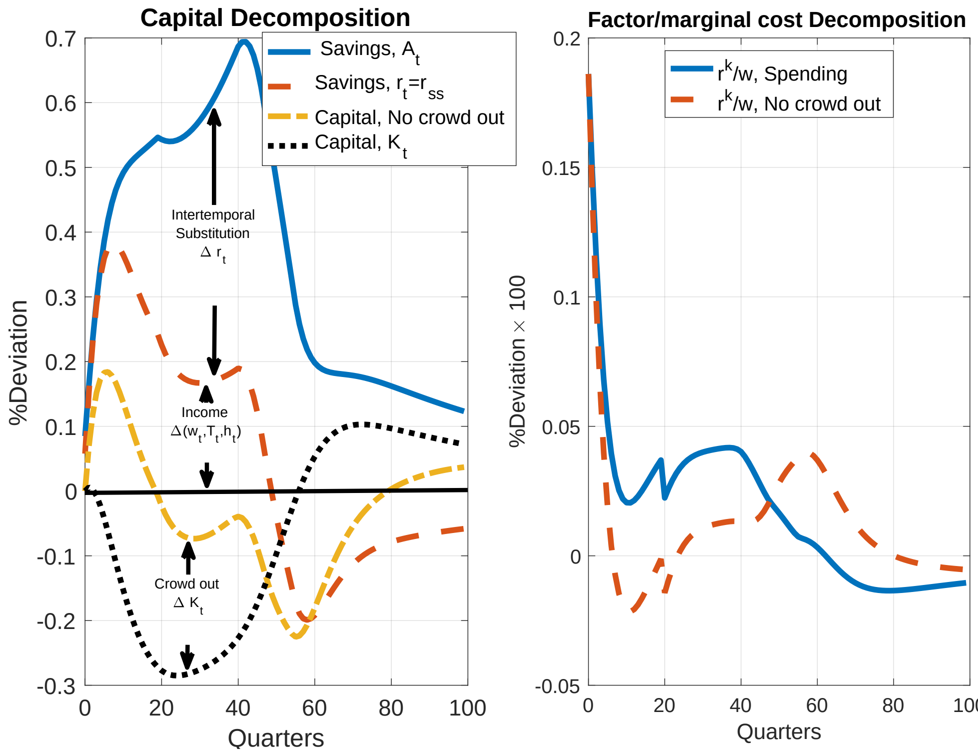


Figure 7: Fiscal Multiplier and Aggregate Investment: Deficit Financing

Another difference between deficit and tax financing is a negative impact of investment demand on the multiplier with deficit financing. Figure 7 illustrates why this is the case. The right panel shows that r^k/w increases, making firms substitute from capital towards labor. The left panel shows that total savings increase (the blue line "Savings, A_t") by more than they would have increased if the real interest rate r^a were to remain constant (the red line "Savings, $r_t = r_{ss}$"), reflecting the "Interest Rate Impact" on savings (Eq. 49). The red line is however not zero, indicating that savings would have increased due to the distributional effects even if the real interest rate did not change. This "Distributional Impact" on savings (Eq. 48) is reflected by the difference between the red line and the solid black horizontal line through zero.

But while total Savings increase, capital still falls below its steady-state level since the increase in savings is smaller than the increase in additional government debt, $K - K^{Crowd} < 0$ (Eq. 46). This crowding out of capital (the difference between the yellow "Capital, No crowd out" and the black dotted line "Capital, K_t") with deficit financing explains the drop in investment and its negative contribution to aggregate demand and the multiplier. The right panel also shows that r^k/w in the counterfactual crowding out experiment where debt is held constant (Eq. 45) is lower than in the benchmark, since aggregate savings have to increase less as no additional debt needs to be absorbed in this experiment.

3.5 Further Analysis

We now extend the analysis in various directions. First, we use a Taylor interest rate rule to describe monetary policy instead of a fixed nominal interest rate in Section 3.5.1. While the literature has primarily focused on studying the effects of an increase in government spending, another commonly used stimulus policy is to increase transfers. Our framework allows us to study the effects of such policies and we do so in Section 3.5.2. Following that, we return to considering spending multipliers and ask how the size of the fiscal multiplier depends on the timing of spending ("forward spending"), on the scale of spending and on the persistence of the stimulus in Sections 3.5.3, 3.5.4 and 3.5.5. In Section 3.5.6 we assess the consequences of cutting government spending G instead of transfers to pay back the debt. Finally, we consider spending and transfer policies in a liquidity trap in Sections 3.5.7 and 3.5.8. We also investigate how the size of the multiplier depends on the degree of price and wage rigidities.

3.5.1 Taylor Rule

We now measure the fiscal multiplier when monetary policy follows a Taylor rule instead of an interest rate peg. Figures 8 and A-3 summarize the main results and the impulse responses, respectively, for deficit-financed spending. Figures 9 and A-4 do so for tax-financed spending.

We find that if monetary policy follows a Taylor rule, the impact multiplier with deficit financing drops to 0.66 and the cumulative multiplier to 0.29. The lower multiplier is a result of a drop in output due to tighter monetary policy. A higher nominal interest rate contracts consumption demand and thus reduces income, which in turn reduces savings and investment. Lower investment reduces income and thus contracts consumption which again reduces income and so on. The consumption and investment responses reinforce each other and together determine an effect of monetary policy on output in our experiment: a 25 base point increase in the real rate (i.e. annually 100 base points) leads on impact to a 1% drop in output. This is an expected magnitude for the drop in output in response to an increase in the real return on bonds and is in line with the findings in e.g., Kaplan et al. (2018). Combining the expansionary output effects of the fiscal stimulus and the contractionary effects of monetary policy yields a

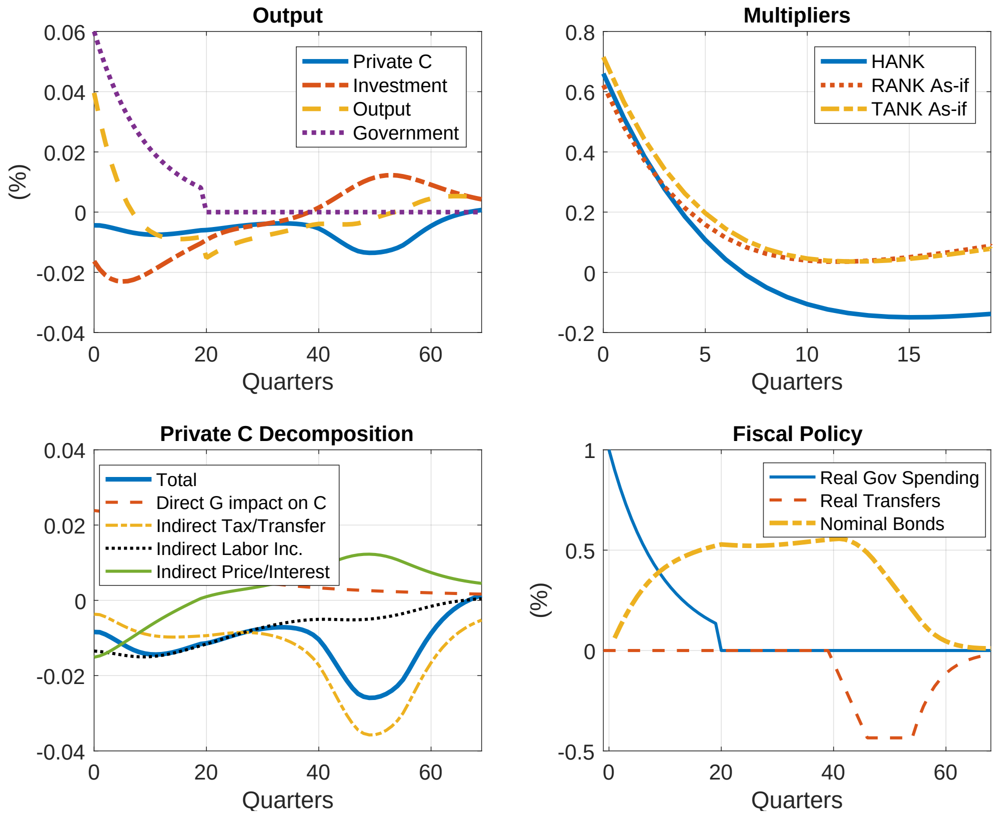


Figure 8: Fiscal Multiplier and Aggregate Consumption: Taylor Rule and Deficit Financing

multiplier of 0.66.

The same arguments apply to tax instead of deficit financing, in which case the multiplier falls to 0.54. The Taylor rule induced decline in the size of the multiplier is smaller with tax than with deficit financing not because monetary policy is less effective but because monetary policy responds less if tax financing is used. Indeed, the real interest rate elasticity of output is unchanged but the smaller price responses with tax financing require a much smaller increase in interest rates.

As a result, while the impact multipliers are very different between financing schemes with fixed nominal interest rates, they become quite similar when a Taylor rule is used. With deficit financing, fiscal stimulus leads to a strong response of monetary policy, which largely undoes the stimulus. With tax financing, fiscal policy is not very stimulative to begin with and monetary policy remains largely unchanged.

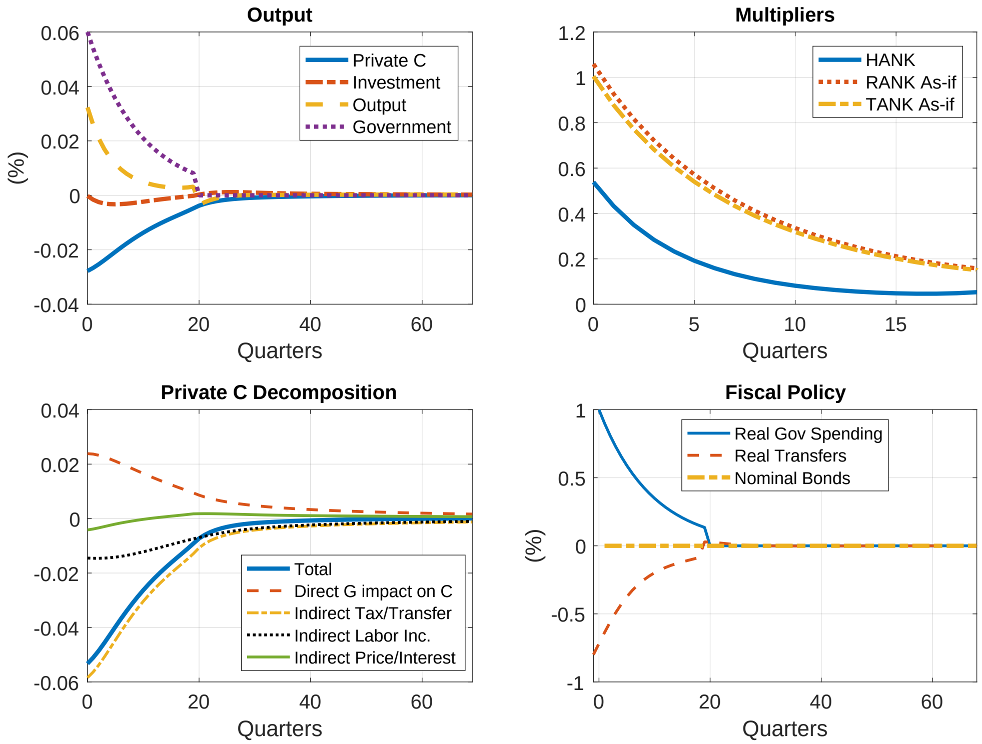


Figure 9: Fiscal Multiplier and Aggregate Consumption: Taylor Rule and Tax Financing

3.5.2 Transfer Multiplier

In this section we consider the multiplier in response to a one percent increase in government *transfers*, again for 20 periods and with persistence $\rho_T = 0.9$. We assume that nominal government spending adjusts to keep real government spending constant in response to the innovation in transfers. We allow the government to deficit-finance the increase in transfers following the same financing scheme as above by first increasing government debt, then by decreasing transfers starting 40 quarters later to pay back the debt.

The impulse responses plotted in Figure A-5 are qualitatively and quantitatively similar to the impulse responses to a deficit-financed increase in government spending. Private consumption rises more, however, when transfers increase than when spending increases. This can be understood because, an increase in transfers benefits low income, high MPC households since their disposable income increases more (in percentage terms) than that of high income, low MPC households. This implies an impact multiplier of 0.66 using the definition (51) of m_0^{IM} but with G replaced by T. Aggregate consumption, the sum of private and government consumption, however, increases less when transfers increase than when spending increases,

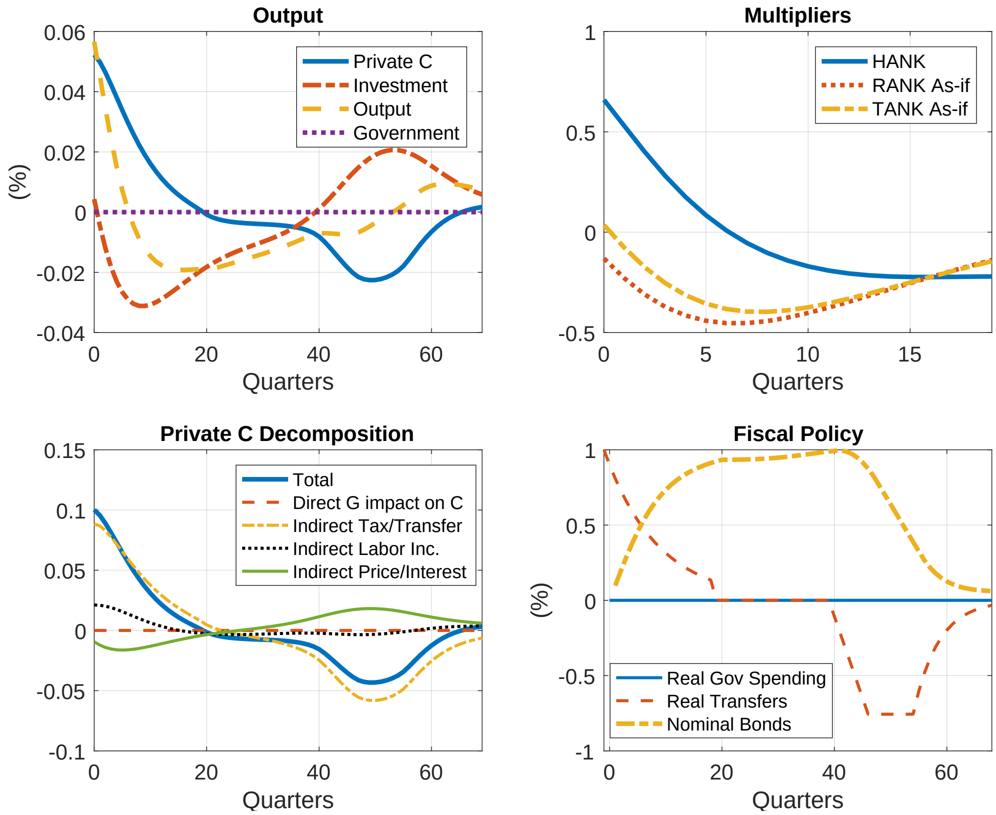


Figure 10: Transfer Multipliers (Deficit Financed)

simply because government spending is unchanged in the former case. As a result, income increases by more in the latter case, implying higher savings and investment such that the output increase is larger.

The cumulative multiplier is now defined as

$$\overline{M} = m_{\infty}^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t (\frac{Y_t}{Y_{ss}} - 1)}{\sum_{t=0}^{19} \tilde{\beta}^t (\frac{T_t P_{ss}}{P_t T_{ss}} - 1)} \frac{Y_{ss}}{T_{ss}/P_{ss}}, \tag{58}$$

where in the denominator we sum only over the first 20 periods since summing to infinity would take the repayment period into account and thus would render the denominator equal or close to zero. Note that this definition does not change the numbers for the cumulative multiplier reported for other experiments. The cumulative multiplier ends up being quite small around 0.1 because the future decrease in transfers needed to return nominal government debt to its steady state level is sufficiently contractionary to almost offset the contemporaneous gains. Measuring the welfare implications of this policy, we find that welfare falls by -0.00067% in consumption equivalents because the welfare gain in the initial periods is eventually (slightly) outweighed by the later welfare losses.[14]

Increasing transfers is purely redistributional and thus has no direct effects in the RANK model. The as-if RANK multiplier is, however, negative due to an increase in real interest rates. The TANK model features a redistributional channel, implying a larger multiplier than in the RANK case. The TANK multiplier is much smaller than our incomplete markets model's one since the TANK model lacks the anticipation and propagation effects operating in a HANK model as we explained in Section 3.3. Figure 10 shows the results.

3.5.3 Forward Spending

Farhi and Werning (2016) show that in complete markets New Keynesian models the further the spending is in the future the larger is the impact, suggesting that "forward-spending" is an effective fiscal policy tool. In contrast, our analysis implies that the multiplier becomes smaller if the spending is pre-announced to occur at a future date. For concreteness, in Figure 11 we assess the effects of stimulus pre-announced to occur 4 quarters ahead (the corresponding impulse responses are plotted in Figure A-6 in the appendix). The additional spending is deficit financed. The price level now increases gradually in anticipation of the future increase in government spending implying that initially output and consumption fall before increasing at the time of the spending increase 4 quarters in the future. However, the initial drop in output makes households shift consumption to these earlier periods which dampens the fall in consumption initially but at the same time lowers their demand at the time of the actual spending increase in quarter 4. As a result, the increase in consumption as well as the multiplier at that time are smaller than the corresponding multiplier in the case when the stimulus occurs immediately and is deficit financed. The investment response is, however, larger since debt only starts to increase when spending actually happens, implying that there is no crowding out and thus higher investment in the first periods with future spending.

[14]For completeness we report the welfare losses for an increase in government spending: -0.0065% for deficit financing and -0.0063% for tax financing. Note that these numbers set the welfare gains from higher government consumption to zero and one should therefore be cautious about using these numbers to assess the welfare consequences of a fiscal stimulus.

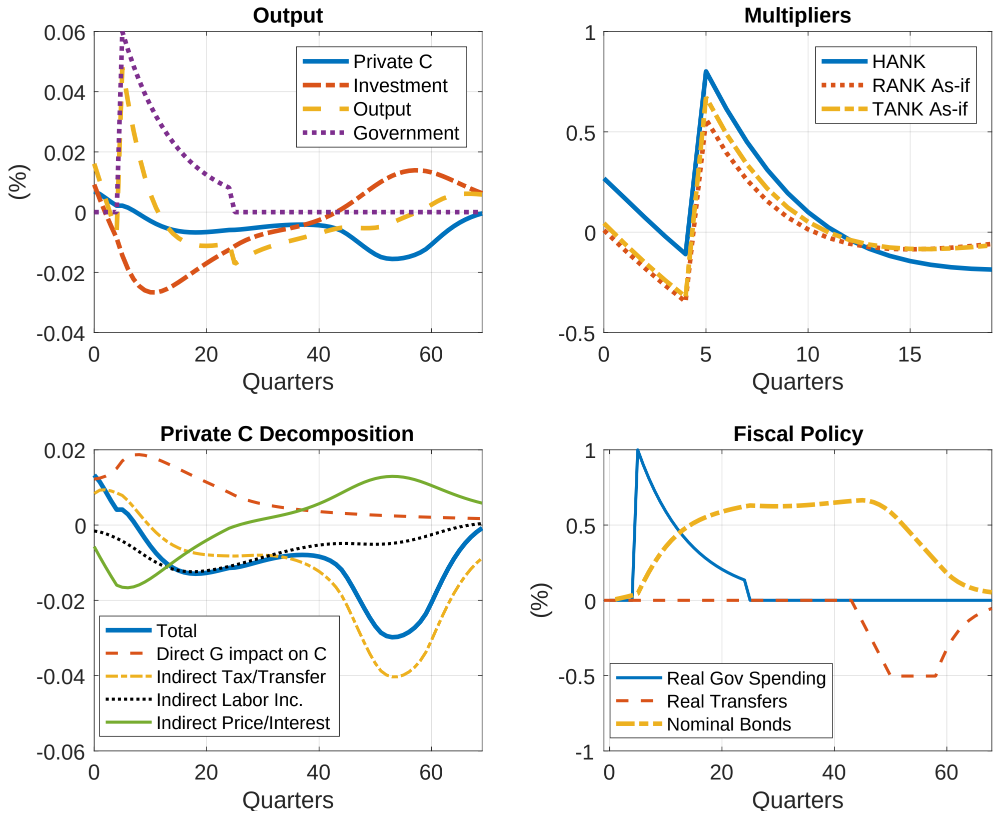


Figure 11: Future (4 Quarters Ahead) Spending: Deficit Financing

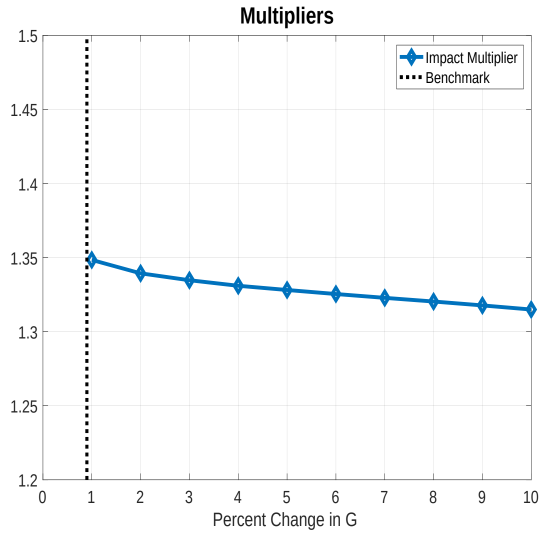
3.5.4 Scale Effects

We now assess the dependence of the size of the multiplier on the scale of the government spending or transfer stimulus. Panel (a) of Figure 12 shows the government spending multiplier for one percent intervals between 1% and 10% increase. Panel (b) of Figure 12 shows the same for the transfer multiplier again for one percent intervals between 1% and 10% increases.

The fall in the multiplier as we increase the scale of spending or transfers is expected in light of our results in Figure 2 in Section 3.3. Those partial equilibrium experiments showed that the propensity to consume falls in the size of the transfer households receive, implying that a larger scale of spending or transfer leads to lower MPCs. The Keynesian cross logic implies that in equilibrium these lower MPCs translate into a lower multiplier.

3.5.5 More Persistent Spending

In this section we study how the persistence of government spending affects the multipliers. As before, we consider a persistent increase in nominal government consumption by one percent



(a) Scale of Spending

(b) Scale of Transfers

Figure 12: Multiplier as a Function of Scale of Spending/Transfers (Deficit Financing)

that decays over the subsequent 19 quarters at varying rates of $1-\rho_g$, where $\rho_g \in [0,1]$, per quarter and then reverts to its steady-state level. Thus, if $\rho_g = 0$, stimulus is purely transitory and takes place in one period only, while if $\rho_g = 1$, nominal government spending is increased by the same amount for all 20 quarters. Regardless of the persistence of spending, the policy is deficit financed according to the same scheme as before with repayment starting 40 quarters after the initial increase in spending.

Figure 13 shows the impact and the cumulative multiplier for various degrees of persistence. As our previous analysis of household demand in Section 3.3 suggests, the impact multiplier m_0^{IM} is increasing in ρ_G. A higher persistence means higher government spending and thus higher income in future periods. This relaxes households' precautionary savings motives and increases their period 0 consumption demand, implying a higher initial multiplier, m_0^{IM}. When the spending becomes temporary, $\rho_G \to 0$, these dynamic interactions are minimized and the multiplier approaches 1. The dynamic effects are maximized when the spending becomes almost permanent, $\rho_G \to 1$, and the multiplier reaches 1.65. Note, however, that the cumulative multiplier moves in the opposite direction as the persistence of spending increases. As the spending becomes almost permanent, the cumulative multiplier falls significantly to a value of less than 0.4. This finding of a deceasing cumulative multiplier is a combination of the two previous results. First, our analysis in Sections 3.3 and 3.5.4 shows that the multiplier is falling

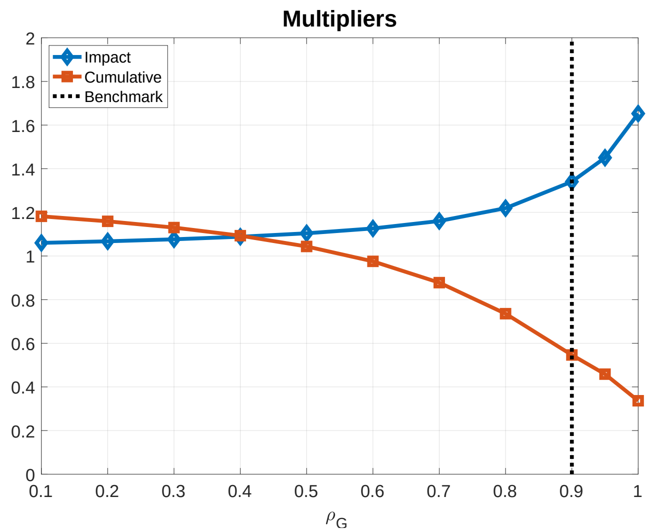


Figure 13: Multipliers: Persistence ρ_G of Spending

in the scale of government spending. As a higher ρ_G translates into a larger increase in total government spending, the multiplier decreases. Second, a higher ρ_G means more spending in future periods while spending in the initial period is unchanged. As Section 3.5.3 shows, future spending is less effective than current spending, making the multiplier smaller again. This effect is reinforced here since a spending increase in, say, period 20 is already paid back starting in period 40. That is the repayment starts 20 periods after the stimulus whereas it starts 40 periods after the stimulus in the benchmark and in Section 3.5.3. Since an earlier repayment is more contractionary, the multiplier falls.

3.5.6 Repayment through G

In this section we consider the multiplier in the deficit-financed case, but instead of cutting transfers in order to bring nominal debt back to its steady-state value, we instead cut government spending G following the same scheme that was previously used for transfers.

The results of this experiment are plotted in Figure 14. They are qualitatively and quantitatively similar to those obtained in Section 3.4.2 for when the deficit-financed increase in government spending is paid back by eventually cutting transfers. Output rises more, however, when spending is used to repay the debt than when transfers are used. The reason is that repayment through G or T has different distributional consequences. A reduction in G reduces labor income and thus hurts workers proportionally to their productivity whereas a reduction in transfers hurts workers independently of their productivity. Since high productivity workers

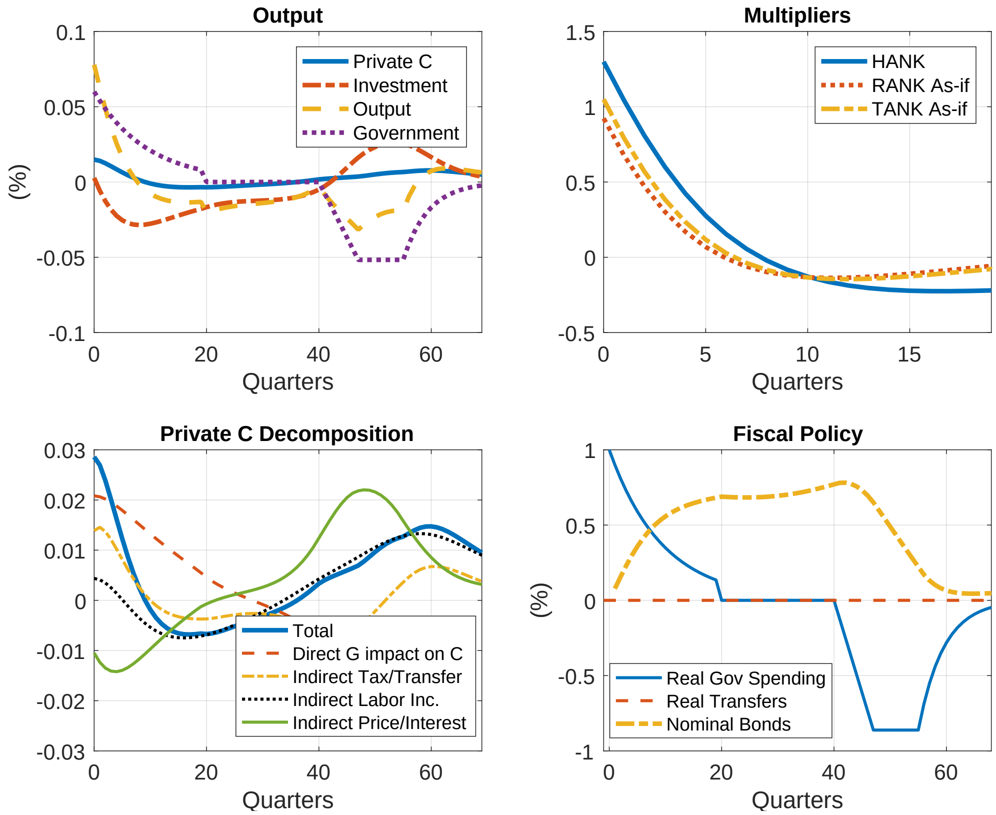


Figure 14: Spending Multipliers (Deficit Repaid Using G)

have high income and low MPCs while low productivity workers have low income and high MPCs, a reduction in T hurts high MPC households more than a reduction in G does. At the time when the repayment starts, these arguments imply that consumption is higher with repayment through G than through T. The flip side is that the low MPC households are the savers and fewer resources in their hands means a fall in savings and investment at the time when repayment through G instead of T starts. Since a reduction in G reduces aggregate demand one-for-one, the negative output effect is much larger when repayment is through G. The differences between the two financing schemes at the time of repayment feed back into earlier periods. The same arguments explain why consumption is again higher but investment is again lower with repayment through G. Combining the effects on consumption and investment leads to a slightly smaller impact multiplier when the repayment is through G. Figure A-7 shows the impulse responses.

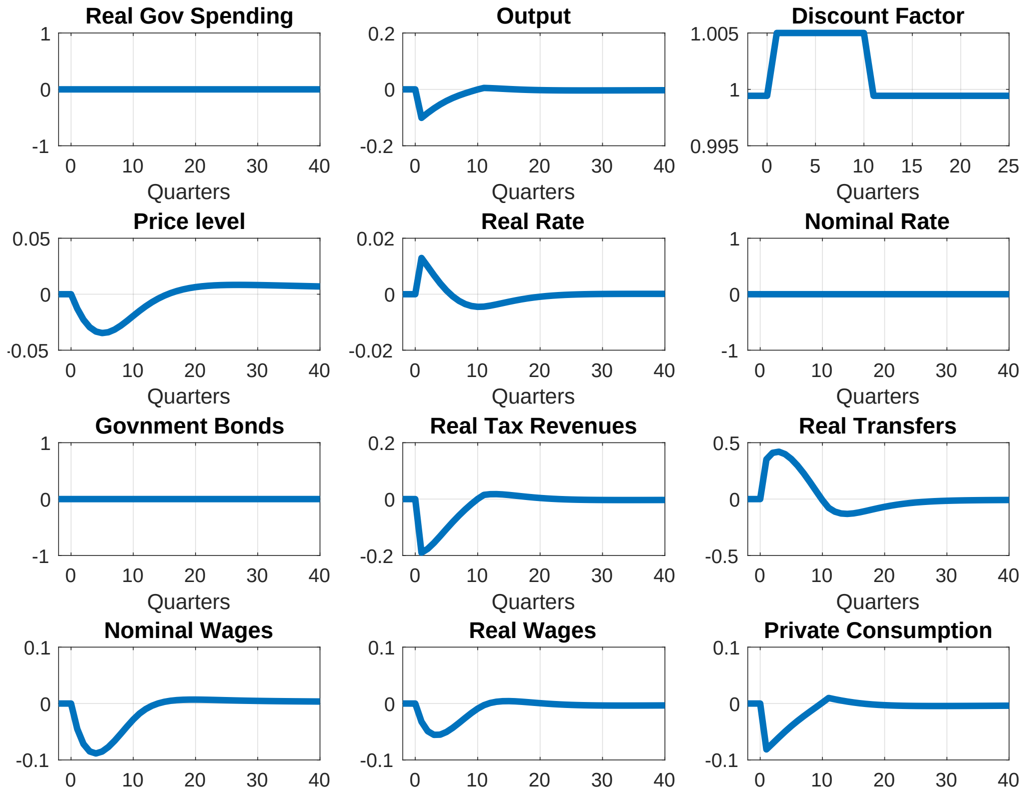


Figure 15: Economy in a Liquidity Trap

Analogously to transfer multipliers, the cumulative multiplier is defined as

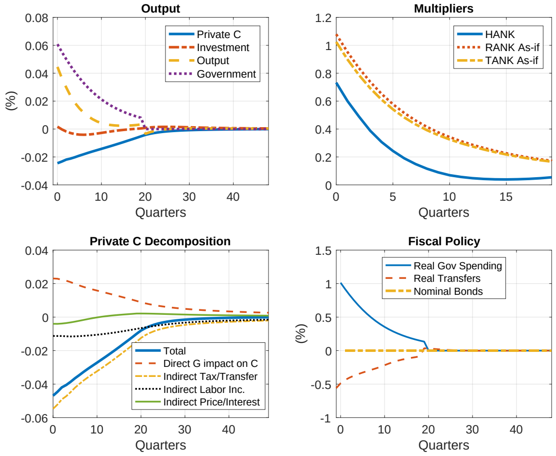
$$\overline{M} = m_{\infty}^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t (\frac{Y_t}{Y_{ss}} - 1)}{\sum_{t=0}^{19} \tilde{\beta}^t (\frac{G_t P_{ss}}{P_t G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}. \tag{59}$$

where in the denominator we again sum only over the first 20 periods. The larger output drop at the time of repayment explains why the cumulative multiplier is negative -0.15 when repayment is through G while it is positive for repayment through T.

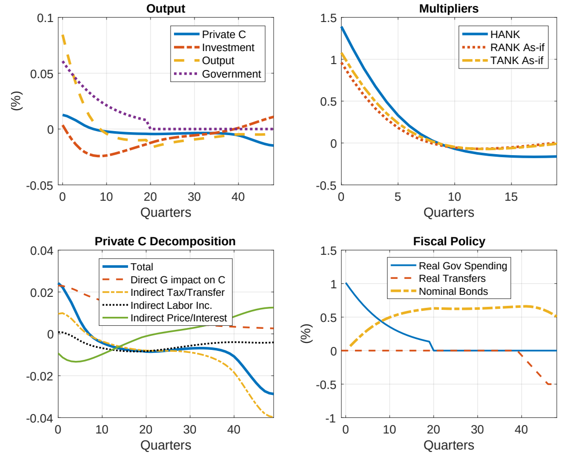
3.5.7 Liquidity Trap

In this section we explore the extent to which the size of the multiplier may vary with other shocks hitting the economy. In particular, we measure the government spending multiplier when economy suffers a large demand shock. Therefore, we generate a liquidity trap in the model, where the ZLB on nominal interest rates is binding. In doing so we follow Cochrane (2017) and increase the quarterly discount factor $\{\beta_t\}_{t=0}^{9}$ by 50 basis points for 10 quarters.[15]

[15]In a linearized complete-markets model this would generate a fall in the natural real rate of interest — the real interest interest rate in a world with flexible prices and wages — of 2 pp (annualized) for 10 quarters



(a) Spending Multiplier (Tax Financed)



(b) Spending Multiplier (Deficit Financed)

Figure 16: Fiscal Multipliers in a Liquidity Trap

In addition, we introduce an investment wedge, τ_t^I, to the mutual fund problem, and set the size of the wedge to 3.6% for 10 quarters, following Christiano et al. (2011).[16] All other parameters are unchanged.

We then feed the $\{\beta_t, \tau_t^I\}_{t=0}^{9}$ series into our model and calculate the response of the economy, which is shown in Figure 15. The resulting recession is quite large as output initially drops by about 7 percent. We assume that the government keeps real value of government spending and the nominal value of government debt constant and adjusts lump-sum transfers period-by-period to satisfy its budget constraint.

Under this scenario, we compute the effect of a simultaneous (at the same time as the liquidity trap starts) 1% increase in nominal government spending under the same balanced-budget and deficit financed schemes outside the liquidity trap. Thus, we can compute the fiscal multiplier as the percent increase in output under this scenario, relative to the benchmark with no increase in spending, divided by the relative percent differences in government spending. The multipliers are plotted in Figure 16. Figure 17 shows the transfer multiplier, where only deficit financing is meaningful. The corresponding impulse responses are reported in Figures A-8 through A-10.

Both the spending and the transfer multipliers in a liquidity trap are not very different from their counterparts in the benchmark experiments outside the liquidity trap when we keep the

and then a return to zero afterwards.

[16] In the liquidity trap if the mutual fund invests in K_{t+1} units of capital it will receive $(1+(1-\tau_{t+1}^I)(r_{t+1}^k-\delta))K_{t+1}$.

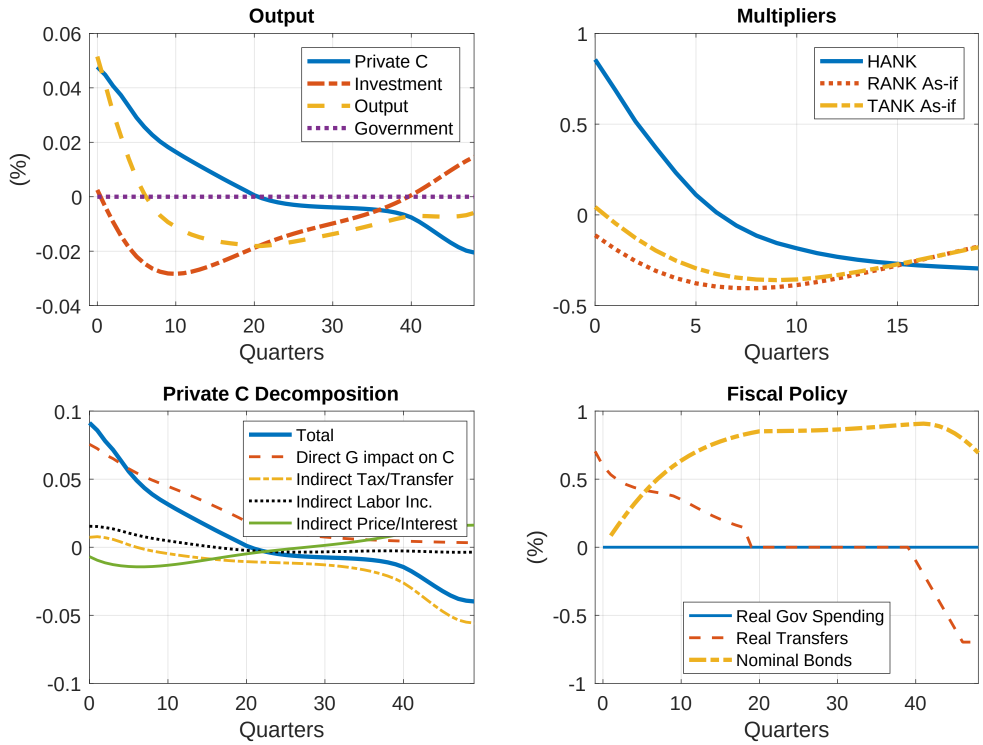


Figure 17: Transfer Multiplier in a Liquidity Trap (Deficit Financed)

nominal interest rate constant. For deficit financing, the impact spending multiplier, $m_0^{IM} = 1.39$ (1.34 in the benchmark) and the cumulative one $\bar{M} = 0.51$ (0.55 in benchmark). For tax financing, the impact multiplier, $m_0^{IM} = 0.73$ (0.61 in the benchmark) and the cumulative one $\bar{M} = 0.48$ (0.43 in the benchmark). The transfer multiplier, $m_0^{IM} = 0.86$ (0.66 in the benchmark) and the cumulative one $\bar{M} = -0.11$ (-0.1 in benchmark). The welfare loss of the transfer stimulus remains quite low, -0.0058% (-0.00067 in benchmark) in consumption equivalents.

Since values are close in a demand-induced liquidity trap and outside a liquidity trap and monetary policy is the same across the two experiments, our findings imply that there is not much state-dependence in the multiplier. However, when outside the liquidity trap a Taylor rule is implemented, we find a much smaller multiplier for deficit financing, on impact $m_0^{IM} = 0.66$ and cumulative $\bar{M} = 0.29$ (see Section 3.5.1). But if one contrasts these multipliers to those obtained in the liquidity trap, this substantial difference is due to a different monetary policy and not due to model non-linearities that could imply state-dependence.

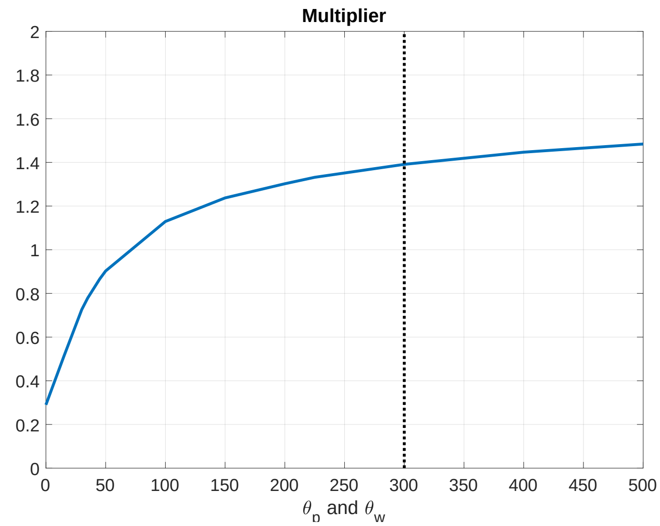


Figure 18: Liquidity Trap and Degree of Rigidities $\theta_p = \theta_w$: Spending Multiplier at $t = 0$.

3.5.8 The Degree of Price and Wage Rigidities

In New Keynesian complete markets models, the size of the multiplier increases if prices become more flexible. There is, however, a discontinuity at fully flexible prices. The multiplier is smaller than one if prices are fully flexible, but is arbitrarily large if the degree of rigidity is close to but not equal to zero. The reason is the response of inflation and thus real interest rates when prices are rigid. The more flexible prices are, the larger is the deflation in a liquidity trap and the larger is the inflation increase in response to the stimulus. Since the inflation response is one-to-one related to the output gain, the multiplier is decreasing while and the inflation response is increasing in $\theta_p = \theta_w$.

In contrast, with fully flexible prices, an increase in the discount factor stimulates savings and thus implies a fall in the real interest rate or equivalently an increase in inflation. A fiscal stimulus raises the real interest rate in a fully flexible price economy that is the inflation rate falls and the multiplier is smaller than one.

Figure 18 shows the multiplier in our HANK economy when we vary $\theta_p = \theta_w$ between 0 (flexible prices and wages) and 500 (very rigid wages and prices). Two stark differences to the complete markets version emerge. The multiplier is now decreasing instead of increasing when rigidities are reduced, i.e. $\theta_p = \theta_w$ falls. And the multiplier converges to the flexible price outcome when rigidities vanish, i.e. $\theta_p = \theta_w$ converges to zero. The discontinuity and the associated exploding multipliers which characterize the standard complete markets model disappear. The reason for these different findings is price level determinacy in our model which implies that for small but non-zero rigidities the real interest rate and inflation responses are close to the flexible price outcome.

4 Conclusions

This paper develops a quantitative laboratory which contains the essential elements needed to evaluate the effects of fiscal stimulus on the economy. At its core is a dynamic Heterogeneous Agent New Keynesian (HANK) model with incomplete markets, capital, price and wage rigidities, and the government budget constraint specified at least partially in nominal terms. These features interact in important ways, but their primary roles are as follows.

1. **Market incompleteness** gives rise to large and heterogeneous across households MPCs as observed in the data. As Ricardian equivalence does not hold, government policies have distributional consequences, which are important in determining their impact. Moreover, the incomplete markets model allows to conduct a meaningful analysis of transfer multipliers, which is important because many stimulus policies take the form of transfers rather than an increase in spending.

2. Matching the empirically relevant degree of **price rigidities** ensures that the model captures the stimulative effect of additional spending on aggregate demand.

3. The model is **fully dynamic** so that demand is also governed by intertemporal motives which has important consequenses for assessing the equilibrium impact of policies. For example, household save some of the additional income induced by current fiscal stimulus for future consumption. This implies higher aggregate demand and thus higher income in the future which relaxes households' precautionary savings motive and allows them to consume even more today. The dynamic model not only captures the equilibrium magnitude of these effects but also allows households to take into account that e.g., a deficit-financed spending stimulus leads to either lower transfers, higher taxes or lower government expenditure in the future.

4. Introducing **nominal government bonds** ensures that the model delivers a uniquely determined price level (Hagedorn, 2016, 2018), implying a unique fiscal multiplier for

arbitrary combinations of monetary and fiscal policies of interest, including an interest rate peg. This significantly expands the set of policies that can be evaluated.

5. Modeling **capital accumulation** is important to capture an investment channel of fiscal stimulus which complements the consumption demand channel. A stimulus increases demand for both employment and investment. A higher investment demand stimulates the production of investment goods which leads to more demand for employment and investment and so on until prices adjust and an equilibrium is reached. Having capital in the model also allows to capture the consequences of crowding-out of investment through higher government debt under deficit-financed stimulus.

6. Introducing **wage rigidities** into the model is important for three reasons. First, it allows us to match the empirically relevant dynamics of labor incomes, important for measuring the dynamics of demand. Second, rigid wages constraint the volatility of firms' profits the distribution of which has first order effects. Finally, the equilibrium increase in investment depends on the elasticity of interest rates and wages as those prices determine firms' substitution between labor and capital.

We then use the laboratory we developed to conduct a number of controlled experiments that shed light on the size of the fiscal multiplier and allow to decompose the economic forces that give rise to it. Specifically, in evaluating the fiscal multiplier it is important to control for the response of monetary policy and for how the fiscal stimulus is financed. Although the effects of any monetary polcy response can be evaluated using our framework, the benchmark experiments are conducted under the assumption that monetary policy holds the nominal interest rate fixed. This helps in isolating the consequences of fiscal policy only and facilitates the comparison of the fiscal multiplier outside and in a liquidity trap so that the comparison reveals the state-dependence of the multiplier and is not obscured by the differential response of the monetary policy. With respect to financing, our baseline analysis considers two benchmarks of (1) contemporaneously financing the expansion in spending through higher taxes or lower transfers and (2) using deficit financing where repayment occurs in the future. The difference in results between these financing benchmarks helps highlight the important role of the timing of expected repayment on the multiplier. For every experiment conducted in our lab, we also perform a parallel experiment using a Representative Agent and Two Agent New Keynesian

(RANK and TANK) models to clarify why the richness of our framework leads to often very different conclusions.

The key lessons that we learned about fiscal stimulus from our experiments are as follows.

1. **The deficit-financed government spending multiplier is significantly larger than the tax-financed one**, 1.34 vs. 0.61, respectively.

2. **In the liquidity trap the multipliers are similar**, 1.39 and 0.73, respectively, implying the lack of strong state-dependence.

3. **Distributional effects are important** as they contribute 0.37 to the 1.34 multiplier when spending is deficit financed and contribute a negative number, -0.5, to the multiplier of 0.61 in the case of tax financing.

4. **The response of monetary policy is crucial for the effectiveness of fiscal stimulus.** When we consider the monetary policy following the Taylor rule and thus translating output and price increases induced by fiscal stimulus outside of the liquidity trap into higher nominal and real rates, the effectiveness of fiscal stimulus falls considerably, with multipliers of 0.54 and 0.66 in cases of tax and deficit financing, respectively. This happens because higher rates contract demand through the intertemporal substitution channel and because they redistribute towards asset-rich low MPC households, implying a further contraction in aggregate demand. Thus, if monetatry policy is described by the Taylor rule, the difference in the size of the multiplier in and outside of the liquidity trap is driven largely by the different response of the monetary policy.

5. **Liquidity trap puzzles documented in RANK models disappear.** Hagedorn et al. (2018a) find that these puzzles carry over to the fully nonlinear RANK model as well as to a HANK model with real debt. We conclude that it is not the nonlinearity of our model which overcomes the puzzles but it is the combination of nominal government debt and market incompleteness. Specifically, we show that in contrast to RANK models, the multiplier in a liquidity trap becomes smaller as prices become more flexible and there is no discontinuity at fully flexible prices.

6. **A pre-announced future spending increase is less effective than an unexpected stimulus**. This is in contrast to RANK models, in which the further the spend-

ing is in the future the larger is its impact today.

7. **The multiplier is decreasing in the size of the spending stimulus.** One should be cautious about proposals that for government spending to be effective it has to be large. Interpreting higher effectiveness as a higher multiplier, we find that scaling up the stimulus decreases its effectiveness.

8. **Transfer multipliers can be assessed.** We find that a deficit-financed increase in lump-sum transfers by 1 dollar increases output on impact by 66 cents. Our framework also allows to measure the welfare consequences of temporary increases in transfer payments which generate winners and losers. We find that the transfer policy leads to a small welfare loss since the initial gains in welfare are outweighed by the losses later on. The same transfer experiment in complete markets does not alter welfare due to Ricardian equivalence.

9. **The intertemporal trade-off of a stimulus: Potentially large initial gains come with potentially larger cost later on.** In addition to reporting the impact multiplier, a standard statistic reported in the literature, we also assess the effectiveness of spending using the cumulative multiplier, i.e., the discounted average multiplier. The two measures do not always point in the same direction. For example, while the impact multiplier increases from 1.06 to 1.65 when we increase the persistence of stimulus spending from 0.1 to 1, the cumulative multiplier falls from 1.18 to 0.33. This difference reflects the short-run/long-run trade-off of a stimulus, which are absent from the standard Keynesian cross logic but which are present and can be measured in our dynamic model. A short-run multiplier larger than one suggests that a fiscal stimulus could be an effective policy tool in a recession and the long-run results suggest that this policy is costly but that these costs only occur when the economy has already recovered. While our paper has focused primarily on the positive dimension of such fiscal policy trade-offs, it can help guide normative analyses as conducted, for example, in Bhandari et al. (2015, 2016, 2017a,b) in a model closely related to ours.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109, 659–84.

——— (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1158–1175.

Auclert, A. (2016): "Monetary Policy and the Redistribution Channel," *American Economic Review*, forthcoming.

Auclert, A. and M. Rognlie (2017): "Inequality and Aggregate Demand," Working Paper 24280, National Bureau of Economic Research.

Auclert, A., M. Rognlie, and L. Straub (2018): "The Intertemporal Keynesian Cross," Working Paper 25020, National Bureau of Economic Research.

Basu, S. and J. G. Fernald (1997): "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105, 249–283.

Baxter, M. and R. G. King (1993): "Fiscal Policy in General Equilibrium," *The American Economic Review*, 83, 315–334.

Bayer, C., R. Lütticke, L. Pham-Dao, and V. Tjaden (2019): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, 87, 255–290.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2015): "Taxes, Debts, and Redistributions with Aggregate Shocks," Working paper.

——— (2016): "Fiscal Policy and Debt Management with Incomplete Markets," *The Quarterly Journal of Economics*, 132, 617–663.

——— (2017a): "Optimal Fiscal-Monetary Policy with Redistribution," Working paper.

——— (2017b): "Public Debt in Economies with Heterogeneous Agents," *Journal of Monetary Economics*, 91, 39 – 51.

Bilbiie, F. O. (2008): "Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," *Journal of Economic Theory*, 140, 162 – 196.

——— (2017): "The New Keynesian Cross: Understanding Monetary Policy with Hand-to-Mouth Households," CEPR Discussion Paper No. 11989.

——— (2018): "A Catch-22 for HANK Models: No Puzzles, No Amplification," CEPR Discussion Paper No. 12601.

Brinca, P., H. A. Holter, P. Krusell, and L. Malafry (2016): "Fiscal Multipliers in the 21st Century," *Journal of Monetary Economics*, 77, 53 – 69.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When Is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119, 78–121.

Cochrane, J. H. (2017): "The New-Keynesian Liquidity Trap," *Journal of Monetary Economics*, 92, 47 – 63.

Den Haan, W. J., P. Rendahl, and M. Riegler (2017): "Unemployment (Fears) and Deflationary Spirals," *Journal of the European Economic Association*, 16, 1281–1349.

Doepke, M. and M. Schneider (2006): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114, 1069–1097.

Dupor, B., M. Karabarbounis, M. Kudlyak, and M. Mehkari (2018): "Regional Consumption Responses and the Aggregate Fiscal Multiplier," Working paper.

Eberly, J., S. Rebelo, and N. Vincent (2008): "Investment and Value: A Neoclassical Benchmark," Working Paper 13866, National Bureau of Economic Research.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281 – 313.

Erosa, A. and G. Ventura (2002): "On Inflation as a Regressive Consumption Tax," *Journal of Monetary Economics*, 49, 761–795.

Farhi, E. and I. Werning (2016): "Fiscal Multipliers: Liquidity Traps and Currency Unions," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and H. Uhlig, Elsevier, 2417 – 2492.

Ferriere, A. and G. Navarro (2018): "The Heterogeneous Effects of Government Spending: It's All About Taxes," Working paper.

Gornemann, N., K. Kuester, and M. Nakajima (2012): "Monetary Policy with Heterogeneous Agents," Working paper 12-21, Federal Reserve Bank of Philadelphia.

Guerrieri, V. and G. Lorenzoni (2017): "Credit Crises, Precautionary Savings, and the Liquidity Trap," *The Quarterly Journal of Economics*, 132, 1427–1467.

Hagedorn, M. (2016): "A Demand Theory of the Price Level," CEPR Discussion Paper No. 11364.

——— (2018): "Prices and Inflation when Government Bonds are Net Wealth," CEPR Discussion Paper No. 12769.

Hagedorn, M., J. Luo, I. Manovskii, and K. Mitman (2018a): "The Liquidity Trap in Nonlinear Models," Working paper.

——— (2019): "Forward Guidance," *Journal of Monetary Economics*, in press.

Hagedorn, M., I. Manovskii, and K. Mitman (2018b): "Monetary Policy in Incomplete-Markets Models: Theory and Evidence," Working paper.

Heathcote, J. (2005): "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *The Review of Economic Studies*, 72, 161–188.

Huggett, M. (1993): "The Risk-free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953 – 969.

İmrohoroğlu, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political economy*, 97, 1364–1383.

——— (1992): "The Welfare Cost of Inflation under Imperfect Insurance," *Journal of Economic Dynamics and Control*, 16, 79–91.

Johnson, D. S., J. A. Parker, and N. S. Souleles (2006): "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, 96, 1589–1610.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary Policy According to HANK," *American Economic Review*, 108, 697–743.

Kaplan, G. and G. L. Violante (2018): "Microeconomic Heterogeneity and Macroeconomic Shocks," *Journal of Economic Perspectives*, 32, 167–194.

Krueger, D., K. Mitman, and F. Perri (2016): "Macroeconomics and Household Heterogeneity," in *Handbook of Macroeconomics*, ed. by J. Taylor and H. Uhlig, Elsevier, 843–921.

Lütticke, R. (2015): "Transmission of Monetary Policy with Heterogeneity in Household Portfolios," Working paper.

McKay, A., E. Nakamura, and J. Steinsson (2016): "The Power of Forward Guidance Revisited," *American Economic Review*, 106, 3133–58.

Oh, H. and R. Reis (2012): "Targeted Transfers and the Fiscal Response to the Great Recession," *Journal of Monetary Economics*, 59, Supplement, 50 – 64.

Ramey, V. A. (2011): "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49, 673–85.

Ravn, M. O. and V. Sterk (2017): "Job Uncertainty and Deep Recessions," *Journal of Monetary Economics*, 90, 125 – 141.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *The Journal of Political Economy*, 1187–1211.

Rupert, P. and R. Šustek (2019): "On the Mechanics of New-Keynesian Models," *Journal of Monetary Economics*, in press.

Woodford, M. (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 3, 1–35.

I Figures

I.1 Impulse Responses: Baseline Experiments, Section 3.4.

Figure A-1: Impulse Response to a 1% Increase in Nominal Government Spending. **Tax Financing** (Constant Nominal Debt).

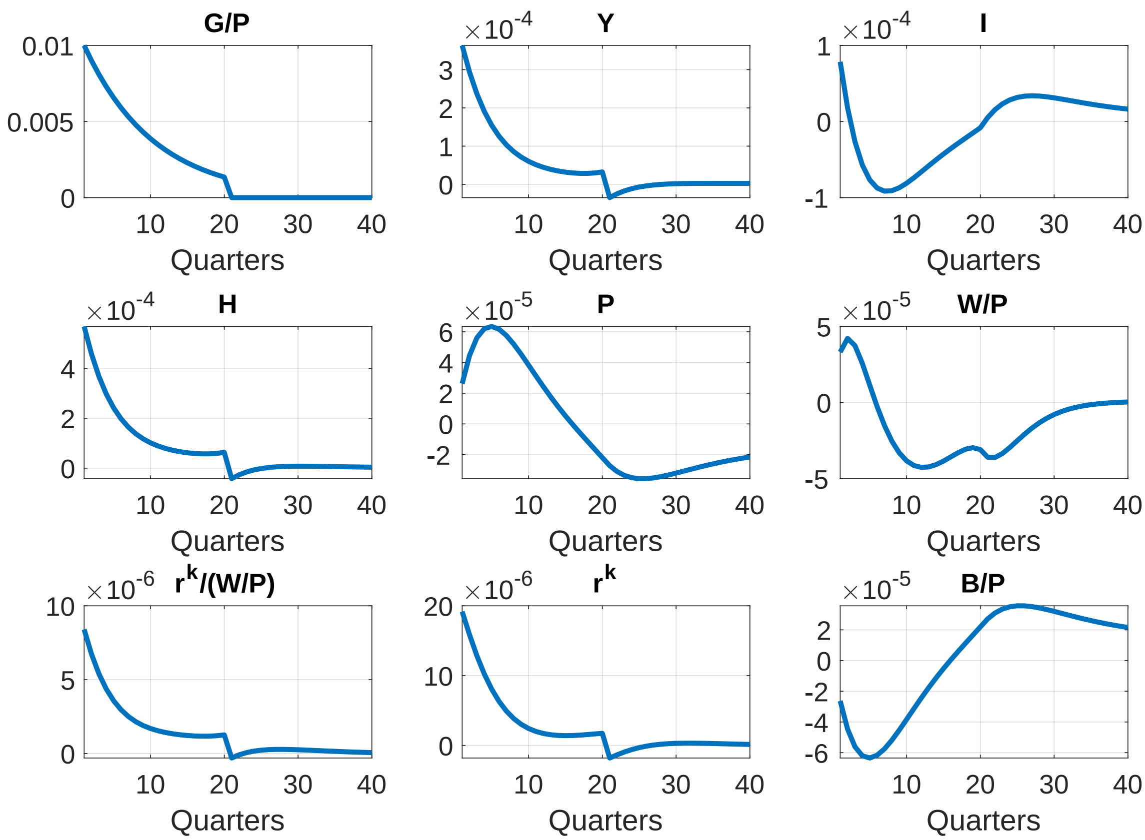
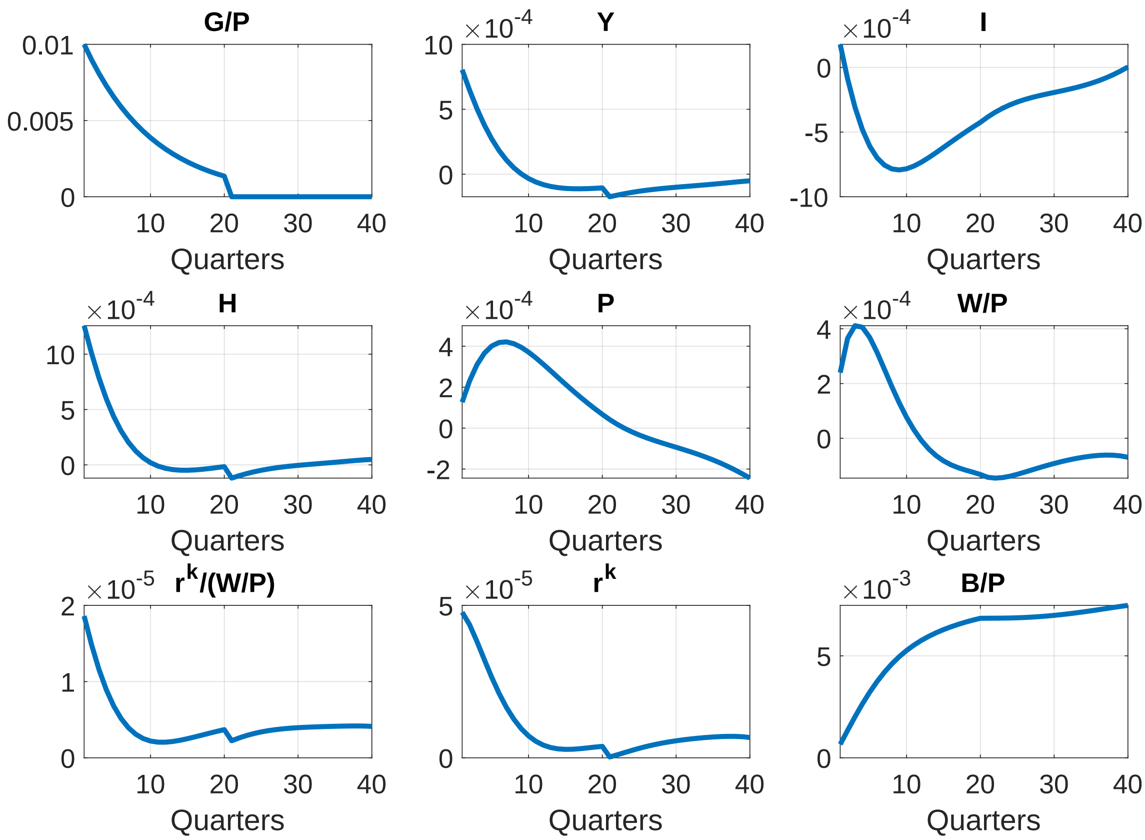


Figure A-2: Impulse Response to a 1% Increase in Nominal Government Spending. **Deficit Financing**.



I.2 Impulse Responses: Taylor Rule, Section 3.5.1.

Figure A-3: Impulse Response to a 1% Increase in Nominal Government Spending.
Taylor Rule, Deficit Financing.

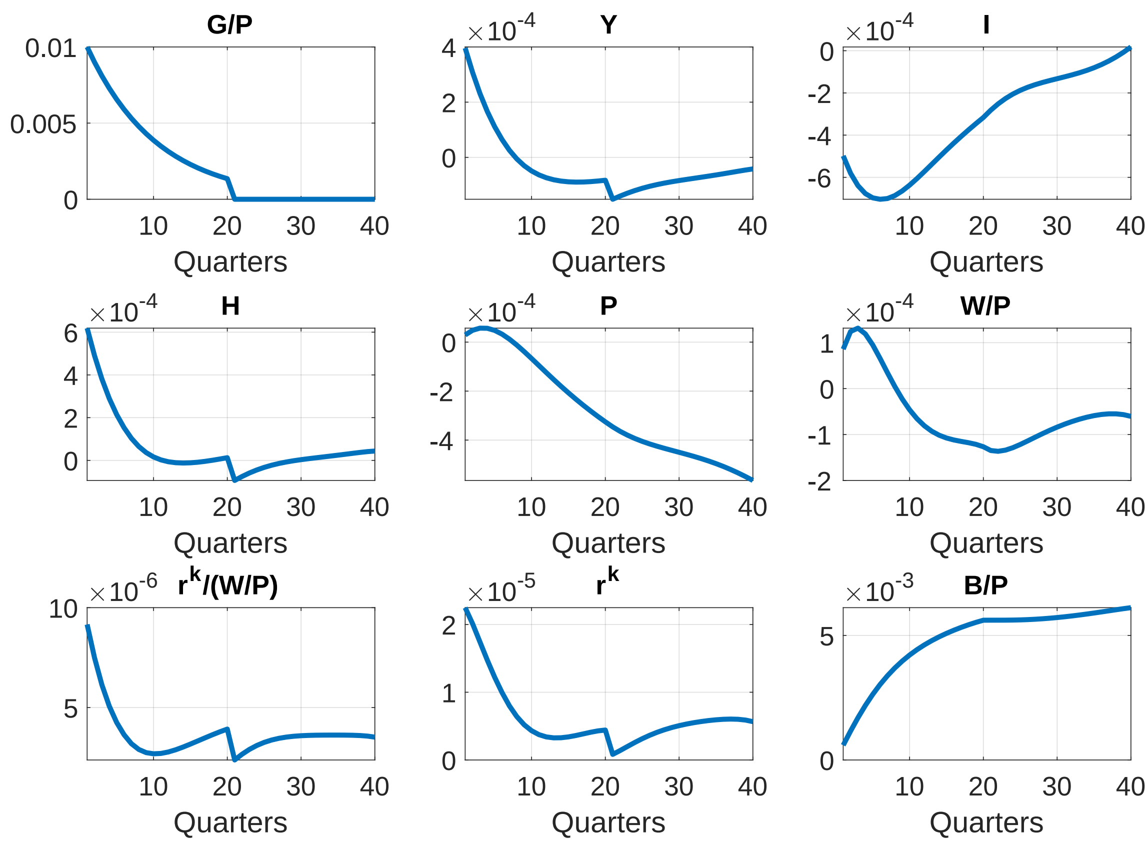
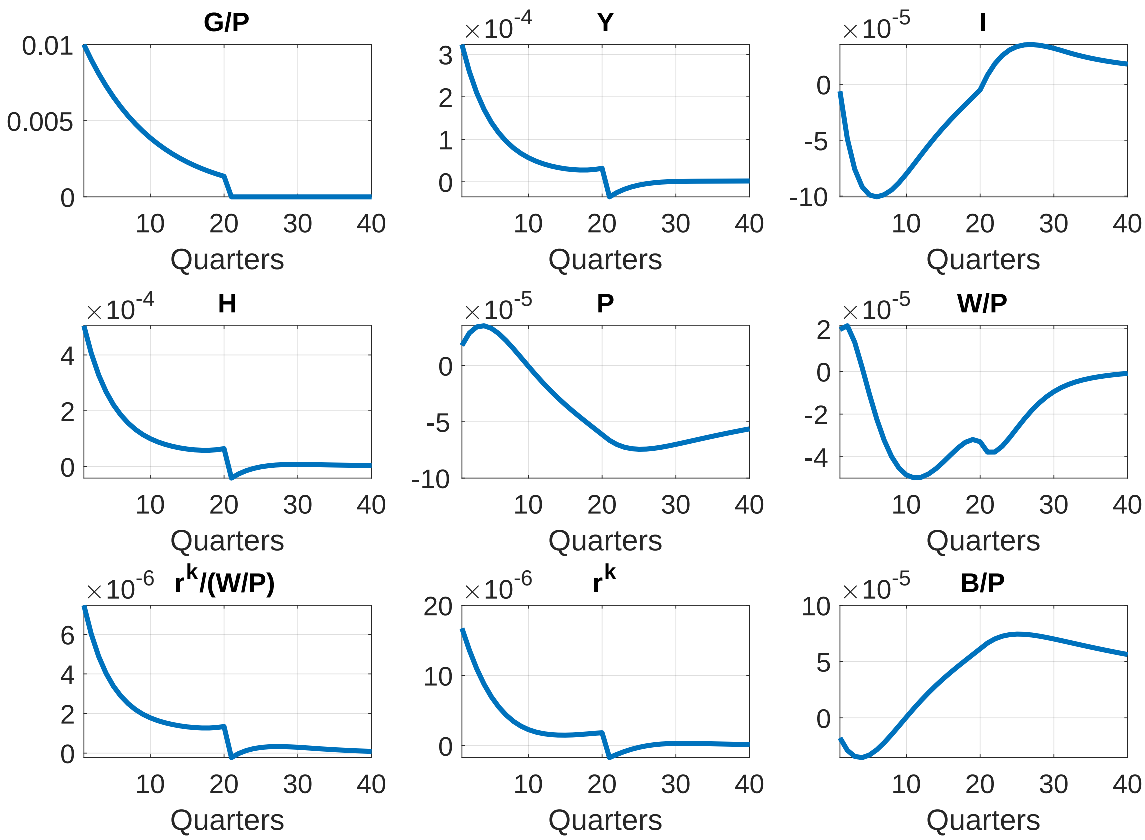
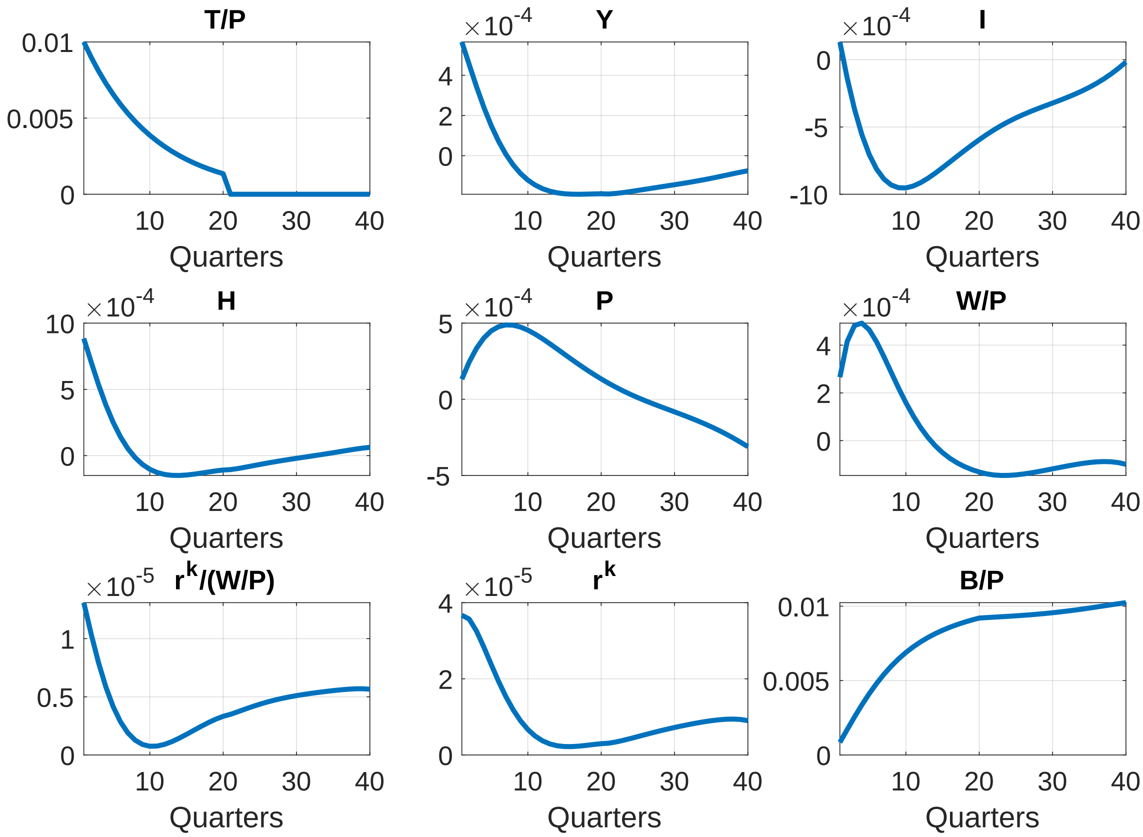


Figure A-4: Impulse Response to a 1% Increase in Nominal Government Spending. **Taylor Rule, Tax Financing** (Constant Nominal Debt).



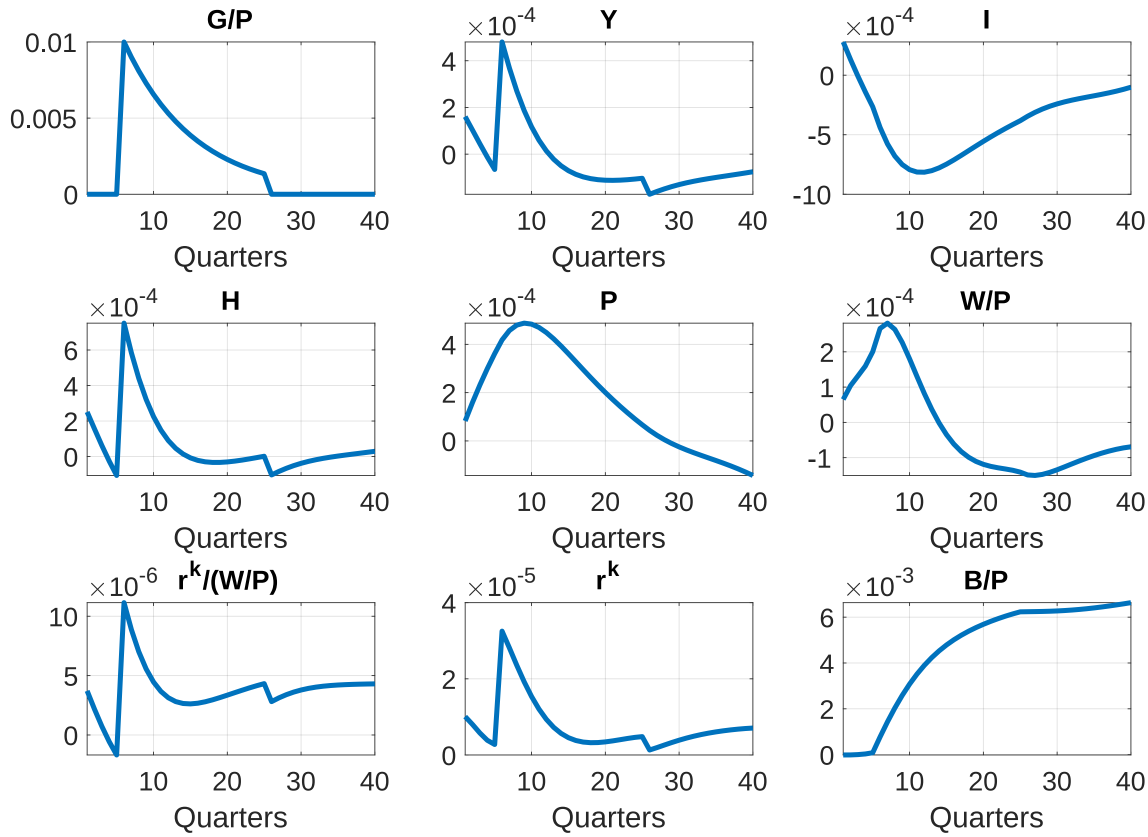
I.3 Impulse Responses: Transfer Multiplier, Section 3.5.2.

Figure A-5: Impulse Response to a 1% Increase in Nominal Government *Transfers*. **Deficit Financing**.



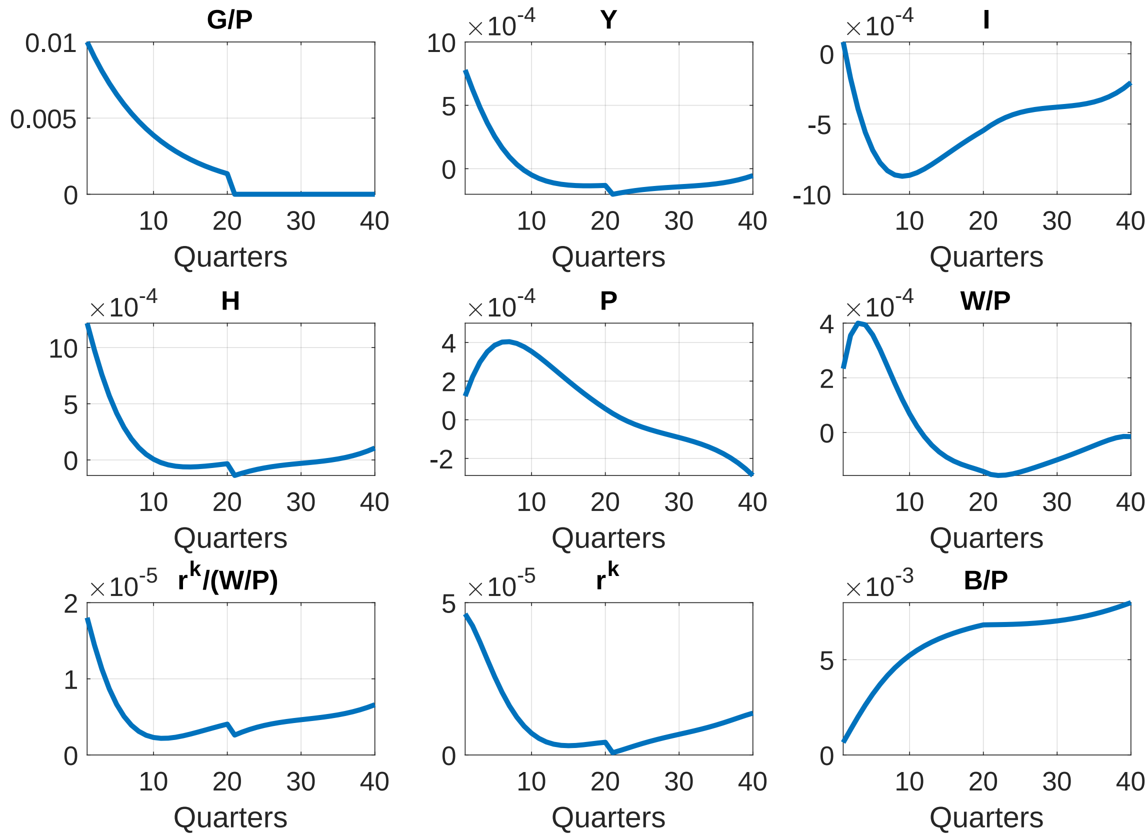
I.4 Impulse Responses: Forward Spending, Section 3.5.3.

Figure A-6: Impulse Response to a *Future* (4 Quarters Ahead) 1% Increase in Nominal Government Spending. **Deficit Financing**.



I.5 Impulse Responses: Deficit Repaid using G, Section 3.5.6.

Figure A-7: Impulse Response to a 1% Increase in Nominal Government Spending. **Deficit Financing, Deficit Repaid using G.**



I.6 Impulse Responses: Liquidity Trap, Section 3.5.7.

Figure A-8: Impulse Response to a 1% Increase in Nominal Government Spending. **Liquidity Trap, Tax Financing** (Constant Nominal Debt).

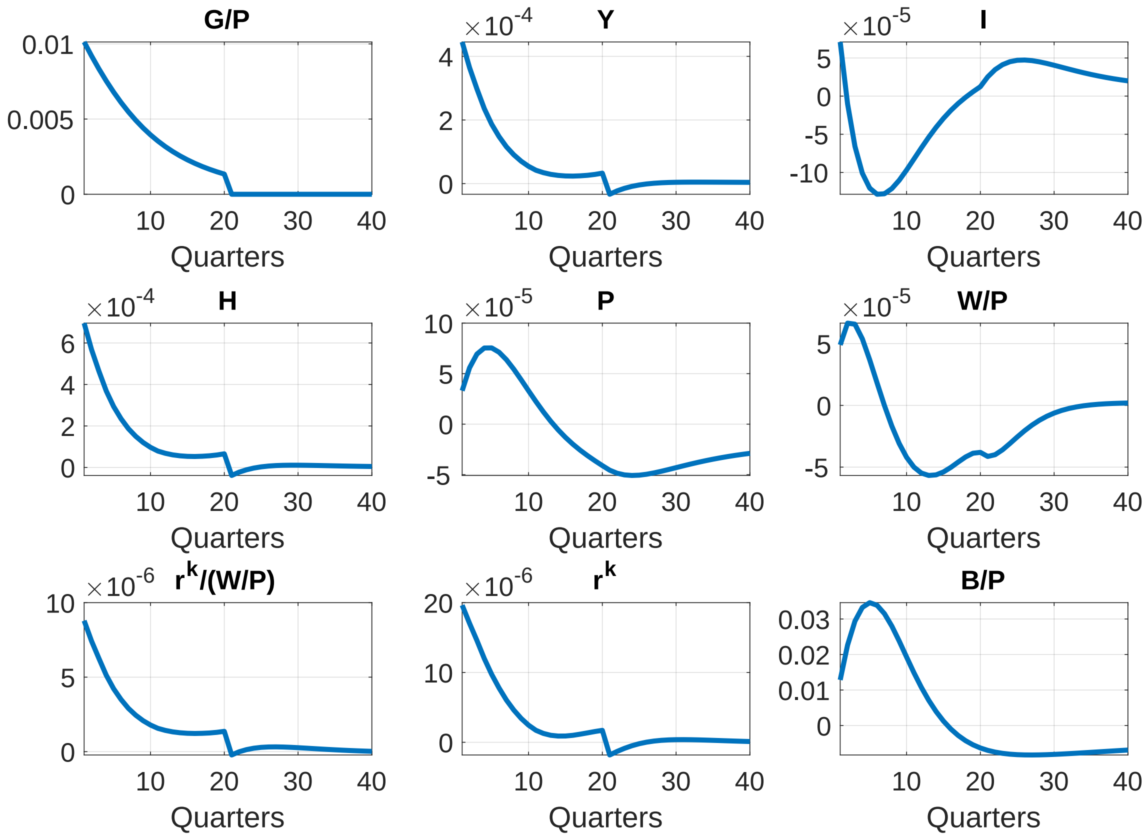


Figure A-9: Impulse Response to a 1% Increase in Nominal Government Spending. **Liquidity Trap, Deficit Financing**.

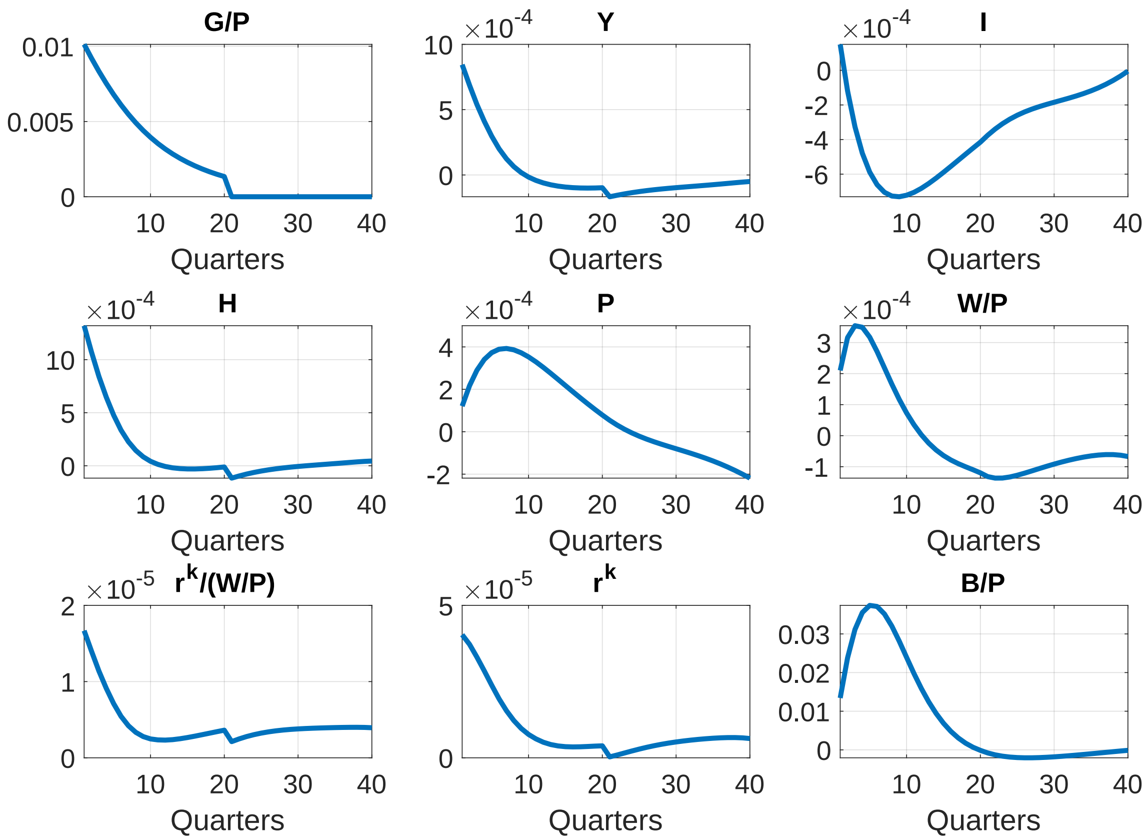
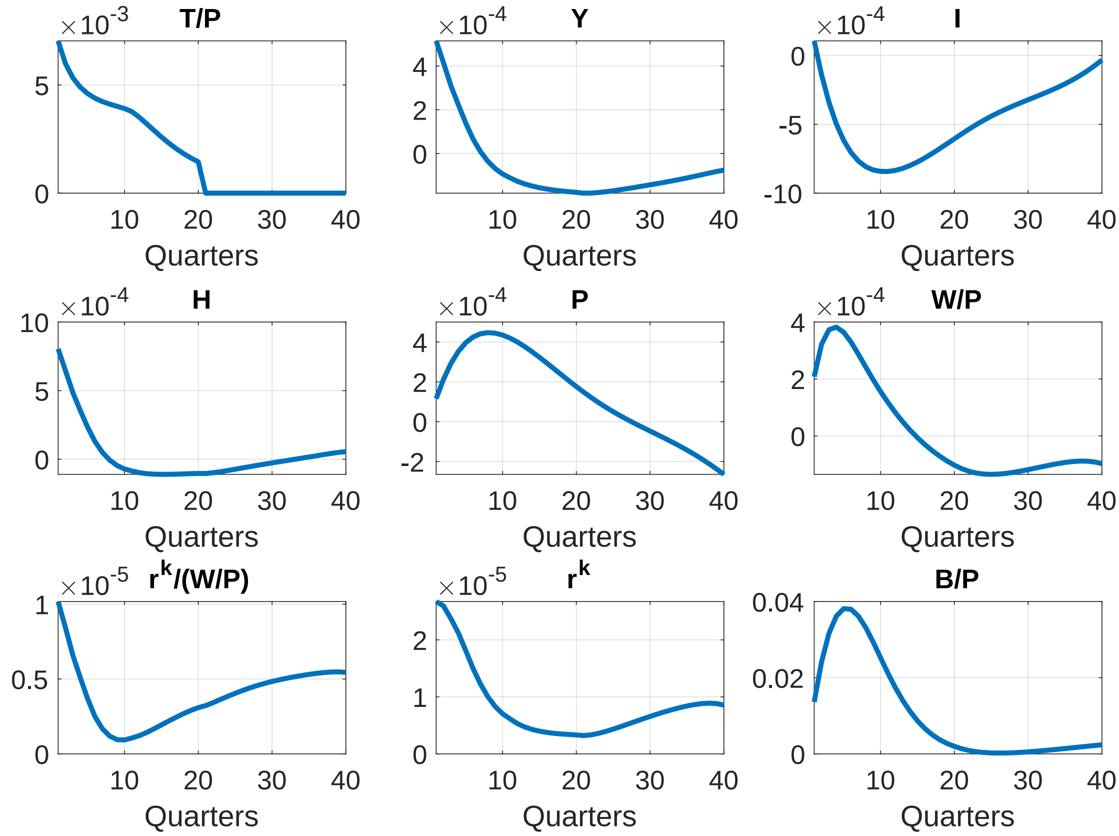


Figure A-10: Impulse Response to a 1% Increase in Nominal Government *Transfers*. **Liquidity Trap, Deficit Financing**.



II Derivations

II.1 Derivation Pricing Equation

The firm's pricing problem is

$$V_t\left(p_{jt-1}\right) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y\left(p_{jt}; P_t, Y_t\right) - \Xi_t(Y_{jt}) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{jt}\right),$$

subject to the constraints $y_{jt} = Z_t K_{jt}^{\alpha} H_{jt}^{1-\alpha}$ and $y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t$.
Equivalently,

$$V_t\left(p_{jt-1}\right) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t}\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t - \Xi_t(Y_{jt}) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{jt}\right).$$

The FOC w.r.t p_{jt} is

$$(1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon mc_{jt} - \theta\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)\frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} V'_{t+1}(p_{jt}) = 0 \tag{A1}$$

and the envelope condition is

$$V'_{t+1} = \theta\left(\frac{p_{jt+1}}{p_{jt}} - \overline{\Pi}\right)\frac{p_{jt+1}}{p_{jt}}\frac{Y_{t+1}}{p_{jt}}. \tag{A2}$$

Combining the FOC and and the envelope condition,

$$\begin{aligned} &(1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t} + \epsilon mc_{jt} \\ -\quad &\theta\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)\frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t}\theta\left(\frac{p_{jt+1}}{p_{jt}} - \overline{\Pi}\right)\frac{p_{jt+1}}{p_{jt}}\frac{Y_{t+1}}{p_{jt}} = 0. \end{aligned} \tag{A3}$$

Using that all firms choose the same price in equilibrium,

$$(1-\epsilon) + \epsilon mc_t - \theta\left(\pi_t - \overline{\Pi}\right)\pi_t + \frac{1}{1+r_t}\theta\left(\pi_{t+1} - \overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} = 0. \tag{A4}$$

II.2 Derivation Wage Equation

$$\Theta\left(s_{it}, W_{it}, W_{it-1}; Y_t\right) = s_{it}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t.$$

The middleman's wage setting problem is to maximize

$$
\begin{aligned}
& V_t^w\left(\hat{W}_{t-1}\right) \\
\equiv\ & \max_{\hat{W}_t} \int \Big(\frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{v(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} di - \int s_{it} \frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t di \\
+\ & \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right), \qquad \text{(A5)}
\end{aligned}
$$

where $h_{it} = h(W_{it}; W_t, H_t) = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon_w} H_t$.

The FOC w.r.t $\hat{W}_t$ is

$$
\begin{aligned}
(1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \qquad \text{(A6)} \\
-\theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t} V'_{t+1}(\hat{W}_t) = 0
\end{aligned}
$$

and the envelope condition is

$$
V'_{t+1} \quad = \quad \theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t}, \qquad \text{(A7)}
$$

where we have used that $\int s = 1$.

Combining the FOC and the envelope condition,

$$
\begin{aligned}
& (1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \\
- \ & \theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t}\theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t} = 0. \qquad \text{(A8)}
\end{aligned}
$$

Using that $\hat{W}_t = W_t$, $\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{\hat{W}_t}{\hat{W}_{t-1}}$ and $h_{it} = H_t$:

$$
\begin{aligned}
& (1-\tau_t)(1-\epsilon_w)\frac{W_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \\
- \ & \theta_w\left(\pi_t^w - \overline{\Pi^w}\right)\pi_t^w + \frac{1}{1+r_t}\theta_w\left(\pi_{t+1}^w - \overline{\Pi^w}\right)\pi_{t+1}^w \frac{H_{t+1}}{H_t} = 0. \qquad \text{(A9)}
\end{aligned}
$$