Arms Sales in Financial Markets

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Abstract

Many financial transactions are of a fixed-sum nature, meaning that any improvement in the terms of trade for one party comes at the expense of another party. We model how the sales of trading advantages (e.g., data or collocation services) affect traders' endogenous participation in a market and vice-versa. We show how the magnitude of the externality that a trading advantage imposes on counterparties impacts financial market conditions. In equilibrium, the optimal sales of trading advantages by a monopolist (e.g., data provider or a securities exchange) may result in inefficiently low levels of market participation and trade.

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1 Introduction

> *By the summer of 2013, the world's financial markets were designed to maximize the number of collisions between ordinary investors and high-frequency traders — at the expense of ordinary investors, and for the benefit of high-frequency traders, exchanges, Wall Street banks, and online brokerage firms. Around those collisions an entire ecosystem had arisen.*
> – Michael Lewis, Flash Boys (p.179)

Many financial transactions are of a fixed-sum nature, meaning that any improvement in the terms of trade for one party comes at the expense of another party. This feature of financial markets is known to promote "arms races", that is, the inefficient acquisition of resources aimed at gaining a relative advantage over rivals and appropriating their surplus.[1] While large financial institutions spend astronomical sums on data and collocation services hoping to take advantage of their counterparties, 41% of individuals who do not participate in financial markets blame the fact that these markets are "rigged" against them.[2] Accordingly, the ecosystem that collects revenues by providing goods and services that benefit a subset of traders at the expense of their counterparties must account for the fixed-sum nature of trading. By providing a substantial advantage to too many traders, a data provider or a securities exchange might push unsophisticated investors to exit the market, thereby reducing the value of the advantage being purchased.

We propose a model to study how the sales of goods and services that impose negative externalities on counterparties affect financial market outcomes, including market participation and volume. In our model, agents differ in their probability of being able to supply liquidity to counterparties. The likelier a market participant is to be asked to supply liquidity, the more valuable gaining a "trading advantage" through superior data and better

[1] See, e.g., Glode, Green, and Lowery (2012), Biais, Foucault, and Moinas (2015), Budish, Cramton, and Shim (2015), Glode and Lowery (2016), and Glode and Ordoñez (2022).

[2] See Royal, James. (March 24, 2021.) "Survey: More than half of investors think the stock market is rigged against individuals." Bankrate.

collocation is. Each agent chooses whether to participate in the market and whether to acquire a trading advantage at a price chosen by a monopolist. We focus our analysis on the interactions among agents' endogenous participation, the pricing of the trading advantage, and the efficiency of trade.

We show that a monopolist maximizes its profits by setting the price of the trading advantage higher than in the standard monopolist problem without externalities. The endogenous market participation of traders who choose not to acquire an advantage creates a second elasticity that the monopolist must consider, in addition to the demand elasticity that is central in the classic monopoly pricing problem. When lowering the price of the advantage, a monopolist increases the quantity demanded but also makes financial markets more "rigged" for traders who do not acquire this advantage. As a result, these unsophisticated traders might decide to exit the market, thereby reducing the frequency at which the buyers of this advantage get to trade. With fewer traders that demand liquidity participating in the market, an advantage that can be used when supplying liquidity becomes less valuable.

Yet, we show that the monopolist's optimal pricing strategy can result in socially excessive sales of goods and services that benefit a subset of market participants, leaving a fraction of the potential gains from trade unrealized due to the non-participation of their disadvantaged counterparties. The welfare losses due to these excessive sales increase in the magnitude of the negative externalities associated with the trading advantage as well as in the share of the surplus that liquidity suppliers can extract when trading. Moreover, the excessive sales of goods and services such as data and collocation services affect agents differently. When the externalities are small, the most likely liquidity suppliers benefit from the excessive sales whereas the most likely liquidity demanders are harmed by them. When the externalities are large, however, all traders are made worse off by the resulting high exit rates of market participants in equilibrium. This case arises because the monopolist maximizes its profits by pricing the advantage low enough to have socially costly exit levels that destroy the surplus collected by *all* traders in equilibrium. Overall, our model highlights that the magnitude of the externality that a trading advantage imposes on counterparties is an important determinant of market outcomes.

Literature review. Our paper contributes to the literature on arms races in financial markets. Glode, Green, and Lowery (2012), Biais, Foucault, and Moinas (2015), Budish, Cramton, and Shim (2015), Glode and Lowery (2016), and Glode and Ordoñez (2022) show that financial firms may have incentives to overinvest (from a social standpoint) in goods and services such as information, expertise, and fast-trading technology that allow their purchasers to take advantage of counterparties. Our paper instead focuses on the pricing of the resources that provide such an advantage to a subset of market participants, from the perspective of a monopolist (e.g., a data provider or a securities exchange) that must consider the impact of its decisions on market participation (and liquidity).

Our paper also relates to the literature on the optimal design of auctions for goods with externalities (e.g., Jéhiel, Moldovanu, and Stacchetti 1996, Eső, Nocke, and White 2010). Unlike in these papers, the monopolist's pricing decision that we model affects the quantity of goods or services produced and, as a result, the magnitude of the externalities imposed on the subset of agents who do not acquire them. Furthermore, we show how the endogenous participation channel affects the optimal pricing, which is a channel that matters in financial markets according to the survey evidence mentioned earlier, but arguably not in many of the settings considered by the existing literature — for example, a country troubled by a neighbor arming up cannot simply "opt out" of a war if attacked. Similar to our approach with quantities in mind, the mechanism design literature (see Segal 1999, Segal 2003, among others) considers the general contracting problem between a principal and multiple agents when allocations can impose externalities. Our setting instead focuses on the interaction between a seller with market power and multiple potential buyers. The full characterization of the monopolist's profit function allows our model to generate sharp predictions about the roles played by the underlying distribution of traders' types, the monopolist's knowledge of agents' types, and the degree of competition that are all missing from more abstract mechanism-design frameworks.

Our insights on the decreasing returns to a trading advantage relate our paper to the literature on information acquisition in financial markets. In models with noise traders or noisy supplies such as Grossman and Stiglitz (1980), Admati and Pfleiderer (1986), and Admati and Pfleiderer (1990) just to name a few, the value of information decreases as more traders acquire it and as asset prices become more informative. In those models, the volume of transactions and the available gains to trade are considered exogenous by traders deciding to acquire information. Our paper, however, focuses on how the sales of trading advantages endogenously reduce trader participation in the market — the more rigged a market appears to be, the less attractive it is for unsophisticated investors. Moreover, the fewer investors agree to participate in the market, the less valuable acquiring a trading advantage is, thereby resulting in a feedback loop between market participation and the sales of trading advantages. We show how the seller of a trading advantage chooses its optimal pricing strategy in light of its effect on market participation and liquidity, which drive how much traders are willing to pay for such advantage.

Roadmap. In the next section, we introduce our model with traders demanding and supplying liquidity and a monopolist offering these traders access to a trading advantage. In Section 3, we analyze traders' decisions whether to acquire a trading advantage and whether to participate in the financial market. In Section 4, we analyze the monopolist's optimal pricing of the trading advantage given traders' equilibrium responses. In Section 5, we study how varying the market structure can affect equilibrium behaviors by traders and by the entity selling a trading advantage. The last section concludes. Proofs of formal results are relegated to the Appendix.

2 Model

We present a simple model to study how a monopolist (e.g., a data provider or a securities exchange) can maximize the profits from providing a good or service (e.g., data or colocation services) that improves the position of a subset of traders at the expense of their counterparties.

Environment. Consider a financial market with a continuum of agents (i.e., potential traders). Each agent i has a type $\theta_i \in [0, 1]$, which denotes the probability that this agent will be in position to supply liquidity to the market at $t = 1$. With probability $(1 - \theta_i)$, however, this agent is hit by a liquidity shock and demands liquidity from the agents in position to supply it (e.g., by exchanging an illiquid asset for cash). We assume that types θ_i are independent and identically distributed across agents. The cumulative distribution function (CDF) of types θ_i is denoted $F : [\underline{\theta}, \overline{\theta}] \to [0, 1]$, where $\underline{\theta} \geq 0$ and $\overline{\theta} \leq 1$, and the probability density function (PDF) is denoted f. We use μ to denote the population mean of θ_i: $\mu \equiv \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)$.

When a transaction occurs between a liquidity demander and a liquidity supplier, a social surplus Δ is created. When acting as a liquidity supplier in a transaction, agent i extracts a payoff $\omega \cdot \Delta + \sigma \cdot a_i$ at $t = 1$ if the transaction occurs, where $a_i \in \{0, 1\}$ denotes whether agent i acquired a trading advantage offered by the monopolist at $t = 0$. Agent i's counterparty thereby extracts a payoff $(1 - \omega) \cdot \Delta - \sigma \cdot a_i$ at $t = 1$ if the transaction occurs.[3] At $t = 0$, the monopolist chooses the price p to charge for the trading advantage and each agent chooses whether to participate in the financial market and, if so, whether

[3] See Glode, Green, and Lowery (2012), Glode and Lowery (2016), and Glode and Ordoñez (2022) for various micro-foundations of this type of payoff functions.

to acquire a trading advantage at that price. For simplicity, we assume that the *marginal* cost of producing a trading advantage such as sharing data already collected or providing collocation access to one more trader is zero.

Matching of traders. The matching of liquidity suppliers and demanders is random and each liquidity supplier can fulfill the needs of as many liquidity demanders as there are. Specifically, each liquidity demander is randomly matched to a liquidity supplier with probability 1 and each liquidity supplier ends up participating in a measure η of transactions, which is determined in equilibrium based on all agents' decisions at $t = 0$ on whether to participate in the market.

Traders' payoff. Prior to knowing whether it will supply or demand liquidity at $t = 1$, agent i expects to collect a payoff:

$$(1 - \theta_i)\mathbb{E}[(1 - \omega)\Delta - \sigma a_{-i}] + \theta_i \eta [\omega \Delta + \sigma a_i], \tag{1}$$

where a_{-i} captures the equilibrium strategies of agent i's potential liquidity suppliers participating in the market (more on this later).

Equilibrium. Since the benefit of acquiring a trading advantage increases with the probability of supplying liquidity, we conjecture that an agent i accepts to pay p in exchange for a trading advantage that boosts its per-transaction payoff by σ whenever $\theta_i \geq \theta_A$. This threshold θ_A will later be determined in equilibrium based on all agents' decisions. We also conjecture that agents decide at $t = 0$ to stay out of the market and collect a normalized payoff of 0 whenever their liquidity type is below θ_P, which again will be determined in equilibrium. For the ease of exposition, the subscripts A and P stand for "Advantage" and "Participation," respectively. Given a price p, the resulting pair (θ_P, θ_A) chosen by traders is defined as the *equilibrium of the traders' subgame*.

Finally, the monopolist would choose the price to maximize its expected profits. The *global equilibrium* is characterized by (p, θ_P, θ_A), where (θ_P, θ_A) is the equilibrium of the traders' subgame when the price of the trading advantage is p.

Welfare. Given an equilibrium (p, θ_P, θ_A), the total welfare is:

$$\int_{\theta_P}^{\overline{\theta}} (1-\theta)(1-\omega)\Delta dF(\theta) + \int_{\theta_P}^{\overline{\theta}} \theta\eta\omega\Delta dF(\theta) = \Delta \cdot \int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta), \tag{2}$$

which is a decreasing function of θ_P. The more agents agree to participate in this market aimed at reallocating liquidity, the more surplus can be created through trade. Since the purchase of a trading advantage is a transfer p from traders to the monopolist, the total welfare is unaffected by this transaction, except through its impact on traders' participation threshold θ_P. While the production of trading advantages such as access to satellite imaging or microwave transmission services might very well represent a socially wasteful use of resources in reality, we abstract away from this type of inefficiency already featured in Glode, Green, and Lowery (2012), Biais, Foucault, and Moinas (2015), and Budish, Cramton, and Shim (2015), among many others, in order to focus our analysis on how the sales of trading advantages affect market participation and liquidity.

3 Traders' Decisions

In this section, we take p, the price of the trading advantage, as given and analyze how traders choose (θ_P, θ_A) as a result. Solving for these quantities involves a fixed-point problem: the participation threshold θ_P depends on the advantage threshold θ_A and the advantage threshold θ_A depends on the matching of liquidity suppliers with demanders, which is captured by η and depends on the participation threshold θ_P.

3.1 Solving for Equilibria of the Traders' Subgame

In order to solve for an equilibrium of the traders' subgame for a given price p, we need to (i) characterize the matching of liquidity demanders and suppliers for a given participation level, (ii) analyze traders' willingness to pay p in exchange for the trading advantage, and (iii) verify that the participation level is consistent with the extent to which traders acquire the trading advantage at a given price p.

Matching liquidity demanders and suppliers. Taking the equilibrium value of θ_P as given, we can derive the transaction volume that each liquidity supplier receives as:

$$\eta = \frac{\int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}. \tag{3}$$

The numerator in the ratio represents the measure of agents demanding liquidity in the market whereas the denominator represents the measure of agents who can supply it. Their ratio thus represents the measure of transactions that each liquidity supplier is randomly matched to (and this ratio can be smaller or larger than 1). Intuitively, a liquidity supplier's transaction volume η is decreasing in θ_P as the higher the type threshold for entering the market is, the lower the measure of transactions a liquidity supplier expects to be needed for, i.e.,:

$$\begin{aligned}
\frac{\partial \eta}{\partial \theta_P} &= \frac{-(1-\theta_P) f(\theta_P) \cdot \int_{\theta_P}^{\overline{\theta}} \theta dF(\theta) + \theta_P f(\theta_P) \cdot \int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\left[\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)\right]^2} \\
&= \frac{-f(\theta_P) \cdot \int_{\theta_P}^{\overline{\theta}} (\theta - \theta_P) dF(\theta)}{\left[\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)\right]^2} < 0. \qquad (4)
\end{aligned}$$

Acquisition of a trading advantage. At $t = 0$, each agent i must decide whether to pay

p to the monopolist in order to gain a trading advantage yielding an extra payoff σ in each transaction for which agent i supplies liquidity. Using equations (1) and (3), we know that agent i agrees to pay p whenever:

$$\begin{aligned} p &\leq \theta_i \eta \sigma \\ &= \theta_i \sigma \cdot \frac{\int_{\theta_P}^{\bar{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)}. \end{aligned} \tag{5}$$

Since a trading advantage can be beneficial only if one participates in the market, agent i pays to acquire a trading advantage whenever:

$$\theta_i \geq \theta_A = \max \left\{ \theta_P, \frac{p}{\sigma} \frac{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} (1-\theta) dF(\theta)} \right\}. \tag{6}$$

This condition shows that more trader types are willing to acquire the trading advantage when the price of this trading advantage is low relative to the payoff boost it provides when supplying liquidity. Also, more trader types are willing to acquire it if the measure of liquidity suppliers relative to liquidity demanders among market participants is low, since a liquidity supplier will receive more transaction volume and thereby use the acquired trading advantage against a larger measure of liquidity demanders.

By assuming that a trading advantage only serves a trader when supplying liquidity, the probability of supplying liquidity acts as a scaling factor for the benefit of acquiring a trading advantage. As a result, agent type θ_i generates the heterogeneity required to have some traders (e.g., market makers and hedge funds) being willing to purchase the trading advantage while others (e.g., retail investors and endowment funds) are not. Alternative sources of heterogeneity that scale up or down the benefit of acquiring the advantage for a given trader (e.g., assets under management, trading volume) could play a similar role in our analysis of the optimal sales of trading advantages with endogenous market participation. As will be clear below, our framework benefits from the tractability of having the same source of heterogeneity driving both the advantage-acquisition and market-participation decisions.

Market participation. At $t = 0$, each agent i must also decide whether to enter the market. If planning to pursue a strategy $a_i \in \{0, 1\}$ upon entering, agent i's expected payoff from entering the market is:

$$\begin{aligned} & (1-\theta_i)\mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}] + \theta_i \eta[\omega\Delta + \sigma a_i] - p a_i \\ = \quad & (1-\theta_i)\left[(1-\omega)\Delta - \sigma \cdot \frac{\int_{\theta_A}^{\bar{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)}\right] + \theta_i(\omega\Delta + \sigma a_i) \cdot \frac{\int_{\theta_P}^{\bar{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)} - p a_i, \end{aligned} \tag{7}$$

where we use the fact that:

$$\mathbb{E}[a_{-i}] = \frac{\int_{\theta_A}^{\bar{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)}. \tag{8}$$

Thus, agent i finds it optimal to participate in the market whenever this expected payoff is greater than zero, or equivalently:

$$(1-\theta_i)\mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}] + \theta_i \eta[\omega\Delta + \sigma a_i] - p a_i \geq 0. \tag{9}$$

The expected payoff (7) is increasing in agent i's liquidity type θ_i whenever:

$$\eta[\omega\Delta + \sigma a_i] \geq \mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}]. \tag{10}$$

If condition (10) holds, having a higher liquidity type is good for an agent and promotes its market participation. This inequality can be interpreted as meaning that the payoff of a liquidity supplier is no less than the payoff earned by a liquidity demander. In other words, imposing this restriction is like assuming that having liquidity is weakly better for a trader than not having liquidity. Condition (10) allows our conjectured equilibrium in which only agents with $\theta_i \geq \theta_P$ participate in the market to work naturally. Since (10) is satisfied when $\omega = 1$, we remark that the condition can be thought of imposing a lower bound on ω. As part of our equilibrium analysis, we will need to check that this condition is satisfied. For the market participation constraint to be relevant, we will typically focus our attention on cases for which the expected payoff associated with demanding liquidity at $t = 1$ is lower than the non-participation payoff (normalized to be zero), in which case (10) will automatically be satisfied.[4]

3.2 Properties of Equilibria of the Traders' Subgame

There are two cases to be considered based on whether the marginal agent whose $\theta_i = \theta_P$ acquires the trading advantage (i.e., $a_i = 1$) or not (i.e., $a_i = 0$) in equilibrium.

The case with $\theta_A > \theta_P$. Adjusting the participation condition (9) by setting $a_i = 0$ yields:

$$\theta_i \geq \frac{-\mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}]}{\eta\omega\Delta - \mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}]}. \tag{11}$$

When the above inequality becomes an equality, it identifies the liquidity type that is indifferent between entering the market or not. By substituting $\mathbb{E}[a_{-i}]$ and η into the condition above and recalling that $\theta_P \geq \underline{\theta}$, we obtain an expression that determines θ_P:

$$\theta_P = \max\left(\underline{\theta}, \frac{-(1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}}{\omega\Delta \cdot \frac{\int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}}\right). \tag{12}$$

[4] If demanding liquidity offered a higher expected payoff than non-participation, each agent would always decide to participate and, as a result, total welfare would necessarily be maximized in equilibrium.

To better understand the above equation, consider two possibilities.[5] First, suppose that the first term in the maximand is smaller than the second term. As a result, θ_P is equal to the second term. Rearranging the equation implies:

$$\frac{\sigma}{\Delta}\int_{\theta_A}^{\overline{\theta}} \theta\, dF(\theta) = \frac{\theta_P}{1-\theta_P}\omega \int_{\theta_P}^{\overline{\theta}} (1-\theta)\, dF(\theta) + (1-\omega)\int_{\theta_P}^{\overline{\theta}} \theta\, dF(\theta). \tag{13}$$

This condition shows that the participation threshold, which affects the right-hand side, and the acquisition threshold, which affects the left-hand side, are mutually related. Intuitively, agents' participation decisions depend on how many agents acquire the trading advantage and vice-versa. Moreover, since the left-hand side of equation (13) is monotonic in θ_A, for a given θ_P, there is at most one θ_A that satisfies (13).

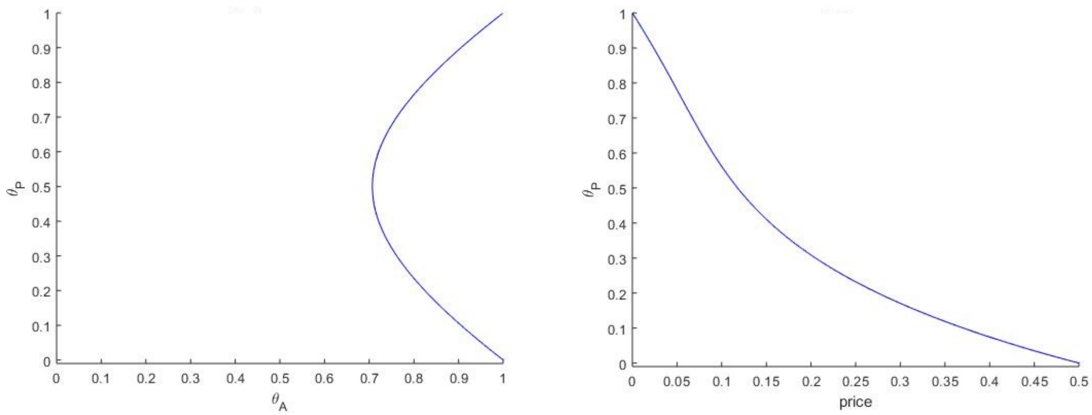
Second, suppose instead that the second term in the maximand is weakly smaller than the first term, in which case the equilibrium $\theta_P = \underline{\theta}$ and we have full participation. Plugging $\theta_P = \underline{\theta}$ into the second term in the maximand implies that θ_A must satisfy the following condition:

$$\frac{\sigma}{\Delta}\int_{\theta_A}^{\overline{\theta}} \theta\, dF(\theta) \leq \frac{\theta_P}{1-\theta_P}\omega \int_{\theta_P}^{\overline{\theta}} (1-\theta)\, dF(\theta) + (1-\omega)\int_{\theta_P}^{\overline{\theta}} \theta\, dF(\theta), \tag{14}$$

where $\theta_P = \underline{\theta}$. Since the right-hand side is decreasing in θ_A, this condition implies a lower bound on θ_A.

Combining the two possible scenarios, we can obtain a pair of possible (θ_P, θ_A) that may arise in an equilibrium. Panel (a) of Figure I plots, for a simple numerical example, the possible (θ_P, θ_A) pairs induced by the participation constraint. We denote the set of

[5] In the case with $\theta_A > \theta_P$, condition (10) becomes: $\frac{\sigma}{\Delta} \geq \frac{\int_{\theta_P}^{\overline{\theta}} (\theta-\omega) dF(\theta)}{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}$, which holds for sufficiently large ω (e.g., when $\omega \geq \overline{\theta}$). An equivalent way to write this condition is: $\omega \geq \frac{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta) - \frac{\sigma}{\Delta}\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{1-F(\theta_P)}$. Since this condition relies on equilibrium values of θ_A and θ_P, we will need to check that this condition is satisfied in any conjectured equilibrium.



(a) (θ_A, θ_P) pairs. **(b)** Implied price.

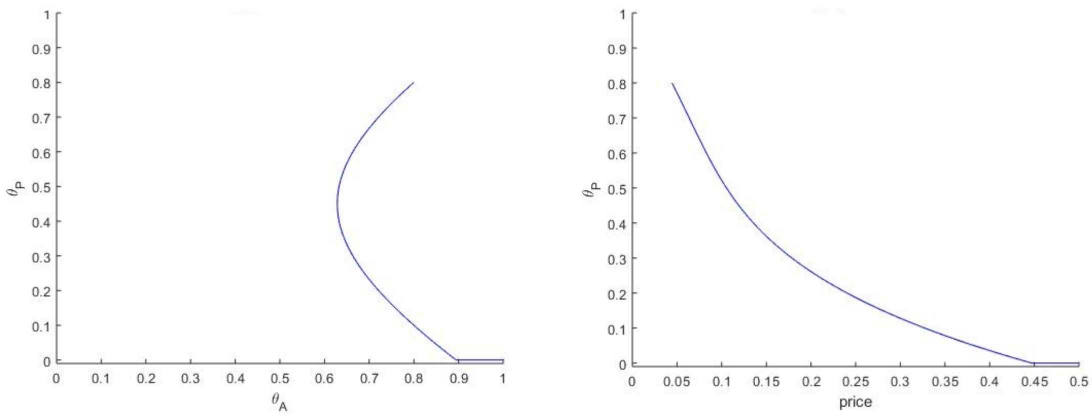
FIGURE I

This figure plots the possible (θ_P, θ_A) pairs and the implied prices p that trigger them. The parameter values are $\omega = 1$, $\sigma/\Delta = 0.5$, and F is the standard uniform distribution $U[0, 1]$.

the possible pairs of (θ_P, θ_A) as Θ and investigate the price that must be charged by the monopolist in order to induce a given pair on Θ. Given that agents would choose $\theta_A = \frac{p}{\sigma\eta}$ and that η is determined by θ_P, we can compute the associated price for a given point on Θ as $p = \sigma\eta\theta_A$.

As we can see in Panel (b) of Figure I, for any given price $p \in (0.0.5)$, there exists a unique θ_P that satisfies the equilibrium participation condition. Another interesting observation from Figure I is that, for small p, for example when it is close to 0.1, locally decreasing the price would increase θ_P, which increases θ_A, meaning it would decrease the demand for the trading advantage. In this case, the trading advantage is a Giffen good: it can generate higher demand when the price rises, creating an upward-sloping demand curve contrary to standard laws of demand. As we show in the alternative numerical example associated with Figure II, there may, however, not be an equilibrium in which $\theta_A > \theta_P$ when p is small enough.[6] Yet, patterns are similar to those in Figure II when p is between 0.05 and 0.1. We formalize this finding in the following lemma.

[6]But there will be an equilibrium where $\theta_A = \theta_P$. See the analysis of this case later in the subsection.



(a) (θ_A, θ_P) pairs. **(b)** Implied price.

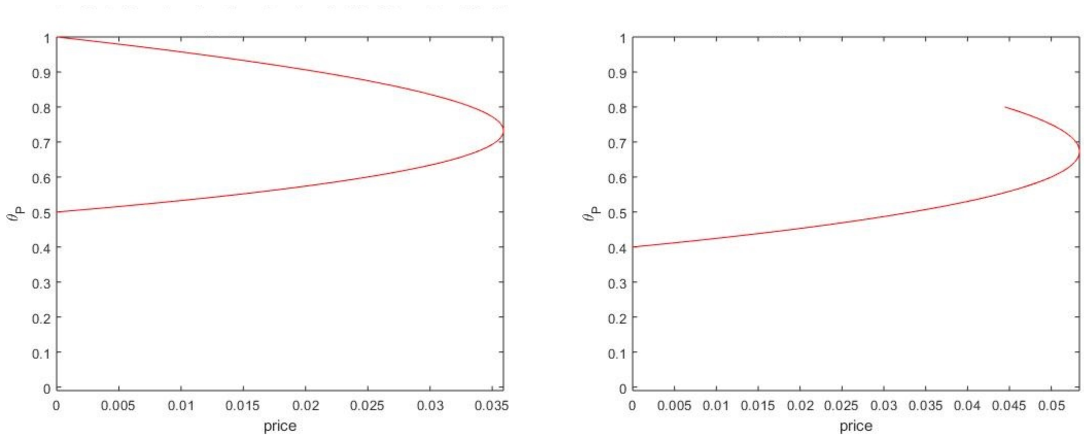
FIGURE II

This figure plots the possible (θ_P, θ_A) pairs and the implied prices p that trigger them. The parameter values are $\omega = 0.9$, $\sigma/\Delta = 0.5$, and F is the standard uniform distribution $U[0, 1]$.

Lemma 1. *When the solution of θ_A from equation* (13) *satisfies $\frac{\partial \theta_A}{\partial \theta_P} > 0$ and the implied price satisfies $\frac{\partial p}{\partial \theta_P} < 0$, then marginally increasing p yields a lower θ_A, i.e., more traders acquire the trading advantage when the price quoted by the monopolist is higher.*

The formal statement of Lemma 1 may not appear to be surprising, but what we want to emphasize here is that the condition imposed in Lemma 1 can be satisfied, as we showed in the numerical analysis. The intuition behind Lemma 1 is that varying the price of a trading advantage has both a direct and an indirect effect on the decision to acquire it. The direct effect is that a lower price implies that more traders find the trading advantage profitable to acquire. The indirect effect is that having more traders acquiring the trading advantage makes the financial market more "rigged", which drives out market participants and in turns lowers the value of the trading advantage. Lemma 1 states that the indirect effect can dominate the direct effect, in which case increasing the price leads to more sales of the trading advantage.

The case with $\theta_A = \theta_P$. In such case, agents either participate in the market while acquiring a trading advantage or do not participate at all. Similar to the previous discussion,



(a) $\omega = 1$.

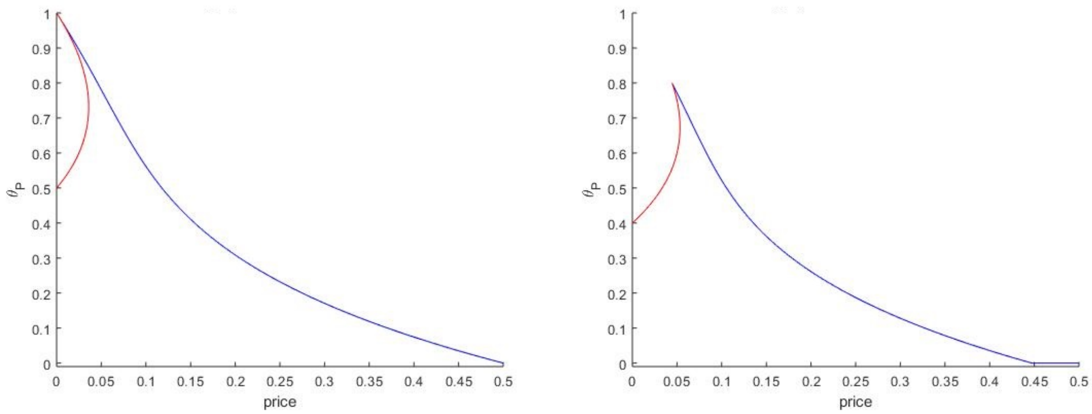
(b) $\omega = 0.9$.

FIGURE III

This figure plots the implied price when $\theta_A = \theta_P$, $\sigma/\Delta = 0.5$, and $F \sim U[0, 1]$.

we can pin down the possible pair of (θ_P, θ_A) and its implied price. We defer the formal analysis of this case to Appendix B. Figure III illustrates the implied price when $\theta_A = \theta_P$ for the two cases where $\omega = 1$ and $\omega = 0.9$, respectively.

Combining both cases. The analyses of cases $\theta_A > \theta_P$ and $\theta_A = \theta_P$ suggest that there can be multiple equilibria of the traders' subgame for a given price p. For example, combining Panel (b) of Figure I with Panel (a) of Figure III yields Panel (a) of Figure IV where the blue curve corresponds to the case with $\theta_A > \theta_P$ and the red curve corresponds to the case with $\theta_A = \theta_P$. Using this figure, we can consider a potential price the monopolist could charge, draw a vertical line corresponding to that price, and identify the location(s) where the line crosses the blue or red curves as the possible equilibria of the traders' subgame that can be sustained by this price. As the figure shows, it is possible to have a unique (θ_P, θ_A) pair, say when $p > 0.1$, but it is also possible to have three (θ_P, θ_A) pairs, say when $p = 0.02$. These scenarios are all possible because of the mix of strategic substitutes and complements that are part of our model: an agent is more willing to participate in a market when more agents participate as market participation increases the liquidity matching ratio η, yet an agent is also less willing to participate in a market when more agents acquire an trading advantage



(a) $\omega = 1$.

(b) $\omega = 0.9$.

FIGURE IV

This figure combines both cases where $\theta_A > \theta_P$ and $\theta_A = \theta_P$, assuming $\sigma/\Delta = 0.5$, and $F \sim U[0,1]$, and shows how to find equilibrium of the traders' subgame for a given price.

as the acquisition of a trading advantage increases how "rigged" the market is.

Given that there may exist multiple equilibria, agents' behavior can appear fragile in the sense that a small change in the price of the trading advantage can lead to a dramatically different equilibrium being played by agents. Another complexity arises when we shift from an equilibrium with $\theta_A = \theta_P$ to an equilibrium with $\theta_A > \theta_P$. In Panel (a) of Figure IV, marginally increasing p from 0.036 (the rightmost point on the red curve) to 0.037 shifts the equilibrium participation threshold θ_P from 0.75 to 0.85. As a result, a tiny increase in the price of the advantage pushes 10% of agents to exit the financial market. However, as we further increase p, the equilibrium remains consistent with the blue curve, so that an increase in p makes the trading advantage less attractive which in turn encourages market participation.

Overall, the analysis of how traders behave for a given price p has highlighted a few interesting phenomena. First, the market for the trading advantage can feature a upward-slope demand curve: as the price of the trading advantage increases, traders may find the trading advantage even more valuable. This happens because a high price can reduce the traders expected to acquire the trading advantage, which in turn promotes financial market liquidity and makes the trading advantage more valuable. Second, the financial market can be fragile in the sense that there can be multiple equilibria. The equilibrium multiplicity is due to the strategic complementary of market participation: more traders participating in the market promotes its liquidity and makes participation more attractive. Third, as the price of the trading advantage changes slightly, we might go from multiple equilibria existing to only one. This is due to a strategic substitution effect coming from the observation that more traders acquiring the trading advantage discourages participation. Thus, a small change in the price of the trading advantage can lead to a dramatic change in financial market conditions.

4 Monopolist's Pricing Decision

Having characterized the possible equilibria of the traders' subgame, we now analyze how a monopolist chooses a price that maximizes the profit from selling a trading advantage when taking into account traders' equilibrium responses.

As stated above, the monopolist's cost of providing an *additional* trader with access to a trading advantage (e.g., by sharing access to already collected data or to an existing high-speed communication network) is assumed to be zero. The monopolist thus chooses a price p to maximize revenues $p \cdot \int_{\theta_A}^{\overline{\theta}} dF(\theta) = p(1 - F(\theta_A))$, subject to each agent's optimal decisions regarding market participation (i.e., condition (9)) and the costly acquisition of a trading advantage (i.e., condition (5)).

Before we can solve for the equilibrium pricing strategy, we need to revisit the two cases we considered in Section 3 with an emphasis on pricing. We first consider the case where the monopolist targets a strict subset of market participants in equilibrium, that is, $\theta_A > \theta_P$.

The case with $\theta_A > \theta_P$. Recall from subsection 3.2 that, for each possible pair $(\theta_P, \theta_A) \in \Theta$, we can find the price charged by the monopolist, which then allows us to compute the monopolist's profit. To compare this optimization problem with the classic monopolist's problem, we write the monopolist's profit as a function of θ_A, which can be interpreted as the marginal buyer type. In our model, the monopolist's problem is to maximize: $p(1 - F(\theta_A)) = \sigma\eta\theta_A(1 - F(\theta_A))$. The first-order condition with respect to θ_A is then given by:

$$\frac{\partial \eta}{\partial \theta_A}\theta_A[1 - F(\theta_A)] + \eta\,[1 - F(\theta_A) - \theta_A f(\theta_A)] = 0, \tag{15}$$

which can be rearranged as:

$$\frac{\partial \eta/\eta}{\partial \theta_A/\theta_A} + 1 - H(\theta_A) = 0, \tag{16}$$

where $H(\theta_A) \equiv \frac{\theta_A f(\theta_A)}{1-F(\theta_A)}$. Note that in the classic monopoly pricing problem (e.g., when η is a constant), the standard FOC is $1 - H(\theta_A) = 0$. However, our pricing problem is different because the level of θ_A affects η. Intuitively, a change in the fraction of agents who acquire the trading advantage affects the buyer types willing to participate in the market, which in turn affect the matching of liquidity suppliers and demanders (i.e., η).

We write ϵ to denote $\frac{\partial \eta/\eta}{\partial \theta_A/\theta_A}$, which can be interpreted as the elasticity of liquidity to the advantages acquired. This elasticity captures how "rigged" a market appears to be for unsophisticated agents — if $\epsilon > 0$, decreasing θ_A leads to more traders acquiring the trading advantage, which may convince agents with low θ_i to stay away and thereby increase θ_P. For ease of exposition, we make the following assumption:

Assumption 1. *$H(\theta)$ is strictly increasing in θ for $\theta \in [0, 1]$.*

Assumption 1 resembles the definition of a strictly regular environment by Fuchs and

Skrzypacz (2015) and the standard assumption in auction theory that bidders' virtual valuation functions are strictly increasing. This regularity condition is satisfied by many well-known distributions, including the uniform distribution.

Our pricing problem can be simplified as setting $H(\theta_A) = 1 + \epsilon$. If the elasticity $\epsilon > 0$, then the optimal θ_A is higher than that in the classic monopolist problem (where η is a constant), meaning there are fewer agents who end up acquiring the trading advantage. Alternatively, if the elasticity $\epsilon < 0$, then the optimal θ_A is smaller than that in the classic problem. We remark that ϵ is a function of θ_A, thus the solution to $H(\theta_A) = 1 + \epsilon$ may be challenging to solve for — there may be multiple solutions. Yet, we can show that at the optimum the elasticity ϵ is weakly positive, as summarized by the following proposition:

Proposition 1. *In an equilibrium where $\theta_A > \theta_P$, the monopolist optimally quotes a price that induces $\theta_A \geq \theta_0$, where θ_0 denotes the smallest $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ such that $H(\theta_0) \geq 1$.*

Note that θ_0 is the marginal buyer type targeted in the classic monopolistic setting. Proposition 1 states that in our current setting with an endogenous η, the monopolist targets a weakly smaller mass of agents to purchase the trading advantage than would be targeted in the classic monopolistic setting with η being a constant.

The case with $\theta_A = \theta_P$. We now explore the pricing of the advantage in the second case we considered above. As discussed in subsection 3.2 and Appendix B, an equilibrium with $\theta_A = \theta_P$ would require the price p charged by the monopolist to be solely a function of θ_P (see equation (B3) in Appendix B). The monopolist's profit would then be given by $p(1 - F(\theta_A)) = p(1 - F(\theta_P))$, which is a function of θ_P. The pricing problem is thus a one-dimensional maximization problem subject to constraint (5) to capture traders' optimal purchasing behavior.

4.1 Efficient/Full-Participation Equilibrium

Having analyzed the monopolist's profit in two possible and mutually exclusive cases, we now compare the monopolist's profit when triggering $\theta_A = \theta_P$ versus $\theta_A > \theta_P$ to identify what the monopolist will want to achieve with its pricing strategy. In this subsection, we derive a sufficient condition under which the externality parameter σ is small enough to allow for the optimal sales of trading advantages that do not prevent efficient market participation in equilibrium. This small-externality "benchmark" highlights how the negative externalities associated with the trading advantage interact with traders' participation decisions.

To start, consider the following arguments. Fixing θ_A, the monopolist would prefer an equilibrium with a lower θ_P as it leads to a higher η and to traders' higher willingness to pay for the trading advantage. However, how low θ_P can go is constrained by agents' unwillingness to participate in "rigged" markets, as captured by equation (12). If the negative externality associated with a trading advantage is sufficiently small, however, we can identify a condition that ensures that all agents are willing to participate in the market and that the pricing problem reduces to the classic monopoly pricing problem (i.e., since η remains constant over a range of θ_A).

When all agents participate and $\theta_P = \underline{\theta}$, no welfare loss is incurred. If we ignore the participation constraint (12) for the time being, it follows that η is a constant when $\theta_P = \underline{\theta}$. The first-order condition of the profit maximization problem with respect to θ_A is precisely $H(\theta_A) = 1$. Given Assumption 1, the monopolist chooses the following θ_A, which we write as $\theta_0 \equiv \max(\underline{\theta}, H^{-1}(1))$. If $H(\underline{\theta}) \leq 1$, then $\theta_0 = \underline{\theta}$; otherwise $\theta_0 > \underline{\theta}$.

Bringing back the participation constraint (12) as part of the analysis means that we need to make sure that all agents are willing to participate even when the monopolist sells the trading advantage to traders whose θ_i is higher than θ_0. Equation (11) becomes

$$\underline{\theta} \geq \frac{-\mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}]}{\eta\omega\Delta - \mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}]}. \tag{17}$$

The above condition must hold for $\theta_P = \underline{\theta}$ and $\theta_A = \theta_0$. As we show in Appendix B, substituting in θ_P and θ_A allows to rearrange condition (17) as:

$$\frac{\sigma}{\Delta} \leq \frac{\frac{\underline{\theta}\omega}{1-\underline{\theta}} + \left(1 - \frac{\omega}{1-\underline{\theta}}\right)\mu}{\int_{\theta_0}^{\overline{\theta}} \theta dF(\theta)} \equiv k. \tag{18}$$

Recall from equation (10) that the conjectured equilibrium also requires:

$$\frac{\sigma}{\Delta} \geq \frac{\int_{\theta_P}^{\overline{\theta}} (\theta - \omega) dF(\theta)}{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}. \tag{19}$$

Substituting in θ_P and θ_A yields:

$$\frac{\sigma}{\Delta} \geq \frac{\int_{\underline{\theta}}^{\overline{\theta}} (\theta - \omega) dF(\theta)}{\int_{\theta_0}^{\overline{\theta}} \theta dF(\theta)} = \frac{\mu - \omega}{\int_{\theta_0}^{\overline{\theta}} \theta dF(\theta)} \equiv k_0. \tag{20}$$

When ω is greater than μ, k_0 is negative, in which case the above condition is not binding given the positivity of σ. Comparing k_0 and k, it is straightforward to verify that $k \geq k_0$ since $\mu = \mathbb{E}[\theta_i] \leq 1$. As a result, we can establish the following proposition:

Proposition 2. *Suppose $\frac{\sigma}{\Delta} \geq k_0$. In equilibrium, we have $(\theta_P, \theta_A) = (\underline{\theta}, \theta_0)$ if and only if $\frac{\sigma}{\Delta} \leq k$, in which case the equilibrium is socially efficient.*

We refer to the equilibrium that features $(\theta_P, \theta_A) = (\underline{\theta}, \theta_0)$ as the small-externality equilibrium. In such equilibrium, the monopolist's pricing decision is locally unaffected by agents' endogenous participation. To understand Proposition 2, recall that σ/Δ can be thought of as how powerful a trading advantage is since it measures the share of trade surplus that a liquidity supplier can expropriate thanks to this advantage. Proposition 2 shows that the equilibrium outcome is socially efficient (i.e., $\theta_P = \underline{\theta}$) when $\sigma/\Delta \leq k$. In that case, the monopolist prefers to induce an equilibrium of the traders' subgame where η is as large as possible, that is, when $\theta_P = \underline{\theta}$.

The condition $\sigma/\Delta \leq k$ is sufficient for the existence of an efficient equilibrium, but it is not a necessary condition. In some cases with $\sigma/\Delta > k$, the monopolist might still find it optimal to induce full participation (i.e., $\theta_P = \underline{\theta}$) by raising θ_A slightly, in which case there is no welfare loss but fewer traders acquire the trading advantage than in a small-externality equilibrium. In fact, when $\sigma/\Delta > k$, either $\theta_P \neq \underline{\theta}$ or $\theta_A \neq \theta_0$. In the former case the equilibrium is inefficient, and in the latter case the monopolist can no longer induce what would have been induced in the classic monopoly pricing problem. As will become clear in the subsection below, as the trading advantage becomes more powerful, inducing an equilibrium of the traders' subgame with full participation requires too few traders to acquire a trading advantage and yields a suboptimal level of profits for the monopolist.

4.2 Inefficient/Partial-Participation Equilibrium

We now write the monopolist's profit as a function of θ_P and study the function's property when θ_P is close to $\underline{\theta}$. This approach implies that a sufficient condition for an inefficient equilibrium is that the profit function is strictly increasing at the point $\theta_P = \underline{\theta}$, in which case the monopolist optimally induces a partial-participation equilibrium with lower aggregate surplus from trade (i.e., total welfare). In this subsection, we derive a sufficient condition for the equilibrium to be inefficient, i.e., $\theta_P > \underline{\theta}$.

We assume that $\theta_A > \theta_P$ for the main text and defer the analysis of $\theta_A = \theta_P$ to

Appendix B. We note that when $\theta_P > \underline{\theta}$, the participation constraint (12) becomes:

$$\frac{\sigma}{\Delta}\int_{\theta_A}^{\overline{\theta}} \theta \, dF(\theta) = \frac{\theta_P}{1-\theta_P}\omega \int_{\theta_P}^{\overline{\theta}} (1-\theta)\, dF(\theta) + (1-\omega)\int_{\theta_P}^{\overline{\theta}} \theta \, dF(\theta). \tag{21}$$

In order to derive our inefficiency condition, a relevant variable we need to keep track of is the solution to θ_A from equation (21) when $\theta_P \to \underline{\theta}$. We denote the solution as θ_1 (assuming it exists). For the equilibrium to be inefficient, θ_1 has to be greater than θ_0, otherwise $(\theta_P, \theta_A) = (\underline{\theta}, \theta_0)$ can be supported, and as argued in the previous subsection, it is then in the monopolist's best interest to induce the efficient equilibrium. From the proof of Proposition 2, the condition $\theta_1 > \theta_0$ is precisely equivalent to $\sigma/\Delta > k$. Since the left-hand side of equation (21) approaches zero as $\theta_1 \to \overline{\theta}$, it follows that, under the condition $\sigma/\Delta > k$, there is a unique θ_1.

A sufficient condition for the equilibrium to be inefficient is that the monopolist's profit function is locally increasing at $\theta_P = \underline{\theta}$. Note that:

$$\frac{\partial[\sigma\eta\theta_A(1-F(\theta_A))]}{\partial\theta_P}\Big|_{\underline{\theta}} = \sigma\left(\frac{\partial\eta}{\partial\theta_P}\theta_A(1-F(\theta_A)) + \eta(1-F(\theta_A)-f(\theta_A)\theta_A)\frac{\partial\theta_A}{\partial\theta_P}\right)\Big|_{\underline{\theta}}. \tag{22}$$

Directly evaluating the right-hand side of the above equation (see the proof of Propostiion 3), we show that it is strictly positive whenever the following condition is satisfied:

$$\frac{\sigma}{\Delta} < \frac{\mu(1-\mu)\left[\frac{\omega}{(1-\underline{\theta})^2}(1-\mu) - \underline{\theta} f(\underline{\theta})\right]}{f(\underline{\theta})(\mu-\underline{\theta})}\frac{H(\theta_1)-1}{\theta_1^2 f(\theta_1)}. \tag{23}$$

This condition ensures that the monopolist's profit function does not peak at $\theta_P = \underline{\theta}$. In particular, locally increasing θ_P above $\underline{\theta}$ can strictly increase the monopolist's profit. The following proposition summarizes the result:

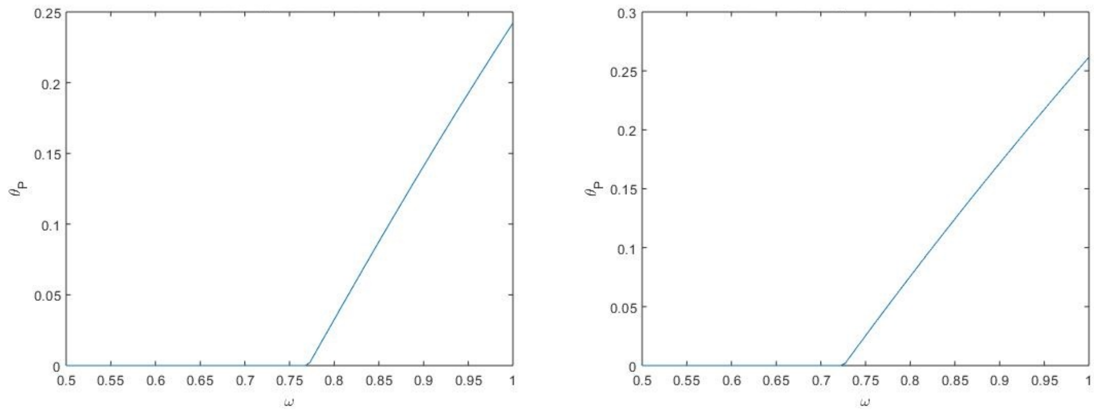
Proposition 3. *Suppose $\theta_A > \theta_P$, $\sigma/\Delta > k$, and condition* (23) *holds, then the monopolist* *optimally induces an equilibrium where $\theta_P > \underline{\theta}$, i.e., a partial-participation equilibrium, in which case the equilibrium is socially inefficient.*

Proposition 3 identifies sufficient conditions for the equilibrium to feature excessive sales of trading advantages — here, the term "excessive" refers to the fact that too many traders acquire the advantage to allow the maximum levels of participation and gains to trade. Condition (23) should, however, *not* be interpreted as requiring that σ/Δ is sufficiently small as θ_1 generally depends on σ/Δ. In Appendix B, we consider the case where F is the standard uniform distribution to help interpret this condition. In that case, we show that condition (23) is satisfied when either σ/Δ or ω is sufficiently large. We discuss these relationships in more detail in the next subsection with the help of numerical examples.

4.3 Parameterization

We now assume that F, the distribution of θ_i, is a standard uniform distribution. Given that we can fully solve the monopolist's problem numerically, we focus on presenting numerical results. First, we study how exogenous parameters ω and σ/Δ affect the participation threshold θ_P. Figures V and VI show that, as we increase ω or σ/Δ, the participation threshold θ_P weakly increases, indicating more welfare losses. Put differently, as liquidity suppliers can extract a higher fraction of the trade surplus or as the externality is stronger, the equilibrium is more likely to feature inefficiently low levels of trader participation.

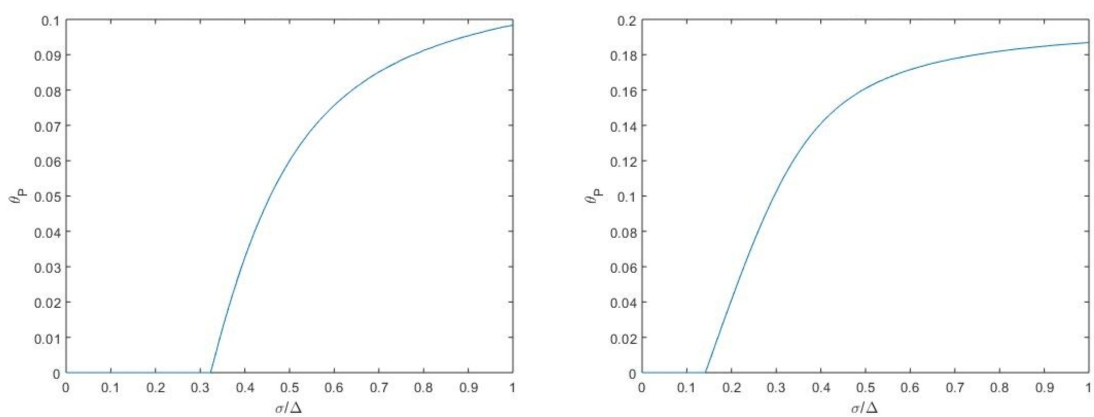
Second, in Figures VII, we plot for different levels of σ/Δ the equilibrium values for θ_A and θ_P in Panel (a), the equilibrium payoffs for three types of agents ($\theta_i = 0.1, 0.5, 0.9$) in Panel (b), and the price charged by the monopolist in Panel (c). We can see that as the relative size of the externality increases, fewer traders acquire the trading advantage, yet the equilibrium becomes more inefficient in the sense that θ_P is weakly increasing. However, the difference in agents' payoff between the $\theta_i = 0.9$ type and the $\theta_i = 0.1$ type is



(a) $\sigma/\Delta = 0.4$. **(b)** $\sigma/\Delta = 0.6$.

FIGURE V

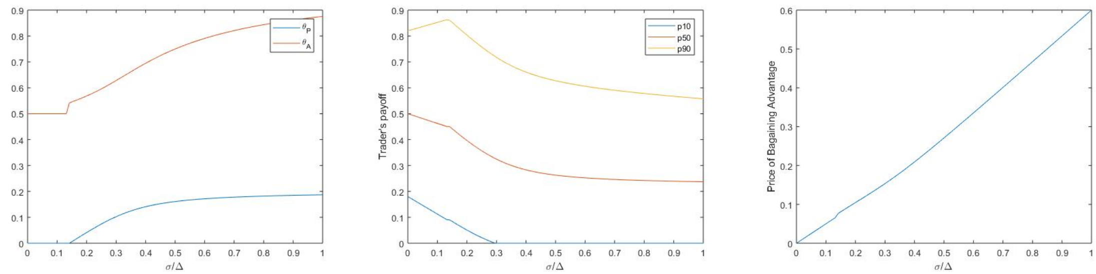
This figure plots the value of θ_P when ω changes.



(a) $\omega = 0.8$. **(b)** $\omega = 0.9$.

FIGURE VI

This figure plots the value of θ_P when σ/Δ changes.



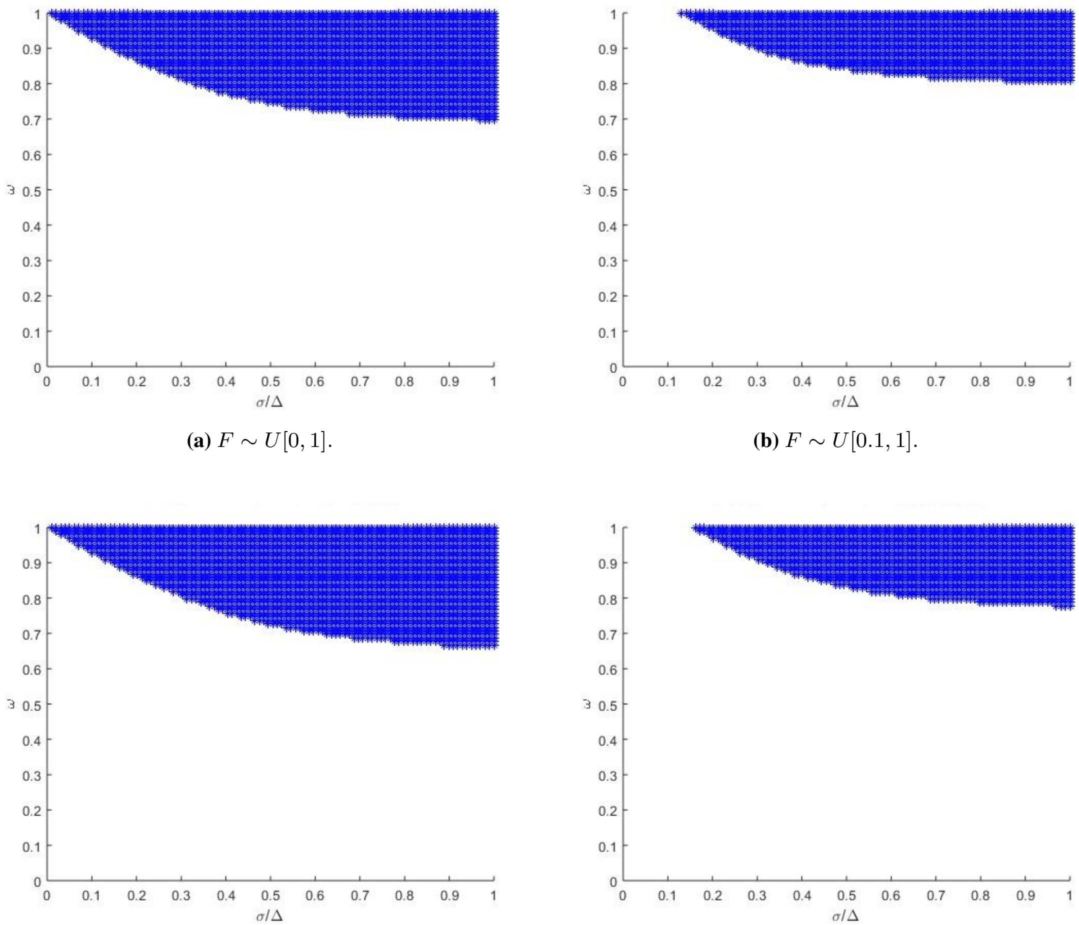
(a) Equilibrium thresholds (θ_P, θ_A). **(b)** Traders' expected payoffs. **(c)** Price charged by the monopolist.

FIGURE VII

This figure plots equilibrium objects as a function of σ/Δ when $\omega = 0.9$

non-monotonic in σ/Δ. Intuitively, when σ/Δ is relatively small, an increase in σ benefits the high-θ_i agents at the expense of the low-θ_i agents. Yet, as the trading advantage gets stronger, the high-θ_i agents are made worse off in equilibrium while the low-θ_i agents are either only slightly affected or exiting the market. When σ/Δ is large, the monopolist optimally induces fewer traders to acquire the trading advantage by charging a higher price for it. For example, the agent with $\theta_i = 0.9$ purchase the trading advantage in equilibrium for all levels of σ/Δ that we plot. Since having fewer agents purchasing the trading advantage benefits those who purchase it, it also allows the monopolist to charge a higher price. The latter channel may dominate, in which case some high-θ_i agents are made worse off by the trading advantage getting stronger.

Third, in Figure VIII, we numerically identify the parameter regions where the equilibrium is efficient and those where the equilibrium is inefficient. As we can see, when we shift the distribution of θ_i to the right (from $U[0,1]$ in Panel (a) to $U[0.1,1]$ in Panel (b)), the inefficient region shrinks, indicating that it is easier to sustain the efficient equilibrium. Similar conclusion can be reached by shifting the distribution to the left (from $U[0,1]$ in Panel (a) to $U[0,0.9]$ in Panel (c)) or by reducing the dispersion of types but preserving the mean (from $U[0,1]$ in Panel (a) to $U[0.1,0.9]$ in Panel (d)). Overall, dispersion in traders' liquidity types leads to more heterogeneity in the value of acquiring an advantage and a



(a) $F \sim U[0, 1]$.

(b) $F \sim U[0.1, 1]$.

(c) $F \sim U[0, 0.9]$.

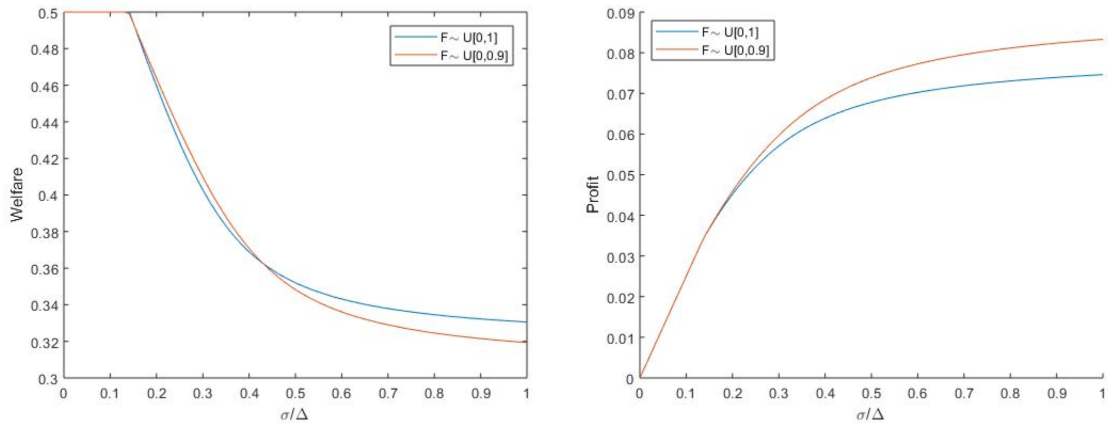
(d) $F \sim U[0.1, 0.9]$.

FIGURE VIII

This figure identifies, in blue, the parameter regions where the equilibrium is inefficient for various levels of trader-type heterogeneity.

higher propensity for inefficient non-participation by a subset of agents.

Lastly, suppose each agent's liquidity-supplying probability decreases by 10%, but Δ is scaled so that the total available surplus $(1 - \mu)\Delta$ remains unchanged. In this case, Figure IX shows that the equilibrium welfare comparison depends on the size of the externality. If σ/Δ is between 0.15 and 0.45, shrinking the liquidity-supplying probabilities is welfare improving. When σ/Δ is higher than 0.45, then the shrinking is welfare destroying. Unambiguously, the monopolist gets a higher profit after the shrinking.



(a) Welfare.

(b) Profit.

FIGURE IX

This figure illustrates the shrinking distribution and changing Δ to match the total available surplus when $\omega = 0.9$.

5 Discussion

In this section, we discuss how varying the market structure can affect equilibrium behaviors by traders and by the entity selling a trading advantage.

5.1 Segmenting Traders Across Trading Venues

Suppose that a second trading venue becomes available and one venue attracts agents whose type is below θ_2 while the other venue attracts agents whose type is above θ_2 (or vice-versa). We suppose that at the time of the sale of a trading advantage, the monopolist does not know the trading venue in which each agent plans to trade — the monopolist must therefore charge the same price to all agents. (We will discuss the possibility of price discrimination later.)

We first consider a situation where all agents above θ_2 participate and purchase the trading advantage. We need to determine θ_A and θ_P for the agents below θ_2. We compute the respective venue-specific η, denoted by η_1 and η_2:

$$\eta_1 = \frac{\int_{\theta_P}^{\theta_2} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)}, \tag{24}$$

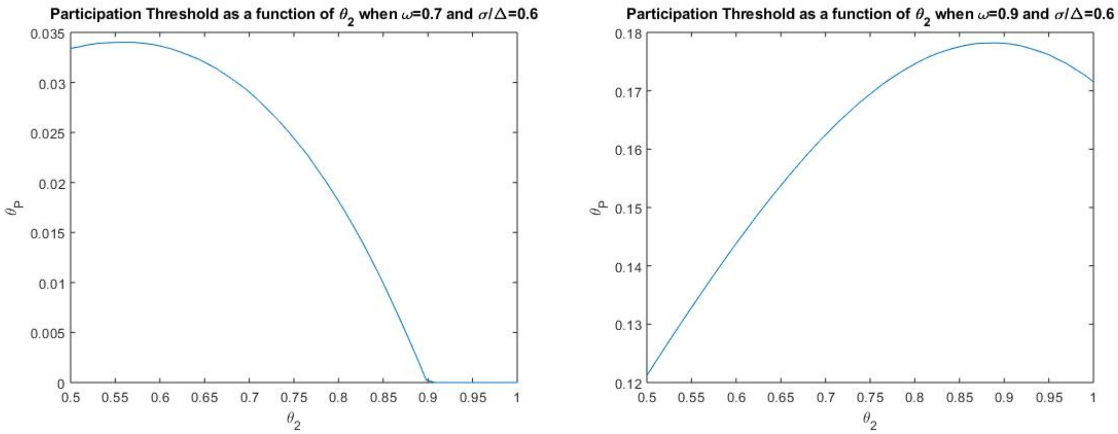
$$\eta_2 = \frac{\int_{\theta_2}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)}. \tag{25}$$

At $t = 0$, each agent $\theta_i < \theta_2$ must decide whether to enter the market. As in the baseline analysis, we can obtain the monopolist's profit as a function of θ_P, with a new constraint on the upper bound of the price such that traders whose type is above θ_2 are willing to pay the price to purchase the trading advantage.

A second situation exists where all agents above θ_2 participate but do not purchase the advantage. This imposes a lower bound on the price charged by the monopolist so that agents above θ_2 are indeed unwilling to purchase the trading advantage. The monopolist's profit is then given by $p(F(\theta_2) - F(\theta_A))$.

By comparing these two cases, we can analyze how the monopolist sets its price. We numerically investigate the impact of adding a second trading venue for the case where F is the standard uniform distribution, and defer the formal analysis to Appendix B.

We first pick parameters such that the baseline model yields an efficient equilibrium. Suppose $\sigma/\Delta = 0.6$ and $\omega = 0.7$. According to Panel (b) of Figure V, the baseline model with only one venue features $\theta_P = 0$, i.e., there is no welfare loss. Now, set $\theta_2 = 0.6$ such that agents above 0.6 trade in one venue while other agents trade in a different venue. Note that segmenting agents across two venues does not affect the total available surplus in the economy per se. However, Panel (a) of Figure X shows that $\theta_P = 0.034$, indicating that there are welfare losses in equilibrium. In this case, it is costlier for the monopolist to induce an efficient equilibrium because an efficient equilibrium requires that only a few traders acquire the advantage. Interestingly, as we increase θ_2, say to $\theta_2 = 0.95$,



(a) $\omega = 0.7$.

(b) $\omega = 0.9$.

FIGURE X

This figure plots the value of θ_P when θ_2 changes and $\sigma/\Delta = 0.6$.

the resulting equilibrium level of participation becomes efficient again. So, when θ_2 is high enough, segmenting agents across two venues can alleviate how "rigged" agents perceive financial markets to be, since those high θ agents are not extracting rents from the low θ agents to the extent that they would have with only one trading venue.

In Panel (b) of Figure X, we pick $\sigma/\Delta = 0.6$ and $\omega = 0.9$. As we increase θ_2 from 0.5 to 0.88, the equilibrium participation threshold θ_P is increasing, implying a larger level of welfare loss. However, as we keep increasing θ_2 from 0.88 to 1, the equilibrium participation threshold θ_P decreases, therefore improving welfare.

Overall, these examples show that segmenting traders across trading venues may increase or decrease equilibrium welfare.

5.2 Giving the Monopolist Less Power through Competition

Our baseline model featured a monopolist selling a trading advantage to prospective traders. We now discuss the potential impact of adding competition in the sales of trading advantages. Specifically, we consider the case where there are at least two sellers engaging in Bertrand competition. Since the marginal cost of the trading advantage is assumed to be zero, the price charged by all sellers is zero in equilibrium. All agents, if they agree to participate, acquire the advantage for free. Does this imply that we have an full-participation equilibrium? The answer lies in equation (12). If $\theta_A = \theta_P = \underline{\theta}$, equation (12) implies that:

$$\frac{-(1-\omega)\Delta + \sigma}{\omega\Delta \cdot \frac{\int_{\underline{\theta}}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}}\theta dF(\theta)} - (1-\omega)\Delta + \sigma} \leq \underline{\theta}. \tag{26}$$

Equivalently,

$$\frac{\sigma}{\Delta} \leq 1 - \omega + \omega\frac{\underline{\theta}}{1-\underline{\theta}}\frac{1-\mu}{\mu} \equiv k_2. \tag{27}$$

If $\sigma/\Delta \leq k_2$, seller competition can ensure that all agents participate in equilibrium, in which case no welfare loss is incurred. However, if $\sigma/\Delta > k_2$, seller competition cannot induce a full-participation equilibrium. Proposition 2 stated that the equilibrium is efficient if $\sigma/\Delta < k$. What is interesting is the fact that k can be bigger than k_2. In that case, when $k_2 < \sigma/\Delta < k$, a monopolist can induce an efficient equilibrium, while competing sellers cannot. For example, when F is a uniform over $[\underline{\theta}, \overline{\theta}]$ with $0 < \underline{\theta} \leq \frac{\overline{\theta}}{2}$, k is given by equation (18). Comparing k and k_2, we find that $k > k_2$ when $(1-\underline{\theta})(\overline{\theta}+\underline{\theta}) > (\overline{\theta}-\underline{\theta})\omega$.

Overall, a market with a monopolist can result in more efficient trader participation than a market with competing sellers. The simple intuition is that the trading advantage can be socially harmful, and competition can induce so many traders to acquire the advantage that it pushes traders with low θ out of financial markets. In subsection 5.3, we however show that giving the monopolist even more power through the possibility of price discrimination can lead to a less efficient level of market participation.

5.3 Giving the Monopolist More Power through Price Discrimination

In subsection 5.2, we showed that reducing the monopolist's power by adding a second seller can make the market participation more inefficient in equilibrium. In this subsection, we show that giving the monopolist even more power can also backfire. Specifically, what we have analyzed so far is a case where the monopolist cannot discriminate traders — the monopolist quotes a unique price to all agents. In this subsection, we consider the possibility of price discrimination by allowing the monopolist to observe a trader's type before quoting a price and thereby engage in first-degree price discrimination.

We now assume that the monopolist sells the trading advantage to all traders whose type is above θ_A, but unlike in the baseline model, the price quoted to each trader θ_i equals its willingness to pay $p_i = \sigma\eta\theta_i$ for all $\theta_i \geq \theta_A$. The monopolist's profit is then: $\sigma\eta \int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)$. Expressing the profit as a function of θ_P and taking the derivative with respect to θ_P yields:

$$\frac{\partial[\sigma\eta \int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)]}{\partial\theta_P} = \sigma \left(\frac{\partial\eta}{\partial\theta_P} \int_{\theta_A}^{\overline{\theta}} \theta dF(\theta) - \eta\theta_A f(\theta_A) \frac{\partial\theta_A}{\partial\theta_P} \right). \tag{28}$$

Setting this expression to be strictly positive, and substituting $\frac{\partial\eta}{\partial\theta_P}$ with (4) and $\frac{\partial\theta_A}{\partial\theta_P}$ with (A9) yields:

$$(1-\mu) \left(\frac{\omega}{(1-\underline{\theta})^2}(1-\mu) - \underline{\theta} f(\underline{\theta}) \right) > \frac{\sigma}{\Delta} f(\underline{\theta})(\mu - \underline{\theta}) \int_{\theta_1}^{\overline{\theta}} \theta dF(\theta). \tag{29}$$

To understand this condition, we suppose that $\underline{\theta} = 0$ (for example, when F is the standard uniform distribution). Condition (29) then reduces to:

$$\omega(1-\mu)^2 > f(\underline{\theta})(1-\omega)\mu^2. \tag{30}$$

Equivalently:

$$\omega > \frac{f(\underline{\theta})\mu^2}{f(\underline{\theta})\mu^2 + (1-\mu)^2}. \tag{31}$$

The above condition is independent of σ/Δ. When F is the standard uniform distribution, the above condition can be further simplified to $\omega > \frac{1}{2}$. In this case, regardless of σ/Δ, as long as $\omega > \frac{1}{2}$, a discriminating monopolist would induce an inefficient equilibrium. In contrast, as Proposition 2 shows, even if $\omega > \frac{1}{2}$, when σ/Δ is small enough a non-discriminating monopolist would optimally induce an efficient equilibrium.

6 Conclusion

Many financial transactions are of a fixed-sum nature, meaning that any improvement in the terms of trade for one party comes at the expense of another party. We model how the sales of trading advantages (e.g., data or collocation services) by a monopolist (e.g., data provider or securities exchange) affects traders' endogenous participation in a market and vice-versa. We show how the magnitude of the externality that a trading advantage imposes on counterparties impacts financial market conditions. While the monopolist accounts for how its pricing strategy affects market participation, we show that its optimal pricing strategy can result in socially excessive sales of goods and services that benefit a subset of market participants, leaving a fraction of the potential gains from trade unrealized due to the non-participation of their disadvantaged counterparties. We show how different types of market participants are heterogenously impacted by this behavior and we study how changing the market structure may solve or worsen the problem of the excessive sales of trading advantages.

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A Proofs of Formal Results

Proof of Lemma 1: Since $\frac{\partial p}{\partial \theta_P} < 0$, it implies that an increase in p leads to a decrease in θ_P. Since $\frac{\partial \theta_A}{\partial \theta_P} > 0$, it implies that θ_A is also lower, meaning more traders acquire the trading advantage hen the price is higher. $\square$

Proof of Proposition 1: We argue by contradiction. Suppose that $\theta_A < \theta_0$. Given the monotonically of H, it implies that $H(\theta_A) < 1$. There are two cases. First, the equilibrium $\theta_P = \underline{\theta}$. Recall from equation (14), the participation constraint implies that there is a lower bound on θ_A. So marginally increasing θ_A is still feasible (i.e., the participation constraint is satisfied). Given that $H(\theta_A) < 1$, marginally increasing θ_A leads to a higher profit. This is a contradiction to the optimality of θ_A.

Second, the equilibrium $\theta_P > \underline{\theta}$. In this case, θ_A is pinned down by equation equation (13). We first show that $\frac{\partial \theta_A(\theta_P)}{\partial \theta_P}|_{\theta_P} \leq 0$. We argue by contradiction. Suppose $\frac{\partial \theta_A}{\partial \theta_P}|_{\theta_P} > 0$. Then marginally decreasing θ_P would decrease θ_A as well. Since decreasing θ_P always increases η. So in this case, not only agents are willing to pay more for the trading advantage, the monopolist also sells to more traders. The monopolist can enjoy a higher profit by decreasing θ_P, contradicted with the optimality of θ_P. Thus, $\frac{\partial \theta_A(\theta_P)}{\partial \theta_P}|_{\theta_P} \leq 0$. Returning to the proof of Proposition 1, together with $\frac{\partial \eta}{\partial \theta_P} \leq 0$, it then follows that $\frac{\partial \eta}{\partial \theta_A} = \frac{\partial \eta}{\partial \theta_P}\frac{\partial \theta_P}{\partial \theta_A} \geq 0$. Thus, the elasticity $\frac{\partial \eta/\eta}{\partial \theta_A/\theta_A} \geq 0$. From the monopolist's FOC $H(\theta_A) = 1+\epsilon$. If the elasticity $\epsilon \geq 0$, then the optimal θ_A is higher than θ_0. This is a contradiction. $\square$

Proof of Proposition 2: For full participation to be an equilibrium response, the fraction of traders who acquire the trading advantage cannot be too large. Specifically, equation (12) implies that for $\theta_P = \underline{\theta}$ we need:

$$\frac{-(1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}}{(\omega\Delta) \cdot \frac{\int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}} \leq \underline{\theta}. \tag{A1}$$

Rearranging (A1) yields:

$$-(1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)} \leq \frac{\underline{\theta}}{1-\underline{\theta}} (\omega\Delta) \cdot \frac{\int_{\underline{\theta}}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)}. \tag{A2}$$

Multiplying both sides by $\int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)$ implies that:

$$-(1-\omega)\Delta \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) + \sigma \cdot \int_{\theta_A}^{\overline{\theta}} \theta dF(\theta) \leq \frac{\underline{\theta}}{1-\underline{\theta}} (\omega\Delta) \cdot \int_{\underline{\theta}}^{\overline{\theta}} (1-\theta) dF(\theta). \tag{A3}$$

Equivalently:

$$\sigma \cdot \int_{\theta_A}^{\overline{\theta}} \theta dF(\theta) \leq \frac{\underline{\theta}}{1-\underline{\theta}} (\omega\Delta) \cdot (1-\mu) + (1-\omega)\Delta\mu. \tag{A4}$$

Recall that we are looking for the condition such that the equilibrium features $\theta_A = \theta_0$ and $\theta_P = \underline{\theta}$, thus:

$$\sigma \cdot \int_{\theta_0}^{\overline{\theta}} \theta dF(\theta) \leq \frac{\underline{\theta}}{1-\underline{\theta}} (\omega\Delta) \cdot (1-\mu) + (1-\omega)\Delta\mu, \tag{A5}$$

which can be rearranged as:

$$\frac{\sigma}{\Delta} \leq \frac{\frac{\underline{\theta}\omega}{1-\underline{\theta}} + \left(1 - \frac{\omega}{1-\underline{\theta}}\right)\mu}{\int_{\theta_0}^{\overline{\theta}} \theta dF(\theta)}. \tag{A6}$$

$\square$

Proof of Proposition 3: Recall that the participation constraint (12) when $\theta_P > \underline{\theta}$ is given by:

$$\frac{\sigma}{\Delta}\int_{\theta_A}^{\overline{\theta}} \theta \, dF(\theta) = \frac{\theta_P}{1-\theta_P}\omega \int_{\theta_P}^{\overline{\theta}} (1-\theta)\, dF(\theta) + (1-\omega)\int_{\theta_P}^{\overline{\theta}} \theta \, dF(\theta). \tag{A7}$$

Taking $\frac{\partial}{\partial \theta_P}$ on both sides of (A7) yields:

$$-\frac{\sigma}{\Delta}\theta_A f(\theta_A)\frac{\partial \theta_A}{\partial \theta_P} = \frac{\omega}{(1-\theta_P)^2}\int_{\theta_P}^{\overline{\theta}} (1-\theta)\, dF(\theta) - \theta_P f(\theta_P). \tag{A8}$$

Evaluating the above equation at $\theta_P = \underline{\theta}$, it follows that:

$$-\frac{\sigma}{\Delta}\theta_1 f(\theta_1)\frac{\partial \theta_A}{\partial \theta_P}\Big|_{\underline{\theta}} = \frac{\omega}{(1-\underline{\theta})^2}(1-\mu) - \underline{\theta} f(\underline{\theta}). \tag{A9}$$

Now returning to the monopolist's profit as a function of θ_P and taking the derivative with respect to θ_P and evaluating at $\theta_P = \underline{\theta}$, we get:

$$\frac{\partial[\sigma\eta\theta_A(1-F(\theta_A)]}{\partial \theta_P}\Big|_{\underline{\theta}} = \sigma\left(\frac{\partial \eta}{\partial \theta_P}\theta_A(1-F(\theta_A)) + \eta(1-F(\theta_A) - f(\theta_A)\theta_A)\frac{\partial \theta_A}{\partial \theta_P}\right)\Big|_{\underline{\theta}}. \tag{A10}$$

A sufficient condition for the equilibrium to be inefficient is that the monopolist's profit function is locally increasing at $\theta_P = \underline{\theta}$. The right-hand side of the above equation is strictly positive whenever this condition is satisfied:

$$\left(\frac{\partial \eta}{\partial \theta_P}\theta_A + \eta(1-H(\theta_A))\frac{\partial \theta_A}{\partial \theta_P}\right)\Big|_{\underline{\theta}} > 0. \tag{A11}$$

Substituting $\frac{\partial \eta}{\partial \theta_P}$ from equation (4), the above condition becomes:

$$\frac{-f(\theta_P) \cdot \int_{\theta_P}^{\overline{\theta}} (\theta - \theta_P) dF(\theta)}{\left[\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)\right]^2} \theta_A (1 - F(\theta_A))\Big|_{\underline{\theta}} + \eta(1 - F(\theta_A) - f(\theta_A)\theta_A) \frac{\partial \theta_A}{\partial \theta_P}\Big|_{\underline{\theta}} > 0. \tag{A12}$$

Plugging $\theta_P = \underline{\theta}$ and substituting $\frac{\partial \theta_A}{\partial \theta_P}$ from equation (A9) yields:

$$\frac{-f(\underline{\theta})(\mu - \underline{\theta})}{\mu^2} \theta_1 (1 - F(\theta_1)) + \frac{1-\mu}{\mu}(1 - F(\theta_1) - f(\theta_1)\theta_1) \frac{\frac{\omega}{(1-\underline{\theta})^2}(1-\mu) - \underline{\theta} f(\underline{\theta})}{-\frac{\sigma}{\Delta}\theta_1 f(\theta_1)} > 0. \tag{A13}$$

Rearranging, we obtain the following condition:

$$\frac{\sigma}{\Delta} < \frac{\mu(1-\mu)\left(\frac{\omega}{(1-\underline{\theta})^2}(1-\mu) - \underline{\theta} f(\underline{\theta})\right)}{f(\underline{\theta})(\mu - \underline{\theta})} \frac{H(\theta_1) - 1}{\theta_1^2 f(\theta_1)}, \tag{A14}$$

which is condition (23) in the main text. □

B Additional Analysis Omitted from the Main Text

B.1 Deriving the Implied Price when $\theta_A = \theta_P$ in the Equilibrium of the Traders' Subgame

The participation condition implies that:

$$\theta_P = \max\left(\underline{\theta}, \frac{p - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}}{(\omega\Delta + \sigma) \cdot \frac{\int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)}}\right), \tag{B1}$$

which means that:

$$\frac{p-(1-\omega)\Delta+\sigma\cdot\frac{\int_{\theta_A}^{\overline{\theta}}\theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}}\theta dF(\theta)}}{(\omega\Delta+\sigma)\cdot\frac{\int_{\theta_P}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_P}^{\overline{\theta}}\theta dF(\theta)}-(1-\omega)\Delta+\sigma\cdot\frac{\int_{\theta_A}^{\overline{\theta}}\theta dF(\theta)}{\int_{\theta_P}^{\overline{\theta}}\theta dF(\theta)}}\leq\theta_P, \tag{B2}$$

and the weak inequality becomes an equality when $\theta_P \neq \underline{\theta}$. Let's focus on the equality case, i.e., $\theta_P > \underline{\theta}$. Rearranging equation (B2) and setting $\theta_A = \theta_P$ implies that the price charged by the monopolist must satisfy the following condition:

$$p=\theta_P\left((\omega\Delta+\sigma)\cdot\frac{\int_{\theta_P}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_P}^{\overline{\theta}}\theta dF(\theta)}\right)+(1-\theta_P)\left((1-\omega)\Delta-\sigma\right). \tag{B3}$$

We also need the last participating agent, whose $\theta_i = \theta_A$, to be willing to pay p to acquire the advantage, that is, $p \leq \eta\theta_A\sigma = \eta\theta_P\sigma$ which is equivalent to imposing that:

$$\theta_P\left(\omega\Delta\cdot\frac{\int_{\theta_P}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_P}^{\overline{\theta}}\theta dF(\theta)}\right)\leq(1-\theta_P)\left(\sigma-(1-\omega)\Delta\right). \tag{B4}$$

Solving condition (B4) leads to a range of θ_P. For any θ_P in this range, there exists a price p, given by equation (B3), such that there exists an equilibrium in which $\theta_A = \theta_P$. We remark that when condition (B4) is slack, the marginal type of buyer, i.e., θ_A, can enjoy a strictly positive rent.

B.2 Sufficient Condition for an Inefficient Equilibrium when $\theta_A = \theta_P$

Analogs to Proposition 3, we analyze the case of $\theta_A = \theta_P$ to derive a sufficient condition for an inefficient equilibrium. Recall that the price charged by the monopolist in this case is given by:

$$p = \theta_P \left((\omega\Delta + \sigma) \cdot \frac{\int_{\theta_P}^{\bar{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\bar{\theta}} \theta dF(\theta)} \right) - (1-\theta_P)\left(-(1-\omega)\Delta + \sigma\right), \tag{B5}$$

and the monopolist's profit is given by $p(1 - F(\theta_P))$.

A sufficient condition for the equilibrium to be inefficient is that the monopolist's profit function is locally increasing at $\theta_P = \underline{\theta}$. Taking $\frac{\partial}{\partial \theta_P}$ on the monopolist's profit function and setting it to be positive yields:

$$\left((1 - F(\theta_P)) \frac{\partial p}{\partial \theta_P} - f(\theta_P) p \right) \Bigg|_{\theta_P = \underline{\theta}} > 0.$$

Equivalently:

$$\frac{\partial p}{\partial \theta_P} \Bigg|_{\theta_P = \underline{\theta}} > f(\underline{\theta}) p.$$

Note that:

$$\begin{aligned} \frac{\partial p}{\partial \theta_P} \Bigg|_{\theta_P = \underline{\theta}} &= (\omega\Delta + \sigma) \frac{1-\mu}{\mu} + \underline{\theta}(\omega\Delta + \sigma) \frac{-f(\underline{\theta})(\mu - \underline{\theta})}{\mu^2} - (1-\omega)\Delta + \sigma, \\ p \Bigg|_{\theta_P = \underline{\theta}} &= \underline{\theta} \left((\omega\Delta + \sigma) \cdot \frac{1-\mu}{\mu} \right) - (1 - \underline{\theta}) \left(-(1-\omega)\Delta + \sigma \right). \end{aligned}$$

Notice that when $p \Big|_{\theta_P = \underline{\theta}} < 0$, then the monopolist would never induce $\theta_P = \underline{\theta}$ because it implies a negative price and a negative profit. We assume $p \Big|_{\theta_P = \underline{\theta}} \geq 0$, which is to say that σ / Δ is sufficient large. Then the condition becomes:

$$(\omega\Delta + \sigma) \left(\frac{1-\mu}{\mu} - \frac{\underline{\theta} f(\underline{\theta})(\mu - \underline{\theta})}{\mu^2} - \underline{\theta} f(\underline{\theta}) \frac{1-\mu}{\mu} \right) + (-(1-\omega)\Delta + \sigma)(1 + (1-\underline{\theta}) f(\underline{\theta})) > 0.$$

If the coefficient on σ is positive, i.e.:

$$\frac{1-\mu}{\mu} - \frac{\underline{\theta} f(\underline{\theta})(\mu - \underline{\theta})}{\mu^2} - \underline{\theta} f(\underline{\theta}) \frac{1-\mu}{\mu} + 1 + (1-\underline{\theta}) f(\underline{\theta}) > 0,$$

then the condition is σ/Δ is sufficiently large. Overall if the above condition holds and σ/Δ is sufficiently large, then the equilibrium is necessarily inefficient.

B.3 Sufficient Condition for an Inefficient Equilibrium when F is a Standard Uniform Distribution

We now assume that F, the distribution of θ_i, is a standard uniform distribution. In this parameterization, condition (23) reduces to:

$$4\left(1 - \frac{1-\omega}{\sigma/\Delta}\right)\left(1 - \sqrt{1 - \frac{1-\omega}{\sigma/\Delta}}\right) < \left(2\sqrt{1 - \frac{1-\omega}{\sigma/\Delta}} - 1\right)\frac{\omega}{\sigma/\Delta}, \tag{B6}$$

which can be rearranged as:

$$g(t) \equiv 6t^3 - 5t^2 - 2\left(1 - \frac{1}{\sigma/\Delta}\right)t + \left(1 - \frac{1}{\sigma/\Delta}\right) > 0, \tag{B7}$$

where $t = \sqrt{1 - \frac{1-\omega}{\sigma/\Delta}}$. We can prove that $g(t)$ crosses zero at exactly one point on the interval $(1/2, 1)$. So there exists a t_0, such that the above inequality becomes $t > t_0$, equivalently:

$$\frac{\sigma}{\Delta} > \frac{1-\omega}{1-t_0^2}, \tag{B8}$$

or:

$$\omega > 1 - \frac{\sigma}{\Delta}(1 - t_0^2). \tag{B9}$$

But note that t_0 is also a function of $\frac{\sigma}{\Delta}$. If ω is sufficiently large and $\sigma/\Delta > k = \frac{4(1-\omega)}{3}$, then the monopolist optimally induces a partial-participation equilibrium, in which case the arms sales are excessive from the social planner's perspective.

B.4 Analysis of the Two Trading Venues

We first consider a situation where all agents above θ_2 participate and purchase the trading advantage. We need to determine θ_A and θ_P for the agents below θ_2. We compute the respective venue-specific η, denoted by η_1 and η_2:

$$\eta_1 = \frac{\int_{\theta_P}^{\theta_2}(1-\theta)dF(\theta)}{\int_{\theta_P}^{\theta_2}\theta dF(\theta)}, \tag{B10}$$

$$\eta_2 = \frac{\int_{\theta_2}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_2}^{\overline{\theta}}\theta dF(\theta)}. \tag{B11}$$

At $t=0$, each agent $\theta_i < \theta_2$ must decide whether to enter the market. If planning to pursue a strategy $a_i \in \{0,1\}$ upon entering, agent i's expected payoff from entering the market is:

$$\begin{aligned} & (1-\theta_i)\mathbb{E}[(1-\omega)\Delta - \sigma a_{-i}] + \theta_i\eta_1[\omega\Delta + \sigma a_i] - pa_i \\ = \;& (1-\theta_i)\left[(1-\omega)\Delta - \sigma\cdot\frac{\int_{\theta_A}^{\theta_2}\theta dF(\theta)}{\int_{\theta_P}^{\theta_2}\theta dF(\theta)}\right] + \theta_i(\omega\Delta + \sigma a_i)\cdot\frac{\int_{\theta_P}^{\theta_2}(1-\theta)dF(\theta)}{\int_{\theta_P}^{\theta_2}\theta dF(\theta)} - pa_i. \end{aligned} \tag{B12}$$

Thus, agent i finds it optimal to participate in the market whenever this expected payoff is greater than zero, or equivalently:

$$\begin{aligned}
\theta_i \quad & \left[(\omega\Delta + \sigma a_i) \cdot \frac{\int_{\theta_P}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\theta_2} \theta dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)}\right] \\
\geq \quad & pa_i - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\theta_2} \theta dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)}. \qquad \text{(B13)}
\end{aligned}$$

Then:

$$\theta_P = \max\left(\underline{\theta}, \frac{-(1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\theta_2} \theta dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)}}{(\omega\Delta) \cdot \frac{\int_{\theta_P}^{\theta_2} (1-\theta) dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_A}^{\theta_2} \theta dF(\theta)}{\int_{\theta_P}^{\theta_2} \theta dF(\theta)}}\right), \qquad \text{(B14)}$$

and $p \leq \theta_A \eta_1 \sigma$. Similarly, to sustain an equilibrium for all agents above θ_2 participate and purchase the advantage, we require that:

$$\frac{p - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)}}{(\omega\Delta + \sigma) \cdot \frac{\int_{\theta_2}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma \cdot \frac{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)}} \leq \theta_2, \qquad \text{(B15)}$$

or equivalently:

$$\frac{p - (1-\omega)\Delta + \sigma}{(\omega\Delta + \sigma) \cdot \frac{\int_{\theta_2}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)} - (1-\omega)\Delta + \sigma} \leq \theta_2. \qquad \text{(B16)}$$

Thus, there is an upper bound in the price that the monopolist can charge:

$$p \leq \theta_2(\omega\Delta + \sigma) \cdot \frac{\int_{\theta_2}^{\overline{\theta}} (1-\theta) dF(\theta)}{\int_{\theta_2}^{\overline{\theta}} \theta dF(\theta)} - (1-\theta_2)\left(-(1-\omega)\Delta + \sigma\right). \qquad \text{(B17)}$$

We now consider the situation where all agents above θ_2 participate but do not purchase the advantage, which requires:

$$\frac{-(1-\omega)\Delta}{(\omega\Delta)\cdot\frac{\int_{\theta_2}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_2}^{\overline{\theta}}\theta dF(\theta)}-(1-\omega)\Delta}\leq\theta_2, \tag{B18}$$

and:

$$p>\sigma\eta_2=\sigma\frac{\int_{\theta_2}^{\overline{\theta}}(1-\theta)dF(\theta)}{\int_{\theta_2}^{\overline{\theta}}\theta dF(\theta)}. \tag{B19}$$

The monopolist's profit is then given by:

$$p(F(\theta_2)-F(\theta_A)). \tag{B20}$$

From here, we can conduct numerical analysis and solve for the monopolist's optimal pricing strategy.