Banking on Uninsured Deposits*  
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Abstract  
Motivated by the regional bank crisis of 2023, we model the impact of interest rates on the liquidity risk of banks. Prior work shows that banks hedge the interest rate risk of their assets with their deposit franchise: when interest rates rise, the value of the assets falls but the value of the deposit franchise rises. Yet the deposit franchise is only valuable if depositors remain in the bank. This creates run incentives for uninsured depositors. We show that a run equilibrium is absent at low interest rates but appears when rates rise because the deposit franchise comes to dominate the value of the bank. The liquidity risk of the bank thus increases with interest rates. We provide a formula for the bank’s optimal risk management policy. The bank should act as if its deposit rate is more sensitive to market rates than it really is, i.e., as if its “deposit beta” is higher. This leads the bank to shrink the duration of its assets. Shortening duration has a downside, however: it exposes the bank to insolvency if interest rates fall. The bank thus faces a dilemma: it cannot simultaneously hedge its interest rate risk and liquidity risk exposures. The dilemma disappears only if uninsured deposits do not contribute to the deposit franchise (if they have a deposit beta of one). The recent growth of low-beta uninsured checking and savings accounts thus poses stability risks to banks. The risks increase with interest rates and are amplified by other exposures such as credit risk. We show how they can be addressed with an optimal capital requirement that rises with interest rates.

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1 Introduction

Why did the recent rise in interest rates destabilize several banks such as Silicon Valley Bank? One view is that these banks were insolvent due to losses on long-term loans and securities, and that this triggered a run by uninsured depositors. Yet alongside these losses, banks were making record profits on deposits by paying deposit rates that were far below market rates. A simple calculation shows that the higher profits on deposits roughly offset the asset-side losses of the banking system (Drechsler et al., 2023a). This is not by chance: prior work finds that banks hedge the interest rate risk of their assets with their deposit franchise (Drechsler et al., 2021). But if banks are hedged, why did the rise in rates destabilize them?

For the deposit franchise hedge to work, depositors must remain with the bank. Conversely, if depositors run, the deposit franchise is destroyed. Therefore, the deposit franchise creates run incentives among uninsured depositors. The run incentives intensify when the value of the deposit franchise rises relative to the value of the bank’s assets. This happens when interest rates go up because that is when deposit profits grow while asset values shrink. A bank that is hedged to interest rates absent a run thus becomes exposed to a run when interest rates rise. The 2023 regional bank crisis can therefore be understood as stemming from the impact of interest rates on the runnability of banks’ uninsured deposit franchise.

We provide a model of this impact and study its implications for bank risk management and capital regulation. Following Drechsler et al. (2021), we model a bank with a low “deposit beta” – a low sensitivity of deposit rates to the market interest rate. The bank then earns a deposit spread that rises with the interest rate. This is the source of its deposit franchise. The deposit franchise does not come for free: the bank pays an operating cost to maintain it. The deposit franchise is effectively an interest rate swap in which the bank pays fixed (the operating cost) and receives floating (the deposit spread). This swap has negative duration. The bank hedges it by investing in assets with positive duration; by holding long-term loans and securities.

We depart from Drechsler et al. (2021) by introducing deposit outflows. They can be due to two reasons. The first is interest rates: as rates rise, the deposit spread widens, and this leads some depositors to seek higher-paying alternatives like money market mutual funds. This is the deposits channel of monetary policy
of Drechsler et al. (2017). The outflows it induces are relatively mild because the bank effectively chooses to have them by choosing a profit-maximizing deposit spread. Nevertheless, we show that they shorten the negative duration of the deposit franchise, leading the bank to shorten the duration of its assets. We provide a formula for how much: the bank should act as if its deposit beta is slightly higher. Using estimates of the sensitivity of outflows to monetary policy from Drechsler et al. (2017), we find that on average banks should use an effective beta that is 0.05 higher than their measured beta. Interest rate-driven outflows can thus be cured by interest rate risk management.

The second reason for outflows is a run by uninsured depositors. Uninsured depositors have an incentive to run if the value of their claims exceeds the value of the bank if they do run. In standard models (Diamond and Dybvig, 1983) this is due to fire sales of the bank’s loans. There are no such fire sales in our model; the bank’s assets are fully liquid. Runs are instead due to the nature of the deposit franchise. When a deposit is withdrawn, the bank loses the stream of deposit spreads net of operating costs it would have earned on that deposit. In effect, the deposit franchise is subject to an extreme form of “fire sale”: its value is fully destroyed in a run. Moreover, since its value is increasing in interest rates, a run is more destructive – and hence more likely – at high interest rates.

Our model thus implies that a bank’s liquidity risk is increasing in interest rates. When interest rates are low, the value of the deposit franchise is small and the value of the bank’s assets is high. A run does not occur because the value of the bank would be unaffected if it did occur. But when interest rates rise and the deposit franchise comes to dominate the value of the bank, a run equilibrium arises. This is true even if the bank is fully hedged to interest rates in the sense that its value is insensitive to interest rate shocks outside the run equilibrium.

The impact of interest rates on liquidity risk again pushes the bank to shorten its asset duration. This limits the fall in the value of the bank’s assets if interest rates rise. Higher asset values deter uninsured depositors from running. Our formula for the bank’s optimal duration again shows that it acts as if its deposit beta is higher. The amount depends on the share of uninsured deposits. When this share is large, a run is more destructive. The greater liquidity risk induces the bank to set a significantly shorter duration ex ante.

In contrast to interest rate-driven outflows, shortening duration to hedge liq-
uidity risk is not a cure. It makes the bank unhedged to interest rates. If interest rates fall instead of rising, the shorter-duration assets fail to appreciate enough to offset the decline in the value of the deposit franchise. In practical terms, the bank’s assets do not generate enough income to cover its operating costs. If the drop in rates is large enough, the bank becomes insolvent. It need not suffer a run given its high asset value and low deposit franchise value. But insolvency turns it into a “zombie bank”. While the literature on zombie banks stresses regulatory forbearance, in our framework they arise from unhedged exposure to a drop in interest rates.

At the heart of our model is a risk management dilemma: the bank can hedge itself to interest rates or liquidity risk but not both. If it hedges to interest rates, it becomes exposed to a run if interest rates rise. If it hedges to liquidity risk, it becomes exposed to insolvency if rates fall. The dilemma disappears only if uninsured deposits do not contribute to the deposit franchise; if they have a deposit beta of one and incur zero operating costs. In this case the bank’s interest rate and liquidity risk management objectives align. The bank invests these deposits in short-term assets.

The bank’s risk management dilemma therefore arises from low-beta uninsured deposits. This is interesting because it explains why previous interest rate cycles did not trigger similar instability. Historically, uninsured deposits were primarily large time deposits and other forms of wholesale funding. These deposits have a beta close to one and do not require expensive branch networks. As Drechsler et al. (2023c) show, this changed after 2012 when uninsured deposits became primarily checking and savings accounts – low beta, high cost.¹ It is this decoupling of interest rate and liquidity risk that creates the bank’s risk management dilemma. Our framework therefore implies that uninsured checking and savings accounts pose an ongoing risk to the banking system.

How can regulation address this risk? First, we show that banks benefit from embedding option-like features such as interest rate floors in their loans, or buying

¹In the case of Silicon Valley Bank (SVB), almost all of its deposits were uninsured corporate checking and savings accounts. These have a low beta and a high operating cost, which calls for a long asset duration. However, the fact that they are uninsured calls for a short duration. One interpretation of SVB’s failure is that it ignored the second consideration. It may have also overdone the first.
interest rate options such as swaptions. While interest rate floors and swaptions are costly up front, their non-linear payoffs ensure that the bank earns enough income to cover its operating costs in case rates fall. The bank can then set its asset duration relatively short to mitigate liquidity risk in case rates rise. Interest rate floors and swaptions can therefore help avoid zombie banks at low interest rates and runs at high rates.

A limitation of option-based risk management is that it can be disrupted by other unhedged shocks such as credit losses on the bank’s investments. These losses reduce the value of the bank’s assets, and this can again trigger a run on the uninsured portion of the deposit franchise. We show that the destabilizing impact of credit losses is itself increasing in interest rates.

A more robust approach to resolving the bank’s risk management dilemma is a capital buffer. A large-enough capital buffer deters a run and allows the bank to set its duration to hedge its interest rate risk. We solve for the optimal capital requirement and find it is increasing in interest rates, again because higher rates lead to more liquidity risk. This result provides a new rationale for a pro-cyclical capital requirement. Different from the literature, it is one purely based on interest rates rather than the state of the economy.

Our final analysis considers the implications for monetary policy. How should the central bank respond to a negative credit risk shock? If banks are hedging liquidity risk and are hence exposed to a drop in interest rates, then an interest rate cut can make them insolvent. In an extreme case, it can destroy their deposit franchise entirely, a form of “reversal rate” (Abadi et al., 2022). This reinforces the case for option-based or capital regulation because it allows banks to fully hedge interest rates. The central bank then does not have to face its own financial stability dilemma in setting interest rates.

2 Related Literature

Our paper belongs to the literature on banks’ risk management (e.g., Freixas and Rochet 2008, English et al. 2018, Nagel and Purnanandam 2020, Di Tella and Kurlat 2020). The buyer of a swaption has the right to enter into an interest rate swap at a fixed rate. The buyer of a receiver option receives the fixed rate and pays floating. The buyer of a payer option pays fixed and receives floating. To implement risk management using swaptions, the bank should buy short-term assets and receiver swaptions or, equivalently, long-term assets and payer swaptions.
Closest is Drechsler et al. (2021), which shows that when banks have deposit market power, their deposit franchise creates a long-term liability that should be hedged with long-term assets. Drechsler et al. (2021) focuses on a simplified setting without deposit outflows. Our paper starts from the observation that there are two kinds of outflows that affect the value of the deposit franchise. First, rate hikes induce outflows when banks do not adjust their deposit rates one-for-one; this is the “deposit channel of monetary policy” (Drechsler et al., 2017). Depositors substitute towards higher-return liquid assets such as money market funds (Xiao, 2018). While these outflows are important to consider and do call for a shorter asset duration, they happen during every monetary policy cycle and are relatively predictable and hence manageable compared to the second kind of outflows: runs on uninsured deposits. As Hanson et al. (2015) argue, these types of outflows have a larger impact on the ability of financial institutions to invest in long-term assets.

Our analysis of the runs on the deposit franchise builds on Diamond and Dybvig (1983) and the subsequent literature. The key point is that the deposit franchise is a large but runnable part of the bank’s value: it relies on depositors remaining with the bank and tolerating low deposit rates. This creates important differences with the standard source of runs in the literature: asset illiquidity as in Diamond and Dybvig (1983). Notably, runs in our model do not rely on fire sales, hence they can happen even when a single bank is affected and when it holds only high-quality liquid assets (as SVB did). Moreover, in our framework liquidity risk increases with the level of interest rates. This has several implications, for instance how banks’ duration mismatch and capital requirements should change with interest rates. Another implication is that bank runs are less likely in a low-rate environment.

In line with recent theoretical work on optimal liquidity regulation (Diamond and Kashyap 2016, Dewatripont and Tirole 2018), we add a hedging perspective by studying what assets the bank should hold to prevent runs, and how liquidity risk management is in tension with interest rate risk management.

Our emphasis on the deposit franchise as an important part of banks’ business model builds on the large literature on the role of banks as private providers of liquid assets, e.g., Gorton and Pennacchi (1990), Kashyap et al. (2002), Stein (2012), DeAngelo and Stulz (2015), Hanson et al. (2015), Dang et al. (2017), Moreira and Savov (2017). Motivated by the zero lower bound period after the 2008 Financial
Crisis, recent work has focused on the harmful effect of persistently low interest rates on the deposit franchise (e.g., Abadi et al. 2022, Ulate 2021, Wang 2022, Wang et al. 2022). This channel is why short asset durations are problematic when interest rates fall.

Our paper is motivated by the 2023 regional bank crisis, which was sparked by asset losses due to an increase in interest rates as opposed to credit shocks. This makes it reminiscent of the Savings & Loans (S&L) crisis of the 1980s. S&Ls held long-duration mortgages, which they hedged with near-zero beta deposits due to the deposit rate ceilings under Regulation Q. When Reg Q was repealed at the end of the 1970s, S&Ls’ deposit betas shot up, making them insolvent (FDIC, 1997, Drechsler et al., 2022).

For the current crisis, Jiang et al., 2023 use detailed bank data and estimate a total decline in asset values of $2.1 trillion. Drechsler et al., 2023b find a similar decline using aggregate data. Drechsler et al., 2023a estimate that the increase in the value of the deposit franchise roughly offsets the decline in asset values, though they emphasize that this valuation is uncertain given the behavioral assumptions on depositors.

Finally, our paper also brings interesting connections to the international finance literature on reserve assets and the Triffin (1961) dilemma (more recently, Farhi and Maggiori, 2017, and He et al., 2019). Triffin (1961) warned that persistent U.S. current account deficits would ultimately lead to a collapse of the dollar, while Despres et al. (1966) argued that the U.S. deficits could remain stable as they were implicitly backed by the present value of risk premia on its external assets and, closer to us, liquidity premia on its liabilities. In a sense, one can think of U.S. debt as the world’s uninsured deposit franchise.

3 A Model of the Deposit Franchise with Outflows

We begin with a bank facing interest rate-driven outflows as in the deposits channel of monetary policy (Drechsler et al., 2017). The bank is able to pay below-market rates on its deposit liabilities, i.e., it has a deposit franchise. However, maintaining the deposit franchise is costly, requiring operating costs $c$ per dollar of deposits each period.
**Timing.** Time is discrete, $t = 0, 1, \ldots$. The initial interest rate is $r$. Then at the end of period $t = 0$ an interest rate shock is realized, the interest rate becomes $r'$ and remains constant forever after. At the beginning of $t = 0$, before the interest rate shock is realized, the bank chooses a portfolio of assets to hedge against the shock, taking into account deposit withdrawals.

**Withdrawal process.** Starting with a deposit base $D_{-1} = D$ at the beginning of $t = 0$, suppose that after the interest rate $r'$ and the deposit rate $r'_d$, depositors withdraw a fraction $\omega(s'_d, r')$, i.e.,

$$D_0 = [1 - \omega(s'_d, r')] D,$$

where $s'_d = r' - r'_d$ is the deposit spread. The outflow rate $\omega$ is increasing in the deposit spread $s'_d$ set by the bank and decreasing in $r'$.

We start by focusing on insured deposits; in Section 4 we extend the framework to uninsured deposits whose withdrawal rate also depends on the bank’s value. The two arguments in the demand function $\omega$ capture the “deposits channel” of monetary policy (Drechsler et al., 2017): depositors withdraw if the deposit rate $r'_d$ is too low relative to $r'$, in order to substitute towards higher-yield assets such as money market funds. Moreover, at higher rates, the opportunity cost of cash is higher, which makes deposit demand less elastic and allows banks to charge a larger spread $r' - r'_d$.

**Deposit pricing.** The bank sets a deposit rate $r'_d$ once and for all after the realization of the $r'$ shock at the beginning of period $t = 0$ according to $r_d(r)$. In principle the bank sets $r_d(r)$ optimally, for instance to maximize the deposit franchise value $DF$, taking into account both the outflows and the operating costs on the remaining deposit base. For now we take as given a pricing function

$$r_d(r') = \beta r'$$

and $\beta \in [0, 1]$ is the deposit beta. The withdrawal rate under the deposit pricing policy is

$$w(r') = \omega ((1 - \beta) r', r').$$

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3Our results can be readily extended to non-linear deposit pricing policies $r_d(r)$, defining a local beta $\beta = \frac{\partial r_d(r)}{\partial r}$. Betas tend to be lower at low rates (Wang, 2022).
Bank valuation. The bank has assets with value $A$ and deposit liabilities that require operating costs $c$ per dollar starting from $t = 1$. The market value of the bank at the start of period $t = 0$ is

$$V = A - L,$$

where $A(r)$ is the total market value of assets such as loans, Treasuries, and mortgage-backed securities (MBS) as a function of the market interest rate $r$, and $L$ is the market value of liabilities that include deposit withdrawals, interest expenses, and operating costs. We adopt the timing convention that withdrawals take place at the start of the period while interest expenses and operating costs are paid at the end. The market to book ratio of liabilities is

$$\frac{L}{D} = w + (1 - w) \sum_{t=1}^{\infty} \frac{\beta r + c}{(1 + r)^t},$$

which is an intangible asset such that

$$V = A - D + DF,$$

where $A - D$ is the bank’s book value. Equivalently, it is the bank’s total value if deposits paid the market rate ($\beta = 1$) and incurred no operating cost ($c = 0$), i.e. if there was no deposit franchise ($DF = 0$).

From the previous expression, we have

$$DF(r) = D (1 - w (r)) \left( 1 - \beta - \frac{c}{r} \right).$$

We normalize outflows before the shock to $w = 0$. Since we consider duration measures which are first-order approximations, we can then evaluate all durations

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4 We assume the cost to acquire and maintain the initial deposit base $D$ has already been paid, which is a convenient normalization.
at \( w = 0 \).

The interesting part is how withdrawals vary with the new interest rate \( r' \). One extreme case is \( w(r') = 1 \), which corresponds to a run at \( t = 0 \). In this case the deposit franchise value falls to 0.

The other extreme case is zero outflows: \( w(r') = 0 \) for any \( r \). This corresponds to a fully captive deposit base \( D \) as in Drechsler et al. (2021). In this case the deposit franchise is simply \( DF = D(1 - \beta - c/r') \). The first part, \( 1 - \beta \), is the present value of deposit spreads, which are equivalent to a floating-rate bond with par value \( 1 - \beta \). The second part, \( -c/r' \), is the cost of a perpetuity paying \( c \) (the operating cost). The formula shows that the deposit franchise is equivalent to an interest rate swap where the bank pays fixed (\( c \)) and receives floating \( (1 - \beta) \).

### 3.1 Hedging Interest Rate Risk without Outflows

We start with the case of no outflows, i.e., \( w'(r) = 0 \). Then locally interest rate risk can be hedged by holding assets whose duration offsets the effective duration of the deposit franchise, i.e.: 

\[
A'(r) + DF'(r) = 0.
\]

This can be accomplished by holding assets with modified duration \( T_A \) such that

\[
T_A = DF'(r).
\]

If we assume that the bank operates with zero initial equity, \( V = 0 \), the value of assets is \( A = D - DF = D(\beta + c/r) \) and therefore

\[
T_A = \frac{1}{r} \left( \frac{c/r}{\beta + c/r} \right),
\]

which decreases with \( \beta \): low-beta banks have a more sensitive deposit franchise value and thus should invest in longer duration assets.

**Quantification: U.S. banks at the end of 2022.** To get a sense of magnitudes, total U.S. deposits \( D \) stood at $17.5 trillion at the end of 2022. For notational simplicity we assume in the model that deposits remaining after the \( t = 0 \) withdrawals have infinite maturity; when quantifying the deposit franchise, we assume instead a
finite maturity $T$ such that the franchise value is

$$DF (r) = D \left( 1 - \beta - \frac{c}{r} \right) \left[ 1 - \frac{1}{(1 + r)^T} \right].$$

We use an average maturity $T = 10$ years, deposit beta $\beta = 0.2$, and operating cost $c = 2\%$. Given these assumptions, the aggregate value of the deposit franchise of all U.S. banks with a long-term interest rate $r = 4\%$ is

$$DF = 17.5 \times (1 - 0.2 - 0.02/0.04) \times \left[ 1 - \frac{1}{1.04^{10}} \right] = $1.7 trillion,

up from $DF = 0$ in early 2021 when $r$ was equal to 2.5\%. This number is very similar to the unrealized losses on the loan and securities portfolio of $1.75$ trillion due to the same increase in rates given an average duration of 3.9 years.

**Remark 1 (Global vs. local hedging.)** The interest rate exposure of the deposit franchise can be perfectly offset by holding on the asset side a perpetuity with fixed coupon $c$ and a floating rate bond with par value $\beta$. This strategy provides a global hedge, whereas duration-matching only ensures a local hedge, in response to small interest rate shocks. Once we allow for outflows there is no simple global hedge and we focus on local immunization.

### 3.2 Hedging Interest Rate-Driven Outflows

Consider now the case with interest-driven deposit outflows with positive “flow beta” $w'(r) > 0$. The sensitivity of the franchise value to $r$ is

$$\frac{DF'}{D} = \frac{c}{r^2} - w' (r) \left( 1 - \beta - \frac{c}{r} \right).$$

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5For $T$, we use a recent FDIC study, see http://fdic.gov/regulations/reform/coredividendstudy.pdf. The number we use, 10 years, is based on the last column of Table 5. Note that it is in the lower range of estimates. For the recent betas, see https://www.gobaker.com/wp-content/uploads/articles/TBG-B2111-Article_Series.pdf. For the operating cost, see Drechsler et al. (2021).
The first part is the same as without outflows. The second part captures the role of outflows. We assume that outflows are not too large

\[
\frac{c}{r^2} \geq w'(r) \left( 1 - \beta - \frac{c}{r} \right)
\]

to ensure that the deposit franchise value remains increasing in interest rates; as we discuss below this condition is easily satisfied in the data.

Taking into account outflows \( w'(r) > 0 \) and assuming again \( A = D - DF = D(\beta + c/r) \), the modified duration for hedging with interest rate-driven outflows is as follows:

**Proposition 1** (Hedging with Interest Rate-Driven Outflows). The modified duration of assets for hedging interest rate-driven outflows is

\[
T_A = \frac{1}{r} \left( \frac{c/r}{\beta + c/r} \right) - w'(r) \left( \frac{1 - \beta - c/r}{\beta + c/r} \right)
\]

Outflows \( w'(r) > 0 \) are equivalent to assuming no outflows \( w' = 0 \) with an effective deposit beta

\[
\tilde{\beta} = \beta + \frac{c}{r^2} - w'(r) \left( \frac{\beta + c/r}{1 - \beta - c/r} \right) \left( 1 - \beta - c/r \right).
\]

The effective beta is higher than the actual beta, \( \tilde{\beta} > \beta \), if and only if the deposit franchise is valuable, \( 1 - \beta > c/r \).

A surprising implication is that the role of outflows is very different at high and low interest rates. When \( 1 - \beta > c/r \) the deposit spread is high enough to make deposit franchise valuable, and therefore deposit outflows \( w'(r) > 0 \) hurt the bank’s value. In that case hedging the extra risk from outflows requires the bank to choose a shorter duration relative to the case without outflows. When \( 1 - \beta < c/r \), the effect of outflows is reversed, and outflows call for a longer duration. The reason is that at low rates, the bank is making losses on deposits once taking operating costs into account, hence an outflow of depositors in response to a rise in rates actually benefits the bank by reducing these losses and increasing the franchise value. It follows that outflows help the bank hedge interest rate risk, hence less hedging is required on the asset side and the optimal duration is longer.
Endogenous deposit pricing and the relation between $\beta$ and $w'$. We treat $\beta$ and $w'$ as exogenous: while they are related through the bank’s deposit-pricing problem, in general $w'$ is not fully determined by $\beta$. The correlation between $\beta$ and $w' (r)$ can take any sign, as $w'$ is given by

$$w' = (1 - \beta) \omega_s + \omega_r,$$

where $\omega_s = \frac{\partial \omega}{\partial s}$, $\omega_r = \frac{\partial \omega}{\partial r}$. Banks facing relatively inelastic depositors (low $\omega_s$) have more market power, which allows them to set a low $\beta$. But given their low $\beta$, these banks could see more or less outflows when rates go up, as $w'$ depends on the product $(1 - \beta) \omega_s$. Theoretically, the correlation between $\beta$ and $w'$ depends on how higher-order properties of the deposit demand function such as the “superelasticity” vary with market power (e.g., Kimball, 1995; Wang and Werning, 2022).

Empirically, the estimates in Drechsler et al. (2017) imply a negative correlation between $\beta$ and $w'$: low $\beta$ banks face stronger rate-induced outflows.

Quantification. Which $w'$ should banks and regulators use to construct effective betas? The strength of normal rate-driven outflows $w'$ can be calibrated using estimates from Drechsler et al. (2017). They show that the typical 400 bps Fed hiking cycle corresponds to a 12% outflow, hence $w' \approx 3$. Using recent average values $\beta = 0.2$, $c = 2\%$ and $r = 4\%$ the normal rate-driven outflows imply an effective beta

$$\tilde{\beta} = 0.25.$$

The average correction $\tilde{\beta} - \beta$ is not negligible. There is also substantial heterogeneity across banks. Most importantly, equation (3) shows that the correction $\tilde{\beta} - \beta$ should be higher for low-$\beta$ banks (as they earn higher deposit spreads hence stand to lose more from outflows) and high $w'$ banks, which also tend to be the low-$\beta$ banks (Drechsler et al., 2017). For instance, their estimates can be approximated by going from $\beta = 0.1$ in high concentration areas to $\beta = 0.3$ in low concentration areas, whereas for outflows $w' = 3.5$ in high concentration areas and $w' = 2.5$ in

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6The numbers for the current hiking cycle are similar. As of March 22, 2023, the Fed funds rate has risen by 4.5% and deposits have shrunk by 4.4% from a year earlier. This gives $w' \approx 1$. Deposit outflows accelerated in March, hence $w'$ is likely to increase closer to the historical norm.
low concentration areas, therefore:

\[ \beta = 0.1 \rightarrow \tilde{\beta} = 0.18 \quad \text{(high concentration)} \]
\[ \beta = 0.3 \rightarrow \tilde{\beta} = 0.33 \quad \text{(low concentration)} \]

Thus correcting for outflows has a modest effect in competitive areas, but corresponds to almost doubling the beta in highly concentrated areas. Interestingly, Drechsler et al. (2021) show that the largest banks (i.e., top 5% or top 10%), which are the economically relevant ones given the highly skewed distribution of bank size, do not match their income and expense betas one-for-one: their income beta is between 5% and 10% higher than their expense beta. One potential explanation is that banks already account partly for the role of outflows \( w' \) as in Proposition 1.

4 Uninsured Deposits and Liquidity Risk

A key assumption so far is that deposits are insured, hence the only outflows are driven by substitution towards high-yield money market funds, but concerns about the strength of the bank play no role. In the presence of uninsured depositors who respond to the bank’s health, however, there is a feedback loop between outflows and the franchise value. A fall in the value of assets \( A \) may trigger run-driven outflows, and the resulting fall in the deposit franchise value can further reduce \( V \), which leads to more outflows, and so on. This means that the effective beta is potentially unstable. In this section we analyze this interaction and its implications for hedging.

Suppose now that there are two kinds of deposits:

\[ D = D_I + D_U, \]

where insured deposits \( D_I \) are exactly as before and \( D_U \) denotes uninsured deposits. The value of the bank is now

\[ V = A - D + DF_I + DF_U, \]

where \( DF_I \) and \( DF_U \) are the deposit franchise values corresponding to insured and uninsured deposits, respectively. The previous analysis applies to insured deposits, whose outflows are given by \( w_I \) as before.
Incentives to run. The key difference is that uninsured deposit outflows also depend on the bank’s solvency. The sensitivity of uninsured depositors’ withdrawals $w_U$ to asset values captures the possibility of runs. We assume that $w_U$ is given by

$$w_U (r) = 1 - [1 - w_U (r)] \lambda (v (r))$$

where $w_U$ is a baseline runoff rate absent any runs as for insured depositors, but with a potentially different sensitivity $w_U' \neq w_I'$ that also satisfies (2), and the increasing function $\lambda$ denotes the fraction of uninsured depositors who do not run as an increasing function of the solvency ratio $v$, given by the ratio of bank value over deposits

$$v (r) = \frac{V (r)}{D}.$$

In the baseline version we assume that incentives to run depend on the total value of the bank $V$, which puts equal weights on liquid assets $A$ and the intangible deposit franchise $DF_I + DF_U$. The implicit assumption is that the bank can issue equity against its deposit franchise to repay depositors. Later we extend the model to allow for a lower pledgeability of the deposit franchise value relative to liquid assets $A$. Therefore the uninsured outflows can be higher than the insured ones both because uninsured depositors are more rate-sensitive (“high $\beta$”) but also because they are concerned about the value of the bank.

We work directly with $\lambda$ and assume a step function around an solvency threshold $\bar{v}$:

$$\lambda (v) = \begin{cases} 
0 & v < \bar{v} \\
1 & v \geq \bar{v}
\end{cases}$$

Figure 1 displays the function $\lambda$. For an uninsured solvency ratio $v$ below $\bar{v}$ (which may be negative) all uninsured depositors run, hence $\lambda = 0$ and $w_U = 1$; if $v$ is above $\bar{v}$ then no uninsured depositor runs, hence $\lambda = 1$ and $w_U = w_U$.

One related interpretation is that the slope of $\lambda$ increases with depositor concentration and access to mobile banking. When the depositor base is diversified across sectors and demographic categories and includes relatively slow-moving depositors, withdrawals are less responsive to news about the bank; our step func-

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7Following the literature on fundamental-based runs (e.g., Rochet and Vives 2004; Goldstein and Pauzner 2005) the model could be extended a step further by deriving the probability of a run as a function $v$ using a richer game with imperfect information among depositors.
Remark 2. Different types of depositors are better defined in terms of their actual withdrawal behavior, but for clarity we label the sleepy depositors “insured” and the flighty ones “uninsured”. In practice the distinction between insured and uninsured depositors is not as clear: some uninsured depositors are “sleepy” while some insured depositors may be flighty because they want to avoid any risk including the burden of going through the FDIC resolution process. Insured depositors’ behavior also depends on the perceived soundness of deposit insurance and the country’s fiscal capacity.

4.1 Runs on the Deposit Franchise

Let \( u = D_{UI}/D \) be the share of uninsured depositors. The solvency ratio of the bank after the interest rate shock as a function of \( \lambda \) around the usual normalization \( w_{UI}(r) = 0 \) is:

\[
v(\lambda, r') = v(0, r') + u \lambda \left( 1 - w_{UI}(r') \right) \left( 1 - \beta_{UI} - c/r' \right).
\]

where \( v(0, r') = \frac{A(r) - D + DF_{I}(r)}{D} \) is the solvency ratio when all uninsured depositors run (\( \lambda = 0 \)). \( DF_{I} \) and thus \( v \) increase with the fraction of remaining depositors \( \lambda \). Inverting this function we get

\[
\Lambda(v, r') = \frac{1}{1 - w_{UI}(r')} \left[ \frac{v - v(0, r')}{u (1 - \beta_{UI} - c/r')} \right].
\]
Λ(ν, r) is defined as satisfying ν(Λ(ν, r), r) = ν, i.e., given r it is the fraction of remaining uninsured depositors that justifies a solvency ratio ν and it increases with ν.

**Definition 1.** Given r, a stable equilibrium (henceforth, equilibrium) is given by a pair of bank value ν and fraction of remaining depositors λ such that

\[ λ(ν) = Λ(ν, r) \] (4)

and Λ crosses λ from below.

Our next main result shows that relying on a large uninsured deposit franchise, for instance at high interest rates, creates the potential for runs:

**Proposition 2** (Interest Rates and Liquidity Risk). *Suppose that the uninsured deposit franchise is profitable: r > r_d^U + c.*

- If ν(0, r) ≥ ν then the unique equilibrium is run-free, i.e., λ = 1.
- If ν(1, r) < ν then the unique equilibrium features λ = 0, i.e., all uninsured depositors run.
- If ν(0, r) < ν ≤ ν(1, r), then both equilibria λ = 0 and λ = 1 coexist.

The run equilibrium is more likely to exist when the share of uninsured deposits u is higher, the interest rate r is higher, and the uninsured deposit beta β^U is lower.

Figure 2 illustrates the shift from a unique run-free equilibrium to equilibrium multiplicity when r increases. The natural interpretation of the condition ν(0, r) > ν is that the bank has enough capital; we return to this issue in Section 5.3 on capital requirements.

### 4.2 Hedging Interest Rate Risk versus Liquidity Risk: the Dilemma

How does r affect the bank’s liquidity risk? We can start from a low r such that ν(0, r) > ν and as r increases, ν(0, r) falls below ν and the run equilibrium appears. A key insight is that this can happen even if the bank is fully hedged against interest rate risk in the good equilibrium.
The bank faces a dilemma, illustrated in Figure 2. Suppose the bank initially hedges interest rate risk, assuming the good equilibrium prevails. Then it chooses assets such that

\[ A'(r) + DF'_I + DF'_U(1, r) = 0 \]

and the optimal duration follows a generalization of Proposition 1. The bank is hedged as long as uninsured depositors do not run, which is shown in Figure 2 by the fact that \( v(1, r) = v(1, r') \) in spite of an increase in rates from \( r \) to \( r' \).

The strategy just described ensures that as \( r \) increases, \( v \) is unchanged hence the good (no-run) equilibrium remains an equilibrium. However, a byproduct is that the value \( v(0, r) \) is then unhedged. Banks face a dilemma as there is no duration that allows them to hedge both the interest rate risk (in the good equilibrium) and the liquidity risk (arising from the bad equilibrium):

**Proposition 3 (Dilemma).** Suppose that initially the uninsured deposit franchise is profitable \( (r > \beta_U r + c) \), only the good equilibrium exists \( (v(0, r) \geq v) \), and for simplicity \( w_U = 0 \).

If the bank perfectly hedges interest rate risk in the good equilibrium by choosing assets \( A(r) \) such that \( v(1, r) \) is constant, i.e., \( A(r) = v^* D - DF_I(r) - DF_U(\lambda = 1, r) \) for some
constant $v^* > v$, then there exists a threshold

$$r = \frac{\frac{v^*-v}{u} + c}{1 - \beta U}$$

decreasing in the uninsured deposit ratio $u$ such that for $r > \bar{r}$, $v(0, r) < v$ hence the run equilibrium exists.

Starting from a run-free equilibrium, hedging liquidity risk requires ensuring that $v(0, r)$ does not fall below $v$. This can be done by choosing assets such that

$$A' + DF'_I = 0$$
effectively ignoring the negative duration provided by the potentially instable uninsured deposit franchise. In terms of duration this means that

$$T_A = \frac{DF'_I}{A}$$

which is lower than the duration that hedges the bank in the good equilibrium, $v(1, r)$: $T_A = \frac{DF'_I + DF'_U(1, r)}{A}$, assuming the uninsured deposit franchise is profitable and hence has a negative duration.

This strategy can again be reinterpreted as hedging without outflows but against an effective beta that exceeds the actual deposit beta. Ignoring the normal rate-driven outflows studied extensively in the previous section, we obtain a very simple formula for the effective beta:

**Proposition 4** (Hedging Liquidity Risk). Suppose there are no normal rate-driven outflows $w'_I = w'_U = 0$, and initially $v(0, r) > v$.

Then the bank can hedge against liquidity risk by using an effective beta

$$\hat{\beta} = \beta + u \cdot \frac{c}{r},$$

where $\beta = (1 - u)\beta^I + u\beta^U$ is the average deposit beta. The effective beta increases with the share of uninsured deposits and is equal to $\beta$ if $u = 0$.

A bank with a larger share of uninsured deposits can hedge against the liquidity risk arising from higher rates by shortening the duration of its assets. Using
\( c = 2\% \) and \( r = 4\% \), the correction \( \hat{\beta} - \beta \) can be potentially very large, going up to

\[
\hat{\beta} - \beta = 0.5
\]

when most deposits are uninsured \((u \to 1)\) as was the case for SVB. This means that the share of long-duration assets should be 50 percentage points lower for such a bank if it wants to hedge its liquidity risk. By contrast, SVB had an unusually high share of long-duration assets.

Going short (by using a high effective beta) does not solve the bank’s dilemma. Given this conservative strategy, the bank ensures that the equilibrium without runs prevails, and thus that its value is \( v(1, r) \). However the duration of its assets is now too short to offset the loss in franchise value if interest rates go down, as depicted in Figure 3.

Shortening duration in order to hedge liquidity risk is not a perfect solution because the bank stands to lose if interest rates fall. This is because at low rates deposits become unprofitable as deposit spreads are insufficient to cover the bank’s operating costs. With a shortened duration, the bank’s assets cannot fully offset the decline in the value of the deposit franchise. This can make the bank insolvent. A run can still happen but is less likely because the value of the liquid assets is relatively high. The bank can thus become a “zombie bank”; the resulting incentives from debt overhang and risk-shifting can have adverse macroeconomic consequences when widespread (Caballero et al., 2008; Acharya et al., 2022).

**Uninsured depositor withdrawals at low rates.** When the interest rate declines by so much that \( v(r, 1) \) falls below \( v \), then uninsured depositors start withdrawing again, i.e., \( \lambda < 1 \). But this kind of mass withdrawal is substantially different from the runs that can happen at high rates. Interestingly, a “low-rates run” can only happen once the deposit franchise has already become unprofitable, i.e., at such a low rate that \( v(\lambda, r) \) is actually decreasing in \( \lambda \), because the profit per dollar of deposits \( r - r_d^U - c \) is negative. In Figure 3 this is the case at the low rate \( r'' \). At the threshold rate \( r' \), the uninsured deposit franchise value is zero, and when the rate falls further it becomes negative. Since the bank is losing money on each uninsured depositor it actually welcomes withdrawals. In that case the bank does not really face liquidity risk: as uninsured depositors withdraw, the bank actually benefits, and starting from \( \lambda = 1 \) uninsured depositors withdraw until the bank value \( v \).
Figure 3: Unique equilibrium as interest rate falls from \( r \) to \( r' \) to \( r'' \).

A resolution: Swaptions. A hedging strategy that takes into account both risks resembles a combination of short duration assets to hedge against increasing liquidity risk as rates go up, combined with receiver swaptions to hedge against interest rate risk from the fall in deposit franchise as rates go down.

Suppose the bank is undercapitalized if \( v \) falls below some threshold \( \bar{v} \) which may be equal to or above the run threshold \( \bar{v} \). Hedging both the liquidity risk on the upside and the interest rate risk on the downside in the sense of ensuring \( v \geq \bar{v} \) requires

\[
\min \{ v(0, r), v(1, r) \} \geq \bar{v},
\]

which implies an asset payoff \( A(r) \) that must satisfy

\[
A(r) \geq A_{\min}(r) = \max \{ \bar{v}D - DF_I(r), \bar{v}D - DF_I(r) - DF_{UI}(\lambda = 1, r) \}.
\]

Figure 4 plots \( A_{\min} \) as a function of \( r \). \( A_{\min} \) must be extremely convex. Below \( r = c/(1 - \beta_U) \), which is the threshold rate above which the uninsured deposit franchise is profitable, \( A_{\min} \) has a long duration. In this example this corresponds to a threshold \( r = 3\% \). Above \( r = 3\% \), \( A_{\min} \) has a short duration. This profile can be implemented in two equivalent ways: by holding long-term assets combined with payer swaptions with strike \( r = 3\% \), or by holding short-term assets com-
bined with receiver swaptions with the same strike. Banks are already active in
the swaption market in order to hedge the negative convexity of their MBS port-
folio induced by prepayment as rates fall. Our model highlights another reason
why banks with a large uninsured deposit base should buy—or be required to
buy—swaptions.

5 Extensions

5.1 High \( \beta \) Uninsured Deposits: a “Divine Coincidence”

As Drechsler et al. (2023c) show, prior to 2012 uninsured deposits consisted pri-
marily of large time deposits, effectively wholesale funding, which have a high
\( \beta^{LI} \) and a low operating cost. As a result, uninsured deposits had a low franchise
value. A byproduct of this, which we dub a “divine coincidence”, was that invest-
ing in short duration assets against the uninsured deposit franchise also prevented
runs. This can explain why SVB-style runs were relatively uncommon.

In the baseline model the operating cost \( c \) is given and common across deposit
types. Operating costs, however, are a result of banks’ optimal investment in ser-
vices (branches, apps, etc.). Therefore operating costs \( c \) and pass-through \( \beta \) should
be negatively related, both across different deposit products and across banks. We
now enrich the model to allow for different, endogenous, operating costs.

Suppose that banks can invest in the quality of their services. The initial invest-
ment determines the future operating costs. Following standard q-theory logic, the bank chooses a higher investment scale and thus higher operating costs c when the deposit franchise is more valuable, for instance if the bank faces less elastic customers (due to higher concentration or other factors). The optimal operating cost is given by a function \( c = \phi(DF) \) with \( \phi(0) = 0 \). Since DF itself depends on the operating cost, we get a solution \( c((1 - \beta)r^*) \) where \( r^* \) is the long-run interest rate and \( c(\cdot) \) is an increasing function with \( c(0) = 0 \). Focusing on uninsured deposits (similar expressions hold for insured deposits), the resulting deposit franchise value is

\[
DF_U = D_U(1 - w_U) \left( (1 - \beta_U) - \frac{c((1 - \beta_U)r^*)}{r} \right)
\]

and its interest-rate sensitivity is

\[
DF'_U = D_U \left[ \frac{c((1 - \beta_U)r^*)}{r^2} - w'_U \left( (1 - \beta_U) - \frac{c((1 - \beta_U)r^*)}{r} \right) \right].
\]

Suppose then that the bank hedges against normal rate-driven outflows, assuming that the good equilibrium in Section 4 holds. We have the simple following result:

**Proposition 5.** There exists a threshold \( \beta_U^* \) increasing in the uninsured deposit ratio \( u \) such that for a high-enough uninsured deposit beta, \( \beta_U \geq \beta_U^* \), the good equilibrium is the unique equilibrium.

As \( \beta_U \to 1 \), the uninsured deposit franchise \( DF_U \) converges to zero and so does the interest rate risk due to the possibility of runs in the bad equilibrium. The “divine coincidence” is that by optimally hedging against normal rate-driven outflows \( w'_U \) using short-duration assets, the bank also stabilizes its solvency ratio \( v \), which deters runs. This is possible because the uninsured franchise \( DF_U \) is small. Note that all this can happen in spite of a large uninsured deposit ratio \( u \), but a higher \( u \) requires a higher \( \beta_U \).

Drechsler et al. (2023c) further show that after 2012 uninsured deposits shifted toward checking and savings accounts. Unlike wholesale funding, these accounts have a low \( \beta \) and a high operating cost \( c \), hence they contribute a lot to the deposit

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8For simplicity we take as given the demand curve for deposits and therefore the resulting \( r_d \) and \( \beta \); of course an even more complete version of this problem would acknowledge that the \( \beta \) is itself endogenous to investments in quality and thus \( c \), but this would only change the function \( \phi \).
franchise. In the case of SVB, almost all of its deposits were uninsured corporate checking and savings accounts. This means that it had a large uninsured deposit franchise. As our model shows, this is the extreme case for a bank’s risk management dilemma. The low $\beta$ and high cost push the bank toward a long asset duration, while the fact that the deposits are uninsured pushes toward a short duration. SVB’s decision to invest heavily in long-term MBS reveals that it ignored the second consideration, leaving it maximally exposed to a run.

A broader implication of our analysis is that uninsured checking and savings accounts, more so than wholesale funding, expose banks to instability.

5.2 Illiquid Deposit Franchise

We assumed that uninsured depositors run based on the solvency ratio $v = V / D$ of the bank. However franchise values are intangible assets that may be difficult to pledge to investors (e.g., to raise equity against them) relative to liquid assets $A$, especially if the bank is stressed. This limited pledgeability increased the liquidity risk of the bank. To see how, suppose that the ratio that enters the function $\lambda(v)$ is

$$v^\rho = \frac{A - D + \rho DF}{D}$$

where $\rho \in [0, 1]$ is a limited pledgeability parameter.\footnote{We could further assume that the insured deposit franchise is more pledgeable than the uninsured deposit franchise.}

One might think, by analogy with Diamond and Dybvig (1983), that as the illiquid asset becomes even more illiquid, runs become more likely. Perhaps surprisingly, as $\rho$ decreases and the deposit franchise becomes harder and harder to sell in case of need, the dilemma between interest rate risk and liquidity risk weakens. A lower $\rho$, uninsured depositors’ withdrawals become less sensitive to the value of the deposit franchise, hence the good and bad equilibria become closer.

5.3 Capital Requirements

In addition to options-based risk management, the bank can solve its risk management dilemma using a capital buffer. This could be costly, in which case a regulator would need to require it. Our model has implications for how such a capital re-
requirement should change with interest rates. Suppose that regulators can require banks to issue equity to achieve a solvency ratio

\[ v \geq \overline{v}. \]

**Proposition 6.** The optimal capital requirement \( \overline{\sigma} \) that prevents runs is

\[ \overline{\sigma}(r, u) = v + u(1 - w_{U}(r)) \left( 1 - \beta^{U} - c/r \right). \]

*It increases with the interest rate \( r \) and the uninsured deposit ratio \( u \).*

Effectively this regulation corresponds to attributing a risk-weight to the uninsured deposit franchise. The reason is that at higher rates, a more valuable deposit franchise makes bank value increase more with the fraction of remaining uninsured depositors \( \lambda \), hence the sensitivity to a run (red line) is steeper, as illustrated in Figure 5. In order to rule out the run equilibrium the bank must issue more equity. Given enough equity to deter runs, the bank no longer aces a dilemma and can focus on interest rate risk management.

The behavior of liquidity risk yields a new rationale for procyclical capital requirements, but unlike standard arguments this procyclicality comes from a pure interest rate effect and not the state of the economy. While we frame the result in terms of capital requirements, depending on the cost of issuing equity even banks themselves may wish to follow this strategy in order to prevent runs.

Figure 5: Capital requirement \( \overline{\sigma} \) at low rate \( r_{low} \) and high rate \( r_{high} \).
5.4 Credit Risk, Amplification, and Monetary Policy

For clarity we focused on interest rate shocks. When banks hold assets exposed to other unhedged sources $s$ of risk, for instance credit risk, then these other shocks can also trigger a run on the uninsured deposit franchise. We show that following the same logic as for interest rate shocks, this amplification is more likely when the uninsured deposit franchise is large, for instance when interest rates are high and when the bank has a large share $u$ of uninsured deposits.

Suppose then that initially only the run-free equilibrium exists and
\[ v(1, r, s) = \frac{A(r, s) - D + DF_I(r) + DF_U(1, r)}{D} \]
where $A$ increases with $s$. In our framework the deposit franchise can only hedge against movements in interest rates if deposit rates are already set to maximize the franchise value.\(^\text{10}\) An adverse shock $s$ can trigger a fall in $A$, which leads some uninsured depositors to withdraw, triggering a negative feedback loop which ends up in the run equilibrium if both the shock and the uninsured deposit franchise $DF_U$ are sufficiently large, as shown in Figure 6. Therefore the runnability of the

\(^{10}\text{For large banks that are considered too-big-to-fail, adverse macroeconomic shocks may lead to an inflow of “flight-to-safety” deposits which can be captured through a net outflow function }w(r, s)\text{. We abstract from this effect as we focus on the liquidity risk of smaller regional banks.}\)
deposit franchise creates a complementarity between negative shocks \( s \) and high interest rates.

Of course the central bank could respond to the shock by lowering interest rates, in order to stabilize \( A(r,s) \). Our framework also highlights that the timing of the policy response and the initial hedging strategy of banks are key. If the bank is initially hedging liquidity risk and thus investing in short-duration assets, then a rate cut may even worsen the impact of the shock \( s \) by hurting the deposit franchise. If the deposit franchise is very valuable, accommodative monetary policy may even remove the good equilibrium, as depicted by the dotted red line in Figure 6. Thus credit risk reinforces the case for strategies such as swaptions and capital requirements that allow banks to keep their deposit franchise valuable and hedge interest rate risk based on the good equilibrium.

An interesting extension outside the scope of our model would include imperfect and dispersed information about \( s \) and characteristics of the deposit franchise such as \( \beta \) and \( w' \): in that case, uninsured depositors could even interpret normal-rate driven withdrawals as a signal of poor asset quality \( s \).

6 Conclusion

We provide a model of the interaction between interest rates and the liquidity risk of banks. The crucial element is the runnability of a deposit franchise built on uninsured deposits. Runs are absent at low interest rates because the deposit franchise brings little value to the bank, but appear when interest rates rise because the deposit franchise comes to dominate the value of the bank. The liquidity risk of the bank thus increases with interest rates. We provide formulas for the bank’s optimal risk management policy but stress that the bank faces a risk management dilemma: it cannot simultaneously hedge its interest rate risk and liquidity risk exposures. This implies that low-beta uninsured deposits (uninsured checking and savings accounts) are a source of instability. We derive implications for optimal capital requirements and study interactions between the deposit franchise and other sources of risk such as credit risk.
References


Freixas, Xavier and Jean-Charles Rochet (2008), Microeconomics of banking: MIT press.


Electronic copy available at: https://ssrn.com/abstract=4411127
Appendix

A  Proofs

A.1  Proof of Proposition 1

\[
\frac{DF'}{D} = (1 - w)\frac{c}{r^2} - w'\left(\frac{(1 - \beta)r - c}{r}\right) 
\]  

(5)

The optimal hedging (modified) duration after normalizing around \( w = 0 \) is

\[
T_A = \frac{DF'}{A} = \frac{1}{r} \frac{c}{\beta r + c} - w'\left(\frac{(1 - \beta)r - c}{\beta r + c}\right) 
\]

and the effective beta \( \tilde{\beta} \) solves

\[
\frac{1}{r} \frac{c}{\beta r + c} - w'\left(\frac{(1 - \beta)r - c}{\beta r + c}\right) = \frac{1}{r} \frac{c}{\beta r + c} 
\]

hence

\[
\tilde{\beta} = \beta + \frac{w'(c + \beta r)}{c - w' r [(1 - \beta)r - c]} [(1 - \beta)r - c]. 
\]

A.2  Proof of Proposition 4

To hedge \( v(0, r) \) need

\[
A' + DF'_l = 0 
\]

\[
T_A = \frac{1}{A} DF'_l 
\]

\[
= \frac{1}{r} \frac{(1 - u) [r_d + c - \beta^l r]}{u(\beta^l r + c) + (1 - u)(\beta^l r + c)} 
\]

This corresponds to the optimal duration for a bank with an effective beta \( \hat{\beta} \) such that

\[
T_A = \frac{1}{r} \frac{[r_d + c - \hat{\beta} r]}{u(\beta^l r + c) + (1 - u)(\beta^l r + c)} 
\]
hence

\[ r_d + c - \hat{\beta}r = (1 - u) \left[ r_d^I + c - \beta^I r \right] \]

\[ \hat{\beta}r = (1 - u) \beta^I r + u (\beta^U r + c) \]

\[ \hat{\beta} = \beta + uc / r \]