The Cost of Intermediary Market Power for Distressed Borrowers

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Abstract

Loan markets for distressed corporate borrowers are characterized by oligopolistic structures in which a few specialized lenders finance a large fraction of loans. In these markets, loan spreads are ultra high even after removing the default- and liquidity-risk component. Borrowers are in desperate need of financing but face limited funding options, while specialized lenders exert substantial market power sustained by their tight and repeated syndication relations and restrained participation. We develop a dynamic game-theoretic model of strategic competition in syndicated lending to distressed borrowers with endogenous entry of specialized lenders, which incorporates these prominent features and sheds novel insights into our understanding of a healthy loan market structure for distressed firms. We estimate the structural model taking into account equilibrium collusion and latent heterogeneity in both demand and supply. We find that lender market power accounts for up to 76 - 96% of the risk-adjusted yield spreads, with a large fraction attributed to collusive lending and restrained participation of specialized lenders. Smaller borrowers are particularly susceptible to lender market power than larger borrowers. Counterfactual analysis shows that both specialized lenders and distressed borrowers would be worse off if no lender collusion is allowed. Further policy analysis suggests that there exists an optimal level of spread-cap that can be imposed by regulators.

Keywords: Collusion in Syndication, Blocking Power, Agency Conflicts, Intermediary Asset Pricing, Maximum Likelihood, Latent Demand Shifts. (JEL: G12, G23, G30, L13)
1 Introduction

Traditional neoclassical asset pricing theories assume that a fully diversified representative investor can trade all assets freely and the asset markets are efficient, typically featuring perfect information, perfect competition, and absence of arbitrage (e.g., Fama, 1965; Ross, 1976). Thus, asset prices are unbiased estimates of the fundamental values of the assets in those theories, primarily determined by the compensations for aggregate fundamental risks borne by the representative investor in equilibrium (e.g., Sharpe, 1964; Merton, 1973). In reality, many important asset markets are, however, mainly intermediated by a relatively small number of highly specialized institutional investors. Market segmentations and the oligopolistic structures of these asset markets, due to high entry barriers, have important asset pricing implications beyond the traditional neoclassical theories. On the one hand, since the pioneered work by Shleifer and Vishny (1997), substantial progress has been made in understanding the unique role of intermediaries in connecting asset prices to economic quantities through the channels of funding liquidity risks, leverage constraints, and fund flow shocks. On the other hand, imperfect competition among highly specialized institutional investors is expected to exert a significant impact on asset prices owing to the market power of these institutional investors. Yet, until recently, the effect of intermediary market power has been relatively understudied in the literature.

Particularly, in contrast to the traditional neoclassical asset pricing theories, we find that the pricing behavior of the specialized institutional lenders across two different loan markets for distressed corporate borrowers — the distressed loan market and the debtor-in-possession (DIP) loan market — is affected by imperfect competition that leads to large lenders’ market power. Loans to distressed but not yet bankrupt firms are known as distressed loans, and loans to firms already in Chapter 11 bankruptcy

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1 Many theoretical studies have shown that intermediaries do not act simply as a sideshow in asset pricing (e.g., Shleifer and Vishny, 1997; Gromb and Vayanos, 2002; Gabaix et al., 2007; Brunnermeier and Pedersen, 2008; Basak and Pavlova, 2013; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Frazzini and Pedersen, 2014; Drechsler et al., 2018; Dou et al., 2022).

2 An investor is said not to have any market power if it is a price-taking agent. An investor is said to have market power if it is not atomistic and can influence asset prices by itself alone.

3 A few influential exceptions include studies that investigate how the market power of intermediaries shapes the security prices in credit card markets (e.g., Knittel and Stango, 2003), certain OTC markets (e.g., Duffie et al., 2005, 2007), bank deposit markets (e.g., Drechsler et al., 2017; Egan et al., 2017), and life insurance markets (e.g., Kojien and Yogo, 2019).

4 The definition of “distressed loans” is consistent with that of “distressed bonds” in bond markets. Specifically, unlike “leveraged loans” and “junk bonds,” which include all debt securities with a credit rating of BB+ or lower, distressed debt securities (including loans and bonds) must have a credit rating of CCC+ or lower. The accurate definition of “distressed loans” for our analysis is elaborated in Section 4.1.
are known as DIP loans. One key empirical finding is that, after removing the default- and liquidity-risk component, there is still a substantial unexplained yield spread, referred to as a “risk-adjusted loan spread,” for both types of loans. Specifically, over 2001 – 2017, the yearly average risk-adjusted loan spread in the distressed loan market remained stable at a very high level, fluctuating roughly between 200 and 350 bps. Similarly, over 2002 – 2019, the yearly average risk-adjusted loan spread in the DIP loan market also remained stable at a very high level, fluctuating between 400 and 750 bps, without obvious cyclical patterns or clear comovement with financial conditions of the economy. This pricing behavior contrasts sharply with that in corporate bond markets in which the credit- and liquidity-risk component accounts for almost all the bond yield spread (e.g., Longstaff et al., 2005; Blanco et al., 2005). In the cross section of syndicated loans, the market power of specialized lenders and the risk-adjusted loan spread are larger for those deals with smaller corporate borrowers; furthermore, the market power of specialized lenders and the risk-adjusted loan spread are larger for those deals with fewer specialized lenders participating in the syndication.

This striking pricing behavior arises naturally from the oligopolistic structures of those loan markets in which a few specialized lenders finance a large fraction of loans and compete imperfectly. Intuitively, in a perfectly competitive financing market, lenders charge interest and fees that are commensurate with (i) the default risk of the borrower, (ii) the liquidity risk of the security, (iii) the ex-post re-contracting costs (Roberts and Sufi, 2009; Berg et al., 2016), and (iv) the marginal costs lenders incur in making debts to firms (such as the monitoring cost, the information generation cost, and the funding cost of lenders). The basic idea is that imperfect competition among specialized institutional lenders in the market of distressed loans and that of DIP loans lead to an economically substantial deviation from the competitive loan pricing behavior. More specifically, specialized lenders are likely to possess significant market power in financing distressed firms for at least the following three reasons. First, distressed firms face a dire liquidity situation and are in desperate need to raise capital to survive with limited funding options, and thus their bargaining position is weak, reflected by their low price elasticity of demand for loans. Second, high entry barriers, caused by specialization and regulation, lead to segmented and concentrated markets in which specialized lenders can tacitly collude in the form of syndication. Third, existing creditors’ blocking power and their information advantage further prevent specialized lenders
from participating in a syndication deal.

This paper examines the role of imperfect (oligopolistic) competition among specialized institutional investors in determining loan prices in the distressed and DIP loan markets.\(^5\) In particular, we estimate the risk-adjusted loan spread attributed to the intermediary market power, and more importantly, we further dissect the intermediary market power into a component that is due to the unilateral market power of specialized lenders, mainly reflecting borrowers’ price elasticity of demand for funding and lenders’ market concentration,\(^6\) versus that which is due to the collusive market power of specialized lenders in the form of syndication.

Dissecting the lender market power in these loan markets, however, is empirically challenging in the following three aspects. First of all, the potential lender collusion, explicit or implicit, is unobservable to econometricians, and constructing empirical proxies to lender collusion is very difficult if not completely impossible. Importantly, the equilibrium outcome in tacit collusion, especially the equilibrium collusion capacity, should depend on the deep structural parameters of the model and co-vary with other equilibrium outcomes in a coherent way, rather than an arbitrary degree of freedom. Secondly, we need to estimate the demand system and the supply structure, without observing actual costs. Particularly, the observed high loan spreads after risk adjustments may not necessarily reflect specialized lenders’ market power; alternatively, they may reflect the substantial private costs incurred by lenders in making loans (such as the monitoring cost, the information generation cost, and the funding cost). Consistently estimating the demand curve is challenging due to the generic omitted variable issue (i.e., the endogeneity issue) — the correlation between loan prices and borrower-specific latent demand shocks, which are included in the econometric error term. Finally, it is highly endogenous whether the distressed borrower would end up getting the syndicated loan by specialized lenders or obtaining the funding from some alternative sources as lenders of last resort (i.e., outside options). Identifying and consistently estimating the size of the outside options is often a challenging task.

\(^5\)Institutional investors, as intermediaries, compete not only in the investment market of assets, but also in the market of delegation products. While this paper focuses on the former, it’s worth noting that there have been recently an increasing number of studies that investigate the latter (e.g., Hortaçsu and Syverson, 2004; Hastings et al., 2017).

\(^6\)The unilateral market power is often referred to as non-collusive (or non-coordinated) market power. It is defined as the ability of an individual agent to act alone to set a price greater than marginal cost and still make positive sales. In the language of game theory, the unilateral market power is obtained in a non-collusive Nash equilibrium (e.g., Borenstein et al., 2002; Rassenti et al., 2003; Wolak, 2003).
To overcome these challenges, we develop and estimate a new structural model that exploits the implications of endogenous (tacit) collusion to dissect the risk-adjusted loan spread into the components attributed to collusive versus unilateral market power of specialized institutional lenders. The model provides a unified description for both the distressed loan market and the DIP loan market. To address the first challenge, we model collusive and non-collusive equilibrium outcomes within a coherent and unified theoretical framework to ensure that the collusion capacity of specialized lenders is determined endogenously and co-vary coherently with other equilibrium outcomes such as the number of specialized lenders and the size of a syndicated loan, and that the collusive equilibrium outcomes that can never go beyond the monopoly or perfect collusion. By doing so, we improve the approach of Nevo (2001), which only considers a model with perfect collusion, even though it is probably not achievable under the estimated parameters. In fact, there have been important efforts made lately in the empirical industrial organization literature (e.g., Ciliberto and Williams, 2014; Miller and Weinberg, 2017) to improve the approach of Nevo (2001) by modeling the departure from non-collusive Nash equilibria exogenously as a parametric function of certain observed characteristics, such as the “multi-route contact” between carriers in Ciliberto and Williams (2014). Our paper takes one step further by endogenizing the relation of the departure from non-collusive Nash equilibrium to the structural parameters and borrowers’ characteristics.

To address the second challenge of omitted latent variables in estimating demand and supply curves, a general strategy is to estimate the demand and then combine the estimated demand system with the specified pricing rules of the supply side to recover the latent costs and the markups. The first step of this strategy usually assumes that observed characteristics present a source of exogenous variation to the choice variable and there exists a vector of instrumental variables (IVs) that include characteristics and cost shifters (Berry, 1994; Berry, Levinsohn, and Pakes, 1995, hereafter referred to as BLP). Our methodology here differs substantially from theirs. We develop a structural model that are general yet analytically tractable. The closed-form solution allows us to estimate the demand system and the supply structure simultaneously based on maximum likelihood estimation when latent demand curve shifts can be summarized by a relatively low-dimensional space. We estimate the model in the two markets separately, using the Markov Chain Monte Carlo (MCMC) estimator. The MCMC estimation methodology is a Bayesian approach and it computes the posterior distribution of the model parameters.
and latent variables conditioning on the observed quantities. The MCMC methodology has seen a quick
growth in finance studies (Sorensen, 2007; Johannes and Polson, 2010; Kortweg, 2010).

To tackle the third challenge of identifying outside options, we highlight that the process of making
a distressed loan naturally follows a three-stage procedure: first, the distressed borrower negotiate with
its existing lenders; second, if the borrower and its existing lenders fail to reach a deal, each specialized
lender chooses whether and how to participate the syndicated loan; and third, if none of the specialized
lenders is willing to make a loan, the distressed borrower has to turn to the lenders of last resort (usually
hedge funds), which is effectively the outside option to the second stage. The last-resort lending behavior
can be well observed in the data, and thus it can be identified and consistently estimated.

Specifically, the structural model has two parts. The first part consists of the demand curves of dis-
tressed corporate borrowers, which specifies the risk-adjusted loan spread that the distressed borrower
is willing to pay for a given amount of loan. The demand system specification is similar to that of Hen-
del (1999). The second part describes the supply side of the loan markets, and it is based on a repeated
game model (e.g., Fudenberg and Maskin, 1986; Rotemberg and Saloner, 1986). This theoretical model
forms the basis for an econometric model of imperfect competition among a clique of specialized in-
stitutional lenders, in which, taking into account any externalities, these lenders endogenously choose
whether to participate in a syndicated loan, and decide how to lend together, cooperatively or not, if
joining the syndication. The repeated game model controls for the endogenous supply and competi-
tion of the lenders, eliminating inconsistency in the estimation of the demand system. Together, the two
parts control for endogeneity for each other to achieve identification. This echoes the important insight
that full information maximum likelihood estimator is equivalent to an instrumental variables estimator
where the instruments embody all the over-identifying model-implied restrictions (e.g., Hausman, 1975).

We assemble a comprehensive dataset that contains 484 distressed loan facilities from 2001–2017 and
297 DIP loans to large U.S. public firms that filed for Chapter 11 bankruptcy from 2002–2019. Our sample
contains detailed information of these loans, including the loan size, spread, the number of lenders in a
syndication, participating lender identities, lender type, as well as characteristics of the borrowers. The

\footnote{Not many studies have been developed on the asset pricing and corporate finance implications of strategic competition in repeated games; a few exceptions include the recent works by Opp et al. (2014), Dou et al. (2021b,c), Chen et al. (2021), and Dou et al. (2021d).}
estimates differ substantially across the two markets, which reveal distinct features that help explain the differentials in the risk-adjusted loan spread in the two loan markets albeit their similarities. Our estimation delivers a few novel findings. First, we find that lenders’ tacit collusion contributes to a sizable component of the risk-adjusted loan spread, ranging from 140 to 160 bps, in both markets. Second, borrowers in both markets exhibit similar and low price elasticity of demand. Taken together, the excess spread present in the DIP loan market cannot be explained by the differentials in collusion capacity or those in price elasticity across the two markets.

According to our estimates, the factors that set the two markets apart are the specialist lenders’ participation cost and their variable cost of lending. Specifically, the estimated participation cost for lenders in the DIP loan market is more than two times larger than that in the distressed loan market. The participation cost accounts for 250 bps in the DIP loan spread but only 80 bps in the distressed loan spread. An important determinant of the participation cost is the existing lender’s blocking power that prevents the borrowers borrowing from alternative lenders, and the other is the existing creditor’s information advantage. Our results therefore suggest that existing lenders in the DIP-loan market have much stronger blocking power and information advantage than those in the distressed loan market. Furthermore, we also find that lenders in the DIP loan market incur significantly higher variable cost of lending. The estimated variable cost is 180 bps in the DIP-loan market while only about 10 bps in the distressed loan market. This finding is quite intuitive since the DIP loan is expected to be more complex.

Our decomposition of the risk-adjusted loan spread in both market allows us to further analyze which types of borrowers are particularly susceptible to the market power of institutional lenders. We partition the borrowers in our sample into size quintiles and repeat our analyses above on the small (bottom quintile) and large (top quintile) borrowers. Our findings are striking: shutting down lender collusion reduces the risk-adjusted loan spread by about 80 (100) bps for large borrowers and by 130 (230) bps for small borrowers in the distressed loan (the DIP loan) market. Our analysis thus suggests that small borrowers are more vulnerable to the market power of institutional lenders, and policies aiming at helping distressed companies in economic and financial crises should target more at small firms given their weak bargaining power and disadvantageous position in the distressed loan markets.

Lastly, we use the estimated model as a laboratory to examine the effect of a widely debated regula-
tory intervention — interest rate cap. We allow the regulator to impose a cap on the specialist lenders’ markup in lending and thus restrict the loan spread that can be charged by these lenders. We analyze the specialist lenders’ strategic responses to the rate-cap policy and the consequence on borrowers’ welfare. Solving the model with rate-cap reveals a few intriguing implications. First, as loan spreads are capped, specialist lenders have less incentive to collude to a small loan size and thus they capture the residual profits by increasing the loan amount. Higher loan amount and lower spread improves the borrowers’ welfare when they borrow from specialist lenders (i.e., a positive intensity margin). But meanwhile, rate-cap reduces the expected profits by specialist lenders and thus discourage their participation in the lending market. As fewer specialists are willing to lend in the market, the likelihood for the borrowers to borrow from the lenders of last-resort rises. Since loans made by lenders of last-resort are much more expensive, rate-cap generates an unintended consequence of reducing the depth of specialist lenders’ market (i.e., a negative extensive margin). Combining the two effects, we demonstrate that the effect of rate-cap on borrower welfare is hump-shaped and there exists an optimal level of rate-cap. The optimal spread averages about 60 bps for distressed loans and 380 bps for DIP loans, compared with 267 bps and 596 bps observed in the data.

Related literature. Our paper contributes to the literature on loan pricing and contracting. Prior studies document the effect of information problems, lending relationship, syndicate structure, and lender specific characteristics on the pricing of corporate loans. A few studies suggest that loans to bankrupt companies are not priced competitively and lenders can capture economic rents (Hasan et al., 2019; Eckbo et al., 2020). Schwert (2020) shows that loan lenders earn a large premium relative to the bondholders of the same company, and documents that the premium is especially large for companies with high default risk. Cai et al. (2018) and Hatfield et al. (2020) suggest that price collusion by syndicate members may exist in syndicated loan markets.

Our study differs from prior papers on a few important fronts. First, we develop a theoretical model that takes into account the real-world frictions presented in the lending markets, including lenders’ private funding costs, borrowers’ downward sloping demand curve, and costs of collusion and monitor-

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8See, Rajan (1992); Petersen and Rajan (1994); Sufi (2007); Ivashina and Scharfstein (2010); Bharath, Dahiya, Saunders, and Srinivasan (2011); Chava and Purnanandam (2011) and Murfin (2012), among others.
ing. Second, fitting this model to our hand-collected novel dataset, we are able to quantify the margins earned by specialized lenders from distressed borrowers as a result of collusion versus private funding costs. More importantly, our study provides policy implications by showing that smaller borrowers are most susceptible to lender collusion and thus adds value to the recent policy debate on whether government should step in to finance bankrupt firms.9

2 Background and Motivating Evidence

2.1 Background

The loan markets for distressed corporate borrowers are among the few most essential asset markets in the economy because of its vital role in determining the ex-ante “financial distress cost” for the whole corporate sector, albeit not the largest by market value. These markets define the survival rate of financially distressed firms and the efficiency of bankruptcy processes. In particular, financially distressed firms seek urgent financing to support their working capital, investment, and debt repayment. Without such financing, their business operation would be in despair and may end up in a premature liquidation. Despite their desperation to fund operations, financially distressed firms face severe financial constraints and other economic frictions in arranging financing.

We emphasize that the importance of an asset market is not equivalent to the size of the market. A useful analogy is the intensive care unit (ICU) in the healthcare and hospital system. The ICU is a small segment of the whole healthcare and hospital system because the ICU admission only accounts for less than 10% of the total hospital admission per annum and the ICU beds only account for about 10% of the hospital beds in total. However, the ICU has the first-order importance to ensure the whole healthcare and hospital system to operate properly. Similar to the ICU segment, the distressed loan and the DIP loan together account for about 10% of the outstanding leveraged loan per annum, but they account for over 45% of the outstanding leveraged loan in the year of 2009 (i.e., the Great Recession).10 Despite the

9In considering the difficulty of smaller distressed borrowers to access financing, the Consolidated Appropriations Act of 2021 (“CAA”) that was signed into law on December 27, 2020 allows small debtors in bankruptcy to access the Paycheck Protection Program (“PPP”) for loans. However, Small Business Administration (“SBA”) holds a different view.

10Financing distressed companies has become an important concern for both policymakers and practitioners as the U.S. enters into a recession during a global pandemic and a record number of large companies filed for bankruptcy in 2020. Albeit not in bankruptcy, many firms suffered large operating losses and became financially distressed. Importantly, as emphasized by
importance of this market to the survival of distressed firms, academic studies on understanding the economic forces that drive the pricing and quantity of distressed loans are scarce. Our study tries to fill this gap by quantifying the effect of lenders’ market power on distressed loan pricing and suggesting implementable policies.

In the distressed loan market and the DIP loan market, distressed corporate borrowers deal with a clique of specialized lenders that finance a large fraction of loans. In Section 2.2, we present a set of novel evidence showing that loan spreads are ultra high even after removing the default- and liquidity-risk components and that the few specialized institutional lenders have large market power in these markets. In fact, not surprisingly, specialized lenders are expected to possess significant market power in financing distressed firms for at least the following three reasons.

First, the distressed borrowers’ bargaining position is weak and their price elasticity of demand for loans is low. Distressed firms face a dire liquidity situation and are in desperate need to raise capital to survive. Importantly, these firms often have very limited “unencumbered” assets to pledge for additional secured credit, and very different from non-distressed ones, they have a few alternative external funding options, such as equity, bonds, or unsecured lines of credit. Moreover, a distressed borrower is reluctant to approach a large number of lenders or even call for an open auction for getting the best priced loan due to concerns of information leakage that could negatively affects its security prices and supply chain.

Second, high entry barriers lead to segmented and concentrated markets in which specialized lenders can tacitly collude. Lenders who are not familiar with or do not have expertise in restructuring of distressed firms and those with regulatory capital concerns choose not to enter the market. Moreover, because specialized lenders have tight and repeated syndication relationships (Cai et al., 2018; Hatfield et al., 2020) and share a considerable amount of multi-market contact beyond the loan markets (Bernheim and Whinston, 1990; Evans and Kessides, 1994; Ciliberto and Williams, 2014), distressed borrowers typically find themselves facing only a clique of specialized lenders, who have strong incentives and capacities to collude tacitly in making loans.

Third, even for the few specialized lenders, there are two forces preventing them from participat-

DeMarzo et al. (2020), there are a large number of firms that suffered from financial distress caused by a “pause” in cash flows. For example, the Basel-III framework requires financial institutions to set aside larger quantity of capital for financing companies with higher default risk because of the higher regulatory risk-weighting of high risk loans.
ing a deal — existing lenders’ blocking power and their information advantage. First, existing lenders of distressed borrowers have a blocking power — they can, to certain extent, prevent outsider lenders from lending to the distressed borrowers, even when the borrowers would prefer to invite more outsider lenders to participate. The existing lenders have strong incentives to present outsider lenders’ participation because they are not only reluctant to share the investment opportunity, but also concerned with ex-post coordination issues and creditor conflicts in bankruptcy processes (Gertner and Scharfstein, 1991; Morris and Shin, 2004; Brunner and Krahnen, 2008; Dou et al., 2021a). In particular, existing lenders can have substantial control over the distressed borrower, and they can threaten the firm of liquidation at the interim date (i.e., early liquidation). With such a power, the existing lenders can dictate an outcome that improves its position at the expense of other new creditors and the firm value. For example, rather than allow the distressed borrower to pursue a syndicated loan with a large number of outsider lenders, the existing lenders can impose a syndication with a small number of outsider lenders by threatening the distressed borrower of a prompt sale of the assets.\footnote{As another example, one common strategy is so-called “roll-up” in which the borrower needs to pay off the earlier loan with proceeds of the newly-added DIP loan, ensuring that the earlier loan is paid in full even if it was not actually fully collateralized (e.g., Tung, 2020).} Second, the existing lenders of distressed borrowers are able to leverage on their advantages in information and control to shy the outsider lenders away from participation. Specifically, distressed borrowers’ existing creditors’ information advantage discourages the participation of the competing outsider lenders who are concerned with adverse selection and allows the existing creditors to hold up borrowers for ultra-high interest rates (e.g., Sharpe, 1990; Schenone, 2010; Santos and Winton, 2008).

\section*{2.2 Motivating Evidence}

\textbf{Concentrated markets dominated by a clique of specialized lenders.} We first construct a comprehensive sample for large public borrowers in the distressed loan market from 2001 to 2017 and a comprehensive sample for large public borrowers in the DIP loan market from 2002 to 2019. We elaborate the details on the construction of the distressed loan sample and that of the DIP loan sample in Sections 4.1 and 4.2, respectively. In our sample, the average maturity of distressed loans is 53 months (i.e., about 4 years), and the average maturity of DIP loans is 10 months (i.e., about 1 year). The average loan-to-
Table 1: Specialized lenders and market concentration

A. Names of specialized lenders

<table>
<thead>
<tr>
<th>Rank</th>
<th>Lender name</th>
<th>Distressed loan market # of deals</th>
<th>DIP loan market # of deals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bank of America</td>
<td>182</td>
<td>Wells Fargo</td>
</tr>
<tr>
<td>2</td>
<td>JP Morgan Chase</td>
<td>175</td>
<td>Bank of America</td>
</tr>
<tr>
<td>3</td>
<td>Wells Fargo</td>
<td>117</td>
<td>JPMorgan Chase</td>
</tr>
<tr>
<td>4</td>
<td>Deutsche Bank</td>
<td>101</td>
<td>GE Capital Corp</td>
</tr>
<tr>
<td>5</td>
<td>Citigroup</td>
<td>100</td>
<td>Citigroup</td>
</tr>
<tr>
<td>6</td>
<td>Credit Suisse First Boston</td>
<td>100</td>
<td>Deutsche Bank</td>
</tr>
<tr>
<td>7</td>
<td>GE Capital Corp</td>
<td>60</td>
<td>Wachovia Bank</td>
</tr>
<tr>
<td>8</td>
<td>Goldman Sachs</td>
<td>58</td>
<td>Wilmington Trust Co</td>
</tr>
<tr>
<td>9</td>
<td>UBS</td>
<td>54</td>
<td>CIT Group</td>
</tr>
<tr>
<td>10</td>
<td>Wachovia Bank</td>
<td>49</td>
<td>Credit Suisse First Boston</td>
</tr>
</tbody>
</table>

B. Three lender types

<table>
<thead>
<tr>
<th>Lender type</th>
<th>Distressed loans # of deals</th>
<th>% # of deals</th>
<th>$ # of deals</th>
<th>% $ of deals</th>
<th>DIP loans # of deals</th>
<th>% # of deals</th>
<th>$ # of deals</th>
<th>% $ of deals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1: Existing lender</td>
<td>47</td>
<td>10.88%</td>
<td>12</td>
<td>5.65%</td>
<td>43</td>
<td>14.48%</td>
<td>3</td>
<td>2.72%</td>
</tr>
<tr>
<td>Type 2: Specialized lender</td>
<td>328</td>
<td>75.93%</td>
<td>194</td>
<td>88.30%</td>
<td>213</td>
<td>86.20%</td>
<td>97</td>
<td>92.73%</td>
</tr>
<tr>
<td>Type 3: Lender of last resort</td>
<td>57</td>
<td>13.19%</td>
<td>13</td>
<td>6.05%</td>
<td>41</td>
<td>13.80%</td>
<td>5</td>
<td>4.54%</td>
</tr>
<tr>
<td>Total</td>
<td>432</td>
<td>100%</td>
<td>219</td>
<td>100%</td>
<td>297</td>
<td>100%</td>
<td>105</td>
<td>100%</td>
</tr>
</tbody>
</table>

Note: The distressed loan sample is from 2001 to 2017, and the DIP loan sample is from 2002 to 2019. The loan size are measured by constant 2019 dollars and presented in the unit of billion dollars.

asset ratios are 13.4% and 16.2% for distressed loans and DIP loans, respectively. Moreover, the average loan spreads after removing credit spreads and liquidity premium are 269 and 596 basis points for distressed and DIP loans, respectively. These spreads are excessively high, and these ultra-high risk adjusted spreads are the primary focus of this paper.

Based on our samples, we define a lender to be a specialized lender in the market if it is one of the top 10 lenders ranked by the number of distressed loans that it financed in our sample, regardless of the number of lenders in the loan syndicate. We can further define three types of deals. The first type contains deals in which loans are provided by only one lender, who is an existing lender and is not a specialized lender. These loans are referred to as “existing-lender loans” and denoted by Type 1. The second type contains deals in which loans are provided by at least one specialized lender. These loans are referred to as “specialized-lender loans” and denoted by Type 2. The third type contains deals in which loans have over 50% of the lenders as hedge funds and private equity funds. These loans are...
referred to as “last-resort loans” and denoted by Type 3.

Panel A of Table 1 presents the names of the top 10 financial intermediaries that are specialized lenders in the distressed loan and DIP loan markets. Panel B of Table 1 reports the breakdown of the number of loans for each of the three types and shows their shares as a fraction of the total number of deals in our samples. There are several important points worth mentioning. First, the number of deals financed by the top intermediaries decays rather fast even among the specialized lenders — the top 1 specialized lender’s involvement in distressed loans or DIP loans is more than 4 times larger than the top 10 specialized lender’s involvement. Second, the specialized-lender loan accounts for 75.93% (88.30%) and 86.20% (92.73%) fraction of the total distressed and DIP loan market, respectively, in terms of the number of deals (in terms of the loan size in dollars). Third, GE Capital Corp is the only specialized lender that is not a large bank with an investment banking arm but a shadow banking lender. In fact, GE Capital Corp is oversight by the Federal Reserve as a “systemically important financial institution” like other large banks, and importantly, it has substantial multi-market contact with large banks in investment banking, real estate, commercial lending & leasing, and healthcare finance services, among others, which makes GE Capital Corp no ordinary shadow banking lender. Although it is unlikely for ordinary shadow banking lenders, such as hedge funds and private equity shops, to be included in the “club” as a reliable member in tacit coordination, GE Capital Corp is unique due to its substantial multi-market contact with large banks. Finally, the specialized lenders are largely overlapped (8 out of 10) between these two loan markets for distressed corporate borrowers, which is consistent with the hypothesis that it requires specialized skills in distress, restructuring, and bankruptcy to become a major player in the distressed and DIP loan markets. All these facts suggest that the loan markets for distressed corporate borrowers feature market segmentations and high concentration, dominated by a few specialized lenders.

**Ultra-high loan spreads.** The loan spread is measured by the all-in spread drawn (AISD) after removing the credit spread component and the liquidity premium component. The AISD is the sum of a loan’s LIBOR spread and its annual fee paid to the lenders. We first estimate the average credit spread component in the yield spread of distressed loans and DIP loans separately. For the DIP loan, we estimate the credit spread component by 20 bps. A DIP loan is short-term with a maturity usually less than 1
Figure 1: Average loan spreads and fraction of deals financed by specialized lenders

year and secured with first lien and it is as safe as high investment-grade short-term bonds whose credit spread component is about 60 – 80 bps, accounting for about 85% of their total bond yield spread (e.g., Chen, Lesmond, and Wei, 2007). And, according to our own calculation using loan facilities in Dealscan and S&P issuer ratings, the median loan yield spread of syndicated loans with ratings of AA- and higher is about 20 bps, making the credit risk component at most 20 bps.

For the distressed loan, we use CDS spreads to back out the implied risk-neutral default probability with 38.02% as the recovery rate of corporate bonds with CCC+ or lower ratings (e.g., Elton, Gruber, Agrawal, and Mann, 2001), and then we assume that recovery rate of the distressed loan is 95% to compute the implied credit spread (e.g., Badoer, Dudley, and James, 2019), which leads to an estimate of 350 bps for the credit spread component of the distressed loan. Our approach closely follows the parity relation between CDS spreads and bond credit spreads derived by Duffie (1999). The details are elaborated in Appendix.

We then estimate the average liquidity premium component in the yield spread of distressed loans

13Blanco et al. (2005) provide supporting evidence for the theoretical parity derived by Duffie (1999) among investment-grade bonds using the swap rate as the risk-free rate.
and DIP loans separately. For the DIP loan, we estimate the liquidity premium by 20 bps. First, a DIP loan is a short-term debt with a maturity usually less than 1 year, and it is a senior and secured loan with first and second lien. As a result, it unlikely cause liquidity concerns to its investors. Second, Longstaff et al. (2005) find that the credit spread component accounts for a major portion of the bond yield spread for bonds. Moreover, Chen et al. (2007) find that the liquidity premium component alone explains about 15% of the bond yield spread for short-term bonds with maturities of 1 – 7 years and ratings of AA- or higher. Taken together, for the DIP loan, the liquidity premium component is unlikely higher than the credit spread component, estimated by 20 bps.

For the distressed loan, we estimate the liquidity premium by 30 bps. First, investors also trade distressed loans in secondary loan markets (e.g., Gande and Sauders, 2012). Particularly, Wittenberg-Moerman (2008) and Bushman et al. (2010) use Loan Trade Database to show that the secondary market of distressed loans is more liquid than that of less risky loans, opposite to the pattern in the secondary markets of bonds. These studies suggest that distressed loans syndicated by more reputable arrangers are likely traded with lower liquidity premia than the distressed bonds. Second, the estimates of Chen et al. (2007) suggest that the liquidity premium is about 34 bps for short-term bonds with maturities of 1 - 7 years and ratings of CCC+ or lower.

Panel A of Figure 1 plots the time series of the average distressed loan spread per year and the fraction of distressed loans financed by specialized lenders per year. Panel B of Figure 1 plots the same yearly time series for the DIP loan market. There are several important points about Figure 1 that worth mentioning. First, the average loan spread after removing the credit spread and the liquidity premium component remain stable over years at a very high level. The distressed loan spread fluctuates mainly between 200 and 350 bps, while the DIP loan spread fluctuates mainly between 400 and 750 bps, without obvious cyclical patterns or clear comovement with financial conditions of the economy. Second, the fraction of loans financed by specialized lenders also stay high persistently, without obvious cyclical patterns or clear comovement with financial conditions of the economy. These facts suggest that the high concentration of the loan markets for distressed corporate borrowers is not due to high concentration clustered around a few point of time, but it is due to the persistent feature in the industry structure of loan markets for distressed corporate borrowers.
Table 2: Variance decomposition: Lender fixed effects on loan spreads

<table>
<thead>
<tr>
<th></th>
<th>A. Distressed loan market</th>
<th>B. DIP loan market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F-tests on lender fixed effects</td>
<td>F-tests on lender fixed effects</td>
</tr>
<tr>
<td>Log loan spread</td>
<td>Lender category</td>
<td>Frequent lender</td>
</tr>
<tr>
<td>ln(R_{i,j,t})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(R_{i,j,t})</td>
<td>3.30 (0.011, 392)</td>
<td>264</td>
</tr>
<tr>
<td>ln(R_{i,j,t})</td>
<td>2.70 (0.000, 372)</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table presents the lender fixed effects on log loan spreads. "Lender category" stands for the fixed effects for lender category (i) through (v), and "Frequent lender" stands for the fixed effects for the lender institution that appears most frequently among all the lenders of a given deal. The reporting format follows Bertrand and Schoar (2003). Reported are F-tests for the joint significance of the lender fixed effects. For each F-test we report the value of the F-static, the p-value, and the number of constraints (the latter two in parentheses). The control variables include loan size, loan type (revolver/term loan), maturity, firm size, ROA, leverage ratio, cash balance, and asset tangibility.

Lender market power and loan spreads. We consider five lender categories according to the lender type and the syndication composition: (i) loan only by existing creditors, i.e., type-1 loan, (ii) loan by 1 specialized lender, (iii) loan by 2-3 specialized lenders, (iv) loan by more than 3 specialized lenders, and (v) loan by lenders of last resort, i.e., type-3 loan. The loans in categories (ii) – (iii) are all type-2 loans.

Importantly, as emphasized our model and estimation, there is no such thing as a random allocation of lenders to borrowers in reality. We want to assess whether there is any evidence that loan spreads systematically change with the lender category based on lender types and syndication compositions. Specifically, similar to Bertrand and Schoar (2003), we propose to estimate the following regression with lender category fixed effects and conduct variance decomposition:

\[
\ln(R_{i,j,t}) = \alpha_t + \gamma_j + \beta X_{i,t} + \lambda_{lender} + \lambda_{frequent} + \epsilon_{i,j,t},
\]

where \(\ln(R_{i,j,t})\) stands for log loan spread of borrower \(i\) of industry \(j\) in year \(t\), \(\alpha_t\) are year fixed effects, \(\gamma_j\) are industry fixed effects for Fama-French 12 industries, \(X_{i,t}\) represents a vector of time-varying borrower
level controls, and $\epsilon_{i,j,t}$ is an error term. The remaining variables in equation (1) are fixed effects for the lender characteristic that we observe in multiple deals. $\lambda_{\text{lender}}$ stands for the fixed effects for lender category, while $\lambda_{\text{frequent}}$ stands for the fixed effects for the lender institution that appears most frequently among all the lenders of deal $i$ in our sample. We separately study the effect of lender category and most frequent lender institution on loan yield spreads. The control variables include loan size, loan type (revolver/term loan), maturity, firm size, ROA, leverage ratio, cash balance, and asset tangibility.

Table 2 reports F-tests and adjusted $R^2$ from the estimation of equation (1) for both the distressed loan sample (panel A) and the DIP loan sample (panel B). We report in the first row the fit of a benchmark specification that includes only industry fixed effects, year fixed effects, and time-varying borrower controls. The next two rows, respectively, report the change in adjusted $R^2$ when we consecutively add the lender category fixed effects and the fixed effects for the lender institution that appears most frequently among all the lenders of a given deal. The second and third rows also report F-statistics from tests of the joint significance of the different sets of lender fixed effects. Overall, the findings in Table 2 suggest that lender effects matter both economically and statistically for the loan spread of distressed corporate borrowers. Including lender fixed effects increases the adjusted $R^2$ of the estimated models significantly. Similarly, we find that the F-tests are large and allow us to reject in most cases the null hypothesis that all the lender fixed effects are zero.

We start with a discussion of the results of the distressed loan market, summarized in panel A of Table 2. The benchmark specification includes controls for borrower fixed effects, year fixed effects, and a rich set of time-varying borrower-specific control variables. The adjusted $R^2$ for this specification is 33.3 percent. The adjusted $R^2$ increases by 5.4 percentage points when we include the lender category fixed effects and by 6.3 percentage points when we include the frequent lender fixed effects. Also, the F-tests are large and statistically significant, comparable to those increases in adjusted $R^2$ of Bertrand and Schoar (2003) for manager fixed effects on firm decisions, leading us to reject the null hypothesis of no joint effect in all cases.

We next discuss the results of the DIP loan market, summarized in panel B of Table 2. The estimation method is exactly the same as the one described for panel A. The adjusted $R^2$ for the benchmark specification is 38.7 percent. For this market, we observe an increase in adjusted $R^2$ of 4.2 percentage points
following the inclusion of the lender category fixed effects. Similarly, we also observe an increase in adjusted $R^2$ of 4.9 percentage points following the inclusion of the frequent lender fixed effects. Moreover, the F-tests are large and statistically significant, leading us to reject the null hypothesis of no joint effect in all cases. Again, the increases in adjusted $R^2$ are comparable to those of Bertrand and Schoar (2003) for manager fixed effects on firm decisions.

3 Game-theoretic model for distressed-loan markets

In this section, we build a novel and flexible game-theoretic model of distressed loan market competition with repeated collaboration relations among specialized lenders and endogenous entries as participants of syndicated lending. We first describe the model’s setup, then explain the predictions that form the basis of our estimation. The model features a financially distressed firm which borrows from the distressed loan market dominated in the hands of a few specialized lenders. The repeated oligopoly lending game features a downward-sloping demand system with heterogeneous borrowers with latent characteristics, tacit collusion among specialized lenders, and endogenous market concentration and market power.

3.1 Generic setup

The model starts with $M$ specialized and repeated major lenders in a specific loan market, who are referred to as “specialists” throughout the paper, and $M_0$ potential lenders, including non-specialized lenders, in the market with $M \ll M_0$. Because of the specialists’ dominant position is highly persistent and there are quite a few large deals arrive in the market every year, the $M$ specialists have strong incentives to form tacit collusion and collaborate in terms of syndicating the distressed loans. “Tacit coordination” need not involve any explicit collusion with direct communication and agreement on joint decisions in the legal sense, and an interchangeable term is “tacit collusion” or “noncooperative collusion” or “conscious parallelism” (e.g., Martin, 2006; Ivaldi et al., 2007; Harrington, 2008; Green et al., 2014; Garrod and Olczak, 2018).

The deals arrive randomly. We assume that the arrivals of deals can be characterized by a Poisson process with intensity $\eta$. Each borrower can be one of the $K$ types, and we label a borrower $i$ by $k_i$ for
The type $k_i$ reflects two fundamental characteristics of borrower $i$ in our model: first, it captures the borrower’s size $A_i$ measured by its total value of asset, and second, as to be specified in the next section, our model allows heterogeneous demand curves across different borrowers, and thus $k_i$ captures the demand curve borrower $i$ is attached to, $d_i$. We denote $k_i = (d_i, A_i)$. Even though size $A_i$ is observable, the demand curve $d_i$ is seen only by the agents inside the model and is latent to the econometricians. We denote the fraction of type $k$ borrowers in the population by $\pi(k)$. Each borrower of type $k$ starts with trying to match with an existing lender with a likelihood, denoted by $\lambda(k)$. If the borrower fails to reach an agreement with the existing lender, then the borrower continues to approach the specialists for funding. It is possible that no specialist is willing to participate in the deal, in which case the borrower has to find the last-resort non-specialized lender. We classify lenders into three types, denoted by $l \in \{1, 2, 3\}$. Particularly, we label the existing lender by $l = 1$, the specialist lender by $l = 2$, and the last-resort non-specialized lender by $l = 3$. Before elaborating the timeline and sequence of lenders’ decisions for each deal in Section 3.1.2, we first introduce the basics about borrowers and lenders below in Sections 3.1.1 and 3.1.2.

3.1.1 Demand specification

When the borrower has type $k$ and faces the lenders of type $l$, the value of the borrower by raising the amount of loan $L$ is

$$W = \underbrace{L^{\beta_l(k)}S_l(k)^{1-\beta_l(k)}}_{\text{PV of running business}} - \underbrace{L \mathbb{E}[e^{-r_f Y}]}_{\text{PV of borrowing loans}},$$

(2)

where $\beta_l(k) \in (0, 1)$ captures the decreasing return to scale of the amount of distressed loans, $r_f$ is the log riskfree rate, $\mathbb{E}[\cdot]$ is the risk-neutral expectation, $Y$ is the total return of the distressed loan, and $S_l(k)$ is the return shifter, a quantity that captures the effect of firm characteristics and incorporates the interactions between investor and asset characteristics (like in Hendel, 1999). The return shifter $S_l(k)$ is specified as follows:

$$S_l(k) \equiv A e^{\nu_l(k)} + \sigma z,$$

(3)

where $A$ is the total asset of the firm, $\nu_l(k)$ captures the effect of the interactions between investor and asset characteristics, and $z$ is a latent firm-specific demand shock.
The total return of the loan $Y$ is $e^y$ with a risk-neutral probability $1 - q^*$ and is $\delta e^y$ with a risk-neutral probability $q^*$, where the probability $q^*$ and the recovery rate $\delta$ are firm specific, and $y$ is log yield on the distressed loan. The risk-neutral default probability $q^*$ and Thus, the firm-level value function can be rewritten as

$$W = L^{\beta_l(k)}S_l(k)^{1-\beta_l(k)} - LR,$$

(4)

where $\ln(R) \equiv y - r_f - d$ with $d \equiv -\ln[1 - q^*(1 - \delta)]$. We refer to $R$ as the risk-adjusted spread of the distressed loan.

The first-order condition for $L$ leads to a demand curve of loan size for a type-$k$ borrower and type-$l$ lenders as follows:

$$\ln\left(\frac{L}{A}\right) = \alpha_l(k) - \epsilon_l(k) \ln(R) + \sigma z,$$

(5)

where

$$\alpha_l(k) \equiv \nu_l(k) + \frac{\ln \beta_l(k)}{1 - \beta_l(k)},$$

(6)

$$\epsilon_l(k) \equiv \frac{1}{1 - \beta_l(k)}.$$  

(7)

The variation in the term $\alpha_l(k) + \sigma z$ captures the demand curve shift across different firms in the population. The coefficient $\alpha_l(k)$ captures the heterogeneous demand level depending on different borrower types $k$ and lender types $l$. The borrower type $k$ is latent to the econometricians, whereas the lender type $l$ is observable to the econometricians.

The demand function in equation (5) is a standard iso-elastic downward-sloping demand curve. The coefficient $\epsilon_l(k)$ is effectively the price elasticity of demand. Consistent with the literature (e.g., Atkeson and Burstein, 2008), we assume that $\epsilon_l(k) > 1$.

We assume that $z$ is i.i.d. distributed according to the standard normal distribution across different cases. The assumption that the demand curve depends on heterogeneous borrowers and differentiated lenders is a natural analog of that in the empirical IO literature emphasizing heterogeneous consumers and differentiated products (e.g., Berry et al., 1995). A higher $\alpha_l(k)$ means that the borrower of type $k$ has higher average demand of loans from the lenders of type $l$. 

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Intuitively, charging higher spread $R$ leads to smaller loan amount because $\varepsilon_l(k) > 1$. The slope coefficient $\varepsilon_l(k)$ mainly captures the price elasticity of the type-$k$ borrower’s loan demand from the type-$l$ lenders. Similar in spirit to Koijen and Yogo (2019), the slope coefficient is determined by multiple important primitive characteristics of borrowers and lenders. Like the approach adopted by Koijen and Yogo (2015, 2019), among others, in their structural estimation, we start with specifying a flexible demand system on top of a structural agent-optimization model, rather than microfoundining the equilibrium relations between the slope coefficient $\varepsilon_l(k)$ and various primitive characteristics of borrowers and lenders. Specifically, $\varepsilon_l(k)$ can reflect the bargaining power of the borrower and the lenders in the loan market, the marginal value of liquidity of the borrower, the intervention style of the lenders, and the non-pecuniary benefits to the firm from borrowing from the relationship lenders. With a larger $\varepsilon_l(k)$, the amount of loans $L$ the firm would like to borrow declines more with an increase in loan spread $R$ required by the lender, meaning that the borrower’s demand for loans is more elastic to the loan spread and thus the borrower has more bargaining power. Take an extreme case as an example to illustrate the role of the slope coefficient $\varepsilon_l(k)$. When $\varepsilon_l(k) \to +\infty$, the borrower has full bargaining power over the lenders since it is not willing to pay any spread on top of the risk-adjusted rate. In such an extreme case, a higher $a_l(k)$ obviously reflects a higher intrinsic loan demand from the borrower.

Demand curves can also vary by lender types. Classic banking theories suggest that banks act as traditional financial intermediaries to provide financing and acting as the delegated monitors (e.g. Diamond (1984, 1991)). Banks rarely seek direct control of the borrower through board representation and other mechanisms due to their concerns of legal liabilities (Fischel, 1989). The loan costs to the borrower are mostly reflected in spreads and fees to be paid to the lender. In contrast, alternative investors such as hedge funds and private equity firms use loan instruments as a tool to engage activism and seek control. They often adopt the “loan-to-own” strategies where they lend to firms with an intention to convert debt into ultimate equity ownership (Jiang et al., 2012). In addition to loan interest and fees, these alternative lenders impose strict covenants that may be tied to management changes and governance, which allow them to directly control the borrower’s business operation. As a result, to borrowers, loans provided by traditional lenders and alternative lenders may be viewed as different products.
We normalize the loan size $L$ by the total asset $A$ in the modeling of the demand system for distressed loans in equation (5). The main reasons or motivations behind such modeling choice are threefold. First, the additional financial distress risk caused by the newly-added leveraged distressed loan $L$ depends on the total asset of the borrower $A$. Second, the leveraged distressed loan $L$ is mainly for covering working capital, which in turn is usually proportional to the firm size and thus the total asset level $A$. Third, in the data, we do find that a strong relationship between the normalized loan size $L/A$ and the spread $R$ within each type of deals.

3.1.2 Supply specification

We first describe the market structure and “technology” of the lenders to make distressed loans. The supply side of distressed loans is characterized by Cournot competition of oligopolistic lenders similar to Berry et al. (1995). The oligopolies can tacit collude usually in the form of syndication, which goes beyond the non-collusive Nash behavior adopted by the BLP framework and captures the highly strategic competition behavior (i.e., tacit coordination). The literature shows that some credit markets are concentrated in the hands of a few leading financial institutions, and importantly, these institutional lenders compete highly strategically including competition in the form of tacit collusion. For example, Knittel and Stango (2003) documents micro-level evidence suggesting that tacit collusion at non-binding state-level ceilings was prevalent in the credit card lending market during the early 1980’s. Moreover, on 5 April 2019, the European Commission published a report – prepared by Europe Economics at the request of the department for competition – on EU loan syndication and its impact on competition in credit markets. The commission had worried that syndicated loan coordination tended to occur tacitly, making anti-competitive collusion on interest rates easier to pull off. The worry is natural because syndicated loans naturally require coordination and communication. The report highlighted the risk that lenders involved in syndicating a loan together would promise future tacit cooperation in exchange for current concessions on limiting loan supply. Nocke and White (2007) and Hatfield et al. (2020) theoretically show that, under certain circumstances, tacit collusion can exist in syndicated markets with repeated interaction of lenders. Further, Carrasco and Manso (2006) theoretically show that syndication is the optimal response of colluding lenders to the communication costs resulting from the negotiations.
between them for a given loan. Our model builds on the important insights of loan syndication and potential tacit collusion of repeated interacted lenders in the distressed loan markets.

**Costs of lending distressed loans.** The lender incurs a fixed cost from learning the deal type and participating in lending, and it also incurs a variable cost that depends on the amount of loan made to the borrower \(L\). Specifically, the lender incurs a one-time fixed cost of \(w\) if it decides to learn the deal type \(k\) and commit to participation. Fixed cost \(w\) is random and privately observed by the lender. However, the distribution of \(w\) is a common knowledge of all agents and is assumed to be the exponential distribution with mean \(\mu\) with the following density function:

\[
f(w; \mu) = \mu e^{-w/\mu}.
\]  

Fixed cost \(w\) captures both direct costs, such as compensation to talents and other labor costs, as well as indirect costs, such as the loss of other investment opportunities because of limited resources. In addition, the lender of type \(i\) incurs variable costs of \(e^{\phi_i + \varsigma u}\) for each unit of lending, where \(u\) is a standard normal random variable that captures the deal-specific latent cost. Taken together, both latent characteristics \(z\) and \(u\) are deal specific (or firm specific). We denote \(x \equiv (z, u)\). Variable cost \(e^{\phi_i + \varsigma u}\) captures both direct costs, such as compensation, as well as indirect costs, such as marginal (shadow) costs of funding for the lender. Consistent with this assumption, we find that the lender explicitly charges proportional fees to cover the variable costs. For any deal, total costs from \(w\) and \(e^{\phi_i + \varsigma u}\) are denoted by

\[
C_i(L) = w 1_{\{L > 0\}} + e^{\phi_i + \varsigma u} L.
\]  

**Timing and sequence of lenders’ decisions within each deal.** Figure 2 illustrates lenders’ choices and possible outcomes in each deal, including lending by existing lenders and last-resort lending possibilities. The time span for each deal is divided into two subperiods, “morning” and “afternoon.” The shocks, such as whether to punish for deviation or not, the type of the deal \(k\), and the private costs \((w_1, \cdots, w_M)\), are realized in the morning, while the lending decisions \(L\) and \(R\) are made in the afternoon. Importantly, specialists must make their decisions whether to participate in the syndicated lending by the end of the
This figure describes the timeline of the model. Each lending period is divided into two subperiods – morning and afternoon. The existing lender first decides whether to lend to the borrower with a probability $\lambda(k)$, if the existing lender does not lend to the borrower, the borrower approaches a group of specialist lenders who can decide whether to participate in a syndicate loan. The specialist lenders need to pay a fixed cost if they participate. If no specialist lenders are willing to participate, the borrower turns to the lenders of last-resort.

In the first step of “morning”, the firm approaches the existing lender, who can be a specialized or non-specialized lender. We assume that each of the $M_0$ potential lenders can be the existing lender with equal chances (i.e., $1/M_0$). The lending agreement with this particular existing lender can only be achieved with a small probability $\lambda(k)$ which depends on case type $k$. If the lending agreement with the existing lender is reached, the monopolistic lender of type $l=1$ chooses the optimal spread and the loan size according to the demand curve:

$$\Pi_1(k, x) = \max_L \left[ \left( e^{\alpha_1(k)+\sigma z} \frac{A}{L} \right)^{1/\epsilon_1(k)} - e^{\phi_1+cz} \right] L,$$  

(10)
where the default probability on the distressed loan does not show up since \( R = \left( e^{\alpha_1(k) + \sigma z A / L} \right)^{1/\varepsilon_1(k)} \) is a risk-adjusted loan spread, and \( x \equiv (z, u) \). It leads to the optimal monopolistic spread and loan size:

\[
R_1(k, x) = \frac{\varepsilon_1(k)}{\varepsilon_1(k) - 1} e^{\phi_1 + \varepsilon u} \quad \text{and} \quad L_1(k, x) = \left[ 1 - \frac{1}{\varepsilon_1(k)} \right]^{\varepsilon_1(k)} e^{\alpha_1(k) - \varepsilon_1(k)(\phi_1 + \varepsilon u) + \sigma z A},
\]

respectively. (11)

Therefore, the optimal profit is

\[
\Pi_1(k, x) = \frac{1}{\varepsilon_1(k)} R_1(k, x) L_1(k, x).
\]

Loan markup is defined as \( [R_1(k, x) - e^{\phi_1 + \varepsilon u}] / e^{\phi_1 + \varepsilon u} \). The loan spread implies that the loan markup is \( 1 / [\varepsilon_1(k) - 1] \), suggesting that the markup ratio decreases with the elasticity coefficient \( \varepsilon_1(k) \). The optimal profit \( \Pi_1(k, x) \) is \( 1 / \varepsilon_1(k) \) fraction of the revenue \( R_1(k, x)L_1(k, x) \). That is, the profit margin is \( 1 / \varepsilon_1(k) \).

When the elasticity coefficient \( \varepsilon_1(k) \) is lower, the borrower’s loan demand is effectively more urgent and pressing, leading to a higher markup and a higher profit margin. The detailed derivations of (11) and (12) are in the appendix.

The expected optimal profit is

\[
\Pi_1(k) = \mathbb{E}^x [\Pi_1(k, x)]
= \frac{1}{\varepsilon_1(k)} \left[ 1 - \frac{1}{\varepsilon_1(k)} \right]^{\varepsilon_1(k) - 1} e^{\alpha_1(k) - \varepsilon_1(k)(\phi_1 + \varepsilon u) + \sigma z^2 / 2 + \sigma^2 / 2 A}.
\]

The game moves to the second stage of “morning” if the firm fails to get a deal from the existing lender. In this case, the competition takes place among the \( M \) specialists (type \( l = 2 \)). We consider tacit collusion of a few specialized lenders. Let \( V^C(k, x, w, m; L^C) \) be the collusive value function of each specialist at the beginning of “afternoon”, if the agreed loan plan is \( L^C(k, x, m) \), there are \( m \) specialists who decided to participate in the deal of syndicated loan, the case is type \( k \), the specialist has private cost \( w \), and the deal-specific characteristics \( x \). There exists an equilibrium threshold \( w^*_C \) such that the specialist would participate in the syndicated loan if and only if \( w \leq w^*_C \).

Note that the privately observed fixed cost \( w \) is a sunk cost when the lender chooses the loan amount \( L \). Because \( V^C(k, x, w, m; L^C) \) is value function of a specialist at the beginning of the “afternoon” when
\( w, k, \) and \( x \) are already observed, the value function has the following functional form:

\[
V^C(k, x, w, m; L^C) \equiv U^C(k, x, m; L^C) - w.
\] (13)

The value function \( U^C(k, x, m; L^C) \) satisfies the following Bellman equation:

\[
U^C(k, x, m; L^C) = \Pi_2(k, x, m; L^C) + \frac{W^C(L^C)}{1 - \delta}, \quad \text{where}
\]

\[
W^C(L^C) = \mathbb{E}^k \left\{ \lambda(k') \frac{\Pi_1(k')}{M_0} + [1 - \lambda(k')] \sum_{m' = 1}^M q(m' | w' \leq w^*_C) \left[ F(w^*_C) \Pi_2(k', m'; L^C) - \int_{w' \leq w^*_C} w' dF(w') \right] \right\},
\]

where \( \mathbb{E}^k[\cdot] \) is the expectation over \( k' \in \{1, \ldots, K\} \) with probability weight \( \pi(k') \) for each \( k' \), and \( \Pi_2(k, m; L^C) \equiv \mathbb{E}^x \left[ \Pi_2(k, x, m; L^C) \right] \) with the profit of the syndicated lending with tacit collusive loan size plan \( L^C \) to be

\[
\Pi_2(k, x, m; L^C) \equiv \left[ \left( e^{\sigma_2(k) + \sigma z} \frac{A}{mL^C(k, x, m)} \right)^{1/\varepsilon_2(k)} - e^{\varphi_2 + \varsigma u} \right] L^C(k, x, m). \] (15)

and the conditional probability \( q(m | w \leq w^*_C) \) is

\[
q(m | w \leq w^*_C) = \frac{\mathbb{P}\{\text{This specialist and other } m - 1 \text{ specialists participate the lending}\}}{\mathbb{P}\{\text{This specialist participates the lending}\}}
\]

\[
= \binom{M - 1}{m - 1} F(w^*_C)^{m - 1} [1 - F(w^*_C)]^{M - m}.
\]

The derivation of this Bellman equation is in the appendix.

The game moves to the last-resort stage of “afternoon” if no specialists would like to participate the lending. In this case, the last resort (a non-specialized lender, i.e., the lender of type \( i = 3 \)) will lend to the firm as a monopoly with a fixed cost \( w \) and variable costs \( e^{\varphi_3 + \varsigma u} \). The last-resort lender optimally chooses \( L_3(k) \) to maximize the profit:

\[
\Pi_3(k, x) = \max_L \left[ \left( e^{\sigma_3(k) + \varsigma z} \frac{A}{L} \right)^{1/\varepsilon_3(k)} - e^{\varphi_3 + \varsigma u} \right] L. \] (16)
It leads to the optimal monopolistic spread and loan size:

$$R_3(k, x) = \frac{\varepsilon_3(k)}{\varepsilon_3(k) - 1} e^{\phi_3 + \zeta u} \quad \text{and} \quad L_3(k, x) = \left[ 1 - \frac{1}{\varepsilon_3(k)} \right]^{\varepsilon_3(k)} e^{\alpha_3(k) - \varepsilon_3(k)(\phi_3 + \zeta u) + \sigma z A},$$

respectively. (17)

Therefore, the optimal profit is $1/\varepsilon_3(k)$ fraction of the revenue $R_3(k, x)L_3(k, x)$ as follows:

$$\Pi_3(k, x) = \frac{1}{\varepsilon_3(k)} R_3(k, x)L_3(k, x).$$

(18)

Similar to the case in which the existing lender supplies all the loan, lower elasticity $\varepsilon_3(k)$ leads to a higher markup $1/[(\varepsilon_3(k) - 1]$ and a higher profit margin $1/\varepsilon_3(k)$. The detailed derivations of (17) and (18) are in the appendix.

### 3.1.3 Collusive Nash Equilibrium

The equilibrium outcomes in the lending of existing creditors and that of last-resort lenders can be explicitly characterized in (11) – (12) and (17) – (18), respectively. The equilibrium collusive loan amount is the central piece of model outcome that needs to be pinned down, which we explain in this subsection.

#### Optimal Lending under Tacit Collusion and Incentive Compatibility Constraint.

We first denote the unconstrained optimal loan amount under tacit collusion by $L^C_{\text{max}}(k, x, m)$, which solves the following profit maximization problem:

$$L^C_{\text{max}}(k, x, m) \equiv \arg\max_{L} \left[ \left( e^{\alpha_2(k) + \sigma z A} \frac{1}{mL} \right)^{1/\varepsilon_2(k)} - e^{\phi_2 + \zeta u} \right] L.$$ 

(19)

The first-order condition gives the solution for $L^C_{\text{max}}(k, x, m)$ as follows:

$$L^C_{\text{max}}(k, x, m) \equiv \frac{1}{m} \left[ 1 - \frac{1}{\varepsilon_2(k)} \right]^{\varepsilon_2(k)} e^{-\varepsilon_2(k)\phi_2 - \varepsilon_2(k)\zeta u} e^{\alpha_2(k) + \sigma z A},$$

(20)

with the corresponding unconstrained optimal loan spread and lending profit under tacit collusion to be

$$R^C_{\text{max}}(k, x, m) = \frac{\varepsilon_2(k)}{\varepsilon_2(k) - 1} e^{\phi_2 + \zeta u} \quad \text{and} \quad \Pi^C_{\text{max}} = \frac{1}{\varepsilon_2(k)} R^C_{\text{max}}(k, x, m)L^C_{\text{max}}(k, x, m),$$

respectively. (21)
Comparing (20) – (21) with (11) – (12), the intuition behind the smallest possible collusive lending outcomes immediately follows. That is, the \( m \) syndication participants first form the strongest coalition that behaves as if it is a monopoly, and then they split the loan equally.

However, the unconstrained optimal collusive loan size \( L^C_{\text{max}}(k,x,m) \), derived in (20), is usually unsustainable in the equilibrium because syndication participants would have strong incentives to deviate and reap additional profits by secretly supplying extra loans to the borrower. To sustain the tacit coordination among the specialized lenders as the club members who repeatedly participate in a particular market of distressed syndicated loans, the specialized lenders have an imperfect capacity of monitoring, communication, and ex-post punishment. Specifically, upon a deviation is detected, the specialized lenders will not tacitly collude anymore starting from the next period with a probability \( \xi \) as the punishment for deviation. This grim trigger punishment strategy is easy to implement and incentive compatible. There is a probability \( 1 - \xi \) that the deviation will not be punished. Thus, the parameter \( \xi \) captures the tacit collusion capacity in a parsimonious way. A lower \( \xi \) reflects a lower tacit collusion capacity, which can be due to more costly monitoring, more costly communications, and higher chances of achieving successful ex-post renegotiation. In our theory and structural estimation, we capture and estimate the tacit collusion capacity by focusing the deep structural parameter \( \xi \) without specifying or estimating various possible economic mechanisms that micro-found the imperfect tacit collusion at a more granular level, which is beyond the scope of this paper.

Now we characterize the set of loan sizes that are sustainable under the tacit collusion scheme with collusion capacity captured by parameter \( \xi \), namely, the set of loan sizes that satisfy the incentive-compatibility constraint. Intuitively, the loan size \( L^C(k,x,m) \) under the tacit collusion cannot be overly small to ensure that the \( m \) participants will have no incentives to deviate. We denote by \( L^C(k,x,m) \) the set of loan sizes for each of the \( m \) syndication participants that satisfy the incentive-compatibility constraint, which can be expressed as follows:

\[
L^C(k,x,m) \equiv \{ L^C : \mathbb{E}^x \left[ U^C(k,x,m;L^C) \right] \geq \mathbb{E}^x \left[ U^D(k,x,m;L^C) \right] \}, \tag{22}
\]

where \( U^C(k,x,m;L^C) \) is the value function when all the \( m \) syndication participants stick to the tacit coordination scheme of specialized lenders in the market as club members, and \( U^D(k,x,m;L^C) \) is the
maximum value of the syndication participant that deviates from the given tacit coordination scheme \( L^C(\cdot, \cdot, \cdot) \), which is described in detail below.

Intuitively, the tacit coordination scheme on loan size \( L^C(\cdot, \cdot, \cdot) \) lies in the set \( L^C(k, x, m) \) if and only if the expected value of not deviating, \( \mathbb{E}^x \left[ U^C(k, x, m; L^C) \right] \), is not strictly dominated by that of deviating, \( \mathbb{E}^x \left[ U^D(k, x, m; L^C) \right] \). As explained in Figure 2, the deviation decision is made after the syndication participants learn their private lending cost \( w \), borrower type \( k \), and the number of syndication participants \( m \), but before the latent case-specific characteristics \( x \) are learned. As a result, each syndication participant compares the two expected values contingent on not deviating or deviating.

**Non-Collusive Nash Equilibrium and Maximum Value of Deviation.** To characterize the maximum value of deviation \( U^D(k, x, m; L^C) \), we need to first characterize the phase of non-collusion competition among specialized lenders, where the outcome is described by the non-collusive Nash equilibrium. This is because the punishment for deviation is to shift into the phase of non-collusive competition.

The game moves to the second stage of “morning” if the firm fails to get a deal from the prepetition lender. In this case, the competition takes place among the \( M \) specialists (i.e. the lenders of type \( l = 2 \)). We consider the non-collusive Nash equilibrium in which the syndication participants never tacitly coordinate on making the loan. Let \( V^N(k, x, w, m) \) be the non-collusive value function of each specialist at the end of “morning,” a function of \( k, x, \) and \( m \), there are \( m \) specialists who decided to participate in the deal of syndicated loan, the case is type \( k \), and the specialist has private cost \( w \). There exists an equilibrium threshold \( w^*_N \) such that the specialist would participate in the syndicated loan if and only if \( w \leq w^*_N \).

Note that the privately observed fixed cost \( w \) is a sunk cost when the lender makes decision on \( L \). Because \( V^N(k, x, w, m) \) is value function of a specialist at the end of the morning when \( w \) is already privately observed, the value function has the following functional form:

\[
V^N(k, x, w, m) \equiv U^N(k, x, m) - w. \tag{23}
\]

The value function \( U^N(k, x, m) \) prior to paying the fixed cost \( w \) and observing the deal-specific char-
acteristics \( x = (z, u) \) satisfies the following Bellman equation:

\[
U^N(k, x, m) = \Pi_2(k, x; L^N) + \frac{W^N}{1 - \delta'} \quad \text{with}
\]

\[
W^N = \mathbb{E}^{k'} \left\{ \lambda(k') \frac{\Pi_1(k')}{M_0} + [1 - \lambda(k')] \sum_{m' = 1}^{M} q(m'|w') \left[ F(w^*_N) \Pi_2(k', m'; L^N) - \int_{w' \leq w^*_N} w'dF(w') \right] \right\},
\]

where \( \mathbb{E}^{k'}[\cdot] \) is the expectation over \( k' \in \{1, \ldots, K\} \) with probability weight \( \pi(k') \) for each \( k' \), and \( \Pi_2(k, m; L^N) \equiv \mathbb{E}^{x} \left[ \Pi_2(k, x; L^N) \right] \) with

\[
\Pi_2(k, x; L^N) \equiv \max_L \left[ \left( \frac{e^{\theta_2(k) + \sigma z}}{L + (m - 1)L^N(k, m)} \right)^{1/\varepsilon_2(k)} - e^{\phi_2 + \varsigma u} \right] L,
\]

and the conditional probability \( q(m|w \leq w^*_N) \) is

\[
q(m|w \leq w^*_N) = \frac{\mathbb{P} \{ \text{This specialist and other } m - 1 \text{ specialists participate the lending} \}}{\mathbb{P} \{ \text{This specialist participates the lending} \}} = \left( \frac{M - 1}{m - 1} \right) F(w^*_N)^{m-1} [1 - F(w^*_N)]^{M-m}.
\]

The derivation of this Bellman equation is in the appendix.

In the non-collusive equilibrium, the equilibrium loan size \( L^N(k, x, m) \) is characterized as follows:

\[
L^N(k, x, m) = \arg\max_L \left[ \left( \frac{e^{\theta_2(k) + \sigma z}}{L + (m - 1)L^N(k, x, m)} \right)^{1/\varepsilon_2(k)} - e^{\phi_2 + \varsigma u} \right] L,
\]

which leads to

\[
L^N(k, x, m) = \frac{1}{m} \left[ 1 - \frac{1}{m \varepsilon_2(k)} \right] ^{\varepsilon_2(k)} e^{-\varepsilon_2(k) \phi_2 - \varepsilon_2(k) \varsigma u} e^{\theta_2(k) + \sigma z A}.
\]

Therefore, the equilibrium revenue of a syndication participant has the following closed-form expression:

\[
\Pi_2(k, x; L^N) = \frac{1}{m^2 \varepsilon_2(k)} \left[ 1 - \frac{1}{m \varepsilon_2(k)} \right] ^{\varepsilon_2(k)} e^{-[\varepsilon_2(k) - 1] \phi_2 + \varsigma u} e^{\theta_2(k) + \sigma z A}.
\]

Finally, we now characterize the maximum value of deviating from an agreed tacit coordination scheme \( L^C(\cdot, \cdot, \cdot) \). We denote by \( V^D(k, x, w, m; L^C) \) the value function for deviation given a fixed tacit coordination scheme \( L^C(\cdot, \cdot, \cdot) \) and state variables \( k, m, \) and \( w \). We define \( V^D(k, x, w, m; L^C) \equiv U^D(k, x, m; L^C) - \)
$w$, and thus, the value function $U^D(k, x, m; L^C)$ satisfies the following Bellman equation:

$$U^D(k, x, m; L^C) = \Pi^D_2(k, x, m; L^C) + \frac{(1 - \zeta)W^C(L^C) + \zeta W^N}{1 - \delta}, \quad (29)$$

where $W^C(L^C)$ is the continuation value with the tacit coordination scheme $L^C(\cdot, \cdot, \cdot)$, defined in (54), $W^N$ is the continuation value in the phase of non-collusive competition, defined in (24), and the contemporaneous profit gained if the syndication participant deviate from the tacit coordination scheme $L^C(\cdot, \cdot, \cdot)$ is

$$\Pi^D_2(k, x, m; L^C) \equiv \max_{L > 0} \left[ \left( e^{a_2(k) + \sigma z} \frac{A}{L + (m - 1)L^C(k, x, m)} \right)^{1/\epsilon_2(k)} - e^{\phi_2 + \xi \mu} \right] L. \quad (30)$$

Thus, according to (29) and (30), the set of loan size schemes $L^C(\cdot, \cdot, \cdot)$ under tacit collusion competition that satisfy the incentive-compatibility constraint can be rewritten as

$$L^C(k, x, m) \equiv \left\{ L^C : \frac{\zeta [W^C(L^C) - W^N]}{1 - \delta} \geq \mathbb{E}^x \left[ \Pi^D_2(k, x, m; L^C) \right] - \mathbb{E}^x \left[ \Pi_2(k, x, m; L^C) \right] \right\}. \quad (31)$$

### 3.1.4 Endogenous Participation Boundaries

The cutoff points $w^*_N$ and $w^*_C$ are determined as in Li and Zheng (2009). Like solving value functions and optimal policies, we solve $w^*_N$, then we solve $w^*_C$. We assume that $w$ is first revealed and specialists choose whether to learn, then $k$ and $m$ are revealed.

We first solve $w^*_N$ according to the condition:

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \pi(k) q(m|w = w^*_N, w^* = w^*_N) \left[ U^N(k, m; w^*_N) - w^*_N \right] = \frac{W^N(w^*_N)}{1 - \delta}, \quad (32)$$

meaning that the marginal specialist with $w = w^*_N$ is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, $W^N(w^*_N)$ is the continuation value if the cutoff is $w^*_N$, $\pi(k)$ is the probability of type $k$, and $q(m|w = w^*_N, w^* = w^*_N)$ is the probability of $m$ participants conditioning on the cost of the specialist is $w^*_N$ and the cutoff $w^*$ is $w^*_N$. $q(m|w = w^*_N, w^* = w^*_N)$ has the fol-
lowing expression:

\[ q(m|w = w^*_N, w^* = w^*_C) = \left( \frac{M - 1}{m - 1} \right) F(w^*_N)^{m-1} [1 - F(w^*_N)]^{M-m}. \tag{33} \]

The equality (32) can be written as

\[ \sum_{m=1}^{M} \sum_{k=1}^{K} \pi(k) q(m|w = w^*_N, w^* = w^*_N) \mathbb{E}^x \left[ \Pi_2(k, x, m; L_N) \right] = w^*_N, \tag{34} \]

It can be numerically solved in the following steps:

(i) Solve \( \mathbb{E}^x \left[ \Pi_2(k, x, m; L_N) \right] \) for each pair of \( (k, m) \).

(ii) Guess an initial \( w^*_N \). If \( \sum_{m=1}^{M} \sum_{k=1}^{K} \pi(k) q(m|w = w^*_N, w^* = w^*_N) \mathbb{E}^x \left[ \Pi_2(k, x, m; L_N) \right] < w^*_N \), we should decrease \( w^*_N \).

(iii) Iterate until the condition is satisfied.

Then, we solve \( w^*_C \) according to the condition:

\[ \sum_{m=1}^{M} \sum_{k=1}^{K} \pi(k) q(m|w = w^*_C, w^* = w^*_C) \left[ U^C(k, m; w^*_N, w^*_C) - w^*_C \right] = \frac{W^C(w^*_N, w^*_C)}{1 - \delta}, \tag{35} \]

where \( w^*_N \) is solved earlier. This means that the marginal specialist with \( w = w^*_C \) is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, \( W^C(w^*_N, w^*_C) \) is the continuation value if the cutoff points are \( w^*_N \) and \( w^*_C \) for non-collusive and collusive Nash equilibria, \( \pi(k) \) is the probability of type \( k \), and \( q(m|w = w^*_C, w^* = w^*_C) \) is the probability of \( m \) participants conditioning on the cost of the specialist is \( w^*_C \) and the cutoff \( w^* \) is \( w^*_C \). \( q(m|w = w^*_C, w^* = w^*_C) \) has the following expression:

\[ q(m|w = w^*_C, w^* = w^*_C) = \left( \frac{M - 1}{m - 1} \right) F(w^*_C)^{m-1} [1 - F(w^*_C)]^{M-m}. \tag{36} \]

The equality (35) can be written as

\[ \sum_{m=1}^{M} \sum_{k=1}^{K} \pi(k) q(m|w = w^*_C, w^* = w^*_C) \mathbb{E}^x \left[ \Pi_2(k, x, m; L_C) \right] = w^*_C, \tag{37} \]
It can also be numerically solved in the following steps:

(i) Solve $E^x \left[ \Pi_2(k, x, m; L^C) \right]$ for each pair of $(k, m)$.

(ii) Guess an initial $w^*_C$. If $\sum_{m=1}^M \sum_{k=1}^K \pi(k)q(m \mid w = w^*_C, w^* = w^*_C)E^x \left[ \Pi_2(k, x, m; L^C) \right] < w^*_C$, we should decrease $w^*_C$.

(iii) Iterate until the condition is satisfied.

3.2 Model solution

There is a theoretical property of the model that can ensure the semi-closed-form solution and help immensely simplify the numerical analysis. Specifically, we show that the equilibrium loan sizes under collusive competition, non-collusive competition, and deviation have the following functional form:

\[
L^C(k, x, m) \equiv \tilde{L}^C(k, m)e^{\alpha_2(k) - \varepsilon_2(k, \phi_2 - \varepsilon_2(k))u + \sigma z} A \\
L^N(k, x, m) \equiv \tilde{L}^N(k, m)e^{\alpha_2(k) - \varepsilon_2(k, \phi_2 - \varepsilon_2(k))u + \sigma z} A \\
L^D(k, x, m) \equiv \tilde{L}^D(k, m)e^{\alpha_2(k) - \varepsilon_2(k, \phi_2 - \varepsilon_2(k))u + \sigma z} A.
\]

Therefore, all equilibrium outcomes only depend on the discrete state variables $k$ and $m$ in a nonparametric way, while their dependence on the continuous state variables in $x$ has a closed form, which is already known.

We solve the model in three steps. We first solve for $U^N$ and $W^N$, together with the endogenous cutoff of participation in a non-collusive equilibrium $\omega^*_N$, which is chosen such that a specialist lender with a participation cost of $\omega^*_N$ is indifferent between participate or not. Then for any given level of colluded loan amount $L^C$, we can solve for $U^C$, $W^C$, and $\omega^*_C$ as functions of $L^C$. $U^D$, $W^D$, and $\omega^*_D$ can be solved accordingly, which are also functions of $L^C$. As the last step, we search for the equilibrium $L^C$ that prevents deviation, as defined in Equation (31).

The model generates solutions for three observable variables of interest, including the number of participating specialist lenders $m$, the loan size $L_A$, and the DIP rate $R$. These variables are functions of the DIP deal type observed in the data (i.e., financed by a monopolistic prepetition lender, or by one or multiple specialist lenders, or by a lender of the last resort) as well as the model parameters.
Lastly, to bring the model to the data, we specify the probability of a borrower $i$ belonging to cluster $k_i = (d, A_i)$ as follows:

$$
\pi(k_i) = \text{Prob}(d | A_i) \cdot \text{Prob}(A_i)
= \frac{e^{\gamma_d + \beta_d(ln(A_i) - E[ln(A)])}}{1 + \sum_{j=2}^{D} e^{\gamma_j + \beta_j(ln(A_i) - E[ln(A)])} \cdot h(A_i)},
$$

where $h(A)$ is the PDF of borrower size and $E[ln(A)]$ is the average log-size. In this specification, borrower size $A_i$ is observable, and we assume that the likelihood of borrower $i$ belonging to demand curve $d \in \{1, 2, ..., D\}$ follows the multinomial logistic distribution with parameter $\gamma_d$ and $\beta_d$ (and $\gamma_1$ and $\beta_1$ are normalized to 1). This specification allows the model to capture possible correlation between borrower size and the demand, as it is plausible that large borrowers and small borrowers differ in their overall demand for the loan and the price elasticity.\footnote{The model can be solved with a continuous-valued size $A$ and any number of demand curve group $g$. In estimation, we divide borrowers into deciles based on their size and assume two heterogeneous demand curve $d \in \{1, 2\}$.}

4 Data Sample

Academics as well as practitioners commonly refer distressed loans as loans provided to firms that are either deeply distressed with the likely prospect of defaulting on its obligations or that are already bankrupt (Altman, Hotchkiss, and Wang, 2019). For our study, we construct a comprehensive data sample that consists of bank loans to both financially distressed but not yet bankrupt U.S. public firms and those public firms already in Chapter 11 bankruptcy from 2001–2019, using two different data filtering methods.

4.1 Loans to distressed firms not yet in bankruptcy

We first assemble a data set of loans to financially distressed firms that are not yet in bankruptcy. The academic literature has proposed a number of measures for identifying financially distressed firms. The existing approaches are broadly categorized as accounting ratio based (e.g. Altman Z-score models) and market price based approaches. The advantage of the market price based measure over accounting ratios...
is that not only it is forward looking but more importantly, it directly measures how costly it is for a firm to raise financing in the financial markets. With fast development and high trading liquidity in the credit derivative markets, a few of recent studies use prices of credit default swaps (CDS) to take a cue on the financial health of firms (e.g., Hortacsu, Matvos, Syverson, and Venkataraman (2013); Brown and Matsa (2016)). Given that credit ratings are also informative indicators for corporate default and many institutions rely on ratings for their investment decisions, for our study, we rely on both CDS prices and credit ratings to identify financially distressed firms.

We retrieve monthly five-year CDS prices on senior unsecured bonds of U.S. public issuers from IHS Markit database and monthly S&P Long-Term Domestic Issuer Ratings of U.S. public firms from Compustat for the period from 2000–2017.\textsuperscript{15} We use two criteria to identify the beginning of a firm’s distressed period, namely, whether the five-year CDS price first hits 1000 bps\textsuperscript{16} or whether a firm’s S&P rating drops to CCC+ or lower\textsuperscript{17}, whichever occurs first. After identifying the initial starting month of a firm’s distress period, we trace the firm’s CDS spreads and credit ratings until the end of 2017. For firms with CDS spreads, the distress period ends when their CDS spreads fall below 500 bps or when they file for bankruptcy.\textsuperscript{18} For firms with credit ratings, the distress period ends when S&P ratings goes up to B- or higher or the firm defaults according to S&P ratings.

Consolidating the distress periods identified using CDS spreads and ratings, removing duplicated time periods, and combining two consecutive periods that have an in-between time gap of less than a year, and removing distressed periods that are shorter than 6 months to avoid capturing transitory periods, we have 637 distressed periods of 520 firms from 2001–2017. We merge the distressed periods with the Dealscan database using the link file by Chava and Roberts to identify loan facilities that have a start date falling into the distress period. After removing facilities that have missing information on lender identities or loan spreads measured by the all-in spread drawn (AISD), which is the sum of LIBOR spread and annual fee, and loans that are unsecured or subordinated, we have a final sample of 520 loan

\textsuperscript{15}Our sample stops in 2017 because Compustat no longer provides S&P ratings of U.S. public firms after 2017.

\textsuperscript{16}Prior studies refer bonds whose yield spread above risk free rate is over 1000 bps as distressed bonds (Altman et al., 2019).

\textsuperscript{17}Loans that are issued by firms with CCC+ or lower ratings can no longer be widely held by institutional investors such as the Collateralized Loan Obligations (CLOs), the most important type of investors in the leveraged loan market, because there are limitations on the amount of CCC-rated loans that can be included in the underlying collateral pool of CLOs (typically 7.5% of the collateral pool). If any, CLOs are net sellers of CCC-rated loans.

\textsuperscript{18}Bankruptcy filings, including both Chapter 11 and Chapter 7 filings, by all U.S. public firms from 2001-2019 from obtained from New Generation Research’s Bankruptcydata.com.
facilities (342 packages) in 185 distressed periods.

We first identify whether a lender is a major lender using their titles (e.g., agent bank, lead arrangers, etc.) following Jiang, Li, and Shao (2010). We consolidate all financial institutions to the parent company. For example, JP Morgan Securities would carry the same unique institution ID as JP Morgan & Co. Moreover, we consider institutions’ M&As that occurred in our sample period and consolidate the target and the acquirer into one entity after the transaction. Moreover, we remove entities that are special purpose investment vehicles and structured products such as CLOs and CDOs, which have little direct involvement in the primary market of distressed loans.

Using historical loan issuance information in Dealscan, we are able to determine whether a major lender is an existing lender to a firm—that is, a lender of the distressed loan is also a lender in an earlier loan that has not yet matured. We also determine whether a lender is a private equity fund or hedge fund by searching the lender’s website and industry publications (Jiang, Li, and Wang, 2012). Next, we identify the CDS prices at the end of the month immediately preceding the loan issuance date. We also collect Treasury Constant Maturity Rate of different maturities from the Federal Reserve at St. Louis and 3-month LIBOR rate from Bloomberg. Using these information, we are able to determine the fraction of loan spreads that are due to the default risk component (See Appendix). Finally, we retrieve firms’ key financial information immediately before loan issuance from Compustat.

We define three types of distressed loans. The first type contains loans provided by only one lender, who is an existing lender and is not a specialist, i.e., existing-lender loans (Type 1). The second type contains loans that are provided by at least one specialist, defined as one of the top 10 lenders (measured by the number of distressed loans that it financed in our sample), regardless of the number of lenders in the loan syndicate, specialist loans (Type 2). The third type contains loans that have over 50% of the lenders as hedge funds and private equity funds i.e., last-resort loans (Type 3). We remove 47 facilities that cannot be classified into any of the above three types. Among the 473 loan facilities in our final sample, the second type of loans it the most dominant type, accounting for 76.7% while Type 1 and Type 3 loans account for 11.2% and 12.1% respectively. Panel A of Table 3 presents the summary statistics of our sample of distressed loans.
This table presents the summary statistics of our sample firms. Our sample consists of 484 distressed loans (in Panel A) and 297 DIP loans (in Panel B) to U.S. public firms between 2001 and 2019. All financial variables are taken from the last fiscal year reported immediately prior to loan initiation, retrieved from Compustat. Assets, Liabilities and Sales are book assets, book liabilities, and revenue measured in millions of dollars, respectively. Leverage is the ratio of book liabilities to book assets. ROA is EBITDA scaled by book assets. PP&E/assets is the ratio of net property, plant and equipment to book assets. Cash/assets is the ratio of cash and short-term securities to book assets. Loan amount (L) is in millions of dollars. L/A is the ratio of DIP amount to book assets. AISD (R) is all-in-spread drawn in basis points. Number of lenders is the number of unique institutions in a syndicate. Loan type 1, Loan type 2 and Loan type 3 are indicator variables for loans provided by an existing lender, specialist lenders, and lenders of last resort, respectively.

### Panel A: Distressed Loans

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>473</td>
<td>8,144</td>
<td>12,888</td>
<td>672</td>
<td>2,558</td>
<td>10,600</td>
</tr>
<tr>
<td>Liabilities</td>
<td>473</td>
<td>8,042</td>
<td>13,812</td>
<td>643</td>
<td>2,515</td>
<td>9,951</td>
</tr>
<tr>
<td>Sales</td>
<td>473</td>
<td>5,647</td>
<td>10,215</td>
<td>450</td>
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<td>6,243</td>
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<tr>
<td>Leverage</td>
<td>473</td>
<td>1,040</td>
<td>0.437</td>
<td>0.797</td>
<td>0.949</td>
<td>1.134</td>
</tr>
<tr>
<td>ROA</td>
<td>467</td>
<td>0.066</td>
<td>0.130</td>
<td>0.039</td>
<td>0.072</td>
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<tr>
<td>PP&amp;E/assets</td>
<td>473</td>
<td>0.385</td>
<td>0.239</td>
<td>0.169</td>
<td>0.355</td>
<td>0.542</td>
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<tr>
<td>Cash/assets</td>
<td>473</td>
<td>0.060</td>
<td>0.065</td>
<td>0.013</td>
<td>0.034</td>
<td>0.085</td>
</tr>
<tr>
<td>Loan amount (L)</td>
<td>473</td>
<td>438.748</td>
<td>649.591</td>
<td>70.000</td>
<td>219.000</td>
<td>500.000</td>
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<tr>
<td>L/A</td>
<td>473</td>
<td>0.133</td>
<td>0.168</td>
<td>0.033</td>
<td>0.079</td>
<td>0.180</td>
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<tr>
<td>AISD (R)</td>
<td>473</td>
<td>434.797</td>
<td>212.441</td>
<td>275.000</td>
<td>400.000</td>
<td>525.000</td>
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<td>473</td>
<td>3.801</td>
<td>2.793</td>
<td>2</td>
<td>3</td>
<td>5</td>
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<tr>
<td>Loan type 1</td>
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<td>0.112</td>
<td>0.316</td>
<td>0</td>
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<tr>
<td>Loan type 2</td>
<td>473</td>
<td>0.767</td>
<td>0.423</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>Loan type 3</td>
<td>473</td>
<td>0.121</td>
<td>0.326</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Loan type 1 (# of lenders)</td>
<td>53</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Loan type 2 (# of lenders)</td>
<td>363</td>
<td>4.273</td>
<td>2.655</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Loan type 3 (# of lenders)</td>
<td>57</td>
<td>3.404</td>
<td>3.438</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Number of specialized lenders</td>
<td>473</td>
<td>2.319</td>
<td>1.646</td>
<td>1</td>
<td>2</td>
<td>3</td>
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### Panel B: DIP Loans

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
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<td>3,383</td>
<td>10,333</td>
<td>412</td>
<td>694</td>
<td>1,916</td>
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<td>Liabilities</td>
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<td>3,220</td>
<td>8,402</td>
<td>438</td>
<td>778</td>
<td>1,883</td>
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<td>Sales</td>
<td>282</td>
<td>2,287</td>
<td>4,700</td>
<td>404</td>
<td>791</td>
<td>1,733</td>
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<td>Leverage</td>
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<td>0.521</td>
<td>0.832</td>
<td>0.972</td>
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<td>0.104</td>
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<td>PP&amp;E/assets</td>
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<td>0.266</td>
<td>0.168</td>
<td>0.354</td>
<td>0.604</td>
</tr>
<tr>
<td>Cash/assets</td>
<td>282</td>
<td>0.052</td>
<td>0.065</td>
<td>0.012</td>
<td>0.028</td>
<td>0.068</td>
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<td>Loan amount (L)</td>
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<td>709.853</td>
<td>41.250</td>
<td>100.000</td>
<td>275.000</td>
</tr>
<tr>
<td>L/A</td>
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<td>0.155</td>
<td>0.052</td>
<td>0.119</td>
<td>0.217</td>
</tr>
<tr>
<td>AISD (R)</td>
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<td>296.998</td>
<td>400.000</td>
<td>600.000</td>
<td>809.091</td>
</tr>
<tr>
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<td>2.128</td>
<td>1.739</td>
<td>1</td>
<td>3</td>
<td>3</td>
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<td>Loan type 1</td>
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<td>0.352</td>
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<td>0</td>
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<tr>
<td>Loan type 2</td>
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<td>0.451</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Loan type 3</td>
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<td>0.138</td>
<td>0.346</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Loan type 1 (# of lenders)</td>
<td>43</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Loan type 2 (# of lenders)</td>
<td>213</td>
<td>2.394</td>
<td>1.912</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Loan type 3 (# of lenders)</td>
<td>41</td>
<td>1.927</td>
<td>1.104</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of specialized lenders</td>
<td>297</td>
<td>1.202</td>
<td>1.007</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
4.2 Loans to Chapter 11 firms

A U.S. firm filing for Chapter 11 bankruptcy can obtain post-petition financing, commonly known as debtor-in-possession (DIP) financing, to support working capital and pay expenses in bankruptcy under Section 364 of the Bankruptcy Code. This law provision permits the bankrupt firm to arrange a DIP loan with super-priority status over all administrative expenses after notice and a court hearing. With existing lenders’ court approval, a DIP loan can be secured by a senior or equal lien on a property that is already subject to a lien (i.e. the “priming lien” provision). These loan contracts are short term and typically contain extensive protective features such as restrictive covenants and milestones (Skeel, 2004; Ayotte and Elias, 2020; Eckbo et al., 2020). With such security and lender protection provision, the default risk of DIP loans are very low, comparable to that of investment-grade loans as shown by Eckbo et al. (2020).

The advantages of including DIP loans for our analysis are threefold. First, given their minuscule default rates, we can use the all-in spreads over an investment-grade benchmark directly to decompose the effect of lender power on loan pricing, compared to distressed loans for which we have to take out the default risk component of distressed loans using CDS spreads. Second, existing lenders’ lien on distressed firms’ assets and their private information about the borrower allow them to have strong bargaining power and even become monopolistic lenders, which is consistent with our model setup. Third, as Eckbo et al. (2020) suggest, the DIP-lending market is quite concentrated. The top 10 lenders financed more than three quarters of their sample firms. Moreover, the lending syndicate is smaller than that for distressed loans. The highly concentrated lending market and small lending syndicate together create an ideal environment for specialized lenders to collude, where monitoring among lenders can be less costly, making punishment on non-collusive lending a more credible threat.

Our initial study sample, similar to that used in Eckbo et al. (2020), includes all DIP loans to Chapter 11 filings by large U.S public firms (with assets above $100 million in constant 1980 dollars) from 2002–2019, compiled from the UCLA-LoPucki Bankruptcy Research Database, Bankruptcydata.com, the Public Access to Court Electronic Record (PACER), and the Dealscan. We calculate the weighted average LIBOR spread and AISD at the loan package level using facility amount as the weight. We have AISD available for 351 loan packages.  

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19 We use loan packages for DIP loans because it is always the same group of lenders that provide different facilities in a DIP-loan package while for other distressed loans, different facilities could be provided by distinct groups of lenders.
We identify the major lenders of DIP loans using DIP financing motions and master credit agreements and determine whether each lender is an alternative investors such as private equity funds or hedge funds. We also determine whether these lenders hold existing debt. Similar to the approach used to identify specialists for distressed loans, we treat a lender as a specialist if it is one of the top 10 lenders measured by the number of DIP loans in our sample. Also similar to distressed loans, we define three types of DIP loans: existing-lender loans (Type 1), specialist loans (Type 2), and last-resort loans (Type 3). We remove 54 DIP loans that cannot be classified into any of the above three types. Table 3, Panel B, presents the summary statistics of our sample firms which obtained DIP-loan packages. In the 297 loan packages in our final sample, the second type of DIP loans is the most dominant type, accounting for 71.7% while Type 1 and Type 3 loans account for 14.5% and 13.8% respectively. Table 3, Panel B, presents the summary statistics of our sample of DIP loans.

5 Estimation

5.1 Likelihood Function

The model solves a few crucial variables for each loan deal based on two deal characteristics. The first characteristics is the cluster \( k_i \) that deal \( i \) belongs to, defined by borrower size and the latent demand curve the deal attaches to. The second characteristics is the deal type \( s_i \) as shown in Figure 1: a loan can be financed by a monopolistic existing lender \((l_i = 1)\), by one or multiple specialist lenders \((l_i = 2)\), or by a lender of the last resort \((l_i = 3)\). Our goal is to estimate the model parameters and identify the latent cluster each deal belongs to based on the observables.

Using the Bayesian approach, we estimate the posterior distribution for the variables of interest, \( P \left( \{k_i\}_{i=1}^N, \Theta | \{l_i, m_i, \ln \left( \frac{L_i}{X_i} \right), \ln (R_i) \}_{i=1}^N \right) \), in which \( \Theta \) is the vector of model parameters, \( m_i \) is the number of specialist lenders in deal \( i \), \( \frac{L_i}{X_i} \) is the loan amount normalized by the borrower’s size, and \( R_i \) is the loan spread after removing the credit premium and liquidity premium component. The observation index \( i = 1, 2, ..., N \) indicates the deals. This posterior distribution describes the estimate of model parameters.

\(^{20}\)These loans include the case of General Motors as its DIP-loan, the largest size history ($33 billion), was provided by U.S. and Canadian governments with a large component of subsidy, and loans that are provided by potential acquirers that use the DIP loan to bridge takeover.
and the augmented latent cluster variable of each deal based on the observables. Based on Hammersley-Clifford Theorem (Besag, 1974), this posterior distribution is fully characterized by two conditional distributions $P \left( \Theta | \{k_i, l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right)$ and $P \left( \{k_i\}_{i=1}^{N} | \Theta, \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right)$, which in turn can be broken down into more lower dimensional conditional distributions.

Conditioning on $\{k_i\}_{i=1}^{N}$, the distribution $P \left( \Theta | \{k_i, l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right)$ is determined by the distribution $P \left( \Theta | \{k_i\}_{i=1}^{N} \right)$ and the likelihood function, $P \left( \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} | \Theta, \{k_i\}_{i=1}^{N} \right)$, implied by the model solution. Specifically,

$$
P \left( \Theta | \{k_i, l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right) \propto P \left( \Theta | \{k_i\}_{i=1}^{N} \right) \cdot P \left( \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} | \Theta, \{k_i\}_{i=1}^{N} \right)$$

$$
= P \left( \Theta | \{k_i\}_{i=1}^{N} \right) \cdot \prod_{i=1}^{N} f_i (\Theta, k_i)
$$

(41)

where

$$
f_i (\Theta, k_i) = P \left( l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) | \Theta, k_i \right)
$$

$$
= P (l_i, m_i | \Theta, k_i) \cdot P \left( \ln(\frac{L_i}{A_i}) | l_i, m_i, \Theta, k_i \right) \cdot P \left( \ln(R_i) | l_i, m_i, \ln(\frac{L_i}{A_i}), \Theta, k_i \right)
$$

(42)

is the likelihood function for deal $i$, conditioning on the model parameters and the cluster it belongs to.

Meanwhile, conditioning on $\Theta$, the distribution $P \left( \{k_i\}_{i=1}^{N} | \Theta, \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right)$ is determined by $P \left( \{k_i\}_{i=1}^{N} | \Theta \right)$ and the likelihood function $P \left( \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} | \Theta, \{k_i\}_{i=1}^{N} \right)$:

$$
P \left( \{k_i\}_{i=1}^{N} | \Theta, \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} \right) \propto P \left( \{k_i\}_{i=1}^{N} | \Theta \right) \cdot P \left( \{l_i, m_i, \ln(\frac{L_i}{A_i}), \ln(R_i) \}_{i=1}^{N} | \Theta, \{k_i\}_{i=1}^{N} \right)$$

$$
= P \left( \{k_i\}_{i=1}^{N} | \Theta \right) \cdot \prod_{i=1}^{N} f_i (\Theta, k_i)
$$

(43)

We summarize below the individual component of the likelihood function in Equation (42) for each type of loan.

First, for loans financed by a monopolistic lender,

$$
P (l_i = 1, m_i | \Theta, k_i) = \lambda(k_i)
$$

39
where $\lambda(k_i)$ is the probability that deal $i$ is financed by an existing lender. $m_i$ becomes irrelevant here because there is only one existing lender involved. The likelihood of loan size and spread is given as below:

$$P \left( \ln(R_i)|l_i, m_i, \Theta, k_i \right) = \Phi \left( \ln(R(k_i, m_i)), \zeta \right)$$  \hspace{1cm} (44)$$

$$P \left( \ln\left( \frac{L_i}{A_i} \right)|l_i, m_i, \ln(R_i), \Theta, k_i \right) = \Phi \left( \ln\left( \frac{L(k_i, m_i)}{A} \right), \sigma \right)$$  \hspace{1cm} (45)$$

where $\Phi$ is the PDF of the normal distribution, $L(k_i, m_i)$ and $R(k_i, m_i)$ are the model-implied loan size and spread when the loan is financed by a monopolistic existing lender, as solved in Equation (11) and the demand curve Equation (5), and $\zeta$ and $\sigma$ are the standard deviation of the normal distribution that captures the lender-side and borrower-side shocks.

Second, for loans financed by a lender of the last resort, it must be the case that no specialist lenders participate and thus $m_i = 0$

$$P \left( l_i = 3, m_i = 0|\Theta, k_i \right) = (1 - \lambda(k_i)) \left( 1 - F(\omega^*_C|k_i) \right)^M$$

where $1 - \lambda(k_i)$ is the probability that deal $i$ is not financed by an existing lender and $\left( 1 - F(\omega^*_C|k_i) \right)^M$ is the probability that none of the $M$ specialist lenders participate. The likelihood of loan size and spread is given as in Equation (44) and (45).

Third, for loans financed by one or multiple specialist lenders, there must be at least one specialist lender ($m_i \geq 1$), and we have

$$P \left( l_i = 2, m_i|\Theta, k_i \right) = (1 - \lambda(k_i)) \left( \begin{array}{c} M \\ m_i \end{array} \right) F(\omega^*_C|k_i)^{m_i} \left( 1 - F(\omega^*_C|k_i) \right)^{M-m_i}$$

where $1 - \lambda(k_i)$ is the probability that deal $i$ is not intermediated by an existing lender and

$$\left( \begin{array}{c} M \\ m_i \end{array} \right) F(\omega^*_C|k_i)^{m_i} \left( 1 - F(\omega^*_C|k_i) \right)^{M-m_i}$$

is the probability that $m_i$ out of $M$ specialist lenders participate in this deal.
5.2 MCMC Estimator

To implement MCMC estimator, for each deal $i$ in the data, we define a vector $z_i$ of dimension $K$ (i.e., the number of possible cluster) so that the $j$th element in the vector describes the probability of this deal belonging to cluster $j$ (i.e., $\pi(k_i = j)$) so that $z_{ij} \geq 0$ and $\sum_{j=1}^{K} z_{ij} = 1$. Then we apply the Metropolis-Hastings algorithm with the following procedure:

**Step 1:** for each deal $i$, update $z^{(g)}_i$ using the EM method based on outputs from the last iteration round $g-1$:

$$z^{(g)}_{ij} = \frac{\pi^{(g-1)}(k_i = j)f_i(\Theta, k_i = j)}{\sum_{\kappa=1}^{K} \pi^{(g-1)}(k_i = \kappa)f_i(\Theta, k_i = \kappa)}$$

(46)

where $\pi^{(g-1)}(k_i = j)$ is the probability of $k_i = j$ (determined by the model parameters obtained from the last iteration), and $f_i(\Theta, k_i)$ is the likelihood function of observables evaluated at the parameter values from the last iteration. Then we follow the SEM algorith (Celeux, 1985, Celeux and Diebolt, 1992) and draw the realization of $k^{(g)}_i$ from the multinomial distributions with weights in Equation (46). We throw out the draw and repeat this step if for any cluster $j$, the total number of deals assigned to this cluster is less than a cutoff (e.g., 3%).

**Step 2:** update other parameters in $\Theta$ using the random walk Metropolis-Hastings algorithm based on the updated draws of $\{k^{(g)}_i\}_{i=1}^N$ from step 1. We compute the acceptance/rejection threshold:

$$\alpha(\Theta^{(g)}, \Theta^{(g-1)}) = \min \left( \frac{P(\Theta^{(g)}|\{k^{(g)}_i\}_{i=1}^N) \prod_{i=1}^N f_i(\Theta^{(g)}, k^{(g)}_i)}{P(\Theta^{(g-1)}|\{k^{(g)}_i\}_{i=1}^N) \prod_{i=1}^N f_i(\Theta^{(g-1)}, k^{(g)}_i)}, 1 \right)$$

where $\Theta^{(g-1)}$ is the parameter vector from the last iteration, and $\Theta^{(g)} = \Theta^{(g-1)} + \Sigma \epsilon$ is the vector of proposed parameters.

**Step 3:** repeat the procedure in step 1 and step 2 and generate a MCMC chain for the classification of cluster and the estimated parameters.

5.3 Estimation Results

In this section, we first discuss what empirical patterns in the data help the MCMC estimator to identify model parameters. We then present the estimation results and our empirical findings for the two dis-
tressed loan markets.

A key parameter in our model, \( \xi \), controls the tacit coordination among specialized lenders. In case of perfect collusion, lenders in a syndicate group coordinate as a cartel, and thus the total loan amount lent by the cartel equals the loan amount lent by a monopolistic lender. However, as collusion intensity falls, syndicated lenders lend more aggressively, leading to larger loan amount by the syndicate group than that by a monopolistic lender. As a result, the ratio of the loan size made by a syndicate lender group (with two or more specialist lenders) to the loan size made by a single monopolistic specialist, \( \frac{L}{A_{\text{m}}}, \frac{L}{A_{\text{m}}} > 1 \), helps pin down \( \xi \). Clearly, as \( \xi \) increases, this ratio declines. Two important things are worth noting here. First, when the profits from deviation are large enough, even the toughest punishment (i.e., \( \xi = 1 \)) cannot fully restore perfect collusion, and therefore it is possible to observe the loan size ratio to remain above one even if \( \xi = 1 \). Second, collusion is harder to achieve in face of large borrowers, because profits from deviation increase as the demand grows. As a result, the model suggests that the loan size ratio is higher for large borrowers than for small borrowers given the level of \( \xi \).

Figure 3 illustrates how the loan size ratio varies with \( \xi \) for small borrowers (left) and large borrowers (right). Panel (a) shows the results for the distressed loan sample while panel (b) shows the results for the DIP sample. In these figures, the solid line depicts the model-implied loan size ratio as a function of \( \xi \), the dash line indicates the average loan size ratio measured in the corresponding samples, and the red circle marks the model prediction with \( \xi \) set to its estimated value (Table 4). In both markets, a low level of \( \xi \) is sufficient to support a high level of tacit coordination among the specialist lenders in face of small borrowers. Specifically, with a \( \xi \) being around 0.2, the loan size ratio is already quite close to one, both in the model and in the data. We, however, observe that the loan size ratio is monotonically decreasing as \( \xi \) increases for loans made to large borrowers, and even with \( \xi = 1 \), the loan size ratio is still above one. In all panels of the figure, the model-predicted loan size ratio (red circle) is very close to their empirical counterparts (dash line), suggesting that the parameter \( \xi \) is well identified in our estimation. Since we estimate \( \xi \) in MCMC, its estimated value is also affected by other observable variables such as loan size, loan spread, and the number of specialized lenders, and thus the loan size ratio is not the only factor that pins down \( \xi \). However, the fact that the model is able to match the loan size ratio so closely even if the loan size ratio is not used explicitly in forming the aggregate likelihood lends strong support to the
Figure 3: Identification of $\xi$

This figure plots the ratio of loan amount by a syndicated lender group ($m > 1$) to loan amount by a single lender ($m = 1$) for small borrowers (left) and large borrowers (right). Small (large) borrowers are the borrowers in the bottom (top) tercile of firm size in our sample. The solid line depicts how this ratio varies as the collusion intensity, $\xi$, changes. The dash line shows the empirical value of this ratio observed in the data for small and large borrowers. The red circle marks the estimated value of $\xi$ and the model-implied loan size ratio. Panel (a) illustrates the results for the distressed loan sample, and panel (b) illustrates the results for DIP sample.
Table 4: Parameter Estimates

This table reports the estimated model parameters together with the standard errors obtained from MCMC. The top panel shows the general model parameters including the likelihood of punishment on deviation $\xi$, the demand and supply shock $\sigma$ and $\varsigma$, the parameters that control the correlation between borrower size and the demand curve, $\gamma$ and $\beta$, as well as the average participation cost $\mu$. The mid panel shows the lender-specific model parameter $\phi$ that captures the lending costs for each type of lenders (i.e., existing lenders, the specialist lenders, and lenders of the last resort). The bottom panel shows the borrower-specific parameters including the demand curve coefficient $\alpha$ and $\epsilon$ and the likelihood for the existing lender to finance the loan $\lambda$.

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<th>Distressed Loans</th>
<th>DIP loans</th>
</tr>
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<td>$\xi$</td>
<td>0.799</td>
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</tr>
<tr>
<td>$\sigma$</td>
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</tr>
<tr>
<td>$\varsigma$</td>
<td>0.960</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>1.304</td>
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</tr>
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</tr>
<tr>
<td>$\mu$</td>
<td>24.42</td>
<td>80.89</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.109)</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.037)</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.023)</td>
</tr>
<tr>
<td></td>
<td>(0.596)</td>
<td>(3.54)</td>
</tr>
<tr>
<td></td>
<td>(0.591)</td>
<td>(0.867)</td>
</tr>
<tr>
<td></td>
<td>(3.54)</td>
<td>(0.235)</td>
</tr>
<tr>
<td></td>
<td>(12.65)</td>
<td></td>
</tr>
<tr>
<td><strong>Lender-specific Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0007</td>
<td>0.0126</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0134</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td></td>
<td>0.0009</td>
<td>0.0167</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td><strong>Borrower-specific Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-7.999</td>
<td>-8.58</td>
</tr>
<tr>
<td></td>
<td>(0.199)</td>
<td>(1.538)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>-6.049</td>
<td>-5.36</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.206)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.058</td>
<td>1.664</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.295)</td>
</tr>
<tr>
<td></td>
<td>1.033</td>
<td>1.254</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.051)</td>
</tr>
<tr>
<td></td>
<td>0.166</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

The fraction of loan deals financed by the existing lender helps identify the parameter $\lambda(k_i)$. This linkage is established exogenously in the model by assuming that there is a probability $\lambda(k_i)$ that the existing lender has the exclusive right to provide the financing and thus block the firm from borrowing from outside lenders.
Different types of lenders (i.e., existing lenders, specialist lenders, and lenders of the last resort) may incur different variable costs (e.g., monitoring costs). Part of the spread R is used to compensate for these variable costs. By comparing R across deals financed by different types of lenders (after controlling for the number of lenders involved), the estimator is able to back out the parameter \( \phi_l \) for existing lenders \((l = 1)\), specialist lenders \((l = 2)\), and lenders of the last resort \((l = 3)\).

Parameters related to the demand curves are estimated based on the classification of clusters, which in turn is achieved as augmented latent variables in MCMC. The relation between \( R \) and \( \frac{L}{A} \) within each cluster helps identify the intercept and elasticity of the corresponding demand curve, \( \alpha(k) \) and \( \varepsilon(k) \). It is worth noting that this approach is different from estimating the demand curve coefficients by simply regressing the quantity \((\ln(\frac{L}{A}))\) on price \((\ln(R))\), which is known to produce biased estimates. The key identification challenge in estimating the demand curve is that, both the demand curve and supply curve can shift simultaneously due to unobservable, fundamental factors, and thus the observed quantity and price are produced by the intersections of multiple demand and supply curves (i.e., in multiple equilibriums). Many empirical studies overcome this challenge by utilizing exogenous shocks or instruments on the supply side and thus fixing the demand curve and shifting the supply, while our approach tackles this issue by characterizing the full joint distribution of both demand and supply in equilibrium. Specifically, our estimator first classifies the observations (loans) to different demand curves based on the model-implied distribution of demand and supply in equilibrium. This classification addresses the identification challenge that both the quantity and price are endogenous by exploring the fundamental factors in the model that simultaneously drive the observed quantity and price. Conditioning on the classification, estimating the coefficients of each demand curve becomes feasible, and it produces unbiased estimates.

MCMC estimator generates Markov chains for each model parameter and the augmented variables. We first present the estimation of cluster classification. For each loan deal \(i\) in our sample, our estimator assigns it a vector \( z_i^{(g)} \) that describes the likelihood of the deal belonging to each cluster in the \(g\)th iteration on the MCMC chain, as specified in Equation (46). We take the average of \( z_i^{(g)} \) over the chain and classify loan \(i\) to the cluster with the highest likelihood. Based on the classification results, Figures 4 depicts the demand curves for the distressed loan sample and DIP loan sample, respectively. Each dot in
the figure represents an observation of loan, and the size of the dots indict the borrower’s size. The two dash lines, in black and gray, depict the two estimated demand curves. Black dots are loans classified to the first demand curve and the white dots are loans classified to the second demand curve. As we discussed above, the classification is performed based on the full likelihood of demand and supply in the model equilibrium and thus it differs from a simple classification based on the observed loan size \( \ln \left( \frac{L}{A} \right) \) and spread \( \ln (R) \). This explains why some dots located closer to one demand curve according to the observed loan size and spread are instead classified to the other demand curve, highlighting the importance of controlling for the endogeneity problem in estimating the demand curve coefficients. In both samples, borrowers on the first demand curve (black dots) have a lower intercept of loan size but a similar (for distressed loans) or higher (for DIP loans) price elasticity of loan demand compared with those classified as the second demand curve (white dots). It indicates that borrowers on the first demand curve have a lower level of overall demand for loan (relative to their firm size) and they are more sensitive to the loan price. Empirically, we find that borrowers on the first demand curve are also larger in size and have higher total revenues, which may explain partly the difference we identify in their demand curves.

Besides the classification of clusters and their the demand curves, the MCMC estimator also delivers the posterior distribution of each model parameter. We report in Tables 4 the point estimate and the standard errors of each parameter for two samples respectively. The point estimate is taken as the parameter set that produces the highest likelihood along the chain, and the standard errors is computed as the standard deviation of the parameter draws along the chain.

The likelihood of punishment on deviation, \( \xi \), is estimated to be 0.799 for distressed loans. It suggests that if a specialist lender deviates from the perceived cartel equilibrium, it will be pushed into a non-collusive equilibrium with 80% of chance. This parameter controls the collusion intensity in the model and a value of 0.799 implies a high level of tacit coordination among specialists. Our estimates show that the punishment on deviation is also large for DIP loans with \( \xi \) estimated to be 0.695. \( \xi \) is estimated with relatively small standard errors in both samples, suggesting that the data pattern strongly rejects a model without tacit coordination among specialist lenders. As we discussed above, the ratio of loan size made by a syndicate group of lenders to the loan size made by a monopolistic lender disciplines the identification of tacit coordination among specialist lenders. Stronger coordination leads to a low loan
size ratio, consistent with the data.

Our estimates of $\zeta$ and $\sigma$ show that unobservable heterogeneity, possibly driven by the lender-side and borrower-side shocks, is important in explaining the cross-sectional variation of the observed loan size and spread. A simple variance decomposition suggests that with the estimated unobservable heterogeneity, our model is able to explain 51% (14%) of variation in $\ln(R)$ and 16% (18%) of variation in $\ln(L)$ for the DIP sample (distressed loan sample).

It is also interesting to note that the lenders’ participation cost, $\mu$, is estimated to differ significantly across the two samples. Specifically, participation cost by DIP lenders is more than three times of that by distressed loan lenders. A crucial determinant of the participation cost is the blocking power by existing lenders (Eckbo et al., 2020): if the existing lender has a strong power in blocking the borrower from reaching out to other lenders even when it does not make the distressed loan to the borrower, then such blocking power can manifest as a large estimated participation cost born by specialist lenders (i.e., it is more difficult for specialist lenders to participate).

The mid panel of Tables 4 reports the lender-specific variable costs. It is worth noting that since we have purged out the risk-free rate and the risk premium in loan spread before estimation, the variable costs here should not be interpreted as funding costs, instead, they reflect other variable costs associated with lending to distressed firms. Monitoring cost and costs of lender participating in the restructuring process can be such variable costs. For example, as a distressed borrower approaches bankruptcy or is already in bankruptcy, lenders need to pay close attention to the borrower’s business operations and legal challenges in the court process. That the estimated variable costs of DIP loans are much higher than those for distressed loans are consistent with more uncertainties faced by lenders in bankruptcy court.

An interesting evidence that stands out in the DIP-loan sample is that the estimated variable costs are about 130 bps for existing lenders and specialist lenders but 170 bps for lenders of last-resort (mostly hedge funds). Unlike the existing lenders who possess information advantage or the specialist lenders who have expertise in this market, hedge funds are rarely frequent players in this market. Most of them appear only once or twice in our sample as the lenders of last-resort, and therefore it is plausible that they face higher variable costs. More importantly, hedge funds often play an active role in the governance of bankrupt firms such as seeking board representation and appointing managers in pursuing a “loan-
This figure shows the classification of borrowers to different demand curves. Each dot in the figure represents an observation in our sample, and the dash lines are the estimated demand curves. The dots in black represent the borrowers belonging to the demand curve marked in black dash line and the dots in white represent the borrowers belonging to the demand curve marked in gray dash line. The intercept and slope of the demand curves represent the constant term and the elasticity of the demand curve function, specified in Equation 5. Panel (a) depicts the classification in the distressed loan market and panel (b) depicts the classification in the DIP market.
to-own” strategy (Jiang et al., 2012; Li and Wang, 2016; Ayotte and Elias, 2020), which requires hedge funds’ significant efforts and resources.

We report the parameters that govern the demand curves in the bottom panel of Tables 4. We estimate the level of demand, $\alpha(\kappa_i)$, and the price elasticity of demand, $\varepsilon(\kappa_i)$. Within each market, the estimated intercept and elasticity just reiterate the two demand curves shown in Figure 4. Across the two market, we find that DIP borrowers exhibit higher elasticity of demand than distressed loan borrowers. Our estimates, therefore, suggest that the strikingly high spread of DIP loans, compared with that of distressed loans, cannot be explained by the price elasticity of demand.

6 Decomposing Loan Spread

Using the estimated model as a laboratory, we quantify the lenders’ market power in the two separate distressed loan markets and decompose their market power into three components: (i) the potential collusion among specialists, (ii) limited participation by these lenders, and (iii) the oligopolistic structure in this market.

Both markets exhibit a few unique and interesting features. First, there is a club of specialist lenders that intermediate a large fraction of loan deals. Indeed, 83% of distressed loans and 70% of DIP deals in our estimated model are financed by the top 10 specialist lenders. This lending market, therefore, resembles an oligopolistic structure. To illustrate the dominant position of top lenders in the two loan markets, Figure 7 shows the loan market network for distressed loans and DIP loans, respectively. It is clear that the top lenders in each market form a strong network with other top lenders in syndicating the loans. Second, most deals are syndicated by a small group of specialist lenders as shown in Table 3. It implies limited participation in lending by specialists, which further reduces lender competition. Third, the lender clique may give rise to possible collusion among specialists that is hard to be detected by outsiders. In this section, we employ the estimated model to perform a few counterfactual benchmarks and decompose the market power of specialist lenders.

Starting from the baseline model with the estimated parameters, we first quantify the effect of lender collusion. To do so, we set the key parameter $\xi$ to zero. In this counterfactual model, all specialist lenders deviate from collusion because no punishment is imposed, and as a result, a non-collusive equilibrium
emerges. We keep other model parameters at their estimated values so that the non-collusive equilibrium still features limited participation by specialist lenders and an oligopolistic market structure. We examine how lender collusion affects the average loan spread, loan size, and the number of participating lenders by comparing the outputs from the two models.

Table 5 shows the results. Columns (1) and (5) report the baseline model predictions for the distressed loan sample and DIP sample respectively, and columns (2) and (6) report the non-collusive model outputs for the two samples. Notably, the average loan spread for specialist lenders declines by 175 bps (from 284 bps in the baseline model to 109 bps in the non-collusive model) for distressed loans, representing a 62% reduction in spread if specialist lenders compete in a non-collusive equilibrium. DIP loan spread declines by 190 bps (from 614 bps to 424 bps) as lender collusion is eliminated, representing a 31% decline. Without collusion, specialists compete more aggressively and lend a larger amount in aggregation. The average loan amount, relative to the borrowers’ size, rises from 0.35 to 4.47 for distressed loans and from 0.24 to 0.57 for DIP loans. Intense competition has a moderate deterrence effect on lender participation, evident by a slight drop in the average number of syndicated lenders.
Table 5: Counterfactual Models and The Decomposition of Lender Market Power

This table reports the model implications for the estimated baseline model and a few counterfactual models. The Estimated model is the baseline model. In the non-collusive model, we shut down collusion by setting $\xi = 0$. In non-collusive, full participation model, we further zero out the participation cost for all specialist lenders and therefore all of them participate in a loan deal. In non-collusive, full participation, unlimited lenders model, we further increase the total number of specialist lenders to positive infinity so that it turns an oligopoly market to a competitive market.

<table>
<thead>
<tr>
<th>Distressed Loans</th>
<th>DIP Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated Model</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Avg. Total Spread (bps)</td>
<td>295</td>
</tr>
<tr>
<td>Frac. Existing</td>
<td>0.109</td>
</tr>
<tr>
<td>Avg. Spread (bps)</td>
<td>356</td>
</tr>
<tr>
<td>Frac. Specialist</td>
<td>0.833</td>
</tr>
<tr>
<td>Avg. Spread (bps)</td>
<td>284</td>
</tr>
<tr>
<td>Frac. Last-Resort</td>
<td>0.059</td>
</tr>
<tr>
<td>Avg. Spread (bps)</td>
<td>336</td>
</tr>
<tr>
<td>Avg. m</td>
<td>2.475</td>
</tr>
<tr>
<td>Avg. L/A</td>
<td>0.351</td>
</tr>
</tbody>
</table>
Next, we examine the effect of limited participation by specialist lenders. In our model, each lender incurs a stochastic participation cost that is assumed to follow an exponential distribution with the mean parameter $\mu$. We construct a counterfactual benchmark of full participation by setting $\mu$ to zero. In this counterfactual model, we still keep $\zeta = 0$ to measure the incremental effect of removing limited participation from the non-collusive model. The results for this counterfactual model are reported in columns (3) and (7) of Tables 5 for the two samples. The direct change is that all specialist lenders now participate in loan syndication and thus the number of lenders reaches the full capacity of 10. Full participation increases competition among specialist lenders, and the loan spread decreases by another 96 bps for distressed loans and a striking 269 bps for DIP loans. This result suggests that limited participation by specialist lenders has a much more pronounced effect in DIP market than in distressed loan market. Because participation costs in our model incorporate both the costs of learning about a loan deal and the existing lenders’ power on blocking other lenders from participating, our estimates indicate that such costs are higher for lenders in the DIP-loan markets. This is consistent with earlier studies that document that new DIP lenders cannot prime “liens” of existing lenders without their consent and thus “prepetition” lenders possess a strong bargaining position in deciding who can participate in a DIP loan syndicate (Skeel, 2004; Ayotte and Morrison, 2009). Moreover, because borrowers tend to be selective in which lenders to approach before bankruptcy filing, it can be costly for some lenders to learn about the deal type (Eckbo et al., 2020). This is generally less of a case for distressed loans.

As the last step, we quantify the effect of an oligopolistic market structure. To turn an oligopolistic market into a competitive market, we increases the total number of specialist lenders from 10 to positive infinity. We build this counterfactual upon the non-collusive, full participation model to capture the incremental effect. In columns (4) and (8) of Tables 5, we find that the average loan spread drops by just 2 bps and 7 bps for distressed loans and DIP loans financed by specialist lenders, respectively. Interestingly, this finding shows that the oligopolistic structure is not the main cause of high loan spread in either market, and the market would be competitive enough if all the 10 specialist lenders can participate.

Moving from the baseline model to the non-collusive, full participation, competitive model, we find that the lenders’ market power accounts for about 96% of distressed loan spread (i.e., $\frac{284-11}{284}$) and 76% of the DIP spread (i.e., $\frac{614-148}{614}$). A further decomposition of the market power suggests that lender
This figure compares different components of loan spread in the distressed loan market and DIP loan market. Collusion represents the component of loan spread that arises from the specialist lenders’ tacit coordination in syndicated lending; Limited participation represents the component of loan spread that arises from the limited competition in small syndicated lending groups, which in turn is a consequence of high participation costs in the estimated model. Oligopoly represents the component of loan spread due to the oligopolistic market structure with a few concentrated lenders; and variable costs represent the component of loan spread used to compensate the lenders for their lending costs (not include funding costs) such as monitoring costs.

collusion contributes to 64% (i.e., \( \frac{284 - 109}{284 - 11} \)) of the market power for distressed loans and 41% of the market power for DIP loans (i.e., \( \frac{614 - 424}{614 - 148} \)), while limited participation contributes to 35% of the market power (i.e., \( \frac{109 - 13}{284 - 11} \)) for distressed loans and 58% of the market power (i.e., \( \frac{424 - 155}{614 - 148} \)) for DIP loans, Finally, the oligopolistic market structure contributes to the remaining negligible 0.7% (i.e., \( \frac{13 - 11}{284 - 11} \)) and 1.7% (i.e., \( \frac{155 - 148}{614 - 148} \)) for the two markets respectively.

We summarize the decomposition of loan spread for the two markets in Figure 5. Specifically, we
compare each component of the loan spread for distressed loans and DIP loans. This figure reveals two main conclusions that are central to our paper. First, tacit coordination among specialist lenders is prevalent in both markets, which allows the lender cartel to extract sizeable rent from borrowers (around 170-190 bps). Second, the excess loan spread in DIP market, compared with that in the distressed loan market, is mainly explained by the existing lenders’ strong power in blocking the borrowers from reaching out to alternative lenders (i.e., high participation costs) and the high monitoring costs in DIP market (i.e., variable costs).

Among the different components that contribute to lender market power, lender tacit coordination often attracts most attention by regulators, and anti-trust policies often target at breaking down such coordination. We then examine which borrowers are more susceptible to lender market power by measuring the effect of lender tacit coordination on borrowers of different size.

In the final part of the analysis, we partition the borrowers into size quintiles and repeat our analyses of the baseline model and the non-collusive model performed above. We report the changes in the average loan spread, loan size, and the number of participating specialist lenders as we shut down lender coordination in Table 6. We compare small borrowers (bottom size quintile) with large borrowers (top size quintile).

When lender coordination is eliminated, we find that small borrowers seem to benefit more. Specifically, the distressed loan spread drops by 214 bps (127 bps) for small (large) borrowers. DIP loan spread drops 230 bps (103 bps) for small (large) borrowers. The increase in loan size due to the elimination of collusion is also more pronounced for small borrowers than large borrowers in both markets. Overall, our analyses here suggest that small borrowers in both markets are more susceptible to lender market power and the effect of collusion on smaller borrowers more than doubles that for large borrowers. Our findings therefore imply that anti-trust policies that deter lender collusion may benefit small lenders more, and in economic or financial crises, financial aids towards distressed companies should target at small firms. The implication is consistent with a recent policy proposal by legal scholars to encourage the U.S. government to provide direct funding to small firms that filed for bankruptcy during the COVID-19 pandemic.21

21See “Use of Chapter 11 and Federal Lending to Help Small Businesses,” by Kathryn Judge and Jared A. Ellias, July 27, 2020, Letter to the Office of Senator Sherrod Brown, as Ranking Member of the Committee on Banking, Housing, and Urban Affairs.
Table 6: Heterogeneous Effects of Lender Collusion

This table reports the effect of shutting down lender collusion on borrowers of different size. We divide borrowers into size quintiles by their book assets and compare counterfactuals between small borrowers (bottom quintile) and the large borrowers (top quintile). The estimated model is the baseline model, and in the non-collusive model, we shut down lender collusion by setting $\xi$ to zero. The effect of shutting down collusion is calculated as the difference between the results from the non-collusive model and those from the baseline model.

<table>
<thead>
<tr>
<th>Borrower size</th>
<th>Distressed loans</th>
<th>DIP loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Non-collusive</td>
</tr>
<tr>
<td>R</td>
<td>Small</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>210</td>
</tr>
<tr>
<td>L/A</td>
<td>Small</td>
<td>0.457</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>0.194</td>
</tr>
<tr>
<td>m</td>
<td>Small</td>
<td>2.497</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>2.418</td>
</tr>
</tbody>
</table>

7 Policy Analysis

Our estimation and quantification results so far suggest that specialized lenders in the distressed loan market and DIP loan market extract rents from borrowers by charging excess risk-adjusted loan spread above their marginal costs; moreover, a major part of these rents is made possible through the market power enjoyed by the few specialized lenders, especially through the tacit collusion in the form of syndication. For such implicit coordination is often hard to detect, let alone make prosecution and enforcement, a simple regulatory intervention is to impose a cap on interest rates. Our findings support the proposals on government intervention by disciplining both markets of distressed loans, reducing the borrowing costs and facilitating borrowers’ credit accessibility, to mitigate the damage of inefficient bankruptcy waves owing to an economy-wide cash-flow pause (e.g., DeMarzo et al., 2020; Conti-Brown and Skeel, 2020). In this section, we focus on interest rate cap regulation (i.e., usury regulation), and use the estimated model to examine the effect of such policies.

Regulations and laws pertaining to interest rate caps have been one of the few most ubiquitous economic legislations historically and geographically (Blitz and Long, 1965). As of today, they still play a vital role in economic activities across different economies. Specifically, our setting is about commercial
loans, defined as loans made primarily for business, commercial, investment, agricultural, or similar purposes, in contrast to consumer loans. For instance, in New York, corporations and limited liability companies (LLCs) cannot be charged more than 16% interest per annum, and specifically, loans to businesses under $2,500,000 are generally exempt from the 16% civil usury cap for consumer loans, but are subject to the 25% cap in 2021.

7.1 Formulation of Polices in the Model

The interest rate cap regulation can be implemented after conditioning on the characteristics of a borrower and adjusting the risk premium to better account for borrower risk heterogeneity and risk pricing. This can substantially improve the effectiveness of the interest rate cap policy because unsophisticated constant interest rate caps have severe limitations on balancing the tradeoff between borrower production and credit access (e.g., Cuesta and Sepúlveda, 2021). Assuming that the regulator can observe or estimate the marginal cost of providing distressed loans $\phi + \varsigma u$ and the risk premium of the loan, we can directly consider the interest rate cap that is imposed on the risk-adjusted spread $R$ in the following form:

$$R_{\text{max}}(x) \equiv R_{\text{max}} e^{\phi + \varsigma u},$$

where $R_{\text{max}}$ is a positive constant.

According to the demand system of the borrowers, the loan amount per specialized lender corresponding to the ceiling on the risk-adjusted spread is

$$L_{\text{min}}(k, x, m) = \frac{1}{m} R_{\text{max}}^{-\varepsilon(k)} e^{[a(k) - \varepsilon(k) \phi] - \varepsilon(k) \varsigma u + \sigma z A},$$

where $m$ is the number of specialized lenders in the syndication, $k$ indicates the type of the borrower, and $x$ contains the characteristics of the deal. In fact, under the interest rate cap specified in (47), the loan amount $L_{\text{min}}(k, x, m)$ is the minimum loan size each specialized lender will offer in equilibrium for the syndication characterized by $(k, x, m)$.

Given that we focus on the risk-adjusted interest rate cap imposed on the spread, the optimal loan sizes for each specialized lender under non-collusive syndication, collusive syndication, and deviation...
have the following respective functional forms:

\[ L^i(k, x, m; R_{max}) \equiv \bar{L}^i(k, m; R_{max})e^{[\alpha(k) - \epsilon(k)\phi] - \epsilon(k)\varsigma u + \sigma z}A, \quad \text{with } i \in \{ N, C, D \}. \quad (48) \]

**Non-collusive equilibrium with interest rate cap** $R_{max}$. Under the interest rate cap regulation, the value function prior to paying the fixed cost $w$ and observing the deal-specific characteristics $x = (z, u)$, denoted by $U^N(k, x, m; R_{max})$, satisfies the following Bellman equation:

\[
U^N(k, x, m; R_{max}) = \Pi_2(k, x, m; L^N, R_{max}) + \frac{W^N(R_{max})}{1 - \delta}, \quad \text{where} \quad (49)
\]

\[
W^N(R_{max}) = \mathbb{E}^{k'} \left\{ \sum_{m' = 1}^{M} q(m' | w' \leq w^*_{N, R_{max}}) \left[ F(w^*_{N, R_{max}}) \Pi_2(k', m'; L^N, R_{max}) - \int_{w' \leq w^*_{N, R_{max}}} w'dF(w') \right] \right\},
\]

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1, \cdots, K\}$ with probability weight $\pi(k')$ for each $k'$, and the cutoff $w^*_{N, R_{max}}$ is determined in the same way as $w^*_N$ but they can be different in the equilibrium.

The symmetric non-collusive Nash equilibrium can be characterized by the following condition:

\[
L^N(k, x, m; R_{max}) = \arg\max_{L \geq L_{min}(k, x, m)} \left[ e^{\alpha(k) + \sigma z} \frac{A}{L + (m - 1)L^N(k, x, m; R_{max})} \right]^{1/\epsilon(k)} - e^{\phi + \varsigma u} L. \quad (50)
\]

Plugging (48) into (50) results in the following relation:

\[
\bar{L}^N(k, m; R_{max}) = \arg\max_{\bar{L} \geq \frac{1}{m}\bar{R}_{max}^{\epsilon(k)}} \left\{ \left[ \bar{L} + (m - 1)\bar{L}^N(k, m; R_{max}) \right]^{-1/\epsilon(k)} - 1 \right\} \bar{L}, \quad (51)
\]

which leads to

\[
\hat{L}^N(k, m; R_{max}) = \max \left\{ \frac{1}{m}\bar{R}_{max}^{-\epsilon(k)}, \frac{1}{m} \left[ \frac{m\epsilon(k)}{m\epsilon(k) - 1} \right]^{-\epsilon(k)} \right\}. \quad (52)
\]

**Collusive equilibrium with interest rate cap** $R_{max}$. Under the interest rate cap regulation, the value function of a specialist at the beginning of the “afternoon” when $w$, $k$, and $x$ are already observed,
denoted by \( V^C(k, x, w, m; L^C, R_{\text{max}}) \), has the following functional form:

\[
V^C(k, x, w, m; L^C, R_{\text{max}}) \equiv U^C(k, x, m; L^C, R_{\text{max}}) - w. 
\]  
(53)

The value function \( U^C(k, x, m; L^C, R_{\text{max}}) \) satisfies the following Bellman equation:

\[
U^C(k, x, m; L^C, R_{\text{max}}) = \Pi_2(k, x, m; L^C, R_{\text{max}}) + \frac{W^C(L^C, R_{\text{max}})}{1 - \delta}, \quad \text{where}
\]

\[
W^C(L^C, R_{\text{max}}) = \mathbb{E}^k \left\{ \lambda(k') \frac{\Pi_1(k', R_{\text{max}})}{M_0} \right\} + \mathbb{E}^k \left\{ [1 - \lambda(k')] \sum_{m' = 1}^M q(m') \left[ F(w_{\text{c}, R_{\text{max}}}^\ast) \Pi_2(k', m'; L^C, R_{\text{max}}) - \int_{w' \leq w_{\text{c}, R_{\text{max}}}^\ast} w' dF(w') \right] \right\},
\]

where \( \mathbb{E}^k[\cdot] \) is the expectation over \( k' \in \{1, \cdots, K\} \) with probability weight \( \pi(k') \) for each \( k' \), and the cutoff \( w_{\text{c}, R_{\text{max}}}^\ast \) is determined in the same way as \( w_{\text{c}}^\ast \) but they can be different in the equilibrium.

For a given scheme of collusive loan size captured by \( \hat{L}^C(k, m) \), the optimal deviation in terms of loan size is the one that maximizes the expected deviation profit, characterized as follows:

\[
\hat{L}^D(k, m; R_{\text{max}}) = \arg\max_{\hat{L} \geq \frac{1}{m} R_{\text{max}}^\ast} \left\{ \left[ \hat{L} + (m - 1)\hat{L}^C(k, m; R_{\text{max}}) \right]^{-1/\epsilon(k)} - 1 \right\} \hat{L}. 
\]  
(55)

The benefit of deviation is the difference between the maximal expected deviation profit and the expected collusive profit without deviation, denoted by \( \Pi^D_2(k, m; \hat{L}^C, R_{\text{max}}) \equiv \mathbb{E}^x \left[ \Pi^D_2(k, x, m; \hat{L}^C, R_{\text{max}}) \right] \) and \( \Pi_2(k, m; \hat{L}^C, R_{\text{max}}) \equiv \mathbb{E}^x \left[ \Pi_2(k, x, m; \hat{L}^C, R_{\text{max}}) \right] \), respectively. Given the collusive scheme \( \hat{L}^C(\cdot, \cdot; R_{\text{max}}) \) and the interest rate cap regulation captured by \( R_{\text{max}} \), the maximal expected deviation profit \( \Pi^D_2(k, m; \hat{L}^C, R_{\text{max}}) \) is achieved at the optimal deviation \( \hat{L}^D(k, m; R_{\text{max}}) \), that is,

\[
\Pi^D_2(k, m; \hat{L}^C, R_{\text{max}})
\]

\[
= \left\{ \left[ \hat{L}^D(k, m; R_{\text{max}}) + (m - 1)\hat{L}^C(k, m; R_{\text{max}}) \right]^{-1/\epsilon(k)} - 1 \right\} \hat{L}^D(k, m; R_{\text{max}})
\]

\[
\times \exp \left\{ \alpha(k) + \frac{1}{2} \theta^2 + [1 - \epsilon(k)] \phi + \frac{1}{2} [1 - \epsilon(k)]^2 \zeta^2 \right\} A. \]
\]  
(56)

Given the collusive scheme \( \hat{L}^C(\cdot, \cdot; R_{\text{max}}) \) and the interest rate cap regulation captured by \( R_{\text{max}} \), the
expected collusive profit $\Pi_2(k, m; \tilde{L}^C, \mathcal{R}_{\text{max}})$ is

$$
\Pi_2(k, m; \tilde{L}^C, \mathcal{R}_{\text{max}})
= \left\{ \left[ m\tilde{L}^C(k, m; \mathcal{R}_{\text{max}}) \right]^{1/\varepsilon(k)} - 1 \right\} \tilde{L}^C(k, m; \mathcal{R}_{\text{max}})
\times \exp \left\{ a(k) + \frac{1}{2} \sigma^2 + [1 - \varepsilon(k)]\phi + \frac{1}{2}[1 - \varepsilon(k)]^2 \xi^2 \right\} A.
$$

We define the IC compatible set of functionals $\tilde{L}^C(\cdot, \cdot; \mathcal{R}_{\text{max}})$ as follows:

$$
\tilde{L}^C(\mathcal{R}_{\text{max}}) \equiv \left\{ \tilde{L}^C : \frac{\xi [W^C(\tilde{L}^C, \mathcal{R}_{\text{max}}) - W^N(\mathcal{R}_{\text{max}})]}{1 - \delta} \geq \Pi_2^D(k, m; \tilde{L}^C, \mathcal{R}_{\text{max}}) - \Pi_2(k, m; \tilde{L}^C, \mathcal{R}_{\text{max}}), \right. \\
\left. \tilde{L}^C(k, m) \geq \frac{1}{m} \mathcal{R}_{\text{max}}^{-\varepsilon(k)}, \ \forall \ k, m \right\}.
$$

In the collusive Nash equilibrium we focus on, the loan size is

$$
\tilde{L}^C(\cdot, \cdot) = \arg\max_{\tilde{L} \in \tilde{L}^C(\mathcal{R}_{\text{max}})} \mathbb{E} \left[ U^C(k, x, m; \tilde{L}, \mathcal{R}_{\text{max}}) \right]. \tag{57}
$$

### 7.2 The Effects of Interest Rate Cap

After we solve the model with interest rate cap, we investigate its effects on borrowers’ welfare. In particular, we are interested in how this interest rate cap influences the loan spread that borrowers pay and the loan amount that they obtain. Since the effects of interest rate cap are similar qualitatively in both markets, we present the results for the DIP loan market as an example.

We start with demonstrating the intended consequences of the interest rate cap policy. In panels (c) and (d) of Figure 6, we plot the loan spread (left) and loan amount (right) as a function of the tightness of interest rate cap controlled by the parameter $\mathcal{R}_{\text{max}}$. As $\mathcal{R}_{\text{max}}$ decreases, the lenders’ markup declines and the equilibrium loan spread drops, indicating a tight interest rate cap control.

As expected, a tighter cap on interest rate reduces the loan spread almost mechanically. Meanwhile, since loan spread is capped, coordination among specialist lenders that aim at restricting the total loan amount for lifting loan spread becomes ineffective, and thus specialist lenders lend more aggressively in order to capture the profits from a larger loan size. Accordingly, we observe a sharp increase in the
loan amount as interest rate cap tightens. Both the decline in loan spread and increase in loan size by specialist lenders benefit the borrowers.

Even though interest rate cap improves borrower welfare through borrowing from the specialist lenders (i.e., the intensive margin), an unintended consequence of this policy is to further discourage the participation of specialist lenders and thus hinders the depth of this market (i.e., the extensive margin). Intuitively, specialist lenders decide to participate only when their expected profits are higher than the participation costs. Interest rate cap reduces the expected profits earned by specialist lenders and therefore excludes more specialist lenders from participating. If no specialist lenders are willing to participate in a specific deal, then the borrower is forced to borrow from the lenders of last-resort who are private investors in the market and may not be restricted by the interest rate cap. Lack of competition among the last-resort lenders and the high variable costs they bear make the loan very expensive. Panel (e) and (f) confirm this model prediction. As $R_{max}$ declines and the spread-cap tightens, the likelihood for the borrowers to borrow from lenders of the last-resort climbs sharply from 10-20% to above 60%. Meanwhile, even if some borrowers can still borrow from specialist lenders, the average number of participating lenders becomes significantly smaller.

Combining the positive effect of interest rate cap on the intensive margin and its negative effect on the extensive margin, Panel (a) and (b) illustrate the net effect. The net effect captures the likelihood of the borrowers borrowing from different types of lenders and the loan spread charged by these lenders. We observe a U-shaped relation between the average loan spread paid by borrowers and the tightness of rate cap and a hump-shaped relation between the average loan amount obtained by these borrowers and the tightness of rate cap. These findings suggest that there exists an optimal level of rate cap that maximizes the borrowers’ welfare. Specifically, the optimal interest rate cap maps to an average loan spread of about 380 bps across all types of lenders in in the DIP loan market.

8 Conclusions

The lending market for distressed loans features an oligopolistic structure, with a few specialist lenders financing a large fraction of loans. It raises the question of how lender market power drives loan pricing in this market. We develop a dynamic game-theoretic model of strategic competition in distress loan
markets with endogenous entry. Our entry and competing model provide several novel implications, including the “entry effect” of collusion capacity on the number of potential specialists and thus the likelihood of ex-post inefficient last-resort lending. Taking into account collusive lending by specialists and latent heterogeneity, we then use a comprehensive data sample that contains both distressed loans to bankrupt firms and those not yet in bankruptcy to estimate the structural model. We find that lender market power accounts for more than 90% of default-risk adjusted loan spread of distressed loans and for up to two-thirds of the DIP-loan spreads. More than half of the lender market power is attributed to collusive lending in distressed loans and limited participation, likely due to lenders’ blocking power, in DIP loans. Smaller borrowers are particularly susceptible to lender market power than larger borrowers, calling for more attention from policy makers towards small borrowers in difficult times. Without lender collusion, a large fraction of distressed borrowers would switch from lenders of last resort such as hedge funds to specialized lenders and benefit from a significantly larger loan amount and a much lower loan spread. Our policy analysis on interest rate cap suggests that such policy has a hump-shaped effect on the borrowers’ welfare due to a positive intensive margin counteracting a negative extensive margin. Specifically, there exists an optimal level of rate-cap that can be imposed by regulators.
Figure 6: The Effects of Interest Rate Cap Policy

This figure shows the effects of the spread-cap policy. The policy details are described in section 7. Panels (a) and (b) illustrate the overall effect of rate-cap on the average loan spread and loan amount; panel (c) and (d) depict the intensive margin of rate-cap on reducing the loan spread charged by specialist lenders and increasing the corresponding loan amount in the DIP-loan market; panel (e) and (f) show the extensive margin of rate-cap on reducing the number of participating specialist lenders and increasing the chance of the borrowers turning to lenders of last-resort.
Appendix

A Solution method:

We conjecture

\[
L^C(k, x, m) = \bar{L}^C(k, m) e^{\alpha_2(k) - \epsilon_2(k) \phi_2 - \epsilon_2(k) \xi u + \sigma z} A \
\]

(58)

\[
L^D(k, x, m) = \bar{L}^D(k, m) e^{\alpha_2(k) - \epsilon_2(k) \phi_2 - \epsilon_2(k) \xi u + \sigma z} A. \
\]

(59)

For each \( \bar{L}^C(k, m) \), we numerically solve

\[
\bar{L}^D(k, m) = \arg\max_L \left[ \left( \frac{1}{L + (m-1)\bar{L}^C(k, m)} \right)^{1/\epsilon_2(k)} - 1 \right] \bar{L}.
\]

(60)

The first-order condition is the maximization problem above is

\[
1 - \frac{1}{\epsilon_2(k)} + \frac{1}{\epsilon_2(k)} \frac{(m-1)\bar{L}^C(k, m)}{\bar{L}^D + (m-1)\bar{L}^C(k, m)} = \left[ \frac{\bar{L}^D + (m-1)\bar{L}^C(k, m)}{\bar{L}^D + (m-1)\bar{L}^C(k, m)} \right]^{1/\epsilon_2(k)}
\]

(61)

Thus, with the optimal deviation \( \bar{L}^D(k, m) \) pinned down in (61), the optimal deviation profit is

\[
\mathbb{E}^x \left[ \Pi_2^D(k, x, m; \bar{L}^C) \right] = \left[ \left( \frac{1}{\bar{L}^D + (m-1)\bar{L}^C} \right)^{1/\epsilon_2(k)} - 1 \right] \bar{L}^D e^{\alpha_2(k) + \frac{1}{2} \sigma^2 + (1-\epsilon_2(k)) \phi_2 + \frac{1}{2} (1-\epsilon_2(k))^2 \xi^2} A
\]

(62)

and

\[
\mathbb{E}^x \left[ \Pi_2(k, x, m; \bar{L}^C) \right] = \left[ \left( \frac{1}{m\bar{L}^C} \right)^{1/\epsilon_2(k)} - 1 \right] \bar{L}^C e^{\alpha_2(k) + \frac{1}{2} \sigma^2 + (1-\epsilon_2(k)) \phi_2 + \frac{1}{2} (1-\epsilon_2(k))^2 \xi^2} A.
\]

(63)

We now try to get \( \hat{L}^C(k, m) \) for \( \bar{L}^C(k, x, m) = \hat{L}^C(k, m) e^{\alpha_2(k) - \epsilon_2(k) \phi_2 - \epsilon_2(k) \xi u + \sigma z} A \). We define the IC compatible set of functionals \( \hat{L}^C(\cdot, \cdot, \cdot) \) as follows:

\[
\hat{L}^C = \left\{ \hat{L}^C : \frac{\xi(W^C(\hat{L}^C) - W^N)}{1 - \delta} \geq \mathbb{E}^x \left[ \Pi_2^D(k, x, m; \hat{L}^C) \right] - \mathbb{E}^x \left[ \Pi_2(k, x, m; \bar{L}^C) \right], \forall k, m \right\}.
\]

(64)
In the collusive Nash equilibrium we focus on, the loan size is

$$\hat{L}^C(\cdot, \cdot) = \arg\max_{L \in \hat{L}^C} E\left[U^C(k, x, m; \hat{L})\right].$$

(65)

Let $$\hat{L}^C_{max}(k, m) \equiv 1_m \left[1 - \frac{1}{\varepsilon_2(k)}\right]^{\varepsilon_2(k)}.$$ According to the definitions, it follows that $$\hat{L}^C_{max}(\cdot, \cdot)$$ is the optimal loan size if $$\hat{L}^C_{max}(\cdot, \cdot) \in \hat{L}^C.$$

**B Estimation of Credit Spreads Based on CDS Prices**

We assume that both credit spreads and CDS premia only mainly reflect the same credit risk. We observe T-year CDS premium, which is paid every period of $$\Delta$$. The frequency $$\Delta = 0.5$$ is semiannual. We also have reliable estimation on the expected recovery rates $$\delta$$ and $$\delta_L$$ for the corporate bond and the distressed loan, respectively. Moreover, we have the information on zero-coupon risk-free bond prices, denoted by $$Z(0, t_i)$$ with $$i = 1, 2, \cdots, T/\Delta$$. Let $$n = T/\Delta$$ and $$t_i = i\Delta$$ for $$i = 1, \cdots, n$$.

**Back out constant hazard rate $$p^*$$**. If CDS only reflects credit risk, the non-arbitrage CDS premium formula is

$$s = \frac{1}{\Delta} \frac{(1 - \delta) \sum_{i=1}^n [P^*(0, t_{i-1}) - P^*(0, t_i)] Z(0, t_i)}{\sum_{i=1}^n P^*(0, t_i) Z(0, t_i)},$$

(66)

where $$P^*(0, t)$$ is the risk-neutral probability of survival up to time $$t$$, modeled as

$$P^*(0, t) \equiv \exp(-p^* \times t).$$

(67)

We can estimate $$p^*$$ using the equations (66) and (67) based on the data of $$s$$, $$\delta$$, and $$Z(0, t_i)$$.

**Estimate loan credit spreads**. If a loan has maturity $$T_L = t_m$$, it holds that

$$1 = P^*(0, t_m) Z(0, t_m) + \sum_{i=1}^m [P^*(0, t_{i-1}) - P^*(0, t_i)] Z(0, t_i) \delta_L + y \Delta \sum_{i=1}^m P^*(0, t_i) Z(0, t_i),$$

where $$y$$ is the annualized loan yield. The equality above recognizes that convention that loans are sold at par, and that the loan yield is exactly equal to the coupon rate.
Then, the annualized yield of the loan is
\[
y = \frac{1}{\Delta} \left( 1 - P^*(0, t_m)Z(0, t_m) - \sum_{i=1}^{m} [P^*(0, t_{i-1}) - P^*(0, t_i)] Z(0, t_i) \delta_L \right) \frac{\sum_{i=1}^{m} P^*(0, t_i)Z(0, t_i)}{\sum_{i=1}^{m} P^*(0, t_i)Z(0, t_i)}.
\]
and the credit spread is \( y - r_f \) with \( r_f = -\ln Z(0, 1) \).
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Figure 7: Loan Market Network

This figure describes the network of the distressed loan markets and the DIP loan market. Each circle represents one unique lender, with the size of the circle depending on the market share of the lender in the corresponding loan market. The tiny circles on the outer layer are for lenders whose market share is negligible. Each line between the lenders represents one coordination in a syndication.