Search in Over-the-Counter Markets

Chaojun Wang*

The Wharton School, University of Pennsylvania

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Abstract

In a tractable model of over-the-counter markets where each investor can arbitrarily distribute her search capacity across other investors, the holdings of an asset are endogenously concentrated among a subgroup of investors. Investors who are more likely to hold the asset search among those less likely to hold it, and vice versa. When directed search is allowed in those existing random search models that endogenize intermediation, intermediation ceases to be an equilibrium outcome and instead the concentration of asset holdings arises endogenously. My model explains the persistent imbalance between banks' funding needs, and contributes novel predictions of asset concentration across investors and asymmetric price dispersion.

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1 Introduction

Investors in over-the-counter markets gear their search toward certain counterparties, not knowing exactly who wishes to buy or sell. In existing models, investors either search randomly or observe exactly who is willing to trade at what terms. This paper offers a simple and tractable model that allows investors to arbitrarily direct their search without observing prices, inventories, or any other investor characteristic. A novel pattern of *asset concentration* arises, whereby the *holdings* of the asset are concentrated into a subset of investors. When directed search is allowed in recent random search models that endogenize intermediation, intermediation ceases to be an equilibrium outcome and asset concentration arises instead. The endogenous asset concentration explains the persistent imbalance between banks' funding needs in the interbank market that cannot be explained by existing theories of intermediation.

The model starts with the random search framework of Duffie, Gârleanu, and Pedersen (2005) and allows investors to allocate their search capacity arbitrarily across other investors. A continuum of infinitely lived investors search among each other to trade an asset of fixed supply. Each investor i has a total "search capacity" that she can choose to distribute across other investors. In particular, investor i can concentrate all her search capacity toward a selected sub-group of investors with a positive measure. Otherwise, the model is identical to that of Duffie et al. (2005). Investors are risk-neutral, can hold up to 1 unit of the asset and cannot short sell. An investor can have a high or low preference for the asset, and the preference type switches over time. Upon meeting, two investors negotiate the terms of trade à la Nash Bargaining.

In equilibrium, investors endogenously split into two groups. Each investor does not search among investors within her own group and searches among all the investors in the other group uniformly at random. I label the two groups as Group B (buyer) and Group S (seller). Whenever an B-investor meets an S-investor with the opposite asset position, *(i)* if the two investors have the opposite preferences, the investor with the high preference buys from the investor with the low preference whenever feasible (fundamental trade); *(ii)* if the two have the same preference, the B-investor buys the asset from the S-investor whenever feasible (asset-concentration trade). In steady state, asset holdings are disproportionally concentrated among the B-investors. Concentrated asset holdings make it optimal for the investors to search across groups and not within groups to increase the likelihood of meeting another investor with the opposite asset position. Conversely, when a B-investor meets an S-investor with the same preference, it is optimal for the B-investor to hold the asset anticipating that she is more likely to meet another investor without the asset in the future than the S-investor. The combination of asset concentration and two-sided directed search thus constitutes an equilibrium.

Recent random search models explain intermediation by letting investors endogenously acquire special technologies to serve as intermediaries, such as superior search technology (Farboodi, Jarosch, and Shimer, 2022) or negotiation skill (Farboodi, Jarosch, Menzio, and Wiriadinata, 2019). Further, Farboodi et al. (2022) shows that in any equilibrium, investors must acquire heterogeneous speed, with the faster investors acting as intermediaries. When investors can direct their search in those models, intermediation likely ceases to be an equilibrium and asset concentration arises instead. The benefit of intermediation trades is the key force in Farboodi et al. (2022) that pushes against homogenous speed. This argument no longer holds when investors are allowed to direct their search. In equilibrium, investors follow a different trading strategy—asset concentration—and do strictly better than in the random search benchmark even in a homogeneous-speed economy. An investor can no longer improve her payoff upon her baseline homogenous-speed value by marginally raising or reducing her speed and making intermediation trades.

I present three empirical implications. First, my model offers a simple explanation to the persistent imbalance between banks' funding surpluses and shortages documented by Craig and Ma (2022), which cannot be explained by any existing models of intermediation. Endogenous fund concentration helps investors mitigate search friction by making it easier for them to locate funding supply and demand. Second, the model predicts that the holdings of an OTC-traded asset are disproportionally concentrated among a subgroup of investors. Asset concentration runs against the benefit of diversification and might cause systemic risk, and cannot be explained by larger transaction costs in OTC markets. Third, the model predicts greater price dispersion when investors with larger holdings of an asset buy from those with smaller holdings of the asset than the other way round. This prediction of asymmetric price dispersion cannot be readily derived from the standard theories of random search, dealer market power, or inventory cost, all of which imply a symmetric price dispersion between buying and selling.

Literature. This paper is most closely related to Chang and Zhang (2021), who to my knowledge are the first to analyze how investors match and trade when they do not observe others' trading needs. In their model, exactly one-half of investors match pairwise with the other half in each of two periods. As a consequence of this rigid setup, its outcome is fragile: Intermediation arises if the asset supply is $1/2$, yet is strictly dominated by asset concentration at any other asset supply and if the game is extended to three periods (Appendix B). I circumvent this rigidity by using the continuous-time search framework instead, to obtain a model that is more tractable and easier to generalize.

This paper is related to three approaches to modeling OTC markets: random search[1], competitive search,[2] and networks.[3] Random search is the most tractable approach to analyze aggregate quantities such as asset prices and trading volume, while the network approach generates the most precise description of strategic interactions between investors (Weill, 2020). Competitive search assumes public posting of binding prices, which describes how limit orders are posted on a transparent central limit order book and perhaps is less realistic when describing opaque OTC markets. In addition, competitive search typically relies on technological or capacity constraints to motivate matching delay, because an investor observes exactly who is willing to trade at what terms through the publicly posted prices. My paper lies in the middle of these three approaches. While it remains as tractable as Duffie et al. (2005), I combine the opacity of OTC markets with investors' ability to choose who they wish to trade with.

I make three contributions: *(i)* My paper proposes an approach of modeling search that encompasses the benefits of the random search, the competitive search, and the network approaches. *(ii)* My model predicts a novel pattern of asset concentration, explains the persistent imbalance of banks' funding needs, and predicts asymmetric price dispersion. More implications are possible under this paper's framework. *(iii)* My results reveal some limit of random search models in deriving intermediation.

The paper is organized as follows. Section 2 sets up a canonical model. Section 3 solves for

[1]Examples include Atkeson, Eisfeldt, and Weill (2015), Bethune, Sultanum, and Trachter (2021), Duffie et al. (2005), Dugast, Üslü, and Weill (2022), Hugonnier, Lester, and Weill (2020), Li, Rocheteau, and Weill (2012), Maurin (2022), Praz (2014), Tsoy (2021), Vayanos and Weill (2008) among many others. Weill (2020) reviews the literature of search models in OTC markets.

[2]Examples include Gabrovski and Kospentaris (2021), Lester, Rocheteau, and Weill (2015), Williams (2021). Wright, Kircher, Julien, and Guerrieri (2021) reviews the literature of competitive search.

[3]Examples include Babus (2016), Babus and Hu (2017), Babus and Kondor (2018), Colliard, Foucault, and Hoffmann (2021), Colliard and Demange (2021), Eisfeldt, Herskovic, Rajan, and Siriwardane (2018), Farboodi (2021), Glode and Opp (2016), Gofman (2014, 2017), Manea (2018), Wang (2016).

investor' equilibrium search and trading strategies. Section 4 derives the model's theoretical and empirical implications. Section 5 concludes.

2 Model

This section sets up the canonical model which starts with the random search framework of Duffie et al. (2005) and allows every investor to distribute her search capacity arbitrarily across other investors.

Time is continuous and runs forever. Infinitely lived investors form a continuum set $[0,1]$ equipped with the Lebesgue measure. Each investor $i \in [0,1]$ has a fixed "search capacity" of λ. Instead of randomly searching among all the other investors at the same intensity λ, investor i can distribute her search capacity across other investors j following any absolutely continuous distribution with a density function $f(i,j)$ such that

$$f \text{ is jointly measurable in } (i,j) \text{ and } \int_{j\in[0,1]} f(i,j)\mathrm{d}j = \lambda, \ \forall i \in [0,1].$$

Every investor i chooses her "search density" function $f(i,j)$ at time $t = 0$ before the realization of any random event. In particular, investor i can concentrate all her search capacity toward a specific measurable sub-group of investors $\operatorname{supp} f(i,\cdot) \subseteq [0,1]$ with a positive measure. Since j also contacts i at the "search density" $f(j,i)$, then i and j meet each other at the total "meeting density" $f(i,j) + f(j,i)$. Duffie, Qiao, and Sun (2018) provide a general guarantee that the directed search model exists and that the exact law of large numbers holds in the model.

I illustrate the search density function f with two examples when $\lambda = 1$.

Example 1: Random search corresponds to the uniform search density function $f(i,\cdot) =$

Unif$[0,1]$ for every $i \in [0,1]$.

Example 2: Investors are partitioned into two measurable sub-groups B and S with positive measures. Each investor in group B searches among investors in the other group S and not within her own group, and vice versa. In terms of search density, $f(i,j) = \mathbb{1}_{\{i\in B, j\in S\}}/|S| + \mathbb{1}_{\{i\in S, j\in B\}}/|B|$ where $|B|$ and $|S|$ are the measures of the subsets B and S. This is two-sided random search, which will turn out to be the equilibrium search pattern.

The remaining setup of the canonical model is identical to that of Duffie et al. (2005). All investors are risk-neutral, with time preferences determined by a constant discount rate $r > 0$. A single nonstorable consumption good is used as a numeraire. Investors have access to a risk-free bank account with interest rate r. A divisible asset pays dividends at the constant rate of 1 unit of consumption good per year. The asset can be traded only when an investor finds another investor according to the directed search model described above.

Investors can hold at most 1 unit of the asset and cannot short-sell. Because investors have linear utility, one can restrict attention to equilibria in which, at any given time and state of the world, an investor holds either 0 or 1 unit of the asset. An investor is characterized by whether he owns the asset or not, and by a preference type that is (h)igh or (l)ow. A low-type investor, when owning the asset, has a holding cost of δ per unit of time; a high-type investor has no such holding cost. In the canonical model, I assume that the asset supply is $A = 1/2$, and that each investor's preference type independently switches between high and low with the same intensity so that the steady-state measure of high-preference investors is equal to the asset supply $1/2$. In a frictionless environment, investors with the high preference hold the asset, while those with the low preference do not. To save one parameter, I normalize the intensity of preference switching to 1 without loss of generality.

Equilibrium concept.

I let $x_{i,t,a}$ denote the steady-state probability that an investor i has preference type $t \in \{h, l\}$ and asset position $a \in \{0, 1\}$. The probabilities $x_{i,t,a}$ add up to 1,

$$\sum_{t \in \{h,l\}, a \in \{0,1\}} x_{i,t,a} = 1. \tag{1}$$

Because the total fraction of investors owning an asset is 1/2,

$$\int_{i \in [0,1]} (x_{i,h,1} + x_{i,l,1}) \mathrm{d}i = \frac{1}{2}. \tag{2}$$

I let $\alpha_{i,t,a}^{j,t',1-a} = \alpha_{j,t',1-a}^{i,t,a}$ denote the endogenous probability that an investor i with preference type t and asset position a trades when she meets an investor j with preference type t' and the opposite asset position $1 - a$. Then the steady-state type distribution $x_{i,t,a}$ of i satisfies:

$$\begin{aligned} 0 = &\int_{j \in [0,1]} (f(i,j) + f(j,i)) \left(-x_{i,t,a} \sum_{t' \in \{h,l\}} x_{j,t',1-a}\, \alpha_{i,t,a}^{j,t',1-a} + x_{i,t,1-a} \sum_{t' \in \{h,l\}} x_{j,t',a}\, \alpha_{i,t,1-a}^{j,t',a} \right) \mathrm{d}j \\ &+ (x_{i,\sim t,a} - x_{i,t,a}), \end{aligned} \tag{3}$$

where $\sim t$ denotes the preference type that is opposite to t. In the equation above, $f(i,j) + f(j,i)$ is the meeting density between i and j. Conditional on a meeting, a trade is feasible between i and j only if the two investors have the opposite asset positions. The term $x_{i,t,a} \sum_{t' \in \{h,l\}} x_{j,t',1-a}\, \alpha_{i,t,a}^{j,t',1-a}$ is the probability that i with preference type t and asset position a trades with j with the opposite asset position $1 - a$, in which case the asset position of i changes to $1 - a$. Likewise, the term $x_{i,t,1-a} \sum_{t' \in \{h,l\}} x_{j,t',a}\, \alpha_{i,t,1-a}^{j,t',a}$ is the probability that i with preference type t and asset position $1-a$ trades with j with the opposite asset position a, in which case the asset position of i changes to a. Finally, $x_{i,\sim t,a} - x_{i,t,a}$ is the intensity of the net flow into type (t, a) of investor i caused by preference switching.

Next, I turn to the value function. When investors i and j trade, I let $P_{i,t,a}^{j,t',1-a}$ denote the endogenous transfer of the numeraire good from investor j of type $(t', 1-a)$ to investor i of type (t, a). The value function of investor i satisfies the HJB equation

$$\begin{aligned} rV_{i,t,a} = &\int_{j\in[0,1]} (f(i,j) + f(j,i)) \sum_{t'\in\{h,l\}} x_{j,t',1-a}\, \alpha_{i,t,a}^{j,t',1-a} \left(V_{i,t,1-a} - V_{i,t,a} + P_{i,t,a}^{j,t',1-a}\right) \mathrm{d}j \\ &+ \mathbb{1}_{a=1} - \delta \mathbb{1}_{a=1,\, t=l}. \end{aligned} \tag{4}$$

In the equation above, $V_{i,t,1-a} - V_{i,t,a} + P_{i,t,a}^{j,t',1-a}$ is net benefit to investor i of type (t, a) when trading with investor j of type $(t', 1-a)$, and $\mathbb{1}_{a=1} - \delta \mathbb{1}_{a=1,t=l}$ is the flow payoff of the asset holding of i.

The probability of trade α and the transfer P satisfy the Nash Bargaining condition (Nash, 1950) with equal bargaining power:

$$\begin{aligned} \alpha_{i,t,a}^{j,t',1-a} &= \begin{cases} 1 & \text{if } V_{i,t,1-a} + V_{j,t',a} \gtrless V_{i,t,a} + V_{j,t',1-a}, \\ 0 & \end{cases} \\ V_{i,t,1-a} - V_{i,t,a} + P_{i,t,a}^{j,t',1-a} &= V_{j,t',a} - V_{j,t',1-a} - P_{i,t,a}^{j,t',1-a}. \end{aligned} \tag{5}$$

To establish the optimality condition for the search density function $f(i, \cdot)$, I consider a deviation of investor i to some alternative search density function $\tilde{f}(i, \cdot)$. Other investors' type distributions and value functions are not affected by this individual investor's deviation.

The steady-state type distribution $\tilde{x}_{i,t,a}$ of investor i solves a new flow balance equation

$$0 = \int_{j\in[0,1]} (\tilde{f}(i,j) + f(j,i)) \left(-\tilde{x}_{i,t,a} \sum_{t'\in\{h,l\}} x_{j,t',1-a}\, \tilde{\alpha}_{i,t,a}^{j,t',1-a} + \tilde{x}_{i,t,1-a} \sum_{t'\in\{h,l\}} x_{j,t',a}\, \tilde{\alpha}_{i,t,1-a}^{j,t',a} \right) \mathrm{d}j$$
$$+ \left(\tilde{x}_{i,\sim t,a} - \tilde{x}_{i,t,a}\right).$$

The value function $\tilde{V}_{i,t,a}$ of investor i solves a new HJB equation:

$$r\tilde{V}_{i,t,a} = \int_{j\in[0,1]} (\tilde{f}(i,j) + f(j,i)) \sum_{t'\in\{h,l\}} x_{j,t',1-a}\, \tilde{\alpha}_{i,t,a}^{j,t',1-a} \left(\tilde{V}_{i,t,1-a} - \tilde{V}_{i,t,a} + \tilde{P}_{i,t,a}^{j,t',1-a} \right) \mathrm{d}j$$
$$+ \mathbb{1}_{\{a=1\}} - \delta \mathbb{1}_{\{a=1,\, t=l\}},$$

where the probability of trade $\tilde{\alpha}$ and the transfer $\tilde{P}$ satisfy the Nash Bargaining condition

$$\tilde{\alpha}_{i,t,a}^{j,t',1-a} = \begin{cases} 1 & \\ & \text{if } \tilde{V}_{i,t,1-a} + V_{j,t',a} \gtrless \tilde{V}_{i,t,a} + V_{j,t',1-a}, \\ 0 & \end{cases}$$
$$\tilde{V}_{i,t,1-a} - \tilde{V}_{i,t,a} + \tilde{P}_{i,t,a}^{j,t',1-a} = V_{j,t',a} - V_{j,t',1-a} - \tilde{P}_{i,t,a}^{j,t',1-a}.$$

Initially, each investor is endowed with an asset with equal probability 1/2 and has the high preference with probability 1/2. Therefore, the optimality condition for investor i is[4]

$$\pi := \frac{1}{4}(\tilde{V}_{i,ho} + \tilde{V}_{i,ln} + \tilde{V}_{i,lo} + \tilde{V}_{i,hn}) \leq \frac{1}{4}(V_{i,ho} + V_{i,ln} + V_{i,lo} + V_{i,hn}). \tag{6}$$

Definition 1. An equilibrium is a profile (f, α, x, V, P) of search density function f, probability of trade α, steady-state type distribution x, value function V and transfer P that

[4] One can also assume that each investor is initially of either type ho or ln with equal probability as in Farboodi et al. (2022), so that investor i maximizes $(\tilde{V}_{i,ho} + \tilde{V}_{i,ln})/2$. The equilibrium outcome remains unchanged.

satisfy

- the total probability equation (1), the market clearing condition (2), the flow balance equation (3),
- the HJB equation (4), the Nash Bargaining condition (5), and
- the optimality condition (6).

Discussion.

Each investor i is assumed to choose an irreversible density function $f(i, \cdot)$ at time $t = 0$. In practice, trading relationships in OTC markets are sticky.[5] Investors rarely change their trading counterparties due to the operational cost and delay of setting up new relationships, the concern of information asymmetry when trading with counterparties with little past interaction, and the value of a repeated interaction in supporting rent sharing. Therefore, I maintain the assumption of "sticky relationship." In my model, if investors were able to change their search density functions at no cost, they would do so as a function of their preference type and asset position. The model might still remain tractable and the equilibrium outcome would feature time varying search relationships. I am not aware of any search models that endogenizes sticky trading relationships. The network approach can endogenize sticky relationships through repeated interactions between a finite number of agents. Assuming a continuum of agents makes search models more tractable than network models at the expense of ruling out repeated interactions. Therefore, search models most likely would need to assume sticky relationships.

[5]Li and Schürhoff (2019) provide evidence on municipal bonds, Di Maggio, Kermani, and Song (2017) on corporate bonds, Bech and Atalay (2010), Afonso, Kovner, and Schoar (2014) on federal funds, Craig and von Peter (2014), in't Veld and van Lelyveld (2014), Craig and Ma (2022) on foreign interbank lending, Peltonen, Scheicher, and Vuillemey (2014) on credit default swaps, Hollifield, Neklyudov, and Spatt (2017) on asset-backed securities and King, Osler, and Rime (2012) on currencies.

Relative to models of competitive search, this paper does not impose the assumption that investors post and commit to publicly observable terms of trade before they are matched. As opposed to on a central limit order book, investors typically do not make binding offers available to all market participants before knowing who she is trading with in OTC markets, and the terms of trade are often up to negotiation after a match is formed. That terms of trade are formed post-match can result from (a) the uncertainty about other investors' default risk, adverse selection risk, and outside option, (b) the lack of a centralized location where terms of trade can be posted together, or (c) the incentive to keep market power. In practice, request for stream allows dealers to stream indicative prices to other investors for parties to negotiate the terms of trade later.

3 Equilibrium

This section establishes the following pattern of trading and directed search as an equilibrium outcome. Proofs are in Appendix A.

Definition 2. Investors endogenously split into two groups of equal size. Each investor do not search among investors within her own group, and searches among all the investors in the other group uniformly at random. I let f denote the search strategy profile, and label the two groups as Group B (buyer) and Group S (seller). Whenever an investor in Group B meets an investor in Group S with the opposite asset position,

1. (Fundamental trade) if the two investors have the opposite preferences, the investor of type h buys from the investor of type l whenever feasible. Such a trade is called a *fundamental trade*;

2. (Asset-concentration trade) if the two investors have the same preference, the B-investor buys the asset from the S-investor whenever feasible. Such as trade is an *asset-concentration trade*.

I let α denote the trading strategy profile.

A fundamental trade is one that directly increases investors' flow payoff as in Üslü (2019), Farboodi et al. (2022, 2019), Neklyudov (2019). Asset concentration is the novel trading pattern of this paper. Such trades do not directly create flow payoff for investors due to their identical preference. Nevertheless, asset concentration generates an indirect benefit by making future trades more likely. A trade is feasible only if the two investors in an encounter have the opposite asset positions. Asset concentration makes B-investors more likely to own the asset than the S-investors. It is therefore optimal for investors to search only across groups and not within groups to increase the likelihood of meeting another investor with the opposite asset position. Conversely, when a B-investor meets an S-investor with the same preference, it is optimal for the B-investor to hold the asset anticipating that she is more likely to meet another investor without the asset in the future than the S-investor. The combination of asset concentration and two-sided directed search thus constitutes an equilibrium.

In Üslü (2019), Farboodi et al. (2022, 2019), Neklyudov (2019), only the fundamental trades occur if investors have homogeneous search speed or bargaining skill, because any non-fundamental trade creates no benefit, direct or indirect, when all homogeneous investors randomly search among each other.

Now, I derive investors' steady-state type distribution under the pattern of trading and directed search of Definition 2. I let the steady-state type distribution of an investor in

Group g ($g = B, S$) be denoted as

$$\begin{pmatrix} x^g & \frac{1}{2} - x^g \\ \frac{1}{2} - y^g & y^g \end{pmatrix},$$

where x^g is the probability that the investor is of type hn, y^g is the probability that the investor is of type lo, $1/2 - x^g$ is the probability that the investor is of type ho, and $1/2 - y^g$ is the probability that the investor is of type ln. First, the total mass of investors owning an asset is $1/2$,

$$\frac{1}{2}\left(\frac{1}{2} - x^B + y^B + \frac{1}{2} - x^S + y^S\right) = \frac{1}{2} \qquad \Longleftrightarrow \qquad y^B - x^B = x^S - y^S. \tag{7}$$

Equating 0 to the net flow of mass into B-investors of type hn gives

$$0 = \underbrace{-\lambda x^B y^S}_{\text{fundamental trades}} - \underbrace{\lambda x^B \left(\frac{1}{2} - x^S\right)}_{\text{asset concentration}} + \frac{1}{2}\left(\frac{1}{2} - y^B - x^B\right). \tag{8}$$

The first term $\lambda x^B y^S$ reflects the flow of mass caused by the fundamental trades as in Duffie et al. (2005). The second term $\lambda x^B \left(1/2 - x^S\right)$ is the flow of mass caused by asset concentration. The B-investors of type hn contact the S-investors of type ho at the rate

$$\underbrace{2\lambda}_{\text{search intensity}} \cdot \underbrace{\left(\frac{1}{2} x^B\right)}_{\text{mass of B-investors of type } hn} \cdot \underbrace{\left[\frac{1}{2}\left(\frac{1}{2} - x^S\right)\right]}_{\text{mass of S-investors of type } ho},$$

while the S-investors of type ho contact the B-investors of type hn at the same rate. Upon each such encounter, the B-investor buys an asset from the S-investor and becomes type ho. The last term $\left(1/2 - y^B - x^B\right)/2$ is the flow of mass caused by preference shocks as in

Duffie et al. (2005).

For the other investor types,

$$0 = -\lambda y^B x^S + \lambda \left(\frac{1}{2} - y^B\right) y^S + \frac{1}{2}\left(\frac{1}{2} - x^B - y^B\right), \tag{9}$$

$$0 = -\lambda x^S y^B + \lambda \left(\frac{1}{2} - x^S\right) x^B + \frac{1}{2}\left(\frac{1}{2} - y^S - x^S\right), \tag{10}$$

$$0 = -\lambda y^S x^B - \lambda y^S \left(\frac{1}{2} - y^B\right) + \frac{1}{2}\left(\frac{1}{2} - x^S - y^S\right). \tag{11}$$

Solving (7) to (11) uniquely pins down the steady-state type distribution (x^B, y^B, x^S, y^S). Without the terms caused by asset concentration in (8) to (11), one obtains the steady-state type distribution under the random-search benchmark:

$$\begin{pmatrix} z(\text{random search}) & \frac{1}{2} - z(\text{random search}) \\ \frac{1}{2} - z(\text{random search}) & z(\text{random search}) \end{pmatrix}.$$

Under the random-search benchmark, *(i)* each investor has the same steady-state probability z(random search) of being a type hn and a type lo, and this probability is identical across all investors; *(ii)* each investor owns an asset with probability $1/2$ in steady-state.

Proposition 1. *(i) The steady-state type distribution satisfies*

$$x^B = y^S < z(\textit{random search}) < x^S = y^B, \textit{ and } x^B + y^B = x^S + y^S < 2z(\textit{random search}).$$

(ii) The steady-state probability $\phi^B := 1/2 - x^B + y^B$ *that a B-investor owns an asset is higher than its random-search benchmark* $1/2$*. That of an S-investor* $\phi^S := 1/2 - x^S + y^S$ *is*

lower than its random-search benchmark

$$\phi^B := \frac{1}{2} - x^B + y^B > \frac{1}{2} > \frac{1}{2} - x^S + y^S =: \phi^S, \ \textit{and} \ \phi^B + \phi^S = 1.$$

Part *(i)* establishes that using asset concentration, an investor's asset position is misaligned with her preference with a lower steady-state probability $x^B + y^B = x^S + y^S$ than that under the random-search benchmark $2z$(random search). Asset concentration reduces an investor's probability of misalignment by increasing her likelihood of meeting another investor with the opposite asset position and thus the chance of a beneficial trade.

Proposition 1 Part *(ii)* confirms that asset concentration indeed makes B-investors more likely to have an asset and S-investors less likely, relative to the random search benchmark.

With these equilibrium type distributions, the investors' equilibrium value functions are solved by the Hamilton–Jacobi–Bellman (HJB) equations

$$rV_{hn}^B = 2\lambda y^S \left(V_{ho}^B - V_{hn}^B - P^{SB}\right) + 2\lambda \left(\frac{1}{2} - x^S\right) \left(V_{ho}^B - V_{hn}^B - P_h^{AC}\right) + \left(V_{ln}^B - V_{hn}^B\right), \tag{12}$$

$$rV_{lo}^B = 2\lambda x^S \left(V_{ln}^B - V_{lo}^B + P^{BS}\right) + \left(V_{ho}^B - V_{lo}^B\right) + (1-\delta),$$

$$rV_{ln}^B = 2\lambda y^S \left(V_{lo}^B - V_{ln}^B - P_l^{AC}\right) + (V_{hn}^B - V_{ln}^B),$$

$$rV_{ho}^B = \left(V_{lo}^B - V_{ho}^B\right) + 1, \tag{13}$$

$$rV_{hn}^S = 2\lambda y^B \left(V_{ho}^S - V_{hn}^S - P^{BS}\right) + \left(V_{ln}^S - V_{hn}^S\right), \tag{14}$$

$$rV_{lo}^S = 2\lambda x^B \left(V_{ln}^S - V_{lo}^S + P^{SB}\right) + 2\lambda \left(\frac{1}{2} - y^B\right) \left(V_{ln}^S - V_{lo}^S + P_l^{AC}\right) + (V_{ho}^S - V_{lo}^S) + (1-\delta),$$

$$rV_{ln}^S = \left(V_{hn}^S - V_{ln}^S\right),$$

$$rV_{ho}^S = 2\lambda x^B \left(V_{hn}^S - V_{ho}^S + P_h^{AC}\right) + \left(V_{lo}^S - V_{ho}^S\right) + 1, \tag{15}$$

where P^{BS} (or P^{SB}) is the price of a fundamental trade when an asset moves from the B-investor to the S-investor (or from the S-investor to the B-investor), and P_h^{AC} (or P_l^{AC}) is the price of an asset-concentration trade when both investors are of type h (or type l). Prices are determined by the Nash bargaining condition with equal bargaining power:

$$\begin{aligned}
V_{ho}^B - V_{hn}^B - P^{SB} &= V_{ln}^S - V_{lo}^S + P^{SB}, \\
V_{ln}^B - V_{lo}^B + P^{BS} &= V_{ho}^S - V_{hn}^S - P^{BS}, \\
V_{ho}^B - V_{hn}^B - P_h^{AC} &= V_{hn}^S - V_{ho}^S + P_h^{AC}, \\
V_{lo}^B - V_{ln}^B - P_l^{AC} &= V_{ln}^S - V_{lo}^S + P_l^{AC}.
\end{aligned} \tag{16}$$

The HJB equations and the Nash Bargaining conditions uniquely pin down the value functions and the prices.

Proposition 2. *The value functions satisfy*

$$V_{ho}^B - V_{hn}^B > V_{ho}^S - V_{hn}^S, \quad V_{lo}^B - V_{ln}^B > V_{lo}^S - V_{ln}^S.$$

Proposition 2 says that when a B-investor and an S-investor have the same preference, owning the asset is more valuable to the B-investor than to the S-investor. This is because the B-investor is more likely to meet another investor without the asset in the future than the S-investor.

Theorem 1. *The profile (f, α, x, y, V, P) is an equilibrium.*

I give a heuristic proof here, which mimics the formal two-step proof in Appendix A. An investor i has two key considerations when deciding how to search and trade:

(1) Investor i is better off searching in a way that makes her more likely to have the

opposite asset position as her counterparties.

(2) In each encounter, investor i and her counterparty j are better off trading in a way that makes them more likely to have the opposite asset positions as their future counterparties.

The second consideration determines the optimal trading strategy for investor i given any search strategy of i through the one-shot deviation principle, and leads to asset concentration when i and j have the same preference. Specifically, say i is a B-investor and fixing any search density function $f(i, \cdot)$ of i. When i and j have the same preference and only one of them have the asset, there are two cases: (a) if j is an S-investor, then i will hold the asset; (b) if j is an B-investor, then j will hold the asset. In case (a) where j is an S-investor, i is more likely to meet another S-investor than j, implying that the two investors' joint continuation value is maximized by letting i hold the asset. In case (b) where j is a B-investor, j is more likely to meet another S-investor than i, implying that the two investors' joint continuation value is maximized by letting j have the asset. This benefit is reflected in the continuation values V of i, in that

$$V_{to}^S - V_{tn}^S \leq V_{to} - V_{tn} \leq V_{to}^B - V_{tn}^B, \quad t \in \{h, l\}.$$

Given the optimal trading strategy, the first consideration of i then determines her optimal search strategy. Specifically, say i is in Group B, then the steady-state probability ϕ that i owns an asset is between 1/2 and that of a non-deviating B-investor ϕ^B

$$1/2 \leq \phi \leq \phi^B \tag{17}$$

under the optimal trading strategy. Upon meeting an S-investor j, the probability that i and j have opposite asset positions is $\phi(1-\phi^S)+(1-\phi)\phi^S$ which is equal to $\phi\phi^B+(1-\phi)(1-\phi^B)$ as $\phi^S = 1-\phi^B$ (Proposition 1). This probability is higher than that if j were a B-investor $\phi(1-\phi^B)+(1-\phi)\phi^B$ by (17). Switching a marginal amount of her search capacity from the B-investors to the S-investors thus makes i more likely to have the opposite asset position as her counterparties. This benefit is reflected in the continuation value of i, in that $V_{ho}+V_{ln}$ and $V_{hn}+V_{lo}$ are maximized when i searches among and only among the S-investors.

Comparison with Chang and Zhang (2021). Using a setup of discrete-time pairwise matching, Chang and Zhang (2021) derives a different matching and trading pattern. Their Lemma 1 states that in each period, matching among investors is positive assortative so that every investor is matched to another investor with the same characteristics in that period. In their model, a summary statistic of an investor's characteristics is the probability that the investor's preference and asset position are misaligned. When two matched investors have the same preference and the opposite asset position, an intermediation trade occurs where one investor, labeled as a "market maker," takes on the misaligned asset position to let the other investor achieve her desired asset position.

The pattern they derived differs from mine in both search/matching and trading: (1) Their matching is positive assortative, whereas mine is negative assortative in that an investor who is more likely to own the asset searches among others who are less likely to own the asset. (2) Their non-fundamental trades consist of intermediation trades, whereas mine consist of asset concentration.

It is possible to show that asset concentration strictly dominates intermediation in a 3-period counter-example, as shown in Appendix B. To rule out the counter-example, one might add an exogenous restriction that every investor must hold the asset with equal probability in every period, or equivalently that the probability of an H-type facing misallocation is equal to the probability of an L-type facing misallocation. Under this restriction, it is possible that their Lemma 1 holds when the asset supply is 1/2. However, it is not clear how this restriction and their solution can be jointly generalized to a situation where the asset supply is not 1/2. Under their setup, I let $A \in (0,1)$ be the asset supply, and assume that the steady-state probability that an investor has the high preference is also A. In period 0, every investor's type distribution is given by

$$\begin{pmatrix} x & A-x \\ 1-A-x & x \end{pmatrix}.$$

After matching and trading using their proposed market making policy in period 1, a market maker's type distribution becomes

$$\begin{pmatrix} y & A-y \\ 1-A-z & z \end{pmatrix}$$

where

$$y = \underbrace{x}_{\text{prior}} - \underbrace{x^2}_{\text{fundamental trades}} + \underbrace{x(A-x)}_{\text{intermediation trades}} = x(1+A-2x),$$

$$z = \underbrace{x}_{\text{prior}} - \underbrace{x^2}_{\text{fundamental trades}} + \underbrace{x(1-A-x)}_{\text{intermediation trades}} = x(2-A-2x).$$

If the asset supply $A \neq 1/2$, $y \neq z$ and hence the restriction that the probability of misalignment is symmetric between the two preference types is violated after one period.

One reason for the difficulty of generalizing the asset supply beyond 1/2 in their setup comes from the rigidity of discrete-time pairwise matching: Exactly one-half of the investors must simultaneously match with the other half in each period. In a continuous-time search environment, investors are "matched" to each other at independent Poisson arrival times, and thus matching does not require two investor groups of equal size to be matched to each other. This flexibility allows a bigger group of investors to match with the remaining, smaller group of investors, which arises in equilibrium when the asset supply is not $1/2$.

In the random search literature, some papers[6] also assume that the asset supply and the steady-state measure of high-preference investors are fixed at $1/2$. Next, I generalize my canonical model to allow for a general asset supply and preference switching.

General Asset Supply and Preference Switching.

I let the asset supply be A for any $A \in (0, 1)$, and assume that each investor's preference type switches from low to high with intensity A while switching back with intensity $1 - A$, so that the steady-state measure of high-preference investors is also A. In a frictionless environment, investors with the high preference hold the asset, while those with the low preference do not.

Investors' equilibrium search pattern generalizes in a straightforward manner.

Definition 3. Investors endogenously split into Group B of size μ and Group S of size $1-\mu$. Each investor searches among all the investors in the other group, and do not search among investors within the same group. I let f_μ denote the search strategy profile.

Investors' equilibrium trading pattern remains unchanged and continues to be α as defined in Definition 2. Under the search and trading strategies (f_μ, α), the steady-state type

[6] Examples include Farboodi et al. (2022, 2019).

distribution of an investor in Group g ($g = B, S$) becomes

$$\begin{pmatrix} x^g & A - x^g \\ 1 - A - y^g & y^g \end{pmatrix},$$

The market clearing equation (7) becomes

$$\mu\left(A - x^B + y^B\right) + (1-\mu)\left(A - x^S + y^S\right) = \mu \qquad \Longleftrightarrow \qquad \mu\left(y^B - x^B\right) = (1-\mu)\left(x^S - y^S\right).$$

The flow balance equations (8) to (11) become

$$\begin{aligned}
0 &= -\lambda x^B y^S - \lambda x^B\left(A - x^S\right) + \mu\left[A\left(1 - A - y^B\right) - (1-A)x^B\right], \\
0 &= -\lambda y^B x^S + \lambda\left(1 - A - y^B\right)y^S + \mu\left[(1-A)\left(A - x^B\right) - Ay^B\right], \\
0 &= -\lambda x^S y^B + \lambda\left(A - x^S\right)x^B + (1-\mu)\left[A\left(1 - A - y^S\right) - (1-A)x^S\right], \\
0 &= -\lambda y^S x^B - \lambda y^S\left(1 - A - y^B\right) + (1-\mu)\left[(1-A)\left(A - x^S\right) - Ay^S\right].
\end{aligned}$$

The above system of linear equations uniquely pins down the steady-state type distribution (x^B, y^B, x^S, y^S).

With these equilibrium type distributions, the HJB equation for investors' equilibrium value functions becomes

$$
\begin{aligned}
rV_{hn}^{B} &= \frac{\lambda}{\mu} y^{S} \left(V_{ho}^{B} - V_{hn}^{B} - P^{SB}\right) + \frac{\lambda}{\mu} \left(A - x^{S}\right) \left(V_{ho}^{B} - V_{hn}^{B} - P_{h}^{AC}\right) + (1 - A) \left(V_{ln}^{B} - V_{hn}^{B}\right), \\
rV_{lo}^{B} &= \frac{\lambda}{\mu} x^{S} \left(V_{ln}^{B} - V_{lo}^{B} + P^{BS}\right) + A \left(V_{ho}^{B} - V_{lo}^{B}\right) + (1 - \delta), \\
rV_{ln}^{B} &= \frac{\lambda}{\mu} y^{S} \left(V_{lo}^{B} - V_{ln}^{B} - P_{l}^{AC}\right) + A(V_{hn}^{B} - V_{ln}^{B}), \\
rV_{ho}^{B} &= (1 - A) \left(V_{lo}^{B} - V_{ho}^{B}\right) + 1,
\end{aligned}
$$

$$
\begin{aligned}
rV_{hn}^{S} &= \frac{\lambda}{1-\mu} y^{B} \left(V_{ho}^{S} - V_{hn}^{S} - P^{BS}\right) + (1 - A) \left(V_{ln}^{S} - V_{hn}^{S}\right), \\
rV_{lo}^{S} &= \frac{\lambda}{1-\mu} x^{B} \left(V_{ln}^{S} - V_{lo}^{S} + P^{SB}\right) + \frac{\lambda}{1-\mu} \left(1 - A - y^{B}\right) \left(V_{ln}^{S} - V_{lo}^{S} + P_{l}^{AC}\right) + A(V_{ho}^{S} - V_{lo}^{S}) + (1 - \delta), \\
rV_{ln}^{S} &= A \left(V_{hn}^{S} - V_{ln}^{S}\right), \\
rV_{ho}^{S} &= \frac{\lambda}{1-\mu} x^{B} \left(V_{hn}^{S} - V_{ho}^{S} + P_{h}^{AC}\right) + (1 - A) \left(V_{lo}^{S} - V_{ho}^{S}\right) + 1,
\end{aligned}
$$

The Nash Bargaining condition (16) for the prices P remain unchanged. The HJB equations and the Nash Bargaining condition uniquely pin down the value functions and the prices.

Finally, the equilibrium size μ^* of B-investors is determined by the condition that a B-investor and an S-investor have the same expected payoff $\pi^B = \pi^S$ where π is defined by (6).

Corollary 1. *The profile $(f_{\mu^*}, \alpha, x, y, V, P)$ is an equilibrium for the economy where the asset supply is A, and each investor's preference type switches from low to high with intensity A while switching back with intensity $1 - A$.*

4 Implications

This section first presents the model's theoretical implications, then its empirical implications.

4.1 Asset Concentration versus Intermediation

A recent theoretical literature focuses on explaining intermediation in OTC markets through positive assortative matching in trading needs (Chang and Zhang, 2021), endogenous search speed (Farboodi et al., 2022) and endogenous bargaining power (Farboodi et al., 2019). In these models, when two matched investors have the same preference and the opposite asset positions, one investor acts as an intermediary by taking on the misaligned asset position to let the other investor achieve her desired asset position. Using the setup of Chang and Zhang (2021), Appendix B shows that asset concentration strictly dominates intermediation in a 3-period counter-example. In the random search models of Farboodi et al. (2019, 2022), intermediation likely ceases to be an equilibrium outcome when investors can direct their search. In the equilibrium of Farboodi et al. (2019), some investors acquire superior, "tough," negotiation skill and endogenously act as intermediaries. However, if directed search was allowed, any investor, soft or tough, has no incentive to search among the tough investors as she can trade at a more favorable price with the soft ones. Farboodi et al. (2022) faces a similar concern, in that no investor has an incentive to search among the fast investors due to their better outside options than the slow ones.

Now, I show that asset concentration continues to arise in equilibrium after endogenizing speed or bargaining skill.

Extended Model: I modify the game as follows: at time $t = 0$, each investor (1) chooses either a search capacity or a bargaining power at some cost, and (2) allocates her search capacity. All investors make these two choices (1) and (2) simultaneously at time $t = 0$.

To ease comparison with Farboodi et al. (2022, 2019), I revert back to the canonical parameter setup where both the asset supply and the steady-state measure of investors with the high preference are $A = 1/2$, and assume that each investor is initially of either type ho or ln with equal probability so that she maximizes $(V_{ho} + V_{ln})/2$.

Endogenous search speed. Each investor i is assumed to incur a cost of $c\lambda_i$ when acquiring a total search capacity of $\lambda_i \in [0, \bar{\lambda}]$, where $c > 0$ and $\bar{\lambda}$ is an exogenous upper bound as in Farboodi et al. (2022).

Definition 4. I let G_λ denote the strategy profile of search capacity acquisition where every investor chooses the same search capacity $\lambda > 0$.

When investors choose the same search capacity λ and follow the directed search and asset concentration strategies (f, α) as given by Definition 2, their steady-state type distribution (x^B, y^B, x^S, y^S), their value function (V^B, V^S), and the prices P remain the same as in the canonical model, and are uniquely pinned down by the market clearing condition and the flow balance equations (7) to (11), the HJB equation (12) to (15), and the Nash Bargaining condition (16) respectively.

Next, I solve for the equilibrium level of search capacity λ^* that every investor chooses. If a B-investor i chooses some alternative search capacity $\tilde{\lambda}$, other investors' type distribution and value function are not affected by this individual investor's deviation. Appendix A shows that it is still optimal for investor i to search only among the S-investors. Then the value function $V(\tilde{\lambda})$ of i solves the HJB equation

$$\begin{aligned}
rV_{hn}(\tilde{\lambda}) &= \left(\lambda + \tilde{\lambda}\right)\left[y^S\left(\frac{V_{ho}(\tilde{\lambda}) - V_{hn}(\tilde{\lambda}) + V_{ln}^S - V_{lo}^S}{2}\right)^+ \right.\\
&\quad \left. + \left(\frac{1}{2} - x^S\right)\left(\frac{V_{ho}(\tilde{\lambda}) - V_{hn}(\tilde{\lambda}) + V_{hn}^S - V_{ho}^S}{2}\right)^+\right] + (V_{ln}(\tilde{\lambda}) - V_{hn}(\tilde{\lambda})),\\
rV_{lo}(\tilde{\lambda}) &= \left(\lambda + \tilde{\lambda}\right)\left[x^S\left(\frac{V_{ln}(\tilde{\lambda}) - V_{lo}(\tilde{\lambda}) + V_{ho}^S - V_{hn}^S}{2}\right)^+ \right.\\
&\quad \left. + \left(\frac{1}{2} - y^S\right)\left(\frac{V_{ln}(\tilde{\lambda}) - V_{lo}(\tilde{\lambda}) + V_{lo}^S - V_{ln}^S}{2}\right)^+\right] + (V_{ho}(\tilde{\lambda}) - V_{lo}(\tilde{\lambda})) + 1 - \delta,\\
rV_{ln}(\tilde{\lambda}) &= \left(\lambda + \tilde{\lambda}\right)\left[y^S\left(\frac{V_{lo}(\tilde{\lambda}) - V_{ln}(\tilde{\lambda}) + V_{ln}^S - V_{lo}^S}{2}\right)^+ \right.\\
&\quad \left. + \left(\frac{1}{2} - x^S\right)\left(\frac{V_{lo}(\tilde{\lambda}) - V_{ln}(\tilde{\lambda}) + V_{hn}^S - V_{ho}^S}{2}\right)^+\right] + (V_{hn}(\tilde{\lambda}) - V_{ln}(\tilde{\lambda})),\\
rV_{ho}(\tilde{\lambda}) &= \left(\lambda + \tilde{\lambda}\right)\left[x^S\left(\frac{V_{hn}(\tilde{\lambda}) - V_{ho}(\tilde{\lambda}) + V_{ho}^S - V_{hn}^S}{2}\right)^+ \right.\\
&\quad \left. + \left(\frac{1}{2} - y^S\right)\left(\frac{V_{hn}(\tilde{\lambda}) - V_{ho}(\tilde{\lambda}) + V_{lo}^S - V_{ln}^S}{2}\right)^+\right] + (V_{lo}(\tilde{\lambda}) - V_{ho}(\tilde{\lambda})) + 1.
\end{aligned}$$

The equilibrium level of search capacity λ^* that every investor chooses satisfies

$$\pi'(\lambda^*) = c, \tag{18}$$

where π is defined in (6). Equation (18) uniquely pins down λ^*. A given B-investor thus has no incentive to deviate from the equilibrium search capacity λ^*. By symmetry, a given S-investor has no incentive to deviate from the equilibrium search capacity λ^* either.

The next corollary establishes that homogenous speed and asset concentration endogenously arise when investors are allowed to direct their search.

Corollary 2. *(Asset concentration with endogenous search speed)*
With the equilibrium speed λ^* *given by* (18)*, there exists some* $\bar{\lambda} > \lambda^*$ *such that, the profile* $(G_{\lambda^*}, f, \alpha, x, y, V, P)$ *is an equilibrium.*

Corollary 2 implies that another main result of Farboodi et al. (2022) is obtained only by not allowing investors to direct their search. Their Proposition 2 establishes that in any "symmetric" equilibrium, investors must acquire heterogeneous speed, with the faster investors acting as intermediaries. However, removing the symmetry restriction and allowing directed search would restore an equilibrium with homogeneous speed, and asset concentration arises in the place of intermediation.

The benefit of intermediation trades is the key force in Farboodi et al. (2022) that pushes against homogenous speed. In any homogenous-speed economy under random search, two investors are indifferent whether to trade or not when they have the same preference (Duffie et al., 2005). When one investor becomes faster or slower, she breaks the tie and the two investors increase their joint surplus by making an intermediation trade in which the faster trader accommodates the slower trader's misallocation (Neklyudov, 2019, Üslü, 2019, Farboodi et al., 2022). Exploiting this feature, Farboodi et al. (2022) shows that a homogenous-speed economy where every investor chooses the same search speed λ cannot be an equilibrium when speed is endogenously chosen, because any given investor would be strictly better off deviating to either a higher or a lower speed to benefit from intermediation trades. In other words, the benefit from intermediation "generates a convex kink in the value function at the mass point, which creates an incentive to choose a different contact rate from everyone else."(Farboodi et al., 2022)

This argument no longer holds when directed search is allowed. In equilibrium, investors follow a different trading strategy—asset concentration—and do strictly better than in the random search benchmark even in a homogeneous-speed economy. Therefore, an investor can no longer improve her payoff upon her baseline homogenous-speed value by marginally raising or reducing her speed and making intermediation trades. The convex kink ceases to exist. Figure 1 plots the numerical value of $V_{ho}(\tilde{\lambda}) + V_{ln}(\tilde{\lambda})$ and its derivative $V'_{ho}(\tilde{\lambda}) + V'_{ln}(\tilde{\lambda})$ as a function of $\tilde{\lambda} \in [0, 10]$, when every other investor chooses the same search speed $\lambda^* = 1$. The derivative $V'_{ho}(\tilde{\lambda}) + V'_{ln}(\tilde{\lambda})$ is strictly decreasing over $\tilde{\lambda} \in [0, 10]$, and the value $V_{ho}(\tilde{\lambda}) + V_{ln}(\tilde{\lambda})$ has no "convex kink" at the mass point $\lambda^* = 1$.

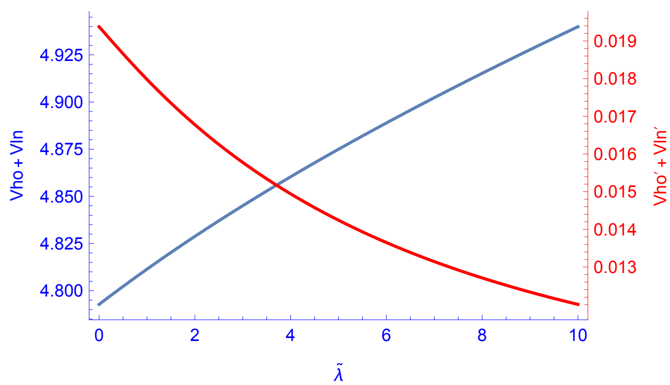

Figure 1: The value $V_{ho}(\tilde{\lambda}) + V_{ln}(\tilde{\lambda})$ and its derivative $V'_{ho}(\tilde{\lambda}) + V'_{ln}(\tilde{\lambda})$

Endogenous bargaining power.

At time $t = 0$, each investor can either incur a cost of c to become "tough" or otherwise become "soft" at no cost. When a tough investor i trades with a soft one j, i extracts all trading rent from j. When the two investors have the same bargaining power, they equally split the trading rent.

Definition 5. I let Φ_{soft} (and Φ_{tough}) denote the strategy profile of bargaining power acquisition where every investor chooses to become soft (and tough).

Next, I show that when the cost c of becoming tough is above a unique threshold c_0, all investors choose to become soft in equilibrium. If $c < c_0$, then all investors choose to become tough in equilibrium. In both cases, asset concentration arrises in the place of intermediation.

When all investors acquire the same bargaining power and follow the directed search and asset concentration strategies (f, α) as given by Definition 2, their steady-state type distribution (x^B, y^B, x^S, y^S), their value function (V^B, V^S), and the prices P remain the same as in the canonical model, and are uniquely pinned down by the market clearing condition and the flow balance equation (7) to (11), the HJB equation (12) to (15), and the Nash Bargaining condition (16) respectively.

Next, I solve for the cost threshold c_0. In the equilibrium where every investor chooses to become tough, if a B-investor deviates to becoming soft, her value function V solves the HJB equation

$$\begin{aligned} rV_{hn} &= V_{ln} - V_{hn}, \\ rV_{lo} &= V_{ho} - V_{lo} + 1 - \delta, \\ rV_{ln} &= V_{hn} - V_{ln}, \\ rV_{ho} &= V_{lo} - V_{ho} + 1. \end{aligned} \tag{19}$$

Then if the cost c of becoming tough is below

$$c_0 \coloneqq V^B_{ho} + V^B_{ln} - (V_{ho} + V_{ln}) = V^B_{ho} + V^B_{ln} - \frac{2 + r - \delta}{r(2 + r)}, \tag{20}$$

the B-investor has no incentive to deviate to soft. Since $V^B_{ho} + V^B_{ln} = V^S_{ho} + V^S_{ln}$ by symmetry, an S-investor has no incentive to deviate to soft either.

In the equilibrium where every investor chooses to become soft, if a B-investor i deviates to becoming tough, Appendix A shows that it is still optimal for i to search only among the

S-investors. Then the value function V for i solves the HJB equation

$$
\begin{aligned}
r\tilde{V}_{hn} &= 2\lambda\left[y^S\left(\tilde{V}_{ho}-\tilde{V}_{hn}+V^S_{ln}-V^S_{lo}\right)+\left(\frac{1}{2}-x^S\right)\left(\tilde{V}_{ho}-\tilde{V}_{hn}+V^S_{hn}-V^S_{ho}\right)\right]+(\tilde{V}_{ln}-\tilde{V}_{hn}),\\
r\tilde{V}_{lo} &= 2\lambda x^S\left(\tilde{V}_{ln}-\tilde{V}_{lo}+V^S_{ho}-V^S_{hn}\right)+(\tilde{V}_{ho}-\tilde{V}_{lo})+1-\delta,\\
r\tilde{V}_{ln} &= 2\lambda y^S\left(\tilde{V}_{lo}-\tilde{V}_{ln}+V^S_{ln}-V^S_{lo}\right)+(V_{hn}-V_{ln}),\\
r\tilde{V}_{ho} &= \tilde{V}_{lo}-\tilde{V}_{ho}+1.
\end{aligned}
$$

Adding up the HJB equation above and the HJB equation (19) for V and comparing the sum to the HJB equation (12) and (13) for V^B, one obtains $V^B = (V+\tilde{V})/2$. Thus, if the cost c of becoming tough is above the threshold c_0 given by (20), the B-investor has no incentive to deviate to tough. By symmetry, an S-investor has no incentive to deviate to tough either.

The analysis above leads to the following corollary.

Corollary 3. *(Asset concentration with endogenous bargaining power)*
(i) If $c > c_0$, where c_0 is given by (20), the profile $(\Phi_{soft}, f, \alpha, x, y, V, P)$ is an equilibrium.
(ii) If $c < c_0$, the profile $(\Phi_{tough}, f, \alpha, x, y, V, P)$ is an equilibrium.
(iii) If $c = c_0$, both profiles $(\Phi_{soft}, f, \alpha, x, y, V, P)$ and $(\Phi_{tough}, f, \alpha, x, y, V, P)$ are equilibria.

4.2 Empirical Evidence and Predictions

This subsection presents three empirical implications on (1) persistent imbalance between banks' funding needs, (2) asset concentration, and (3) price dispersion.

Persistent imbalance between banks' funding needs.

Craig and Ma (2022) document a novel empirical pattern that a stable set of banks consistently borrow and another set consistently lend in the German interbank market. The persistence of interbank lending and borrowing is at odds with the pattern of intermediation where intermediaries make offsetting trades and cannot be explained by existing models of intermediation. In other words, certain banks persistently have funding shortages, whereas some others have funding surpluses. What causes the systematic imbalance between banks' funding needs, despite the latter raiseing systemic risk (Craig and Ma, 2022)?

This paper offers a simple explanation to this puzzle. In my model, while all investors are ex-ante identical, the B-investors have a higher probability of having excess funding, whereas the S-investors are more likely to be short of funds. Fund concentration helps investors mitigate search friction by making it easier for them to locate funding supply and demand.

Asset concentration.

The model predicts asset concentration beyond the interbank market. Other OTC markets, such as those for corporate bonds, have greater search friction due to their assets being non standardized (Pinter and Üslü, 2022). The model predicts that asset holdings are more concentrated in the bond market than in the stock market where trades are centralized and search friction is not a concern. Coppola (2021) offers preliminary evidence of asset concentration among mutual funds and insurance companies in the U.S. corporate bond market, finding that the largest 10 insurers hold 34 percent of the sector's assets, while the largest 50 own 71 percent.[7]

The benefit of diversification would predict portfolio dispersal as opposed to portfolio concentration. Further, larger transaction costs cannot explain greater portfolio concentra-

[7]These measures are only preliminary for the purpose of testing asset concentration. One should compute every investor's portfolio weight. Under the null hypothesis that every investor holds the same market portfolio, the portfolio weight would be the same across investors.

tion in the bond market. Investors face a "size penalty" in bond markets, in that a given investor incurs a greater per-unit transaction cost on a larger trade than on a smaller one (Pinter, Wang, and Zou, 2021). The size penalty is inconsistent with portfolio concentration.

Like fund concentration, asset concentration also has the potential of causing systemic risk.[8] Reducing search friction would therefore have the benefit of reducing the need to concentrate asset holdings and thus systemic risk. This result suggests that asset standardization and centralizing OTC trades not only directly promote trading efficiency, but also lower systemic risk by encouraging investors to hold more diversified portfolios.

Asymmetric price dispersion.

The literature has identified different types of price dispersion in OTC markets, including price dispersion between larger and smaller trades,[9] between dealers with larger and smaller inventory size (Ho and Stoll, 1983, Friewald and Nagler, 2019, Colliard et al., 2021), between sophisticated and unsophisticated clients (Hau, Hoffmann, Langfield, and Timmer, 2021), and between more and less active clients (O'Hara, Wang, and Zhou, 2018).

This paper contributes a novel prediction, that price dispersion is larger when a B-investor buys from an S-investor than when a B-investor sells to an S-investor. Three different prices P^{SB}, P_h^{AC}, P_l^{AC} are possible when a B-investor buys from an S-investor, two of which are associated with asset concentration. When a B-investor sells to an S-investor, however, only one price P^{BS}—associated with a fundamental trade—is possible.

Empirically, this prediction can be tested as follows: Given an asset, first, one can identify

[8] A different literature shows that asset commonality raises the risks of bank run and fire-sale (Shaffer, 1994, Goldstein and Pauzner, 2004, Stiglitz, 2010, Wagner, 2010, 2011, Allen, Babus, and Carletti, 2012, Goldstein, Kopytov, Shen, and Xiang, 2022).

[9] Examples include Schultz (2001), Bessembinder, Maxwell, and Venkataraman (2006), Harris and Piwowar (2006), Edwards, Harris, and Piwowar (2007), Goldstein, Hotchkiss, and Sirri (2007), Green, Hollifield, and Schürhoff (2007a,b).

a group of B-investors who have disproportionally more asset holdings and a group of S-investors with less asset holdings. One would then compare the price dispersions between when the B-investors buy from and sell to the S-investors.

This prediction of asymmetric price dispersion cannot be readily derived from the standard theories of random search, dealer market power, or inventory cost, all of which imply a symmetric price dispersion between buying and selling. Finding an asymmetric price dispersion in the data could thus serve as an indirect evidence of search friction in OTC markets.

5 Conclusion

In a tractable model of over-the-counter markets where each investor flexibly distributes her search capacity across other investors, asset holdings are endogenously concentrated among a subgroup of investors. Investors who are more likely to hold an asset search among those less likely to hold the asset, and vice versa. When directed search is allowed in those existing random search models that endogenize intermediation, intermediation ceases to be an equilibrium outcome whereas asset concentration continues to arise endogenously. My model explains the persistent imbalance between banks' funding needs, and contributes novel predictions of asset concentration across investors and asymmetric price dispersion.

Appendices

A Proofs

Proof of Proposition 1. *Part (i)*: By symmetry, $x^B = y^S$ and $y^B = x^S$. It follows from (8) and (9) that $2\lambda \left(x^B\right)^2 < 2\lambda \left(y^B\right)^2$. Thus, $x^B = y^S < x^S = y^B$.

Next, I show that $x^B < z(\text{random search}) < y^B$. The probability $z(\text{random search})$ satisfies the flow balance equation

$$0 = -\lambda z^2(\text{random search}) + \frac{1}{2}\left(\frac{1}{2} - 2z(\text{random search})\right), \tag{21}$$

If $z(\text{random search}) \leq x^B$, then the equation above contradicts (8). If $z(\text{random search}) \geq y^B$, then the equation above contradicts (9). Therefore,

$$x^B = y^S < z(\text{random search}) < x^S = y^B.$$

If $z(\text{random search}) \leq \left(x^B + y^B\right)/2$, then the flow equation (21) for $z(\text{random search})$ contradicts (8) + (9). Therefore, $z(\text{random search}) > \left(x^B + y^B\right)/2$.

Part (ii) is an immediate consequence of Part *(i)*. □

Proof of Proposition 2. I let $\Delta_h^B = V_{ho}^B - V_{hn}^B$, $\Delta_h^S = V_{ho}^S - V_{hn}^S$, $\Delta_l^B = V_{lo}^B - V_{ln}^B$, and $\Delta_l^S = V_{lo}^S - V_{ln}^S$. Then (13) − (12) and (15) − (14) yield

$$r\Delta_h^B = \Delta_l^B - \Delta_h^B + 1 - \lambda y^S \left(\Delta_h^B - \Delta_l^S\right) - \lambda \left(\frac{1}{2} - x^S\right)\left(\Delta_h^B - \Delta_h^S\right),$$

$$r\Delta_h^S = \Delta_l^S - \Delta_h^S + 1 - \lambda y^B \left(\Delta_h^S - \Delta_l^B\right) + \lambda x^B \left(\Delta_h^B - \Delta_h^S\right).$$

Subtracting the above two equations gives

$$\left(r+1+\lambda\left(\frac{1}{2}+x^B\right)\right)\left(\Delta_h^B-\Delta_h^S\right)=\left(1-\lambda y^B\right)\left(\Delta_l^B-\Delta_l^S\right)+\lambda\left(y^B-y^S\right)\left(\Delta_h^B-\Delta_l^S\right).$$

Likewise, one can obtain

$$\left(r+1+\lambda\left(\frac{1}{2}+x^B\right)\right)\left(\Delta_l^B-\Delta_l^S\right)=\left(1-\lambda y^B\right)\left(\Delta_h^B-\Delta_h^S\right)+\lambda\left(y^B-y^S\right)\left(\Delta_h^B-\Delta_l^S\right).$$

It follows from the above two equations that

$$\Delta_h^B-\Delta_h^S=\Delta_l^B-\Delta_l^S=\frac{\lambda\left(y^B-y^S\right)\left(\Delta_h^B-\Delta_l^S\right)}{r+\lambda\left(\frac{1}{2}+y^B+x^B\right)}>0. \tag{22}$$

□

PROOF OF THEOREM 1. I consider a B-investor who allocates a search capacity $\lambda' \leq \lambda$

toward the S-investors. Her value function V solves the HJB equation

$$
\begin{aligned}
rV_{hn} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(\frac{V_{ho}-V_{hn}+V^S_{ln}-V^S_{lo}}{2}\right)^+ \\
& \qquad + y^S\left(\frac{1}{2}-x^S\right)\left(\frac{V_{ho}-V_{hn}+V^S_{hn}-V^S_{ho}}{2}\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad y^B\left(\frac{V_{ho}-V_{hn}+V^B_{ln}-V^B_{lo}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(\frac{V_{ho}-V_{hn}+V^B_{hn}-V^B_{ho}}{2}\right)^+\Bigg] + (V_{ln}-V_{hn}) \\
rV_{lo} = \quad & (\lambda+\lambda')\Bigg[\quad x^S\left(\frac{V_{ln}-V_{lo}+V^S_{ho}-V^S_{hn}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^S\right)\left(\frac{V_{ln}-V_{lo}+V^S_{lo}-V^S_{ln}}{2}\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad x^B\left(\frac{V_{ln}-V_{lo}+V^B_{ho}-V^B_{hn}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^B\right)\left(\frac{V_{ln}-V_{lo}+V^B_{lo}-V^B_{ln}}{2}\right)^+\Bigg] + (V_{ho}-V_{lo}) + 1 - \delta \\
rV_{ln} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(\frac{V_{lo}-V_{ln}+V^S_{ln}-V^S_{lo}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^S\right)\left(\frac{V_{lo}-V_{ln}+V^S_{hn}-V^S_{ho}}{2}\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad y^B\left(\frac{V_{lo}-V_{ln}+V^B_{ln}-V^B_{lo}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(\frac{V_{lo}-V_{ln}+V^B_{hn}-V^B_{ho}}{2}\right)^+\Bigg] + (V_{hn}-V_{ln})
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
rV_{ho} = \quad & (\lambda+\lambda')\Bigg[\quad x^S\left(\frac{V_{hn}-V_{ho}+V^S_{ho}-V^S_{hn}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^S\right)\left(\frac{V_{hn}-V_{ho}+V^S_{lo}-V^S_{ln}}{2}\right)^+ \Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad x^B\left(\frac{V_{hn}-V_{ho}+V^B_{ho}-V^B_{hn}}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^B\right)\left(\frac{V_{hn}-V_{ho}+V^B_{lo}-V^B_{ln}}{2}\right)^+ \Bigg] + (V_{lo}-V_{ho}) + 1.
\end{aligned}
$$

Step 1: Given any search strategy λ', I conjecture and verify that asset concentration is the optimal trading strategy. That is,

$$
\begin{aligned}
&\text{Fundamental trades:} && V_{ho}-V_{hn}+V^g_{ln}-V^g_{lo} \geq 0, \quad V_{ln}-V_{lo}+V^g_{ho}-V^g_{hn} \geq 0, \quad g \in \{S,B\} \\
&\text{Asset concentration:} && V_{ho}-V_{hn}+V^S_{hn}-V^S_{ho} \geq 0, \quad V_{lo}-V_{ln}+V^S_{ln}-V^S_{lo} \geq 0, \\
& && V_{ho}-V_{hn}+V^B_{hn}-V^B_{ho} \leq 0, \quad V_{lo}-V_{ln}+V^B_{ln}-V^B_{lo} \leq 0.
\end{aligned}
$$

Under these conjectures, the HJB equation for the value function V becomes

$$
\begin{aligned}
rV_{hn} = \; & (\lambda+\lambda')\left[y^S\frac{V_{ho}-V_{hn}+V^S_{ln}-V^S_{lo}}{2} + \left(\frac{1}{2}-x^S\right)\frac{V_{ho}-V_{hn}+V^S_{hn}-V^S_{ho}}{2}\right] \\
& + (\lambda-\lambda')y^B\frac{V_{ho}-V_{hn}+V^B_{ln}-V^B_{lo}}{2} + (V_{ln}-V_{hn}), && (24) \\
rV_{lo} = \; & (\lambda+\lambda')x^S\frac{V_{ln}-V_{lo}+V^S_{ho}-V^S_{hn}}{2} + (V_{ho}-V_{lo}) + (1-\delta) \\
& + (\lambda-\lambda')\left[x^B\frac{V_{ln}-V_{lo}+V^B_{ho}-V^B_{hn}}{2} + \left(\frac{1}{2}-y^B\right)\frac{V_{ln}-V_{lo}+V^B_{lo}-V^B_{ln}}{2}\right], && (25) \\
rV_{ln} = \; & (\lambda+\lambda')y^S\frac{V_{lo}-V_{ln}+V^S_{ln}-V^S_{lo}}{2} + (V_{hn}-V_{ln}), && (26) \\
rV_{ho} = \; & (\lambda-\lambda')x^B\frac{V_{hn}-V_{ho}+V^B_{ho}-V^B_{hn}}{2} + (V_{lo}-V_{ho}) + 1. && (27)
\end{aligned}
$$

I let $\Delta_h(\lambda') = V_{ho} - V_{hn}$ and $\Delta_l(\lambda') = V_{lo} - V_{ln}$, and show that $\dot{\Delta}_h(\lambda') > 0$ and $\dot{\Delta}_l(\lambda') > 0$

for any $\lambda' \in [-\lambda, \lambda]$ to complete Step 1. Then (27) − (24) and (25) − (26) yield

$$r\Delta_h(\lambda') = (\lambda - \lambda')x^B \frac{\Delta_h^B - \Delta_h(\lambda')}{2} - (\lambda + \lambda')\left[y^S \frac{\Delta_h(\lambda') - \Delta_l^S}{2} + \left(\frac{1}{2} - x^S\right)\frac{\Delta_h(\lambda') - \Delta_h^S}{2}\right]$$
$$- (\lambda - \lambda')y^B \frac{\Delta_h(\lambda') - \Delta_l^B}{2} + \Delta_l(\lambda') - \Delta_h(\lambda') + 1,$$
$$r\Delta_l(\lambda') = (\lambda - \lambda')\left[x^B \frac{\Delta_h^B - \Delta_l(\lambda')}{2} + \left(\frac{1}{2} - y^B\right)\frac{\Delta_l^B - \Delta_l(\lambda')}{2}\right] - (\lambda + \lambda')y^S \frac{\Delta_l(\lambda') - \Delta_l^S}{2}$$
$$+ (\lambda + \lambda')x^S \frac{\Delta_h^S - \Delta_l(\lambda')}{2} + \Delta_h(\lambda') - \Delta_l(\lambda') + 1 - \delta.$$

When $\lambda' = \lambda$, $V(\lambda) = V^B$, because the HJB equation (24) to (27) for the investor's value function $V$is identical to that (12) and (13) for a B-investor's value function V^B. By symmetry, $V(-\lambda) = V^S$. Thus, $\Delta_h(\lambda) = \Delta_h^B$, $\Delta_l(\lambda) = \Delta_l^B$, $\Delta_h(-\lambda) = \Delta_h^S$, $\Delta_l(-\lambda) = \Delta_l^S$. Taking the derivatives with respect to λ' for $\lambda' = \lambda$, one obtains a linear system of equations for $\dot{\Delta}_h(\lambda)$ and $\dot{\Delta}_l(\lambda)$. Using (22), one can show that $\dot{\Delta}_h(\lambda) > 0$ and $\dot{\Delta}_l(\lambda) > 0$. Symmetrically, one can show that $\dot{\Delta}_h(-\lambda) > 0$ and $\dot{\Delta}_l(-\lambda) > 0$.

Next, taking the derivatives with respect to λ' twice gives

$$\left[r + 1 + \lambda\left(\frac{1}{4} + x^B\right) + \lambda'\left(\frac{1}{4} - y^B\right)\right]\ddot{\Delta}_h(\lambda') - \ddot{\Delta}_l(\lambda') = -\left(\frac{1}{2} - 2x^S\right)\dot{\Delta}_h(\lambda'),$$
$$\left[r + 1 + \lambda\left(\frac{1}{4} + x^B\right) - \lambda'\left(\frac{1}{4} - y^B\right)\right]\ddot{\Delta}_l(\lambda') - \ddot{\Delta}_h(\lambda') = \left(\frac{1}{2} - 2x^S\right)\dot{\Delta}_l(\lambda').$$

A solution to the above system of linear differential equations is in the form of $\dot{\Delta}_h(\lambda') = a_h e^{\alpha\lambda'} + b_h e^{\beta\lambda'}$ and $\dot{\Delta}_l(\lambda') = a_l e^{\alpha\lambda'} + b_l e^{\beta\lambda'}$ for some real numbers a_h, b_h, a_l, b_l, and α, β not dependent on λ'. There are three cases:

- If $\dot{\Delta}_h(\lambda')$ is monotone in $\lambda' \in [-\lambda, \lambda]$, then $\dot{\Delta}_h(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$.
- If $\dot{\Delta}_h(\lambda')$ is not monotone in $\lambda' \in [-\lambda, \lambda]$, and if $\alpha\beta > 0$, then $\dot{\Delta}_h(\lambda') \to 0$ when

$\lambda' \to \infty$ or $\lambda' \to -\infty$. Since the equation $\ddot{\Delta}_h(\lambda') = 0$ admits at most one solution $\lambda' \in \mathbb{R}$, it must be that $\dot{\Delta}_h(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$. Otherwise, if $\dot{\Delta}_h(\lambda') \leq 0$ for some $\lambda' \in [-\lambda, \lambda]$, $\ddot{\Delta}_h(\lambda') = 0$ for at least two different values of $\lambda' \in \mathbb{R}$.

- If $\dot{\Delta}_h(\lambda')$ is not monotone $\lambda' \in [-\lambda, \lambda]$, and if $\alpha\beta < 0$, then $a_h b_h > 0$ and thus $\dot{\Delta}_h$ is either convex or concave. Hence, $\dot{\Delta}_h(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$.

Altogether, $\dot{\Delta}_h(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$. Likewise, $\dot{\Delta}_l(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$. Figure 2 plots the numerical values of $\Delta_h(\lambda')$ and $\Delta_l(\lambda')$ as a function of $\lambda' \in [-\lambda, \lambda]$.

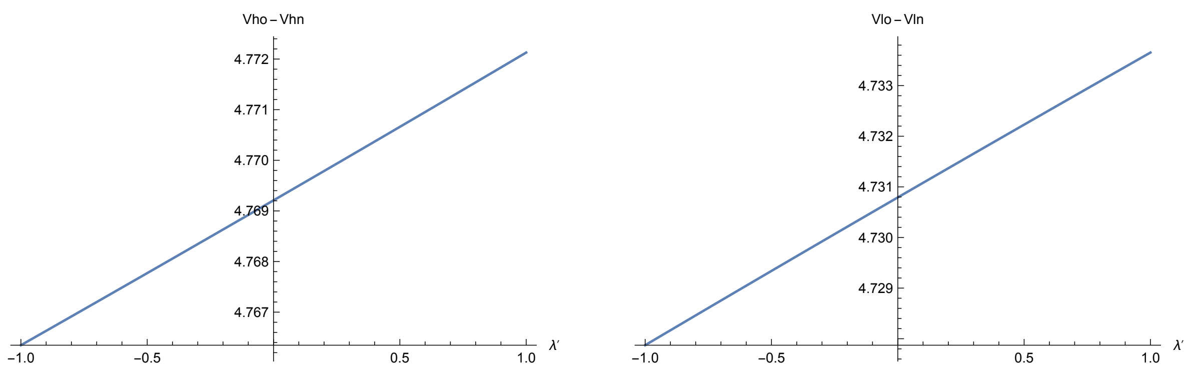

Figure 2: This figure plots $\Delta_h(\lambda') = V_{ho}(\lambda') - V_{hn}(\lambda')$ (left) and $\Delta_l(\lambda') = V_{lo}(\lambda') - V_{ln}(\lambda')$ (right) as a function of $\lambda' \in [-\lambda, \lambda]$.

Step 2: I show that over $\lambda' \in [-\lambda, \lambda]$, $V_{ho}(\lambda') + V_{ln}(\lambda')$ and $V_{hn}(\lambda') + V_{lo}(\lambda')$ are maximized when $\lambda' = \pm\lambda$. Since $V_{ho}(\lambda') + V_{ln}(\lambda')$ and $V_{hn}(\lambda') + V_{lo}(\lambda')$ are even in λ' by symmetry, it suffices to show that $V_{ho}(\lambda') + V_{ln}(\lambda')$ and $V_{hn}(\lambda') + V_{lo}(\lambda')$ are convex in $\lambda' \in [-\lambda, \lambda]$. Taking the second derivatives of (27) + (26) and (24) + (25) with respect to λ' yield

$$c_1(\ddot{V}_{ho}(\lambda') + \ddot{V}_{ln}(\lambda')) - c_2(\ddot{V}_{hn}(\lambda') + \ddot{V}_{lo}(\lambda')) = c_3\dot{\Delta}_h(\lambda') + c_4\dot{\Delta}_l(\lambda'),$$
$$c_5(\ddot{V}_{ho}(\lambda') + \ddot{V}_{ln}(\lambda')) - c_6(\ddot{V}_{hn}(\lambda') + \ddot{V}_{lo}(\lambda')) = c_7\dot{\Delta}_h(\lambda') + c_8\dot{\Delta}_l(\lambda'),$$

for some $c_1, \ldots, c_8 > 0$ not dependent on λ'. Solving the system of linear equations above gives $\ddot{V}_{ho}(\lambda') + \ddot{V}_{ln}(\lambda') = a\dot{\Delta}_h(\lambda') + b\dot{\Delta}_l(\lambda')$ and $\ddot{V}_{hn}(\lambda') + \ddot{V}_{lo}(\lambda') = c\dot{\Delta}_h(\lambda') + d\dot{\Delta}_l(\lambda')$ for some $a, b, c, d > 0$. Step 1 established that $\dot{\Delta}_h > 0(\lambda')$ and $\dot{\Delta}_l(\lambda') > 0$ for every $\lambda' \in [-\lambda, \lambda]$, therefore, $V_{ho}(\lambda')+V_{ln}(\lambda')$ and $V_{hn}(\lambda')+V_{lo}(\lambda')$ are both convex in $\lambda' \in [-\lambda, \lambda]$. Figure 3 plots the numerical values of $V_{ho}(\lambda') + V_{ln}(\lambda')$ and $V_{hn}(\lambda') + V_{lo}(\lambda')$ as a function of $\lambda' \in [-\lambda, \lambda]$.

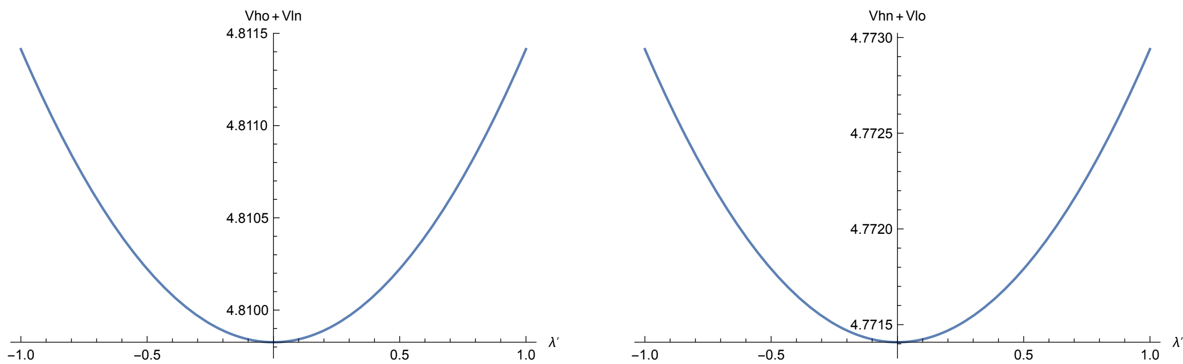

Figure 3: This figure plots $V_{ho}(\lambda')+V_{ln}(\lambda')$ (left) and $V_{hn}(\lambda')+V_{lo}(\lambda')$ (right) as a function of $\lambda' \in [-\lambda, \lambda]$.

□

PROOF OF COROLLARY 1. The proof of Theorem 1 generalizes in a straightforward manner to prove Corollary 1. □

PROOF OF COROLLARY 2. I consider a B-investor i who acquires an alternative total search capacity $\tilde{\lambda}$, and allocates $\lambda' \leq \tilde{\lambda}$ toward the S-investors. The value function V of investor i

solves the HJB equation

$$
\begin{aligned}
rV_{hn} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(\frac{V_{ho}-V_{hn}+V_{ln}^S-V_{lo}^S}{2}\right)^+ \\
& \qquad + y^S\left(\frac{1}{2}-x^S\right)\left(\frac{V_{ho}-V_{hn}+V_{hn}^S-V_{ho}^S}{2}\right)^+\Bigg] \\
& + \left(\tilde{\lambda}-\lambda'\right)\Bigg[\quad y^B\left(\frac{V_{ho}-V_{hn}+V_{ln}^B-V_{lo}^B}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(\frac{V_{ho}-V_{hn}+V_{hn}^B-V_{ho}^B}{2}\right)^+\Bigg] + (V_{ln}-V_{hn}) \\
rV_{lo} = \quad & (\lambda+\lambda')\Bigg[\quad x^S\left(\frac{V_{ln}-V_{lo}+V_{ho}^S-V_{hn}^S}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^S\right)\left(\frac{V_{ln}-V_{lo}+V_{lo}^S-V_{ln}^S}{2}\right)^+\Bigg] \\
& + \left(\tilde{\lambda}-\lambda'\right)\Bigg[\quad x^B\left(\frac{V_{ln}-V_{lo}+V_{ho}^B-V_{hn}^B}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^B\right)\left(\frac{V_{ln}-V_{lo}+V_{lo}^B-V_{ln}^B}{2}\right)^+\Bigg] + (V_{ho}-V_{lo}) + 1 - \delta \\
rV_{ln} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(\frac{V_{lo}-V_{ln}+V_{ln}^S-V_{lo}^S}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^S\right)\left(\frac{V_{lo}-V_{ln}+V_{hn}^S-V_{ho}^S}{2}\right)^+\Bigg] \\
& + \left(\tilde{\lambda}-\lambda'\right)\Bigg[\quad y^B\left(\frac{V_{lo}-V_{ln}+V_{ln}^B-V_{lo}^B}{2}\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(\frac{V_{lo}-V_{ln}+V_{hn}^B-V_{ho}^B}{2}\right)^+\Bigg] + (V_{hn}-V_{ln})
\end{aligned}
$$

$$
\begin{aligned}
rV_{ho} = \quad & (\lambda + \lambda') \Bigg[\quad x^S \left(\frac{V_{hn} - V_{ho} + V_{ho}^S - V_{hn}^S}{2} \right)^+ \\
& \qquad + \left(\frac{1}{2} - y^S \right) \left(\frac{V_{hn} - V_{ho} + V_{lo}^S - V_{ln}^S}{2} \right)^+ \Bigg] \\
& + \left(\tilde{\lambda} - \lambda' \right) \Bigg[\quad x^B \left(\frac{V_{hn} - V_{ho} + V_{ho}^B - V_{hn}^B}{2} \right)^+ \\
& \qquad + \left(\frac{1}{2} - y^B \right) \left(\frac{V_{hn} - V_{ho} + V_{lo}^B - V_{ln}^B}{2} \right)^+ \Bigg] + (V_{lo} - V_{ho}) + 1.
\end{aligned}
$$

The above HJB equation differs from the HJB equation (23) only in the search capacity allocated to the B-investors, which changes from $\lambda - \lambda'$ to $\tilde{\lambda} - \lambda'$. The same two-step proof as in the proof of Theorem 1 works to show that (1) asset concentration is the optimal trading strategy, and (2) over $\lambda' \in [0, \tilde{\lambda}]$, $V_{ho}(\lambda', \tilde{\lambda}) + V_{ln}(\lambda', \tilde{\lambda})$ and $V_{hn}(\lambda', \tilde{\lambda}) + V_{lo}(\lambda', \tilde{\lambda})$ are maximized when $\lambda' = \tilde{\lambda}$. That is, it is optimal for investor i to allocate all her search capacity $\tilde{\lambda}$ toward the S-investors.

The remaining proof is given by the analysis preceding Corollary 2. $\square$

PROOF OF COROLLARY 3. In the equilibrium where every investor chooses to become soft, I consider a B-investor i who deviates to becoming tough and allocates a search capacity

$\lambda' \leq \lambda$ toward the S-investors. The value function V of investor i solves the HJB equation

$$
\begin{aligned}
rV_{hn} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(V_{ho}-V_{hn}+V_{ln}^S-V_{lo}^S\right)^+ \\
& \qquad + y^S\left(\frac{1}{2}-x^S\right)\left(V_{ho}-V_{hn}+V_{hn}^S-V_{ho}^S\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad y^B\left(V_{ho}-V_{hn}+V_{ln}^B-V_{lo}^B\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(V_{ho}-V_{hn}+V_{hn}^B-V_{ho}^B\right)^+\Bigg] + (V_{ln}-V_{hn}) \\
rV_{lo} = \quad & (\lambda+\lambda')\Bigg[\quad x^S\left(V_{ln}-V_{lo}+V_{ho}^S-V_{hn}^S\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^S\right)\left(V_{ln}-V_{lo}+V_{lo}^S-V_{ln}^S\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad x^B\left(V_{ln}-V_{lo}+V_{ho}^B-V_{hn}^B\right)^+ \\
& \qquad + \left(\frac{1}{2}-y^B\right)\left(V_{ln}-V_{lo}+V_{lo}^B-V_{ln}^B\right)^+\Bigg] + (V_{ho}-V_{lo}) + 1 - \delta \\
rV_{ln} = \quad & (\lambda+\lambda')\Bigg[\quad y^S\left(V_{lo}-V_{ln}+V_{ln}^S-V_{lo}^S\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^S\right)\left(V_{lo}-V_{ln}+V_{hn}^S-V_{ho}^S\right)^+\Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad y^B\left(V_{lo}-V_{ln}+V_{ln}^B-V_{lo}^B\right)^+ \\
& \qquad + \left(\frac{1}{2}-x^B\right)\left(V_{lo}-V_{ln}+V_{hn}^B-V_{ho}^B\right)^+\Bigg] + (V_{hn}-V_{ln})
\end{aligned}
$$

$$
\begin{aligned}
rV_{ho} = \quad & (\lambda+\lambda')\Bigg[\quad x^S \left(V_{hn} - V_{ho} + V_{ho}^S - V_{hn}^S\right)^+ \\
& \qquad + \left(\frac{1}{2} - y^S\right) \left(V_{hn} - V_{ho} + V_{lo}^S - V_{ln}^S\right)^+ \Bigg] \\
& + (\lambda-\lambda')\Bigg[\quad x^B \left(V_{hn} - V_{ho} + V_{ho}^B - V_{hn}^B\right)^+ \\
& \qquad + \left(\frac{1}{2} - y^B\right) \left(V_{hn} - V_{ho} + V_{lo}^B - V_{ln}^B\right)^+ \Bigg] + (V_{lo} - V_{ho}) + 1.
\end{aligned}
$$

The above HJB equation differs from the HJB equation (23) only in the gains from trade, which are no longer divided by 2 as investor i has all the bargaining power and keeps all the trading surplus. The same two-step proof as in the proof of Theorem 1 works to show that (1) asset concentration is the optimal trading strategy, and (2) over $\lambda' \in [0, \lambda]$, $V_{ho}(\lambda') + V_{ln}(\lambda')$ and $V_{hn}(\lambda') + V_{lo}(\lambda')$ are maximized when $\lambda' = \lambda$. That is, it is optimal for investor i to allocate all her search capacity λ toward the S-investors.

The remaining proof is given by the analysis preceding Corollary 3. □

B A Counter Example to Chang and Zhang (2021)

In a 3-period example ($N = 3$), I show that the two-sided matching and asset concentration policy strictly dominates the positive assortative matching and intermediation policy in the Lemma 1 of Chang and Zhang (2021).

In period 0, the type distribution of every investor is

$$
\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}.
$$

I first consider the policy of Chang and Zhang (2021).

In period 1, investors are split into two groups: those who received immediacy and those who provided immediacy. The type distribution of an investor who received immediacy becomes

$$\begin{pmatrix} \frac{1}{4}-\frac{1}{16}-\frac{1}{16} & \frac{1}{4}+\frac{1}{16}+\frac{1}{16} \\ \frac{1}{4}+\frac{1}{16}+\frac{1}{16} & \frac{1}{4}-\frac{1}{16}-\frac{1}{16} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{8} \end{pmatrix}.$$

In each matrix cell, the first number is the prior probability. The second number is the probability of a fundamental trade, the third number is the probability of a non-fundamental trade.

The type distribution of an investor who provided immediacy becomes

$$\begin{pmatrix} \frac{1}{4}-\frac{1}{16}+\frac{1}{16} & \frac{1}{4}+\frac{1}{16}-\frac{1}{16} \\ \frac{1}{4}+\frac{1}{16}-\frac{1}{16} & \frac{1}{4}-\frac{1}{16}+\frac{1}{16} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}.$$

In period 2, the 2 groups of investors are further split into 4 groups, with type distributions

$$\begin{pmatrix} \frac{1}{8}-\frac{1}{64}-\frac{3}{64} & \frac{3}{8}+\frac{1}{64}+\frac{3}{64} \\ \frac{3}{8}+\frac{1}{64}+\frac{3}{64} & \frac{1}{8}-\frac{1}{64}-\frac{3}{64} \end{pmatrix} = \begin{pmatrix} \frac{1}{16} & \frac{7}{16} \\ \frac{7}{16} & \frac{1}{16} \end{pmatrix},$$

$$\begin{pmatrix} \frac{1}{8}-\frac{1}{64}+\frac{3}{64} & \frac{3}{8}+\frac{1}{64}-\frac{3}{64} \\ \frac{3}{8}+\frac{1}{64}-\frac{3}{64} & \frac{1}{8}-\frac{1}{64}+\frac{3}{64} \end{pmatrix} = \begin{pmatrix} \frac{5}{32} & \frac{11}{32} \\ \frac{11}{32} & \frac{5}{32} \end{pmatrix},$$

$$\begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{8} \end{pmatrix},$$

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}.$$

Since only fundamental trades directly generates surplus, and non-fundamental trades are beneficial only by creating more future fundamental trades, it is sufficient to compute the total volume of fundamental trades.

Period 1: $\frac{1}{2}\left(2\frac{1}{4}\frac{1}{4}\right) = 2^{-4}$ (That is, measure $\frac{1}{2}$ of pairs, in each pair two possibilities for fundamental trades, each has probability $\frac{1}{4} \times \frac{1}{4}$.)

Period 2: $\frac{1}{4}\left(2\frac{1}{8}\frac{1}{8}\right) + \frac{1}{4}\left(2\frac{1}{4}\frac{1}{4}\right) = 2^{-7} + 2^{-5}$.

Period 3: $\frac{1}{8}\left(2\frac{1}{16}\frac{1}{16}\right) + \frac{1}{8}\left(2\frac{5}{32}\frac{5}{32}\right) + \frac{1}{8}\left(2\frac{1}{8}\frac{1}{8}\right) + \frac{1}{8}\left(2\frac{1}{4}\frac{1}{4}\right) = 2^{-10} + 25 \cdot 2^{-12} + 2^{-8} + 2^{-6}$.

Next, I consider the two-sided matching and asset concentration policy: In period 0, investors are divided into two equal groups, B and S. In each period, match an B-investor with a S-investor. If a fundamental trade is feasible between the two investor, I let it occur. If instead the two investors have the same preference, and only one of them owns the asset, I let the B-investor holds the asset. Then in period 1, the type distribution of a B-investor becomes

$$\begin{pmatrix} \frac{1}{4} - \frac{1}{16} - \frac{1}{16} & \frac{1}{4} + \frac{1}{16} + \frac{1}{16} \\ \frac{1}{4} + \frac{1}{16} - \frac{1}{16} & \frac{1}{4} - \frac{1}{16} + \frac{1}{16} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}.$$

The type distribution of an S-investor becomes

$$\begin{pmatrix} \frac{1}{4} - \frac{1}{16} + \frac{1}{16} & \frac{1}{4} + \frac{1}{16} - \frac{1}{16} \\ \frac{1}{4} + \frac{1}{16} + \frac{1}{16} & \frac{1}{4} - \frac{1}{16} - \frac{1}{16} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{8} & \frac{1}{8} \end{pmatrix}.$$

In period 2, the type distributions of a B-investor and an S-investor become

$$\begin{pmatrix} \frac{1}{8}-\frac{1}{64}-\frac{1}{32} & \frac{3}{8}+\frac{1}{64}+\frac{1}{32} \\ \frac{1}{4}+\frac{1}{16}-\frac{1}{32} & \frac{1}{4}-\frac{1}{16}+\frac{1}{32} \end{pmatrix} = \begin{pmatrix} \frac{5}{64} & \frac{27}{64} \\ \frac{9}{32} & \frac{7}{32} \end{pmatrix},$$
$$\begin{pmatrix} \frac{1}{4}-\frac{1}{16}+\frac{1}{32} & \frac{1}{4}+\frac{1}{16}-\frac{1}{32} \\ \frac{1}{8}+\frac{1}{64}+\frac{1}{32} & \frac{1}{8}-\frac{1}{64}-\frac{1}{32} \end{pmatrix} = \begin{pmatrix} \frac{7}{32} & \frac{9}{32} \\ \frac{27}{64} & \frac{5}{64} \end{pmatrix}.$$

The volume of fundamental trades is

Period 1: $\frac{1}{2}\left(2\frac{1}{4}\frac{1}{4}\right) = 2^{-4}$.

Period 2: $\frac{1}{2}\left(\frac{1}{64}+\frac{1}{16}\right) = 2^{-7}+2^{-5}$.

Period 3: $\frac{1}{8}\left(\frac{5}{64}\frac{5}{64}+\frac{7}{32}\frac{7}{32}\right) = 25\cdot 2^{-13}+492^{-11}$.

Hence the asset concentration policy has the same volume of fundamental trades in Periods 1 and 2 as the intermediation policy, but a Period-3 volume of

$$(25+196)\cdot 2^{-13} = 221\cdot 2^{-13},$$

which is greater than the Period-3 volume of the intermediation policy of

$$(8+50+32+128)\cdot 2^{-13} = 218\cdot 2^{-13}.$$

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