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THE MACROECONOMICS OF TRADE CREDIT

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ABSTRACT

In most countries, suppliers of intermediate goods and services are also the main providers of short-term financing to firms. This paper studies the macroeconomic implications of these financial links. In our model, trade credit is the outcome of a long-term contract between firms linked in the production process, and it is sustained in equilibrium by reputation forces as customers lose the relationship with their suppliers in case of a default. These financial links give rise to a credit multiplier: suppliers can enforce repayment of these IOUs, and they can discount these bills with banks to obtain liquidity. This process can either dampen or amplify the output effects of financial shocks, depending on the borrowing capacity of suppliers. Using Italian data, we find that the credit multiplier is sizable and show that trade credit amplified the output costs of the Great Recession by 45%.

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1 Introduction

After the Great Recession, a significant amount of research was dedicated to determining the role of financial factors in overall economic instability. Most of the models used by economists to gauge the impact of financial shocks—for instance, those building on the pioneering work of Kiyotaki and Moore (1997b) and Bernanke, Gertler, and Gilchrist (1999)—put capital markets and financial intermediaries at the forefront. In these models, companies' funding needs are solely met by these institutions, implying, by construction, direct spillovers of financial shocks to the demand for capital and labor by businesses.

In reality, however, most companies around the world address the bulk of their liquidity needs using trade credit—the financing that suppliers of intermediate inputs provide in the form of extended payment terms. Figure 1 shows the significance of this phenomenon for non-financial businesses for a selection of advanced economies. Noticeable in the figure is that trade credit claims are usually as large—and in countries such as Spain and Italy, even larger than—short-term debt securities and loans issued by non-financial corporations combined. In addition, trade credit is often more volatile than these other debt instruments, underscoring the importance of understanding its drivers and the economic effects of these movements.

In this study, we propose a framework for examining the macroeconomic significance of this phenomenon and reevaluate the question of how financial shocks spread through the rest of the economy. In our model, firms can borrow from banks and from their suppliers of intermediate inputs. Unlike bank debt, which is partially upheld by law, trade credit relies on a reputation mechanism for enforcement, with customers having an incentive to repay it to avoid being cut off from future supplies of the goods.[1] Suppliers can then discount these claims with banks, enhancing in this fashion the liquidity of the overall system. We show that this *credit multiplier* reduces the economic distortions due to financial frictions on average, but it also makes the economy potentially more exposed to financial market disruptions—in that credit supply shocks not only affect firms bank financing but also hinder the ability of their suppliers to provide trade credit. Indeed, when calibrating our model to Italian data, we show that the presence of trade credit amplified the output costs of the Great Recession by 45%.

We consider an economy in which households consume a basket of differentiated goods. The production of these goods is organized on different supply chains (or sectors), with upstream firms producing intermediate goods using labor and downstream firms using

[1]We introduce this asymmetry in the legal enforcement of bank and trade credit because, in most legislations, trade credit is one of the most junior forms of credit. See Cuñat and García-Appendini (2012) and Jacobson and Von Schedvin (2015) for a discussion.

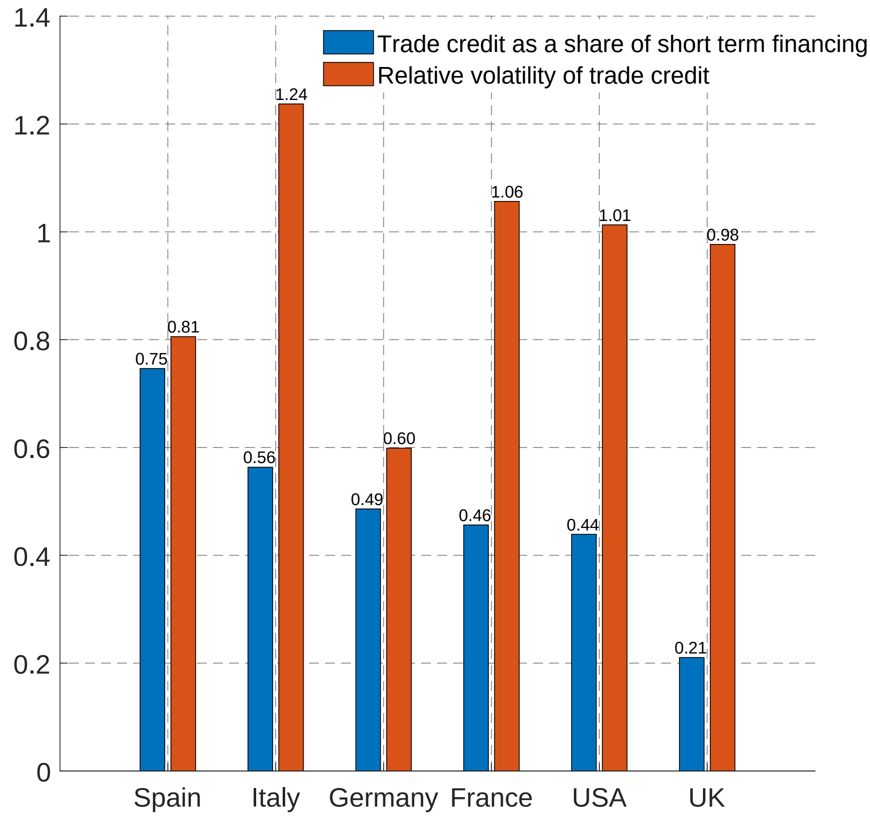


Figure 1: Relative size and volatility of trade credit

Note: Our calculations from annual Eurostat Institutional Sector Accounts for the 1995-2017 period. Trade credit is "Trade credit and other receivables" on the (unconsolidated) liability side of non-financial corporations. Short-term debt is the sum of "Short-term debt securities" and "Short-term loans" on the liability side of non-financial corporations. We define short-term financing as the sum of short-term debt and trade credit. The blue bar reports for each country the sample mean of trade credit as a fraction of short-term financing. To construct the red bar, we first HP-filter the log of trade credit and short-term debt, compute the standard deviations of both series, and determine their ratio. A value above 1 indicates that the cyclical component of trade credit is more volatile than that of short-term debt.

these inputs to produce consumption goods. We follow Bigio and La'o (2020) and assume that part of the revenues of the final good firms are realized only after they purchase inputs, so these firms need credit to operate. Banks can lend to firms, but because of limited commitment, they can lend only up to a fraction of the firms' revenues. As in Jermann and Quadrini (2012), this debt limit is time-varying and stochastic, and we will think about an unexpected tightening as a negative credit supply shock. Importantly, we allow the downstream firms to borrow also from their suppliers within the context of a long-term contract. These contracts specify not only the quantity of the good supplied and its price but also the financing—whether payments occur at the beginning or at the end of the period. We refer to the payments that occur at the end of the period as trade credit.

Trade credit can emerge in equilibrium as part of the optimal contract between downstream and upstream firms. The enforcement of these credit relationships is sustained by a reputation mechanism: if the final good firm defaults on the trade credit, the supplier will stop providing the good in the future. So, the higher the value of a relationship between a supplier-customer pair, the more trade credit they will be able to support in equilibrium. This value is endogenous, and it depends on characteristics of both suppliers and customers. For example, a low elasticity of substitution across intermediate goods or a high degree of concentration in the suppliers' market makes the supplier-customer relationship more valuable. This is because, in such cases, it is costly for downstream firms to be excluded from an intermediate good. As a result, the pair can sustain a higher level of trade credit. Similarly, a lower need for liquidity by downstream firms reduces trade credit. We allow some of these characteristics to differ across sectors in order to obtain a rich set of predictions that we can test empirically.

We use this framework to study the macroeconomic implications of trade credit. To do so, we compare the behavior of our economy to that of a counterfactual "spot economy," which is identical in all respects to the former, with the exception that all economic entities engage in spot transactions. Focusing on a special case of the model that is analytically tractable, we present two sets of results. First we show that trade credit reduces the economic distortions due to financial frictions and brings the economy closer to the first best.[2] Second, we show that the presence of trade credit can make the economy more sensitive to financial shocks, amplifying their effects on output.

To understand the first result, let us consider the spot economy. Here, downstream firms are the only borrowers, as they need cash at the beginning of the period to purchase intermediate inputs. Ultimately, bank credit in the economy is used for two purposes: paying for the production costs and paying for the rents of suppliers. When bank credit is scarce, the rents effectively crowd out expenditures on production costs and reduce the overall output produced.

In contrast, in the economy with trade credit, the suppliers' rents are in part paid at the end of the period, which alters the equilibrium outcome in two ways. First, a larger share of bank credit is used toward the payment of production costs. Second, suppliers can discount their accounts receivable with banks if they need more liquidity. As a result, the total available quantity of bank credit increases. Both factors act as a multiplier on the credit available to pay for production costs and, ultimately, on the output produced. We show that this credit multiplier allows the economy to achieve the first-best level of production in steady state when credit markets are not too frictional—that is, when the fraction of revenues that firms can pledge to banks is sufficiently large.

[2]By "first best," we mean the allocation achieved when credit markets are frictionless.

We then move on to study the response to financial shocks. While in the spot economy, a tightening of the firms' financial constraints decreases output, in the trade credit economy, the effects of the same shock are more subtle as suppliers face two opposing forces. On the one hand, they have an incentive to provide trade credit to their customers when credit supply falls. On the other hand, the financial shock tightens the suppliers' borrowing constraint as well, increasing the costs of providing trade credit when the latter binds. When suppliers have sufficient borrowing capacity, trade credit will end up substituting for the fall in bank credit supply, and the effects of the financial shocks on output will be dampened. When suppliers are financially constrained, trade credit may fall in response to the financial shock. The fall can then amplify the effects of the credit supply shock on output. That is, the financial shock in this scenario triggers a fall not only in bank credit but also in trade credit, and the overall output losses can be larger relative to what happens in the spot economy.

We quantify the model using Italian firm-level balance sheet data from the historical ORBIS dataset for the 2007-2015 period. We aggregate this dataset at the sectoral level and collect data on the size of trade credit claims as well as other factors that in our model shape the cross-sectoral heterogeneity in trade credit: the share of intermediate inputs costs over sales, the accounts receivable of the sector, and the market concentration of suppliers.[3]

Consistent with the predictions of the model, we find that industries that have high liquidity needs (high intermediate-input share and accounts receivable) and that purchase inputs from more concentrated sectors have a larger share of accounts payable over their sales. Indeed, these factors account for the bulk of the cross-sectoral variation in trade credit present in our data. The calibrated model matches quantitatively these cross-sectional facts, and it predicts well the sectors that ended up suffering the most during the financial shocks associated with the Great Recession. Specifically, we back out from the calibrated model an indicator of the tightness of the financial constraints of suppliers of a given sector and use it to sort the different sectors in our dataset. In a dynamic difference in differences specification, we then find that firms operating in sectors with more financially constrained suppliers in 2007 experienced a substantially larger fall in trade credit and sales during the Great Recession compared to other sectors. This finding is consistent with the key mechanism of our model: suppliers' credit constraints are key to understanding the behavior of trade credit and output in response to aggregate financial shocks.

Finally, we use the calibrated model to quantify the aggregate implications of trade credit for the Italian economy. For that purpose, we perform two exercises. First, to assess the size

[3]To compute this object, we use our dataset to construct the sales Herfindahl-Hirschman index (HHI) for each sector in the economy and use sales share from the Italian input-output table to construct an HHI for the suppliers of a given sector.

of the credit multiplier, we compare the steady state of our economy with that of the spot economy. We find that the benchmark economy has three times more bank credit than the counterfactual spot economy. This allows the economy with trade credit to support 14% more output. Second, we study the response to a financial shock that mimics the depth and persistence of the Great Recession. We show that absent trade credit, output would have fallen 6 percentage points rather than the observed 11. Thus, trade credit was responsible for 45% of the output losses due to the financial shock.

Literature. There is a large literature on trade credit in corporate finance. Several papers have focused on understanding the reasons why we observe borrowing and lending between firms despite the existence of developed financial markets; see Petersen and Rajan (1997) and Cuñat and García-Appendini (2012) for surveys. These theories build on the hypothesis that suppliers may have some advantages over financial institutions in lending to their customers. In Biais and Gollier (1997) and Burkart and Ellingsen (2004), for example, suppliers can better monitor their customers, while in Frank and Maksimovic (1998), they can liquidate unused inputs more effectively because of their established network of buyers. The paper closest to ours in this literature is Cuñat (2007), who proposes a theory in which trade credit is supported in equilibrium by suppliers' threat of cutting off customers from all future provisions of the input. The main contribution of our paper is to incorporate this idea in a business cycle model with aggregate shocks and study the macroeconomic implications of trade credit.

In this respect, we contribute to a large literature studying the macroeconomic effects of financial frictions. Jermann and Quadrini (2012) demonstrate the importance of shocks to firms' short-term financing for macroeconomic volatility. Up to now, however, only a few papers have focused on the role of trade credit. The early contribution of Kiyotaki and Moore (1997a) develops a model with endogenous trade credit relationships and shows that firm-specific shocks can be amplified through these credit chains.[4] Bigio (2023) proposes a model in which changes in the network of payments between firms generate fluctuations in total factor productivity. More recently, researchers have investigated the role of trade credit in propagating aggregate shocks; see, for instance, Hardy, Saffie, and Simonovska (2022) and Mateos-Planas and Seccia (2021). Closer to our work, Altinoglu (2021) and Luo (2020) introduce trade credit relationships in the production network economy of Bigio and La'o (2020). In these papers, trade credit is a deep parameter that does not respond to changes in the economic environment. In our work, instead, trade credit is endogenous and responds to aggregate shocks. These movements, in turn, are critical to understanding

[4] See Boissay and Gropp (2013), Jacobson and Von Schedvin (2015), Costello (2020), and Amberg, Jacobson, Von Schedvin, and Townsend (2021) for evidence on the importance of these spillovers.

how financial shocks propagate to the economy.

The work closest to ours in this literature is Reischer (2020), which builds on Altinoglu (2021) and Luo (2020) but allows the price and quantity of trade credit to respond to shocks. In her model, as in ours, trade credit can potentially dampen or amplify the effects of credit supply shocks. We view the two papers as complementary. While Reischer (2020) deals with a richer production network than we do, our framework microfounds trade credit and its relationship with bank finance, leading to novel insights. For example, the core mechanism in our model is the spillover of trade credit on the allocation and quantity of bank credit—which we label, the credit multiplier. In Reischer (2020), this interdependence is absent because bank credit is modeled via an exogenous rule.[5]

The concept of a credit multiplier is related to the seminal work of Holmstrom and Tirole (1997). In that paper, a moral hazard problem limits the amount of funds that investors can give to firms. Banks—who have access to a monitoring technology—can borrow from the first group and lend to the second, a process that increases overall credit available to firms in the economy. In our framework, credit is multiplied because suppliers can enforce repayment via reputation, and they can discount their accounts receivable with banks.

Finally, our paper is related to studies that have introduced optimal contracts in general equilibrium models with aggregate shocks.[6] The paper closest to ours is Cooley, Marimon, and Quadrini (2004), which studies the implications of limited enforcement of financial contracts for the propagation of aggregate technological shocks. We instead focus on trade credit and its role for the amplification of financial shocks. From a technical point of view, our model features permanent sectoral heterogeneity, which makes the distribution of promised values a (high-dimensional) aggregate state variable. In the numerical algorithm, we deal with this issue by nesting the solution of the optimal contract within a fixed point problem à la Krusell and Smith (1998).

2 The model

We consider a closed economy populated by infinitely lived households, firms, and banks. Households supply labor and consume a bundle of imperfectly substitutable goods. These

[5] Another important difference is that trade credit in our framework is a forward-looking variable as it depends on the value of future relationships between firms, while in Reischer (2020) it is the outcome of a static decision. This leads to different predictions regarding the role of expectations in determining trade credit relationships and potentially different policy prescriptions when dealing with payments' crises.

[6] See, for instance, the work of Kehoe and Perri (2002), Dovis (2019), and Aguiar, Amador, and Gopinath (2009) for applications to international capital flows; Boldrin and Horvath (1995) and Souchier (2022) for the role of long-term wage contracts for labor market fluctuations; and Di Tella (2017), who studies how the optimal state contingency of financial contracts affects the financial amplification mechanism.

final goods are produced by competitive firms that combine capital with intermediate inputs, while the intermediate inputs are produced by monopolists using labor. We will refer to the production process for a specific final good as a *production line* or sector. There is a lag between production and the full receipt of payments, so final good firms need to obtain credit to pay for intermediate inputs. Credit is provided by competitive banks and by the suppliers of intermediate inputs. We describe the environment in detail in Section 2.1, define an equilibrium in Section 2.2, and discuss some of the simplifying assumptions made in Section 2.3.

2.1 Environment

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. Uncertainty is described by a Markov process that takes finite values in the set $\mathcal{S}$. We denote by s_t the state of the process at time t and by $s^t = (s_0, s_1, \ldots, s_t)$ the history of states up to period t. The process for s_t is given by the transition matrix $\pi(s_{t+1}|s_t)$. All equilibrium variables are in general functions of the history s^t, but whenever no confusion is possible, we leave this dependence implicit and only use a subscript t.

Households. Households supply labor to firms at the competitive wage W_t and every period receive the profits from firms in the economy, Π_t. They use this income to purchase a continuum of imperfectly substitutable final goods $\{y_{i,t}\}$ at prices $\{p_{i,t}\}$. So, the budget constraint of the representative household is given by

$$\int p_{i,t} y_{i,t} di = W_t L_t + \Pi_t.$$

Households choose labor and final goods to maximize their lifetime utility,

$$U = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[C_t - \chi \frac{L_t^{1+\psi}}{1+\psi} \right] \right\},$$

where β is the rate of time preference, ψ is the inverse Frisch elasticity of labor supply, and C_t is a CES aggregator of final goods:

$$C_t = \left(\int y_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}.$$

The household's optimization problem yields two familiar optimality conditions, one for labor supply and one for the demand for final good i,

$$\chi L_t^{\psi} = W_t \tag{1}$$

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\gamma} C_t, \tag{2}$$

where P_t is the price index for the consumption bundle C_t. We normalize P_t to 1 in our analysis.

Production. The production of a final good of type i, $y_{i,t}$, is carried out by a continuum of competitive final good firms. These firms are endowed with capital k and purchase N_i intermediate inputs, which we denote by $\{x_{ij,t}\}$, in order to produce the final good $y_{i,t}$. The technology to produce the final good is

$$y_{i,t} = k^{1-\eta_i} \left\{ \left[\sum_{j=1}^{N_i} x_{ij,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i},$$

where σ is the elasticity of substitution across intermediate inputs and η_i governs the intermediate-input share of production. As in Atkeson and Burstein (2008), we assume that $\sigma > \gamma$, that is intermediate inputs are more substitutable than final goods.

The intermediate inputs $x_{ij,t}$ are produced by monopolists using a linear technology with labor as the sole production input,

$$x_{ij,t} = l_{ij,t}.$$

Thus, each good of type i takes place in a production line where the "upstream" firms are monopolists and produce imperfectly substitutable goods, and the "downstream" firms are perfectly competitive.

Financial markets. Each period is split into two stages. In the first stage (the morning), final good firms start the production process and obtain a fraction of their sales. In the second stage (the afternoon), final good firms finish production and receive the remainder of their sales. The lag between production and the receipt of sales is due to two factors. First, we assume that production is not instantaneous. Firms produce a fraction δ of output in the morning and the remainder in the afternoon. Second, a fraction π_i of the final goods sold in the morning are paid for in the afternoon. Therefore, final good firms receive only a fraction $\delta(1-\pi_i)$ of their sales in the morning. Because of these assumptions, final good firms may need credit in order to purchase intermediate inputs. Credit can be obtained from two sources: competitive banks and the suppliers of intermediate inputs.

Competitive banks collect wages from households in the morning and offer loans to downstream and upstream firms. We denote by $b_i(s^t)$ the amount borrowed by final good firms that produce final good i at time t for history s^t. Similarly, $b_{ij}(s^t)$ denotes the amount borrowed by the intermediate good monopolist that sells variety j to final good firms of type i. As in Bocola and Lorenzoni (2022), firms cannot commit to repaying the debt in the afternoon, and if they default, they suffer a penalty equal to a fraction $1-\theta_t$ of their afternoon revenues. The parameter θ_t depends on the state of the Markov process according to the function $\theta_t = \theta(s_t)$ and is the only exogenous source of uncertainty in the model. There are no further penalties for a defaulting firm besides the static loss of revenues.

Given these assumptions, final good firms repay their debt with banks in the afternoon as long as

$$b_i(s^t) \leq [1-\theta(s_t)]\,[1-\delta(1-\pi_i)]\, rev_i(s^t), \tag{3}$$

where $rev_i(s^t) \equiv p_i(s^t)y_i(s^t)$ is the revenue of firm i in history s^t. As in Jermann and Quadrini (2012), we interpret $\theta(s_t)$ as an index of the health of the banking sector, with a high value implying more frictional credit markets.

In addition to borrowing from banks, final good firms can borrow from their suppliers. That is, when selling a good $x_{ij}(s^t)$ to a final good firm, the monopolist specifies a spot payment to be made in the morning, $p^{\mathrm{s}}_{ij}(s^t)$, and a payment to be made in the afternoon, $p^{\mathrm{tc}}_{ij}(s^t)$. Effectively, $p^{\mathrm{tc}}_{ij}(s^t)$ is the *trade credit* offered by the monopolist. We will discuss momentarily how the terms of the contract are determined.

Depending on the terms of the contract, the upstream producers may also need to borrow from banks. This happens when the payments they receive in the morning are smaller than the wages they need to pay to workers, $p^{\mathrm{s}}_{ij}(s^t) \leq W(s^t)x_{ij}(s^t)$. In that case, the upstream firm borrows the difference from banks. Also, the upstream firms cannot borrow more than a fraction $[1-\theta(s_t)]$ of their afternoon revenues (their accounts receivable), or they would default on the banks. This implies the borrowing constraint

$$b_{ij}(s^t) \leq [1-\theta(s_t)]p^{\mathrm{tc}}_{ij}(s^t). \tag{4}$$

Trade credit. The trade credit contract is the outcome of a long-term relationship between upstream and downstream firms operating in the same production line i. We assume that upstream firms make a take-it-or-leave-it offer to the downstream firms specifying the terms of the contract, $\{x_{ij}(s^t), p^{\mathrm{s}}_{ij}(s^t), p^{\mathrm{tc}}_{ij}(s^t)\}$ for all $\{s^t\}$. That is, the offer not only includes current prices and quantities but also commits to future prices and quantities for each possible history, s^t.

Differently from bank credit, firms that default on their trade credit do not suffer any direct loss of revenues.[7] The enforcement of these contracts is instead guaranteed by a reputation mechanism. A final good firm that defaults at time t on its supplier is permanently excluded from purchasing the intermediate good produced by that supplier from time $t+1$ onward.

We denote by $J_i(s^t)$ the time t expected discounted value of a final good firm operating in industry i at time t under the contract,

$$J_i(s^t) = \sum_{\tau=0}^{\infty} \sum_{s^{t+\tau}} \beta^\tau \pi\left(s^{t+\tau} \middle| s^t\right) \left[rev_i(s^{t+\tau}) - \sum_{j=1}^{N_i} \left(p_{ij}^{\mathrm{s}}(s^{t+\tau}) + p_{ij}^{\mathrm{tc}}(s^{t+\tau}) \right) \right],$$

where $\pi\left(s^{t+\tau} \middle| s^t\right) = \prod_{l=1}^{\tau} \pi\left(s_{t+l}|s^{t+l-1}\right)$ is the probability of arriving at history $s^{t+\tau}$ from s^t. We denote by $J_i^{(-j)}(s^t)$ the corresponding value when the firm cannot purchase intermediate inputs from firm j starting at period t,

$$J_i^{(-j)}(s^t) = \sum_{\tau=0}^{\infty} \sum_{s^{t+\tau}} \beta^\tau \pi\left(s^{t+\tau} \middle| s^t\right) \left[rev_i^{(-j)}(s^{t+\tau}) - \sum_{j' \neq j} \left(p_{ij'}^{\mathrm{s}}(s^{t+\tau}) + p_{ij'}^{\mathrm{tc}}(s^{t+\tau}) \right) \right],$$

where $rev_i^{(-j)}$ denote the revenues of a final good firm in production line i when it does not purchase from supplier j.[8]

We denote by $\tilde{J}_i^j(s^t) \equiv J_i(s^{t+1}) - J_i^{(-j)}(s^{t+1})$ the expected discounted surplus of the match between final-goods producer i and the upstream firm j. Since a permanent break in the trade relationship is the only cost of defaulting on trade credit, final good firms do not have an incentive to default on the trade credit contract with j as long as

$$p_{ij}^{\mathrm{tc}}(s^t) \leq \beta \mathbb{E}\left[\tilde{J}_i^j(s^{t+1}) | s^t \right]. \tag{5}$$

The above is a key constraint in the model. It says that the greater the value of the downstream firm being in a relationship with supplier j, the more trade credit the supplier can extend. In the analysis that follows, we will refer to (5) as the trade credit constraint.

Each supplier j in production line i chooses the optimal contract to maximize the present

[7] This asymmetry between bank credit and trade credit captures the fact that, in many legislative statutes around the world, trade credit is a junior claim relative to bank credit in bankruptcy proceedings.

[8] In what follows, we will consider a symmetric equilibrium in which all upstream suppliers to firm i offer the same trade credit contract. Under this assumption, we obtain that $rev_i^{(-j)}(s^{t+\tau}) = \left[\frac{N_i-1}{N_i}\right]^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}} rev_i(s^{t+\tau})$.

discounted value of its profits,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^{\tau} \pi \left(s^{t+\tau} \middle| s^t \right) \left[p^{\mathrm{s}}_{ij}(s^t) + p^{\mathrm{tc}}_{ij}(s^t) - W(s^t) x_{ij}(s^t) \right] \right\} \tag{6}$$

taking as given the wage $W(s^t)$, the consumers' demand for industry i goods in equation (2), the no-default constraints (3) and (4), the trade credit constraint (5), the participation constraint of the final good producer, $\tilde{J}^j_i(s^{t+1}) \geq 0 \ \forall s^{t+1}$, and the feasibility requirement that $rev_i(s^t) - p^{\mathrm{s}}_{ij}(s^t) - p^{\mathrm{s}}_{ij}(s^t) \geq 0 \ \forall s^t, j$.[9] The suppliers of intermediate goods take as given aggregate quantities and the trade credit contracts of all other suppliers but internalize how their trade credit contract affects $y_i(s^t)$ and $p_i(s^t)$.

2.2 Equilibrium

A symmetric equilibrium is a set of aggregate variables $\{L(S^t), C(s^t), W(s^t), \Pi(s^t)\}$ for all s^t, final good firm quantities and prices $\{y_i(s^t), p_i(s^t)\}$ for all i and s^t, and intermediate-input firm quantities and prices $\{x_i(s^t), p^{\mathrm{s}}_i(s^t), p^{\mathrm{tc}}_i(s^t)\}$ for all i and s^t, such that:[10]

1. Given aggregate prices and profits, aggregate consumption and labor solve the household's problem.
2. Given aggregate prices and quantities as well as the trade credit contracts of all other suppliers, the set $\{y_i(s^t), p_i(s^t), x_i(s^t), p^{\mathrm{s}}_i(s^t), p^{\mathrm{tc}}_i(s^t)\}$ solve the problem of the intermediate goods firm supplying to firm i.
3. The labor market clears for every history s^t:

$$L(s^t) = \int_i N_i x_i(s^t) di.$$

2.3 Discussion

Before moving on, let us discuss some of the simplifying assumptions we made.

[9] In principle, we should also include the incentive constraints that allow for multiple deviations (e.g., final good firms defaulting on both banks and firms or defaulting on multiple firms at the same time). If final good firms were to default on banks and supplier j, they would lose $[1-\theta(s_t)]\,[1-\delta(1-\pi_i)]\, rev_i(s^t)$ and the expected discounted surplus of the match with supplier j. So, if (3) and (5) are satisfied, there is no incentive to default on both. It is possible to show that in a deterministic steady state, the trade credit constraints (5) also imply that firms have no incentives to default on the other suppliers as long as $\sigma > \gamma$. This is also true outside the steady state in all our numerical experiments.

[10] We do not include a j subscript for intermediate-input goods because in a symmetric equilibrium, all suppliers to firm i will have the same quantities and prices.

First, our model assumes that defaulting on bank loans results in revenue losses only in the current period. That is, we assume that a defaulting firm is not excluded from financial markets and that the bank cannot recoup a fraction of the firm's future profits. This assumption is made for tractability as it results in a simple borrowing constraint. Assuming different punishments would change the nature of the constraint, but we do not expect it would alter the key predictions of our model. Specifically, our model will feature a role for trade credit as long as the costs of defaulting on banks and on a supplier are larger than the costs of defaulting on the banks in isolation. This would be true, for instance, if final good firms could keep operating the business conditional on a bank default and if it would be costly for the firms to substitute the intermediate input provided by the supplier. We believe that both are reasonable assumptions.

Second, our model assumes that suppliers of intermediate inputs can commit to the entire path of quantities and prices offered in their contract. It is worth noting that this makes the problem of intermediate-input suppliers *time inconsistent*, as suppliers have an incentive to promise higher quantities and lower prices in the future in order to be able to extend more trade credit to the firm in the current period. Alternatively, we could have assumed that suppliers cannot commit to the entire path of future actions and solve for the Markov-perfect equilibrium. The key difference is that without commitment the supplier takes the trade credit limit as given and cannot affect it, while with commitment the suppliers have a motive to promise better terms in the future to the final good firms when the trade credit constraints bind today. We believe it is not unreasonable to assume that suppliers engage in this type of behavior in the context of a long-term relationship with their customers. Aside from this difference, the two models would be identical.

Third, we assume that firms are not able to accumulate cash holdings. In making this assumption, we stay close to the literature that studies the misallocation of production due to financial frictions. Bigio and La'o (2020), which studies the misallocation of production in a multi-sector framework with a production network, also considers within-period financing frictions in which firms cannot accumulate cash holdings. While allowing firms to accumulate cash holdings reduces the costs of financial frictions, this channel can be muted by adding to the model the tax advantage of debt or by making firm owners impatient.

3 Macroeconomic implications of trade credit

We now study the macroeconomic implications of trade credit in our economy. For this purpose, we will characterize some key properties of the model—the deterministic steady state and the response of endogenous variables to a financial shock—and compare these predictions to those of a *spot economy*. The spot economy is identical to the benchmark with the exception that intermediate good producers are banned from extending trade credit to their customers, $p^{\text{tc}}(s^t) = 0$. The difference between the benchmark and the spot economy isolates the macroeconomic implications of trade credit.

We start in Section 3.1 by considering a simplified version of our environment that is analytically tractable. We provide two main results. First, we show that trade credit relationships allow the economy to support more credit and to better allocate it toward productive uses, making it possible to sustain a higher level of output on average. Second, we study the response of the economy to a financial shock. We show that trade credit can either dampen or amplify the macroeconomic effects of financial shocks depending on the borrowing capacity of suppliers. Section 3.2 extends some of these results to the fully fledged model.

3.1 A special case

Consider a special case with only one production line and one supplier of intermediate inputs, $N_i = 1$. We normalize the level of capital for final good firms to 1 so that the production function is $y = x^{\eta}$.[11] We further assume that $\delta = 0$ so that all revenues of the final good firms are realized in the afternoon. In addition, we set the inverse of the Frisch elasticity to zero, $\psi = 0$. This last assumption implies that the wage is constant over time and given by $W = \chi$. Given these assumptions, the equilibrium is fully characterized by solving the decision problem of the monopolist.

We start by studying the spot economy and later move on to the benchmark.

The spot economy. In the spot economy final good firms borrow from banks in order to settle their transactions with the monopolist in the morning. Given that $\delta = 0$, the borrowing constraints for the final good firms are $p^{\text{s}}(s^t) \leq [1 - \theta(s_t)]x^{\eta}(s^t)$. The monopolist chooses $\{x(s^t), p^{\text{s}}(s^t)\}$ to maximize the present discounted value of profits subject to the borrowing constraints of the final good firms and their participation constraints. Since $N_i = 1$, the participation constraints boil down to $J(s^{t+1}) \geq 0$ for all s^{t+1}. This problem is equivalent to solving a static profit maximization problem, which yields the first-order condition

$$(1 - \theta)\eta x^{\eta - 1} = W. \tag{7}$$

To understand the behavior of the monopolist, suppose first that credit markets are

[11] In a slight abuse of notation, we do not index variables by the production line i and the identity of the supplier j as we did in Section 2 because there is only one production line and one supplier in this example.

frictionless, $\theta = 0$. In this case, the monopolist chooses the scale of production so that the marginal product of labor equals the wage, and extracts all the rents from the final good producers by setting $p^s = x^\eta$.[12] When $\theta > 0$, this solution cannot be attained because the borrowing constraint of the final good firms binds. In this case, the economy features a "labor wedge" and output is distorted down relative to the first best, a standard result for models with working capital constraints.

Using equation (7), we can solve for labor as a function of the financial shock θ and for the elasticity of labor to the financial shock,

$$x^{\text{spot}} = \left[\frac{(1-\theta)\eta}{W}\right]^{\frac{1}{1-\eta}}, \qquad \varepsilon^{\text{spot}}_{x,\theta} \equiv \frac{dx^{\text{spot}}/x^{\text{spot}}}{d\theta/\theta} = \frac{\theta}{(1-\theta)(1-\eta)}. \tag{8}$$

The economy with trade credit. In the benchmark economy, the supplier can also offer trade credit to final good firms, up to the limit defined by the trade credit constraint (5). The presence of this constraint makes the decision problem of the monopolist dynamic because future rents promised to final good firms affect how much trade credit the monopolist can support at date t. This decision problem can be written recursively. Let J be the rents that the monopolist has promised to a final good firm after history s^t, and let θ be the realization of the financial shock at s_t. The monopolist chooses labor x, the morning and afternoon payments $\{p^{\text{s}}, p^{\text{tc}}\}$, as well as the future continuation values for final good firms conditional on any realization of the financial shock next period, $J'(\theta')$, to maximize the present discounted value of profits ,

$$V(J,\theta) = \max_{x,p^{\text{s}},p^{\text{tc}},J'(\theta')} \left(p^{\text{s}} + p^{\text{tc}} - Wx\right) + \beta\mathbb{E}\left[V(J'(\theta'),\theta')|J,\theta\right].$$

subject to the debt limits that final good firms and the monopolist face when borrowing from banks,

$$p^{\text{s}} \leq (1-\theta)x^\eta, \tag{9}$$

$$\max\{Wx - p^{\text{s}}, 0\} \leq (1-\theta)p^{\text{tc}}, \tag{10}$$

the debt limit that final good firms face when borrowing from the monopolist,

$$p^{\text{tc}} \leq \beta\mathbb{E}[J'(\theta')|J,\theta], \tag{11}$$

[12] In this case, there are no output distortions despite the monopoly power of the supplier because the latter can set prices non-linearly.

the promise-keeping constraint,

$$J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta \mathbb{E}[J'(\theta')|J, \theta], \tag{12}$$

the participation constraints $J'(\theta') \geq 0$, and the feasibility requirement that $p^{\mathrm{s}} + p^{\mathrm{tc}} \leq x^{\eta}$.

The outcome of this problem is a policy function $\{x(J,\theta), p^{\mathrm{s}}(J,\theta), p^{\mathrm{tc}}(J,\theta)\}$ and a law of motion for future promises for each possible state θ', $J'(\theta'|J,\theta)$. These objects fully characterize the behavior of the economy with trade credit in this example.

Before moving on to study the properties of this economy, it is useful to derive the first-order conditions. Let μ, ι, and κ be the Lagrange multipliers associated, respectively, with the constraints (9), (10), and (11), and let λ be the multiplier associated with the promise-keeping constraint and ω the multiplier on the feasibility requirement. After some rearrangement, we obtain

$$\begin{aligned} x &= \left[\frac{\eta(1-\theta\mu+\iota)}{W(1+\iota)}\right]^{\frac{1}{1-\eta}}, && (13) \\ 1+\iota &= \mu+\lambda+\omega, && (14) \\ 1+(1-\theta)\iota &= \kappa+\lambda+\omega, && (15) \\ \lambda'(\theta') &= \lambda+\kappa. && (16) \end{aligned}$$

Equation (13) is the optimality condition for labor. This choice is distorted down relative to the first-best allocation when the borrowing constraint of the final good firms binds.

Equations (14) and (15) are the optimality conditions with respect to p^{s} and p^{tc}. A marginal increase in p^{s} raises the profits of the monopolist by one unit and relaxes his borrowing constraint with the banks (when it binds), at the cost of tightening the borrowing constraint of the final good firms, the promise-keeping constraint, and the feasibility requirement. The trade-off for p^{tc} is very similar. The key difference between p^{s} and p^{tc} is that the latter does not relax the borrowing constraint of the supplier as much because only a fraction $(1-\theta)$ of its accounts receivable can be pledged with the bank. When the borrowing constraint of the monopolist does not bind, $\iota = 0$, we have that $\mu = \kappa$ and the supplier is indifferent between being paid spot or in credit. When the borrowing constraint of the supplier binds, $\iota > 0$, issuing trade credit to customers becomes more costly relative to being paid spot. So, a tightening of this constraint reduces the supplier's incentives to extend trade credit to his customers.

Equation (16) combines the first-order condition of the problem with respect to $J'(\theta')$ and the envelope condition. This equation describes a law of motion for the multiplier on the promise-keeping constraint, λ. We can see that the multiplier grows over time whenever the trade credit constraint binds, $\kappa > 0$. This is quite intuitive: the supplier has an incentive to increase future rents for his customers when the trade credit constraint binds because this allows the supplier to extend more trade credit today. As the rents of the final good firms grow over time, so does the multiplier on the promise-keeping constraint.

3.1.1 The steady state and the credit multiplier

We start by considering the case in which θ_t is deterministic and equal to $\tilde{\theta}$ for all t. The next proposition characterizes the limit of the optimal contract as $t \to \infty$.

Proposition 1. *Let $\bar{\theta}$ be such that $(1-\bar{\theta})(1+\beta\bar{\theta}) = \eta$. If $\tilde{\theta} \leq \bar{\theta}$, the borrowing constraint of the monopolist does not bind in steady state and the optimal contract converges to*

$$x = \left[\frac{\eta}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = (1-\tilde{\theta})x^\eta, \qquad p^{tc} = \beta\tilde{\theta}x^\eta, \qquad J' = \tilde{\theta}x^\eta. \tag{17}$$

If $\tilde{\theta} > \bar{\theta}$, the borrowing constraint of the monopolist binds in steady state and the optimal contract converges to

$$x = \left[\frac{(1-\tilde{\theta})(1+\beta\tilde{\theta})}{W}\right]^{\frac{1}{1-\eta}} \qquad p^s = (1-\tilde{\theta})x^\eta \qquad p^{tc} = \beta\tilde{\theta}x^\eta \qquad J' = \tilde{\theta}x^\eta. \tag{18}$$

Proposition 1 identifies two regions of the parameter space that feature different behavior of the steady state of the model. The first region, which occurs when $\tilde{\theta}$ is smaller than the threshold $\bar{\theta}$, is characterized by undistorted labor choices—with the marginal product of labor equating the wage. In this region, total output is independent of $\tilde{\theta}$. The second region, instead, features a level of output that is distorted downward relative to this level, with the size of the distortion increasing in $\tilde{\theta}$.

The solid line in Figure 2 presents a numerical illustration of the deterministic steady state as a function of $\tilde{\theta}$. The figure plots the steady state level of output, bank credit to final good firms, and trade credit offered by the supplier. We can see that when $\tilde{\theta} \leq \bar{\theta}$, output is independent of $\tilde{\theta}$, while higher $\tilde{\theta}$ induces final good firms to obtain less credit from banks and more credit from their supplier. As the supplier is paid more in credit when $\tilde{\theta}$ is high, he is also more dependent on bank credit to finance his cost of production. When $\tilde{\theta} > \bar{\theta}$, the demand for credit by the monopolist is so large that his borrowing constraint binds in steady state. From that point on, higher levels of $\tilde{\theta}$ are associated with less bank *and* trade credit for the final good firms, and with lower output.

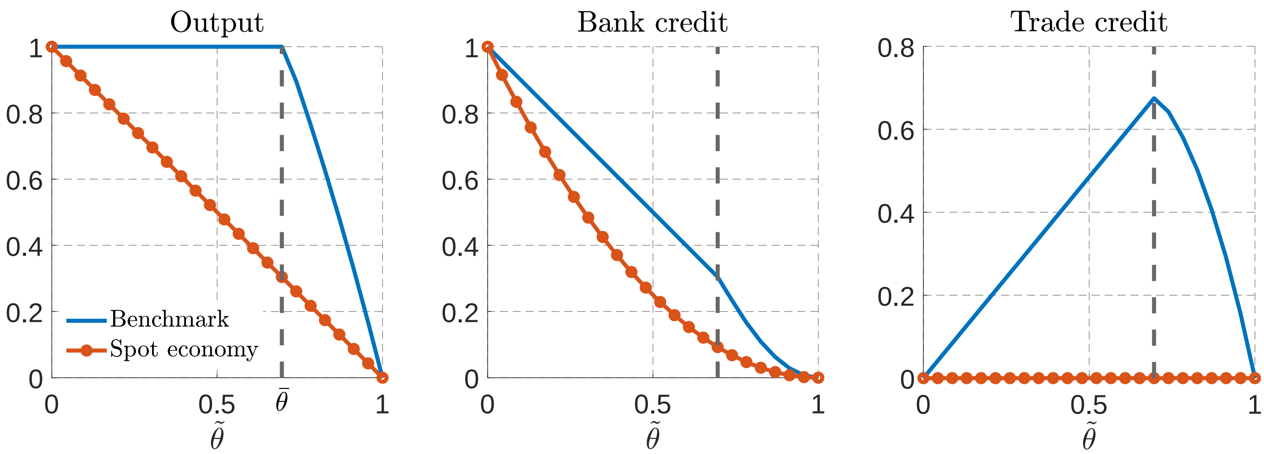


Figure 2: The deterministic steady state

Note: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.7$, and $\chi = 0.7$. The solid line reports the steady state level of output, x^{η}, bank credit to final good firms, p^{s}, and trade credit to final good firms, p^{tc}, in the benchmark economy for different values of $\tilde{\theta}$. The dashed line reports the same values for the spot economy.

The figure also compares the steady state of the benchmark economy to that of the spot economy (line with circles). We can see that the economy with trade credit always features a higher level of output relative to the spot economy, with the distance between the two economies increasing when $\tilde{\theta} \leq \bar{\theta}$ and decreasing otherwise. Thus, trade credit relationships alleviate the economic costs of credit market frictions, especially when the monopolist has untapped borrowing capacity.

Why can the economy with trade credit support a higher steady state level of production than the spot economy? To answer this question, let's first consider the spot economy. There, bank credit flows to the final good firms and from there to the monopolist. The monopolist then uses these funds for two distinct purposes: paying for the labor costs necessary to produce the intermediate good and paying for his rents.[13] The payment of these rents crowds out expenditures on productive inputs, and it contributes to keeping production below the first best. The economy with trade credit can instead direct more funds toward the payment of labor costs in the morning for two reasons. First, part of the supplier's rents are now paid in the afternoon, which implies a lower degree of crowding out of productive expenditures. Second, the supplier can pledge his accounts receivable with banks up to $(1-\tilde{\theta})$ of their value. This allows the economy as a whole to obtain more credit from banks in the morning: in the spot economy, bank credit is at most $(1-\tilde{\theta})x^{\eta}$, while it is at most $(1-\tilde{\theta})x^{\eta} + (1-\tilde{\theta})\beta\tilde{\theta}x^{\eta}$ in the benchmark economy.

[13]Indeed, from the first-order condition (7), we can see that a fraction $(1-\tilde{\theta})\eta$ of the revenues of final good firms is directed toward labor payments, while a fraction $(1-\tilde{\theta})(1-\eta)$ goes toward the supplier's rents.

Summarizing this discussion, we note that trade credit has two main effects: it increases the overall *quantity* of bank credit in the economy, and it improves its allocation. Both forces facilitate the remuneration of productive inputs, and they allow the economy to sustain a higher level of production.

3.1.2 Response to financial shocks

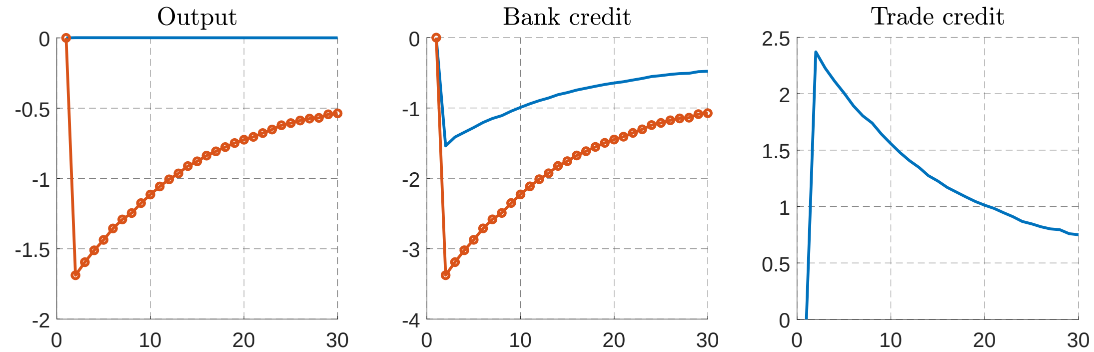
We now turn to studying how the spot and trade credit economy respond to an increase in θ. This shock tightens the credit supply from banks, so the comparison will be informative about whether the presence of trade credit dampens or amplifies the economic effects of financial shocks. To this purpose, we assume that θ can take two values: $\{\tilde{\theta}, \tilde{\theta} + \varepsilon\}$ with $\epsilon > 0$ but small and transition matrix $p(\theta'|\theta)$. We then study how output responds when there is a switch from the low to the high-θ state.

The output effect of this shock in the spot economy can easily be studied using equation (8), and the line with circles in panel (a) of Figure 3 displays a numerical illustration: in response to an increase in θ, credit supply shrinks, final good firms demand less intermediate inputs, and the economy produces less output. In the benchmark economy, the output effects of the same shocks are more subtle, and they depend on whether or not the monopolist is financially constrained.

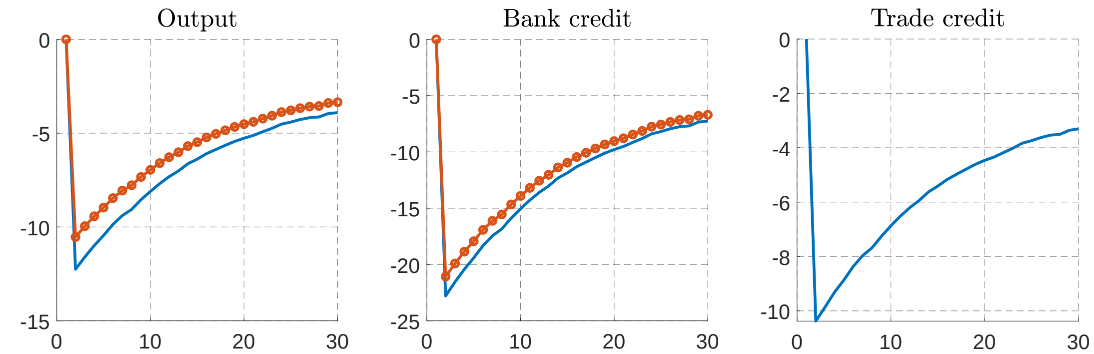
Proposition 2. *Suppose that $\tilde{\theta} < \bar{\theta}$. Let $\theta = \tilde{\theta}$ for a sufficiently long time, and consider a switch to $\theta = \tilde{\theta} + \varepsilon$ where ε is small enough so that $\tilde{\theta} + \varepsilon \leq \bar{\theta}$. Then, output does not change in response to the shock.*

From the previous analysis, we know that the borrowing constraint of the monopolist does not bind in a deterministic steady state when $\tilde{\theta} \leq \bar{\theta}$. This also turns out to be the case in the stochastic economy, as long as the support of $\tilde{\theta} \leq \bar{\theta}$. Proposition 2 then establishes that when the constraint of the monopolist never binds, output eventually becomes independent of the financial shock. The solid lines in panel (a) of Figure 3 provides a numerical illustration of this case. We can see that following the shock, bank credit falls, but the supplier extends more trade credit. This substitution between bank credit and trade credit is so strong that it completely dampens the effects of the financial shock on output.

When $\tilde{\theta} > \bar{\theta}$, the borrowing constraint of the monopolist binds in steady state. In this region, output responds to the financial shock because the supplier cannot fully absorb it. The following result provides a partial characterization of the impact effect of the financial shock.



(a) Dampening of financial shocks



(b) Amplification of financial shocks

Figure 3: Impulse response function to a financial shock

Note: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.7$, $\varepsilon = 0.01$, $p(\tilde{\theta}|\tilde{\theta}) = 0.99$, and $p(\tilde{\theta}+\varepsilon|\tilde{\theta}+\varepsilon) = 0.95$. For the top panel, we set $\tilde{\theta} = 0.4$, while for the bottom panel we set $\tilde{\theta} = 0.9$. The solid line reports the response of output, bank credit to final good firms, and trade credit to final good firms in log-deviations from steady state in the benchmark economy. The line with circles reports the same information for the spot economy. Impulse response functions are computed by simulations following the methodology in Koop, Pesaran, and Potter (1996).

Proposition 3. *Suppose that the borrowing constraint of the monopolist binds, and let $\varepsilon_{x,\theta}$ be the elasticity of labor to the financial shock θ. We then have*

$$\varepsilon_{x,\theta} = \alpha \varepsilon_{x,\theta}^{spot} + \frac{p^{tc}}{p^{tc} + (1-\eta)x^{\eta}} \varepsilon_{p^{tc},\theta}, \tag{19}$$

where $\alpha < 1$, $\varepsilon_{x,\theta}^{spot}$ is the elasticity of labor to θ in the spot economy defined in equation (8), and $\varepsilon_{p^{tc},\theta}$ is the elasticity of trade credit to the financial shock.

To obtain the above expression, we differentiate the borrowing constraint of the supplier, $Wx = (1-\theta)x^{\eta} + (1-\theta)p^{\text{tc}}$, with respect to θ and rearrange terms. Equation (19) provides a useful comparison for the effect of a financial shock in the two economies. From here we can see that $\varepsilon_{p^{\text{tc}},\theta} < 0$ is a necessary condition for having $\varepsilon_{x,\theta} < \varepsilon_{x,\theta}^{\text{spot}}$. That is, trade credit *amplifies* the effects of financial shocks on output only when it falls sufficiently in response to the shock.

The response of trade credit, however, is typically ambiguous in this region and it depends on model parameters: on the one hand, the supplier would like to increase trade credit after a negative financial shock in order to substitute for the fall in spot payments; on the other hand, the financial shock also tightens the borrowing constraint of the supplier, making trade credit expensive from its viewpoint. The solid line in panel (b) of Figure 3 provides a numerical example in which this second motive prevails and $\varepsilon_{p^{\text{tc}},\theta} < 0$. Following a financial shock, the benchmark economy features not only a fall in bank credit to final good firms but also a fall in the trade credit supplied by the monopolist. Consistent with Proposition 3, the complementarity between bank and trade finance in this region amplifies the effects of the shock on output relative to what happens in the spot economy.

3.2 The full model

The special case studied in this section and the economy of Section 2 have two key differences. First, rather than having just one production line with one supplier, the full model has many production lines with heterogeneous characteristics that face potentially many suppliers. Second, the fully fledged model has general equilibrium forces that operate via the demand schedule of each type of final good i and the wage—mechanisms that were muted in the special case studied so far. Despite these differences, most of the results we discussed in this section extend to the full model.

First, the decision problem of a supplier operating in a production line i in partial equilibrium is very similar to the monopolist case studied in the previous subsection, and it can be written recursively in terms of promised value for the final good firms $\tilde{J}_i$ and the aggregate financial shock θ. While we leave the analysis of that problem to Appendix A.1, let us discuss here some properties of the optimal contract, starting from the deterministic steady state.

Proposition 4. *Fix (W, C) and let*

$$rev_i(x) = C^{\frac{1}{\gamma}} k^{\frac{\gamma-1}{\gamma}(1-\eta_i)} (x)^{\eta_i \frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}}$$

be the revenues of final good firms operating in production line i as a function of x. Let

$$x_i^{unc} = \left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} \eta_i \frac{\gamma-1}{\gamma} C^{\frac{1}{\gamma}}}{W N_i^{1-\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}} \right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}},$$

and define x_i^{con} implicitly from the expression

$$\begin{aligned} W x_i^{con} &= \left\{ \left[1-\tilde{\theta}(1-\delta+\delta\pi_i)\right] + (1-\tilde{\theta})\beta N_i \left[N_i \left(1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma}{\gamma-1}} \right) \right.\right. \\ &- \left.\left. \frac{1}{N_i}\left[1-\tilde{\theta}(1-\delta+\delta\pi_i)\right] \right] \right\} \frac{rev_i\left(x_i^{con}\right)}{N_i}. \end{aligned} \tag{20}$$

There exist two thresholds, $\underline{\theta}_i$ and $\overline{\theta}_i$, with $\underline{\theta}_i$ defined as

$$\underline{\theta}_i = \left\{ 1 - N_i \left[1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] \right\} \frac{1}{(1-\delta)+\delta\pi_i} \tag{21}$$

and $\overline{\theta}_i$ being the θ guaranteeing that equation (20) holds with equality when $x_i^{con} = x_i^{unc}$, such that

1. *If $\tilde{\theta} \leq \underline{\theta}_i$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{unc} \\ p_i^s &= N_i \left[1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] rev_i(x_i^{unc}) \\ p_i^{tc} &= 0 \end{aligned}$$

2. *If $\tilde{\theta} \in (\underline{\theta}_i, \overline{\theta}_i]$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{unc} \\ p_i^s &= \{1-\theta(1-\delta+\delta\pi_i)\}\, rev_i(x_i^{unc}) \\ p_i^{tc} &= \beta N_i \left\{ \left[1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] - \frac{1}{N_i}\left[1-\theta(1-\delta+\delta\pi_i)\right] \right\} rev_i(x_i^{unc}) \end{aligned}$$

3. *If $\tilde{\theta} > \overline{\theta}_i$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{con} \\ p_i^s &= \{1-\theta(1-\delta+\delta\pi_i)\}\, rev_i(x_i^{con}) \\ p_i^{tc} &= \beta N_i \left\{ \left[1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma}{\gamma-1}} \right] - \frac{1}{N_i} \left[1 - \theta(1-\delta+\delta\pi_i) \right] \right\} rev_i(x_i^{con}) \end{aligned}$$

There are three regions to consider. When $\tilde{\theta} \leq \underline{\theta}_i$, final good firms in industry i produce the financially unconstrained level of output without the need for trade credit. This possibility is also present in the special case of Section 3.1, but it occurs only for $\tilde{\theta} = 0$.[14] When $\tilde{\theta} \in (\underline{\theta}_i, \overline{\theta}_i]$, production line i still implements the financially unconstrained level of output, but suppliers need to extend trade credit for that purpose. This corresponds to region 1 in Proposition 1. When $\tilde{\theta} > \overline{\theta}_i$, the borrowing constraint of suppliers in production line i binds in steady state, and output is distorted downward relative to the first best—as it was in the case in region 2 in Proposition 1.

Proposition 4 is also useful for deriving some cross-sectional implications of the model.

Lemma 1. *Suppose that the economy is in steady state, and consider a production line i. If $\theta > \underline{\theta}_i$, we have the following comparative static results:*

$$\frac{\partial \left(p_i^{tc} / rev_i \right)}{\partial \eta_i} > 0, \qquad \frac{\partial \left(p_i^{tc} / rev_i \right)}{\partial \pi_i} > 0, \qquad \frac{\partial \left(p_i^{tc} / rev_i \right)}{\partial N_i} < 0.$$

Lemma 1 tells us that production lines with high intermediate inputs share (high η_i), a high share of accounts receivable over sales (high π_i), and that source intermediate goods from concentrated markets (low N_i) will tend to have more trade credit as a fraction of their sales. These results are quite intuitive. A high η_i and high π_i mean that final good firms have a large need for liquidity, as they need to purchase many intermediate goods in the morning while having less cash flow. A low N_i means that there are few suppliers providing intermediate goods; this reduces the outside option for final good firms in case they default on their suppliers—a factor that allows production line i to sustain more trade credit in equilibrium.

[14] In the special case, the monopolist extracts all the rents from final good producers when the latter are financially unconstrained. The spot payments required to achieve that are equal to the revenues of the final good firms, but this is feasible only when $\tilde{\theta} = 0$. In the full model, two ingredients mitigate this force. First, when $N_i > 1$, the suppliers cannot extract all rents from final good producers because of competitive forces. Second, when $\delta > 0$, final good firms have some resources in the morning to pay the suppliers. For this reason, the full model features a range of $\tilde{\theta}$ in which the first-best level of output is implemented without the need for trade credit.

The analysis of how the economy adjusts to financial shocks in the full model is somewhat different from the special case considered in Section 3.1 because financial shocks now trigger general equilibrium effects that were absent in the previous analysis. These forces are also present in the spot economy, and they can be studied analytically in this case. After some manipulations of the optimality condition for labor, we can derive the elasticity of x_i in sector i to a marginal increase in θ in the spot economy:

$$\varepsilon_{x_i,\theta} = \frac{-\theta\left(1-\delta+\delta\pi_i\right)}{\left[1-\theta\left(1-\delta+\delta\pi_i\right)\right]\left(1-\eta_i\right)} + \frac{1}{1-\eta_i}\left(\frac{1}{\gamma}\varepsilon_{C,\theta} - \varepsilon_{W,\theta}\right).$$

The first part of the right hand side of this expression is identical to (8), after factoring in the fact that we set $\delta = 0$ in the special case. The second part of the expression isolates the general equilibrium forces. When θ increases, it reduces the demand for all the other goods in the economy, $\varepsilon_{C,\theta} < 0$: to the extent that varieties are not perfectly substitutable ($\gamma < \infty$), the demand for variety i will fall as a result. This "aggregate demand" channel amplifies the impact of the shock on the output produced by sector i, and it is stronger the smaller is γ. In addition, an increase in θ lowers labor demand and depresses wages, $\varepsilon_{W,\theta} < 0$. This reduces the cost of production and dampens the effect of the shock on output.

These two general equilibrium forces are at play in the benchmark model too, and they will contribute to shaping the response of the economy to financial shocks. In the next section we will solve numerically a calibrated version of the fully fledged economy and use it to assess whether trade credit dampened or amplified the financial shocks associated with the Great Recession in our application. To solve for an equilibrium, we need to conjecture a law of motion for the wage W and for the demand for the consumption basket C as a function of the state variables in the economy—the financial shock θ and the *distribution* of promised values $\{\tilde{J}_i\}$ for all production lines in the economy. To deal with the associated curse of dimensionality, we follow Krusell and Smith (1998) and approximate the dependence of $\{W, C\}$ on this distribution with simple autoregressive terms. See Appendix A.4 for a description of the algorithm.

4 Quantitative analysis

We now move on to a quantitative analysis of the macroeconomic implications of trade credit. Section 4.1 describes the data, and Section 4.2 presents the calibration and discusses the in-sample and out-of-sample properties of the model. Section 4.3 presents the main counterfactuals—one aimed at assessing the size of the credit multiplier and one studying the role of trade credit during the Great Recession. Section 4.4 concludes with a discussion of some policy implications.

4.1 Data

We use annual firm-level data on Italian firms between 2007 and 2015 from the historical ORBIS dataset compiled by Bureau van Dijk. We study a balanced panel of non-financial corporations. Appendix A.3 lays out the cleaning procedure. For each firm in our panel, we measure operating revenues, sales, short-term bank loans, a measure of expenditures on intermediate inputs, the sector in which the firm operates, as well as accounts payable and receivable. Accounts payable is the amount that the firm owes for goods it already received, while accounts receivable is the amount that the firm needs to receive for goods that it has already sold.[15]

We map the data to our model as follows. Using the classification from the Italian input-output tables, we partition the firms in our panel into 58 different sectors. We then average the firm-level balance sheet items for firms within each sector, obtaining the sectoral-level data. Each sector corresponds to a different production line in our model, and the balance sheet data are mapped to the corresponding item for the final good firms.

Given the above assumption, we map the share of accounts payable over sales for sector i to $p^{\text{tc}}_{i,t}/rev_{i,t}$. In addition, the sectoral balance sheet items provide information on key parameters of the model. The share of expenditures on intermediate inputs (materials and services) over operating revenues—$(p^{\text{s}}_{i,t} + p^{\text{tc}}_{i,t})/rev_{i,t}$ in the model—is informative about η_i.[16] The share of accounts receivable over sales, $\delta\pi_i$ in the model, is informative about the distribution of π_i. The ratio of short-term bank loans to sales provides information about θ_t, as it is equal to $[1 - \theta_t(1 - \delta + \delta(1 - \pi_i)]$ in the model when the financial constraint of final good firms in industry i binds.

We also use the sectoral data to construct an empirical counterpart to N_i, the degree of concentration of the suppliers' market for production line i. To do so, we exploit the fact that in the symmetric equilibrium, suppliers have the same sales shares, so $1/N_i$ corresponds to the Herfindahl-Hirschman index (HHI) of the suppliers' market for sector i. To construct this object in the data, we use ORBIS and obtain the HHI for each of the 58 sectors in our data. We then use Italy's input-output tables to construct a weighted average HHI of a sector's suppliers. The weight of sector j in this calculation is equal to the share

[15]The Italian ORBIS dataset does not contain an explicit variable with expenditures on intermediate inputs. To construct this variable, we subtract earning before income and tax (EBIT), the wage bill, and depreciation from operating revenues.

[16]From the expressions in Proposition 4, we can see that in a deterministic steady state, this ratio is equal to $\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}$ when N_i is large.

of inputs provided to sector i by sector j.[17]

Table 1 provides descriptive statistics on the sector-level variables. The ratio of accounts payable to sales is 23% on average, which is 40% higher than the average ratio of short-term loans to sales. The ratio of accounts receivable to sales is 29% on average. There is large variation across sectors in all three of these ratios. The average ratio of expenditures on intermediate inputs to sales is 67%. Finally, the average $\text{HHI}^{\text{supplier}}$ is 0.05. This value corresponds to a market operated by 20 identical suppliers.

Table 1: Sector-level descriptive statistics

	Mean	St. deviation	Min	Max
Accounts payable/sales	0.23	0.05	0.10	0.39
Accounts receivable/sales	0.29	0.08	0.07	0.59
Short-term bank loans/sales	0.16	0.09	0.07	0.62
Intermediate inputs /sales	0.67	0.08	0.41	0.84
$\text{HHI}^{\text{supplier}}$	0.05	0.02	0.02	0.14

Notes: This table presents descriptive statistics on sector-level variables of interest. The sample includes the 58 sectors in Italy's input-output table in 2007. The value of supplier's HHI is computed in 2010 because of data limitations.

Before moving on to the calibration, we use the dataset to test some of the key predictions of the model. We start by studying the cross-sectoral relations described in Lemma 1. There, we showed that in a deterministic steady state, sectors with high η_i, π_i and a low N_i are characterized by a larger share of accounts payable over sales. We can evaluate these predictions by performing simple linear regressions, with results reported in Table 2.

The dependent variable in all specifications is the ratio of accounts payable to sales for each sector at time t. Specification (1) shows that accounts receivable are positively associated with accounts payable, with a quite strong relationship (an adjusted R^2 of 0.33). Specification (2) shows that there is also a significant positive relationship between expenditures on intermediate inputs—a proxy for η_i—and accounts payable, as our model would predict. Specification (3) shows a significant and positive relationship between suppliers' HHI and accounts payable. That is, firms that source intermediate inputs from more concentrated sectors tend to pay more in credit. This is also consistent with the predictions of our theory, as the more concentrated is that sector, the greater the incentives firms have to pay the suppliers. Specification (4) reports the regression with all three control variables. All coefficients are significant, and their signs are consistent with Lemma 1. These three factors jointly account for 48% of the variation in accounts payable across sectors.

[17] We construct this index using 2010 data, as it is the only year in our sample where we have Italian data on input-output relationships at a fine sectoral level.

Table 2: Cross-sectional sector-level regressions

	Dep. variable: Accounts payable/sales			
	(1)	(2)	(3)	(4)
Accounts receivable/sales	0.297^{***}			0.326^{***}
	(0.029)			(0.027)
Intermediate inputs /sales		0.154^{***}		0.239^{***}
		(0.028)		(0.020)
$\text{HHI}^{\text{supplier}}$			0.655^{***}	0.303^{***}
			(0.111)	(0.085)
Adj. R^2	0.329	0.092	0.116	0.477
Obs.	522	522	522	522

Notes: This table presents the sector-level regression results. Consistent with the model's prediction, we find that a high ratio between accounts payable and sales is associated with: (i) a high ratio of accounts receivable to sales, (ii) a high ratio of intermediate inputs to sales, and (iii) a high degree of HHI among the sector's suppliers. All regressions include year fixed effects. Robust standard errors are in parentheses. *** - significant at the 1% level.

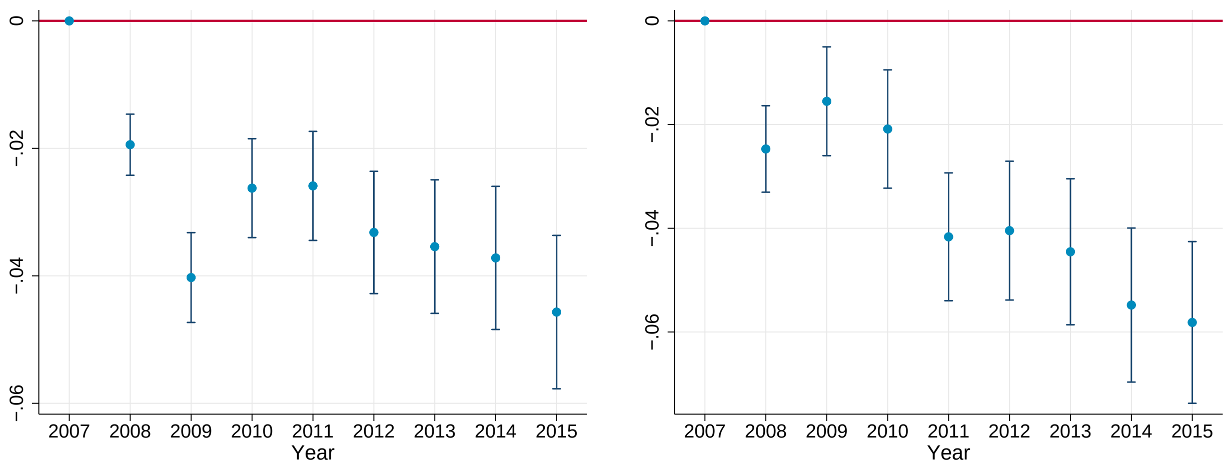
A second important prediction of the model is that firms' production choices are more sensitive to financial shocks the tighter the financial constraints of their suppliers. To test this prediction, we follow a diff-in-diff approach: we use the calibrated model (see the next subsection) to sort sectors according to the borrowing capacity of their suppliers in 2007 and check whether those with more financially constrained suppliers experienced a deeper fall in trade credit and output in response to the financial shocks associated with the Great Recession. Specifically, we compute for each sector in our dataset the variable $\overline{\theta}_i$—the threshold level of θ such that the financial constraints of suppliers in sector i bind in a deterministic steady state. Sectors differ in their value of $\overline{\theta}_i$, and those with a relatively low value will tend to have more constrained suppliers. Our approach consists of dividing the 58 sectors into two equally sized groups depending on the value of $\overline{\theta}_i$ and then estimating the regression

$$y_{f,i,t} = \alpha_f + \beta_t \times \mathbb{1}\left[\overline{\theta}_i < \text{median}\left(\overline{\theta}_i\right)\right] + \Gamma_t \times X_f + \epsilon_{f,i,t}, \tag{22}$$

where $y_{f,i,t}$ is the dependent variable of interest (accounts payable or sales) of firm f operating in sector i at time t. The regression includes firm-level fixed effects and time fixed effects, as well as other controls that interact with time fixed effects.[18]

[18]The controls include indicators for whether the average sales of the firm are larger than those of the median firm in the sample, whether the capital intensity proxied by the assets-to-sales ratio is higher than the median, whether the firm operates in manufacturing, and whether it operates in the service sector.

Figure 4 displays the estimation results. Panel (a) presents the estimates for β_t when the dependent variable is the (log) of firms' sales. Consistent with the theory, firms in sectors that were closer to the financially constrained region experienced a larger drop in sales after 2007. Two drops are noticeable. The first is in 2008–2009, when the sales of these sectors dropped between 2% and 4% relative to other sectors, and the second drop is during the intensification of the sovereign debt crisis in 2011–2012. Panel (b) of Figure 4 presents the same differential for firms' accounts payable. We can see that accounts payable for low-$\overline{\theta}_i$ sectors fell 2% relative to those of high-$\overline{\theta}_i$ sectors in the aftermath of the Great Recession. This differential grew to 6% after the sovereign debt crisis.



(a) Sales (b) Accounts payable

Figure 4: Dynamic diff-in-diff results

Notes: These figures present the point estimates and 99% confidence intervals for β_t in (22). Panel (a) reports the results for log-sales as the dependent variable, while the dependent variable in panel (b) is the log of accounts payable. Standard errors are clustered at the firm level.

4.2 Calibration

We can classify the structural parameters into four groups: preference parameters $[\beta, \psi, \chi, \gamma]$, common financial and technology parameters, $[\delta, \sigma]$, sector-specific parameters, $\{\eta_i, \pi_i, N_i\}_i$, and parameters governing the stochastic process of the financial shock. For the quantitative analysis, we assume that θ_t follows a two-state Markov process $\theta_t \in \{\theta_L, \theta_H\}$. We interpret θ_L as the state of the financial sector in "normal times" and a switch to θ_H as a financial crisis. To be consistent with our data, a time period in the model corresponds to one year, and we consider 58 different sectors.

We set $\beta = 0.98$ and $\psi = 1.00$, standard values in the macroeconomic literature. In addition, we set $p(\theta_L|\theta_L) = 0.99$ to be consistent with the notion that financial crises are rare events in advanced economies. The remaining parameters are chosen simultaneously so that the model matches a set of sample moments computed using our dataset. Table 3 reports the calibrated parameters along with the empirical targets and their model counterpart in simulations. Below, we describe the sample moments and discuss heuristically which model parameters they help us discipline.

We target the ratio of accounts receivable over sales, expenditures on intermediate inputs/services over sales, and the $\mathrm{HHI}_i^{\mathrm{supplier}}$ for each of the sectors in 2007 and match it to the sample average in model simulations conditional on $\theta = \theta_L$. As we discussed previously, these variables provide information on the sector-specific parameters $\{\pi_i, \eta_i, N_i\}_i$. We also target the average ratio of accounts payable to sales in 2007. Given the other model parameters, this moment provides information about δ, as higher δ reduces the need for trade credit for all sectors. In our calibration, δ is equal to 0.54. A value of 3.5 for χ guarantees that the level of worked hours in our simulations equals one-third in normal times.

We use the behavior of the average ratio of firms' short-term bank loans over sales to discipline the stochastic process for θ_t. This ratio went from 0.16 in 2007 to 0.12 in 2008, and by 2011 it was back to 0.14. We choose the parameters of the stochastic process so that on average our model replicates this behavior when there is a switch from θ_L to θ_H. This yields $\theta_L = 0.78$, $\theta_H = 0.84$, and $p(\theta_H|\theta_H) = 0.86$.

The remaining parameters, γ and σ, have poorly measured empirical counterparts. To discipline the former, we target the peak-to-trough fall in average log sales during the Great Recession (0.12) and match it in the model to the average fall in firms' log sales conditional on a switch from θ_L to θ_H. As we explained in Section 3.2, a higher value of γ dampens the general equilibrium effects and thus is associated with a smaller response of aggregate output to the financial shock. To discipline σ, we include the diff-in-diff coefficient β_{2008} for log sales, reported in Figure 4. Holding the other parameters fixed, changes in σ affect $\bar{\theta}_i$, the boundary of the region in which the suppliers' borrowing constraint binds, and it thus affects the size of this differential response. Our calibration yields $\gamma = 3$ and $\sigma = 5$.

Besides having good in-sample fit, the model reproduces quite well the features of the distribution of accounts payable that are not targeted in the calibration. Panel (a) of Figure 5 plots the ratio of accounts payable over sales for all 58 sectors in the data (vertical axes) and in the model (horizontal axes). The 45-degree line in this graph indicates a perfect fit for the model, in the sense that the model would exactly predict the accounts receivable for each sector. We can see from the graph a strong, positive association, meaning that the

Table 3: Model parameters and targeted moments

Parameter	Value	Moment	Data	Model
Mean(π_i)	0.52	Distribution of accounts receivable/sales		
Stdev(π_i)	0.14			
Mean(η_i)	0.81	Distribution of intermediate inputs/sales		
Stdev(η_i)	0.09			
Mean(N_i)	23.83	Distribution of HHI$^{\text{supplier}}$		
Stdev(N_i)	10.96			
δ	0.55	Mean(Accounts payable/sales)	0.23	0.23
χ	3.50	Mean(Worked hours/total hours)	0.33	0.33
θ_L	0.78	Mean(Bank loans/sales) in 2007	0.16	0.16
θ_H	0.84	Mean(Bank loans/sales) in 2008	0.12	0.12
$p(\theta_H \vert \theta_H)$	0.86	Mean(Bank loans/sales) in 2011	0.14	0.14
γ	3.00	% Fall in average firms' sales	0.12	0.12
σ	5.00	β_{2008} in eq. (22), log-sales as dep. var.	-0.02	-0.02

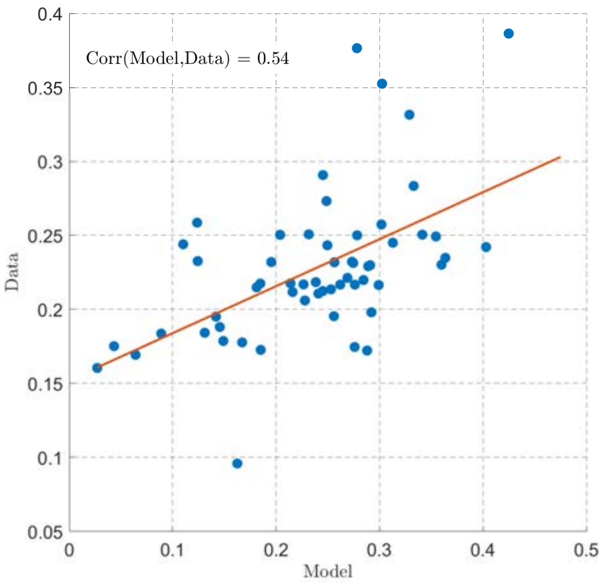
Notes: The table reports the numerical values of model parameters along with the empirical targets used in the calibration. The first six rows report statistics for the distribution of $\{\pi_i, \eta_i, N_i\}$ across the 58 sectors of our analysis. These parameters are chosen to replicate exactly the value of accounts receivable/sales, intermediate input costs/sales, and the HHI$^{\text{supplier}}$ for each of the 58 sectors. The other rows report the exact numerical values for the remaining parameters along with the moments used to discipline them. The sample moments in the model are computed using simulations.

calibrated model predicts well the differences in trade credit across sectors.

Panel (b) of Figure 5 reports the correlation across sectors between accounts payable and other key ratios in the data and in the model. The table shows that the quantitative model displays cross-sector correlations that are consistent with the data. The correlation between accounts payable and accounts receivable is 0.48 in the model, very close to its data counterpart of 0.59. The correlation between payables and the share of intermediate inputs is positive in the data (0.21) and in the model (0.68), although it is substantially higher in the latter. The model implied correlation between accounts payable and the supplier HHI is also close to the data (0.30 vs. 0.24). Finally, the model reproduces a positive correlation across sectors between accounts payable and bank loans.

4.3 The macroeconomic implications of trade credit

In Section 3, we focused on two main macroeconomic implications of trade credit. First, we showed that *on average*, trade credit relationships allow for a better allocation of credit and increase its overall size, thus reducing the output costs of financial frictions. Second,



(a) Trade credit across sectors

	Data	Model
corr(payable/sales, receivable/sales)	0.59	0.48
corr(payable/sales, intermediate-inputs/sales)	0.21	0.68
corr(payable/sales, HHI$^{\text{supplier}}$)	0.24	0.30
corr(payable/sales, bank debt/sales)	0.35	0.59

(b) Cross-sector correlation patterns

Figure 5: Trade credit across sectors

Notes: Panel (a) displays a scatter plot of the model's prediction for accounts payable to sales in the model (*x-axis*) rekatuve ti the data (*y-axis*) in all 58 sectors. The correlation between the model and data is 0.54. Panel (b) reports the cross-sector correlations of the accounts payable to sales ratio with different variables in the data and in the model. In both panels, the data refer to 2007. In the model, the statistics are averages from a long simulation conditioned on $\theta_t = \theta_L$.

we have seen that an economy with trade credit can be more or less sensitive to *changes* in the financial conditions of the banking sector, depending on suppliers' borrowing capacity. We now use the calibrated model to quantify these two aspects.

We start by measuring the size of the credit multiplier and quantifying its implications for output. For that purpose, Table 4 compares the average behavior of credit and output in the benchmark economy to those of two counterfactual economies. The first economy is identical in all respects to the benchmark, except that final good firms cannot issue trade credit to their suppliers, ($p_i^{\text{tc}} = 0$ for all i). In the second economy, final good firms cannot issue trade credit to suppliers *and* they are paid on the spot by their customers ($p_i^{\text{tc}} = 0$ and $\pi_i = 0$ for all i). We can think of the latter as the *spot economy* because none of the transactions between the different economic entities—intermediate good producers, final good producers, and households—involve the issuance of IOUs.

Let us start by comparing the benchmark with the $p_i^{\text{tc}} = 0$ economy. Total bank credit in the latter is on average 43% the size of bank credit in the benchmark. This is due to two effects. First, in the benchmark economy, final good firms have higher revenues, so they can mechanically obtain more credit from banks.[19] Second, in the benchmark economy, sup-

[19]In both economies, firms can pledge $(1 - \theta_t)$ of their afternoon revenues.

Table 4: Quantifying the credit multiplier

		Benchmark	$p_i^{\text{tc}} = 0$	$p_i^{\text{tc}} = 0$ and $\pi_i = 0$
Bank credit		1.00	0.43	0.33
	To final good firms	0.73	0.43	0.33
	To suppliers	0.27	0.00	
	% allocated to wages	0.96	0.02	0.03
Output		1.00	0.60	0.86

Notes: This table reports bank credit and output in the benchmark economy and in the two counterfactual economies. The figure reports time averages conditional on the economy being in good times ($\theta_t = \theta_L$). We normalize the bank credit and output by their value in the benchmark economy.

pliers borrow from banks by pledging their accounts receivable—something that doesn't happen in the counterfactual economy because suppliers do not borrow from banks. This latter mechanism is quite sizable in the benchmark economy, as credit backed by the accounts receivable of suppliers represents approximately 27% of total bank credit to firms. Aside from increasing its quantity, the benchmark economy also has a better allocation of credit relative to the counterfactual. To see that, Table 4 also reports the fraction of bank credit that is directed toward the payments of productive inputs—the wages of workers. To that purpose, we compute the equilibrium wage payments by suppliers and subtract the cash that final good firms have available in the morning. We then scale this indicator by total bank credit to firms. A value of 1 means that all bank credit in the economy is allocated to the payment of productive inputs. We can see that in our benchmark economy, this ratio is indeed close to 1, while in the spot economy, most of the bank credit is used to pay for suppliers' rents. The combination of these two factors—lower credit and a worse allocation—implies that output in the $p_i^{\text{tc}} = 0$ economy is 60% that of the benchmark, as the last row of Table 4 shows.

In the counterfactual we just discussed, final good firms cannot issue IOUs to their suppliers but they do accept IOUs from their customers, the households. In the last column of Table 4, we report what happens to credit and output in an economy in which households also pay on the spot, $\pi_i = 0$ for all i. We can see that output is higher while credit is lower relative to the previous counterfactual. This happens because the fraction of cash that final good firms receive in the morning is higher when $\pi_i = 0$, which reduces credit demand. Qualitatively, however, the comparison with the benchmark is similar to that of the previous counterfactual: the spot economy features substantially less bank credit relative to the benchmark and 14% lower output.

The second quantitative exercise consists of assessing how trade credit shaped the response of the Italian economy to the financial shocks of the Great Recession. To do so, we study the response of the benchmark economy to a tightening of aggregate financial conditions—a switch from θ_L to θ_H—and compare it to what happens in the counterfactual economies.

Figure 6 reports the response of output to the financial shock in the benchmark economy and in the two counterfactuals. In the benchmark economy (solid line), output falls 11% on impact. In the two counterfactual economies, the output effects of the same shock are much smaller: in the $p_i^{\text{tc}} = 0$ economy, output falls by 7.7% on impact, while in the spot economy, it falls by only 5.9%. This means that the presence of trade credit substantially amplified the macroeconomic implications of financial shocks during the Great Recession—between 30% and 45% of the total response, depending on which counterfactual we consider in the comparison. The large amplification is a product of the fact that in our calibration, most of the sectors (50 of 58) were already in the financially constrained region in 2007. In that region, the aggregate financial shock not only had the effect of reducing bank credit to final good firms, but also depressed the supply of trade credit in the economy.

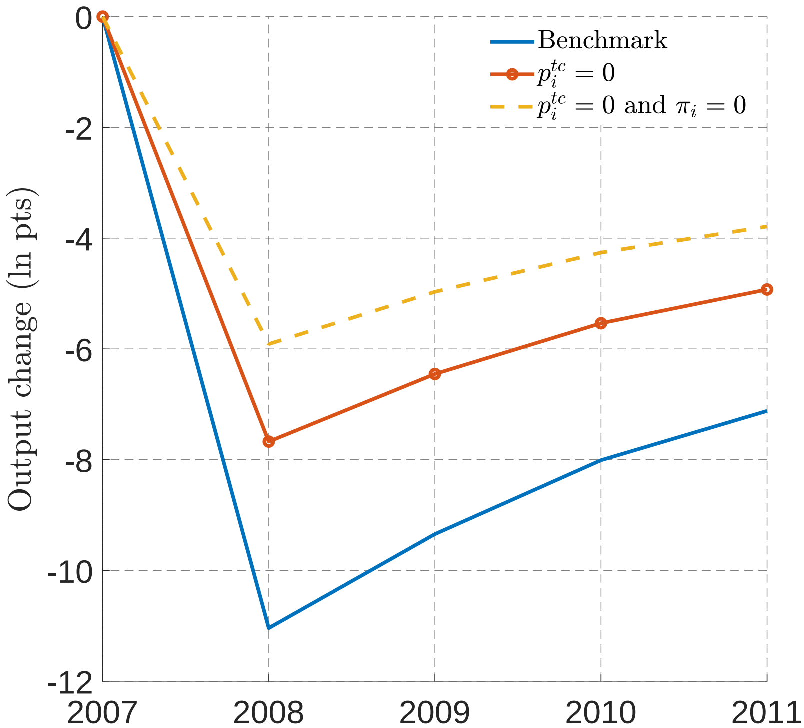


Figure 6: The implications of trade credit for the Great Recession

Note: This figure displays the response of output to a financial shock in the benchmark economy (solid line), the counterfactual economy with $p_i^{\text{tc}} = 0$ (line with circles) and the counterfactual economy with $p_i^{\text{tc}} = 0$ and $\pi_i = 0$ (dashed line).

4.4 The effects of corporate subsidies

In this section, we study the effectiveness of corporate subsidies in alleviating financial constraints and stimulating output during financial crises. It is immediate to see that such subsidies raise output in our environment. Our focus, however, is on studying which firms would benefit the most: would subsidies be more effective in stimulating output if they were directed toward downstream or upstream firms? We start by analyzing this question in the special case of Section 3.1 and show that the subsidy is more effective when targeted toward the financially constrained supplier. We then move on to the quantitative model and compare alternative types of interventions.

To illustrate the difference between a lump-sum subsidy to final good producers and one to suppliers, we first consider the special case economy studied in Section 3.1, which features a single production line with a single supplier. We further assume that there are no aggregate shocks and that the degree of financial frictions is such that the borrowing constraint of the monopolist binds in the steady state absent corporate subsidies ($\theta > \bar{\theta}$). The problem of the supplier with lump-sum subsidies is as follows:

$$
\begin{aligned}
V(J) = \max_{\{x, p^{\mathrm{s}}, p^{\mathrm{tc}}, J'\}} \quad & \left(p^{\mathrm{s}} + p^{\mathrm{tc}} - Wx\right) + T_s + \beta V(J'), \\
\text{s.t.} \qquad & p^{\mathrm{s}} \leq (1-\theta)x^{\eta} + T_f, \\
& Wx - p^{\mathrm{s}} \leq (1-\theta)p^{\mathrm{tc}} + T_s, \\
& p^{\mathrm{tc}} \leq \beta J', \\
& p^{\mathrm{s}} + p^{\mathrm{tc}} \leq x^{\eta} + T_f, \\
& J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta J',
\end{aligned}
$$

where T_f is the periodic lump-sum subsidy to final good firms, and T_s is the one to the monopolist. Notice that J, the customer value of its relationship with the supplier, does not include T_f because the subsidy is paid to the customer independently of its relationship with the supplier.

The following proposition characterizes the effects of the two subsidies in steady state when the supplier is financially constrained.

Proposition 5. *Suppose that $\theta > \bar{\theta}$ where $\bar{\theta}$ is defined implicitly by $(1-\bar{\theta})(1+\beta\bar{\theta}) = \eta$. Then, the effects of corporate subsidies on employment in the steady state around the point without corporate subsidies are given by*

$$
\left.\frac{\partial x}{\partial T_s}\right|_{T_s = T_f = 0} = \frac{1}{(1-\eta)W}, \tag{23}
$$

and

$$\left.\frac{\partial x}{\partial T_f}\right|_{T_s=T_f=0} = \frac{1-\beta(1-\theta)}{(1-\eta)W}. \tag{24}$$

Proposition 5 states that when the monopolist's borrowing constraint binds, a subsidy to the monopolist is more effective in stimulating output relative to a subsidy to the final good producer. While both types of subsidies stimulate employment, a corporate subsidy to final good firms is only a fraction $[1-\beta(1-\theta)]$ as effective.

To understand this result, let's substitute the borrowing constraint of the final good producers with that of the supplier, both binding around the *laissez-faire* steady state, to obtain

$$Wx = (1-\theta)x^{\eta} + (1-\theta)p^{\text{tc}} + T_s + T_f.$$

We can then use this expression to study the effects of corporate subsidies on output. The two subsidies enter this expression in a similar fashion, so they will end up having different output effects only if they have differential effects on p^{tc}. This is precisely what happens, with T_f crowding out trade credit more than T_s. Indeed, substituting the borrowing constraint of the final good producer into the expression for J, we obtain that in steady state

$$p^{\text{tc}} = \beta J = \beta[\theta x^{\eta} - T_f].$$

Holding x constant, a one dollar subsidy to final good firms reduces one-for-one the value of their relationship with the supplier, and it therefore depresses the steady state level of trade credit by β dollars. This effect doesn't happen when the subsidy is given to the supplier. So, T_f ends up crowding out trade credit more relative to T_s, explaining why the latter is a better tool for stimulating output.

We now return to the quantitative model. We assume that during a financial crisis, the government has limited resources to support financially constrained firms.[20] We denote by T_i^{supplier} and T_i^{final} the lump-sum corporate subsidies to suppliers and final good producers in production line i, respectively. The total sum of these subsidies is levied as a lump-sum tax on households in the economy. We assume that these subsidies are given at the start of the period as long as $\theta = \theta_H$—that is, as long as the economy remains in a financial crisis.

In addition to directly changing the profits of final good producers and suppliers, corporate subsidies affect the borrowing constraints of both types of firms. In Appendix A.1.1,

[20]Note that we assume that there is perfect information so that the government can perfectly observe which sectors are financially constrained. Dávila and Hébert (2023) study the optimal design of corporate taxation with private information so that the government cannot directly observe which firms are financially constrained. Unlike their paper, our focus is on where on the supply chain a corporate subsidy is more effective when trade credit is available.

Table 5: The effects of corporate subsidies during the Great Recession

Subsidies	Output gain	Effectiveness
Uniform across firms	0.91	1
Uniform across producers	0.85	0.93
Uniform across suppliers	0.98	1.07
Uniform across firms with $\bar{\theta}_i < \theta_H$	0.98	1.08
Uniform across suppliers with $\bar{\theta}_i < \theta_H$	1.05	1.15

Notes: This table reports the counterfactual increase in output (in log points) during 2008 relative to our benchmark specification for different allocations of subsidies across firms. The total amount of subsidies is set to be 1% of output in normal times. Effectiveness represents the relative contribution to output of the subsidies scheme with respect to a uniform subsidy to all firms. The larger the effectiveness, the larger is the contribution to output.

we lay out the supplier's problem with corporate subsidies. We assume that the size of government subsidies is equal to 1% of GDP in normal times. We consider five configurations of corporate subsidies: (i) a uniform subsidy to all firms, (ii) a uniform subsidy to all producers, (iii) a uniform subsidy to all suppliers, (iv) a uniform subsidy to all sectors with $\theta > \bar{\theta}_i$, and (v) a uniform subsidy to all suppliers in sectors with $\theta > \bar{\theta}_i$. Table 5 presents the results.

The first row of Table 5 shows that a uniform subsidy to all firms during the Great Recession raises output by about 0.9%. That is, instead of output declining by 11% in 2008, a uniform subsidy to all firms would reduce the decline to 10.1%. The second and third rows display the effects of corporate subsidies when given uniformly to only producers and only suppliers, respectively. As Proposition 5 suggests, a uniform subsidy to suppliers is more effective in stimulating output during a financial crisis. While a uniform subsidy to producers raises output by 0.85 log points, a uniform subsidy to suppliers raises output by 0.98 log points. That is, a uniform subsidy to suppliers in the economy is about 15% more effective than a uniform subsidy to all producers and about 7% more effective than a uniform subsidies to all firms in the economy.

The final two rows of Table 5 display the effects of corporate subsidies when targeted toward sectors which are more likely to be financially constrained. Overall, there are 53 sectors in the economy with $\bar{\theta}_i < \theta_H$. Corporate subsidies are more effective at stimulating output when targeted toward these sectors. A uniform subsidy to all firms in these financially constrained sectors raises output by 0.98 log points—an increase in 8% relative to an untargeted subsidy configuration toward all sectors. A uniform subsidy toward only suppliers in financially constrained sectors has the largest effect on output from the configurations we've analyzed. Under such subsidy configuration, output goes up by 1.05

log points—15% more effective than a uniform subsidy to all firms in all sectors in the economy.

Our paper is not the first to study the effectiveness of corporate subsidies in stimulating output in the context of production networks. Liu (2019) shows that optimal corporate subsidies in a production network should be targeted toward sectors that are more *central* to market imperfections—sectors that supply a disproportionate fraction of output to other sectors with severe market imperfections. He finds that these sectors are typically upstream sectors. Glode and Opp (2021) studies the effectiveness of corporate subsidies in a production chain when firms can default on their trade credit in equilibrium. They find that corporate subsidies can be more effective when targeted toward downstream producers, as such subsidies can help prevent *default waves*. Relative to these papers, our analysis sheds light on another motive that shapes the design of optimal corporate subsidies—their effects on trade-credit linkages among firms.

5 Conclusion

This paper has proposed an equilibrium model to explain the prevalence of trade credit as a form of short-term financing for companies around the world and used it to understand the macroeconomic implications of this phenomenon. In our theory, trade credit is enforced in equilibrium by reputational forces, as customers have an incentive to repay their suppliers out of fear of losing that relationship. This mechanism allows the economy to increase credit provided by the financial system because suppliers can enforce these IOUs and at the same time discount these claims with financial institutions to obtain liquidity. We provide cross-sectional evidence consistent with this theory and fit the model to Italian data in order to quantify the macroeconomic implications of the credit multiplier. We show that this process allows the economy to support 14% more output on average, but it also makes the economy more vulnerable to financial shocks, as the presence of trade credit amplified the output costs of the Great Recession by 45%.

We believe this framework could be used to address a number of important questions. For example, we could apply it to study the role of trade credit relationships in the propagation of firm-specific shocks throughout the production network. In addition, the forward-looking aspect of these relationships makes them particularly vulnerable to self-fulfilling confidence crises, something that could rationalize the sudden disruptions in the payments' chain observed for certain countries during the Great Recession. We plan to address these and other exciting questions in future research.

References

Aguiar, Mark, Manuel Amador, and Gita Gopinath. 2009. "Investment Cycles and Sovereign Debt Overhang." *The Review of Economic Studies* 76 (1):1–31.

Altinoglu, Levent. 2021. "The origins of aggregate fluctuations in a credit network economy." *Journal of Monetary Economics* 117:316–334.

Amberg, Niklas, Tor Jacobson, Erik Von Schedvin, and Robert Townsend. 2021. "Curbing shocks to corporate liquidity: The role of trade credit." *Journal of Political Economy* 129 (1):182–242.

Atkeson, Andrew and Ariel Burstein. 2008. "Pricing-to-market, trade costs, and international relative prices." *American Economic Review* 98 (5):1998–2031.

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." *Handbook of macroeconomics* 1:1341–1393.

Biais, Bruno and Christian Gollier. 1997. "Trade credit and credit rationing." *The review of financial studies* 10 (4):903–937.

Bigio, Saki. 2023. "A Theory of Payments-Chain Crises." Tech. rep., National Bureau of Economic Research.

Bigio, Saki and Jennifer La'o. 2020. "Distortions in production networks." *The Quarterly Journal of Economics* 135 (4):2187–2253.

Bocola, Luigi and Guido Lorenzoni. 2022. "Risk-sharing externalities." Tech. rep., National Bureau of Economic Research.

Boissay, Frederic and Reint Gropp. 2013. "Payment defaults and interfirm liquidity provision." *Review of Finance* 17 (6):1853–1894.

Boldrin, Michele and Michael Horvath. 1995. "Labor contracts and business cycles." *Journal of Political Economy* 103 (5):972–1004.

Burkart, Mike and Tore Ellingsen. 2004. "In-kind finance: A theory of trade credit." *American economic review* 94 (3):569–590.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini. 2004. "Aggregate consequences of limited contract enforceability." *Journal of political Economy* 112 (4):817–847.

Costello, Anna M. 2020. "Credit market disruptions and liquidity spillover effects in the supply chain." *Journal of Political Economy* 128 (9):3434–3468.

Cuñat, Vicente. 2007. "Trade credit: suppliers as debt collectors and insurance providers." *The Review of Financial Studies* 20 (2):491–527.

Cuñat, Vicente and Emilia García-Appendini. 2012. "Trade Credit and Its Role in Entrepreneurial Finance." In *The Oxford Handbook of Entrepreneurial Finance*. 526–557.

Dávila, Eduardo and Benjamin Hébert. 2023. "Optimal Corporate Taxation under Financial Frictions." *The Review of Economic Studies, forthcoming* .

Di Tella, Sebastian. 2017. "Uncertainty shocks and balance sheet recessions." *Journal of Political Economy* 125 (6):2038–2081.

Dovis, Alessandro. 2019. "Efficient sovereign default." *The Review of Economic Studies* 86 (1):282–312.

Frank, Murray and Vojislav Maksimovic. 1998. "Trade credit, collateral, and adverse selection." *Unpublished manuscript, University of Maryland* .

Glode, Vincent and Christian C Opp. 2021. "Private renegotiations and government interventions in debt chains." *Available at SSRN 3667071* .

Hardy, Bryan, Felipe E Saffie, and Ina Simonovska. 2022. "Economic Stabilizers in Emerging Markets: The Case for Trade Credit." Tech. rep., Working paper.

Holmstrom, Bengt and Jean Tirole. 1997. "Financial intermediation, loanable funds, and the real sector." *the Quarterly Journal of economics* 112 (3):663–691.

Jacobson, Tor and Erik Von Schedvin. 2015. "Trade credit and the propagation of corporate failure: An empirical analysis." *Econometrica* 83 (4):1315–1371.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic effects of financial shocks." *American Economic Review* 102 (1):238–71.

Kehoe, Patrick J and Fabrizio Perri. 2002. "International business cycles with endogenous incomplete markets." *Econometrica* 70 (3):907–928.

Kiyotaki, Nobuhiro and John Moore. 1997a. "Credit Chains." Tech. rep., Unpublished Manuscript.

———. 1997b. "Credit cycles." *Journal of political economy* 105 (2):211–248.

Koop, Gary, M Hashem Pesaran, and Simon M Potter. 1996. "Impulse Response Analysis in Nonlinear Multivariate Models." *Journal of Econometrics* 74 (1):119–147.

Krusell, Per and Anthony A Smith, Jr. 1998. "Income and wealth heterogeneity in the macroeconomy." *Journal of political Economy* 106 (5):867–896.

Liu, Ernest. 2019. "Industrial policies in production networks." *The Quarterly Journal of Economics* 134 (4):1883–1948.

Luo, Shaowen. 2020. "Propagation of financial shocks in an input-output economy with trade and financial linkages of firms." *Review of Economic Dynamics* 36:246–269.

Mateos-Planas, Xavier and Giulio Seccia. 2021. *Trade Credit Default*. CFM, Centre for Macroeconomics.

Petersen, Mitchell A and Raghuram G Rajan. 1997. "Trade credit: theories and evidence." *The review of financial studies* 10 (3):661–691.

Reischer, Margit. 2020. "Finance-thy-neighbor: Trade credit origins of aggregate fluctuations."

Souchier, Martin. 2022. "The Pass-through of Productivity Shocks to Wages and the Cyclical Competition for Workers." Tech. rep., Working Paper.

Appendix

A.1 Full model - recursive formulation

In Section 2, we've presented the problem of the intermediate-good producer in sequence form. In this section, we lay out the recursive formulation of the problem. We will use the recursive formulation to derive optimality conditions and to prove the different propositions in the paper.

Before writing down the recursive formulation, it is useful to briefly discuss the state variables in the firm's problem. First, the exogenous state variable is θ, the degree of financial frictions in the economy. In addition, each intermediate-good producer carries over a promise made in the previous period to final good firms. This promise guarantees the final good producer a discounted surplus of the match, $\tilde{J}_i^j$. We study a symmetric equilibrium, under which the promise made by suppliers in production lines in sector i are all identical and we can denote them by $\tilde{J}_i$. The aggregate state of the economy therefore consists of θ as well as the distribution of $\tilde{J}_i$ across all sectors, which we denote by Ω. We will denote the wage that arises in general equilibrium by $w(\theta, \Omega)$ and the revenue of final good firm i by $rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega)$. The latter is given by

$$rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega) = P(\theta, \Omega) C(\theta, \Omega)^{\frac{1}{\gamma}} \left(k^{1-\eta_i} \left\{ \left[\sum_{n=1}^{N_i} x_n^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i} \right)^{\frac{\gamma-1}{\gamma}} \tag{A.1}$$

with $P(\theta, \Omega)$ and $C(\theta, \Omega)$ denoting the aggregate price index and consumption in the economy.

The recursive problem of the intermediate-good producer supplying goods to final good firms in sector i is given by

$$\begin{aligned}
V\left(\tilde{J}(\theta), \theta, \Omega\right) = & \max_{p_j^s, p_j^{tc}, x_j, \tilde{J}'(\theta')} p_j^s + p_j^{tc} - w(\theta, \Omega) x_j + \beta \mathbb{E}\left[V\left(\tilde{J}'(\theta'), \theta', \Omega'(\theta') \right) \right] \\
& s.t. \quad p_j^s + \sum_{n \neq j} p_n^s \leq \left[1 - \theta\left(1 - \delta(1 - \pi_i)\right)\right] rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\mu] \\
& \qquad p_j^{tc} \leq \beta \mathbb{E}\left[\tilde{J}'(\theta')\right], && [\gamma] \\
& \qquad w(\theta, \Omega) x_j - p_j^s \leq (1 - \theta) p_j^{tc}, && [\iota] \\
& \qquad \tilde{J}(\theta) = rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) - rev_i(\{0, \vec{x}_{-j}\}, \theta, \Omega) - (p_j^s + p_j^{tc}) + \beta \mathbb{E}_\theta\left[\tilde{J}'(\theta')\right], && [\lambda] \\
& \qquad p_j^s + p_j^{tc} + \sum_{n \neq j} (p_n^s + p_n^{tc}) \leq rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\rho]
\end{aligned}$$

where the red letters correspond to the Lagrange multipliers associated with each equation. The first-order conditions are given by

$$
\begin{aligned}
&[p^s] && 1+\iota = \mu + \lambda + \rho, && \text{(A.2)}\\
&[p^{tc}] && 1+(1-\theta)\iota = \gamma + \lambda + \rho, && \text{(A.3)}\\
&[x] && \left[(1-\theta\left(1-\delta(1-\pi_i)\right))\mu + \rho + \lambda\right]\frac{\partial rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega)}{x_j} = w(\theta,\Omega)(1+\iota), && \text{(A.4)}\\
&[J'(\theta')] && \beta P(\theta'|\theta) V_J\left(J'\left(\theta'\right), \theta'\right) = -\beta P(\theta'|\theta)\left(\gamma+\lambda\right), && \text{(A.5)}
\end{aligned}
$$

The envelope condition is given by

$$
V_{\tilde{J}(\theta)}\left(\tilde{J}\left(\theta\right), \theta\right) = -\lambda(\theta), \tag{A.6}
$$

Combining the two sets of conditions above we obtain the following optimality condition

$$
\lambda'(\theta') = \lambda(\theta) + \gamma(\theta). \tag{A.7}
$$

In the symmetric equilibrium, we have that

$$
rev_i(x,\theta,\Omega) = P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta\frac{\gamma-1}{\gamma}},
$$

and[21]

$$
\frac{\partial rev_i(x,\theta,\Omega)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta\frac{\gamma-1}{\gamma}-1}. \tag{A.8}
$$

A.1.1 The recursive problem with corporate subsidies

We assume that the government uses lump-sum subsidies to support financially constrained firms during financial crises. In particular, $N_i T_i^{\text{final}}$ denotes the total lump-sum subsidy to final-good producers and T_i^{supplier} the lump-sum subsidies to each supplier in production line i. The recursive problem of the intermediate-good producer supplying goods to final good firms is then given by

[21] Note that the second expression is not obtained by differentiating the first with respect to x, but rather by imposing symmetry in the expression of $\frac{\partial rev_i(\{x_j, \vec{x}_{-j}\})}{\partial x_j}$.

$$
\begin{aligned}
V\left(\tilde{J}(\theta),\theta,\Omega\right) = & \max_{p_j^s,p_j^{tc},x_j,\tilde{J}'(\theta')} p_j^s + p_j^{tc} - w(\theta,\Omega)x_j + T_i^{\text{supplier}}\mathbb{1}\left(\theta=\theta^H\right) + \beta\mathbb{E}\left[V\left(\tilde{J}'(\theta'),\theta',\Omega'(\theta')\right)\right] \\
\text{s.t.} \quad & p_j^s + \sum_{n\neq j} p_n^s \leq \left[1-\theta\left(1-\delta(1-\pi_i)\right)\right] rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) + N_i T_i^{\text{final}}\mathbb{1}\left(\theta=\theta^H\right), && [\mu] \\
& p_j^{tc} \leq \beta\mathbb{E}\left[\tilde{J}'(\theta')\right], && [\gamma] \\
& w(\theta,\Omega)x_j - p_j^s \leq (1-\theta)p_j^{tc} + T_i^{\text{supplier}}\mathbb{1}\left(\theta=\theta^H\right), && [\iota] \\
& \tilde{J}(\theta) = rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) - rev_i(\{0,\vec{x}_{-j}\},\theta,\Omega) - (p_j^s+p_j^{tc}) + \beta\mathbb{E}_\theta\left[\tilde{J}'(\theta')\right], && [\lambda] \\
& p_j^s + p_j^{tc} + \sum_{n\neq j}(p_n^s+p_n^{tc}) \leq rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) + N_i T_i^{\text{final}}\mathbb{1}\left(\theta=\theta^H\right), && [\rho] \\
& \tilde{J}'(\theta') \geq 0. && [\zeta(\theta')]
\end{aligned}
$$

A.2 Proofs

Proof of Proposition 1

Proof. In the steady state, $\lambda' = \lambda$ so that equation (16) implies $\kappa = 0$. Combining equations (14) and (15), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint boils down to

$$J = \frac{1}{1-\beta}\left(x^\eta - p^s - p^{tc}\right).$$

Consider first the case where $\mu = 0$. In that case $\iota = 0$ and the first order condition with respect to x implies

$$x = \underbrace{\left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}}}_{x_{fb}}.$$

From the promise keeping constraint, we obtain that $p^s + p^{tc} = x_{fb}^\eta - (1-\beta)J$. For a given level of J, the borrowing constraints do not bind if

$$
\begin{aligned}
J &\geq \tilde{\theta}x_{fb}^\eta, \\
J &\leq \frac{1}{(1-\tilde{\theta})(1-\beta)}\left[(1+\tilde{\theta})(1-\tilde{\theta})x_f b^\eta - W x_{fb}\right].
\end{aligned}
$$

Therefore, any level of J that satisfies both conditions, if such exists, can be sustained as a steady state. We will restrict attention to the lower bound of such support ($J = \tilde{\theta}x_{fb}^\eta$), which guarantees the supplier the most surplus in the steady state.[22] This level of J is supported by $p^{tc} = \beta\tilde{\theta}x^\eta$ and $p^s = (1-\tilde{\theta})x^\eta$.

Depending on parameter values, there could potentially be no level of J such that both

[22]Note that such equilibrium selection does not affect output in the economy as $x = x^{fb}$ when $\iota = \mu = 0$ regardless of the level of J.

borrowing constraints do not bind in the steady state. That is, for some parameters, the borrowing constraints bind in the steady state. Such case occurs when

$$(1-\tilde{\theta})(1-\beta)\tilde{\theta}x_{fb}^{\eta} > \left[(1+\tilde{\theta})(1-\tilde{\theta})x_f b^{\eta} - W x_{fb}\right].$$

Rearranging using the definition of x_{fb}, we obtain that the borrowing constraints bind in the steady state if and only if

$$\eta > \left[(1-\tilde{\theta})(1+\beta\tilde{\theta})\right].$$

Note that the RHS is decreasing in $\tilde{\theta}$. Let $\bar{\theta}$ be the level of $\tilde{\theta}$ which makes the equation above hold with equality. For any $\tilde{\theta} \geq \bar{\theta}$, the condition above is satisfied and both borrowing constraints bind in the steady state. In this case, we have that $p^s = (1-\tilde{\theta})x^{\eta}$ and the promise keeping constraint implies $(1-\beta)J = \tilde{\theta}x^{\eta} - p^{tc}$. The supplier's borrowing constraint is

$$(1-\tilde{\theta})(1-\beta)J = \left[(1+\tilde{\theta})(1-\tilde{\theta})x^{\eta} - Wx\right].$$

For the trade credit constraint to be satisfied, we must have $J \geq \tilde{\theta}x^{\eta}$. As in the unconstrained case, we will restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\tilde{\theta}x^{\eta}$. Plugging $J = \tilde{\theta}x^{\eta}$ into the supplier borrowing constraint we obtain:

$$(1-\tilde{\theta})(1-\beta)\tilde{\theta}x^{\eta} = \left[(1+\tilde{\theta})(1-\tilde{\theta})x^{\eta} - Wx\right].$$

Rearranging we obtain that when $\tilde{\theta} > \bar{\theta}$ we have

$$x = \left[\frac{(1+\beta\tilde{\theta})(1-\tilde{\theta})}{W}\right]^{\frac{1}{1-\eta}}.$$

□

Proof of Proposition 2

Proof. We will show that when $\theta < \bar{\theta}$, the equilibrium level of output does not change with θ. We conjecture that

$$J(\theta) = \theta\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}},$$

and that the borrowing constraints as well as the trade credit constraint are not binding so that $\mu(\theta) = \omega(\theta) = \kappa(\theta) = \iota(\theta) = 0$. From equation (13), this implies

$$x = \left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}}.$$

We want to verify that our conjecture for J_θ constitutes an equilibrium, in which the borrowing constraints as well as the trade credit constraint are not binding. We set

$$p^s = (1-\theta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}}, \tag{A.9}$$

$$p^{tc} = \beta J = \beta\theta\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}}. \tag{A.10}$$

When the spot price and trade credit price are equal to these values, we confirm that the promise keeping equation holds with equality for the value of $J(\theta)$ we conjectured. We then check whether the supplier borrowing constraint (10) is satisfied:

$$W\left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}} - (1-\theta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}} \leq \beta\theta(1-\theta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}},$$

Rearranging we obtain

$$\eta \leq (1+\beta\theta)(1-\theta). \tag{A.11}$$

Note that the RHS is decreasing in θ, and that for $\bar{\theta}$ the equation above holds with equality. So for all $\theta \leq \bar{\theta}$, this inequality holds, and we have that the supplier's borrowing constraint is not binding. Thus, we confirm that our conjecture for $J(\theta)$ consists of an equilibrium in which the borrowing constraints as well as the trade credit constraint are not binding.

Since the value of x is independent of θ, any change to θ which doesn't move it above $\bar{\theta}$, does not lead to a change in x. That is, output does not change in response to a small shock that raises θ as long as $\theta < \bar{\theta}$.

□

Proof of Proposition 3

Proof. Recall from equation (8) that the elasticity of labor to the financial shock in the spot economy is given by:

$$\varepsilon_{x,\theta}^{\text{spot}} \equiv \frac{dx^{\text{spot}}/x^{\text{spot}}}{d\theta/\theta} = \frac{\theta}{(1-\theta)(1-\eta)}.$$

Given that the monopolist borrowing constraint is binding, we have

$$wx = (1-\theta)x^{\eta} + (1-\theta)p^{tc}. \tag{A.12}$$

Totally differentiating with respect to θ we obtain

$$w\frac{\partial x}{\partial \theta} - \left[-x^{\eta} + (1-\theta)\eta x^{\eta-1}\frac{\partial x}{\partial \theta}\right] - \left[-p^{tc} + (1-\theta)\frac{\partial p^{tc}}{\partial \theta}\right] = 0$$

So, rearranging we have

$$\frac{\partial x}{\partial \theta}\left[w - (1-\theta)\eta x^{\eta-1}\right] = -x^{\eta} - p^{tc} + (1-\theta)\frac{\partial p^{tc}}{\partial \theta}$$

Substituting the supplier's borrowing constraint (A.12), we have

$$\frac{\partial x}{\partial \theta}\left[(1-\theta)x^{\eta-1} + (1-\theta)\frac{p^{tc}}{x} - (1-\theta)\eta x^{\eta-1}\right] = -x^{\eta} - p^{tc} + (1-\theta)\frac{\partial p^{tc}}{\partial \theta}$$

So, rearranging we get

$$\frac{\partial x}{\partial \theta}\left[(1-\theta)(1-\eta)x^{\eta-1} + (1-\theta)\frac{p^{tc}}{x}\right] = -x^{\eta} - p^{tc} + (1-\theta)\frac{\partial p^{tc}}{\partial \theta}$$

Rearranging using $\varepsilon_{x,\theta} = \frac{\partial \ln x}{\partial \ln \theta}$, we have

$$\varepsilon_{x,\theta}\frac{1}{\theta}\left[(1-\theta)(1-\eta)x^{\eta} + (1-\theta)p^{tc}\right] = -x^{\eta} - p^{tc} + (1-\theta)\frac{\partial p^{tc}}{\partial \theta},$$

Rearranging, we have

$$\varepsilon_{x,\theta} = -\frac{\theta(x^{\eta} + p^{tc})}{(1-\theta)(1-\eta)x^{\eta} + (1-\theta)p^{tc}} + \frac{p^{tc}}{p^{tc} + (1-\eta)x^{\eta}}\varepsilon_{p^{tc},\theta}.$$

Rearranging the first term in the RHS we obtain

$$\varepsilon_{x,\theta} = -\frac{\theta}{(1-\theta)(1-\eta) + \eta(1-\theta)\frac{p^{tc}}{x^{\eta}+p^{tc}}} + \frac{p^{tc}}{p^{tc} + (1-\eta)x^{\eta}}\varepsilon_{p^{tc},\theta}.$$

Using the definition of $\varepsilon^{\text{spot}}_{x,\theta}$, the equation above simplifies to

$$\varepsilon_{x,\theta} = \alpha\varepsilon^{spot}_{x,\theta} + \frac{p^{tc}}{p^{tc} + (1-\eta)x^{\eta}}\varepsilon_{p^{tc},\theta}.$$

where $\alpha < 1$ as $\eta(1-\theta)\frac{p^{tc}}{x^{\eta}+p^{tc}} > 0$. $\square$

Proof of Proposition 4

Proof. We shall study the deterministic steady state when the wage level is W and aggregate consumption is C. In the steady state, the promise-keeping values as well as the Lagrange multipliers do not change over time. Equation (A.7) then implies $\gamma = \zeta = 0$, and combining equations (A.2)–(A.3), we get $\mu = \theta\iota$ and $\rho + \lambda = 1$. In a symmetric equilibrium, all suppliers in a production line will supply the same quantity of intermediate inputs. To ease notation, we will use $rev_i(x)$ to denote $rev_i(\{x,\ldots,x\})$. In equilibrium we obtain

$$rev_i(x) = C^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta\frac{\gamma-1}{\gamma}},$$

and[23]

$$\frac{\partial rev_i(x)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta P C^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta\frac{\gamma-1}{\gamma}-1}. \tag{A.13}$$

Finally, in the steady state of a symmetric equilibrium, the promise keeping constraint is:

$$\tilde{J}(\theta) = A_i rev_i - \frac{1}{N_i}p^s - \frac{1}{N_i}p^{tc} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')], \tag{A.14}$$

where

$$A_i \equiv 1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

Let's first analyze an equilibrium in which $\iota = 0$ and study the conditions under which such equilibrium exists. Using Equation (A.4) and Equation (A.13), we can solve for x_i and obtain:

$$x_i = \underbrace{\left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}}\eta_i\frac{\gamma-1}{\gamma}C^{\frac{1}{\gamma}}}{WN_i^{1-\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}}}_{\equiv x_i^{unc}}. \tag{A.15}$$

Let's further conjecture that $p^{tc} = 0$ and $\tilde{J} = 0$. From the promise keeping constraint it then follows that

$$p^s = N_i A_i rev_i(x_i^{\text{unc}}).$$

[23]Note that the second expression is not obtained by differentiating the first with respect to x, but rather by imposing symmetry in the expression of $\frac{\partial rev_i(\{x_j,\vec{x}_{-j}\})}{\partial x_j}$.

The final producer's borrowing constraint must hold for this allocation to be sustained in equilibrium, so the above characterizes the solution as long as

$$N_i A_i \leq 1 - \theta\left(1 - \delta(1 - \pi_i)\right) \iff \theta \leq \underbrace{\frac{1 - N_i A_i}{1 - \delta(1 - \pi_i)}}_{\underline{\theta}_i}. \tag{A.16}$$

That is, if $\theta \leq \underline{\theta}_i$ then the borrowing constraints for both the final-good producer as well as the supplier do not bind in the steady state allocations and $x_i = x_i^{unc}$. For $\theta > \underline{\theta}_i$, it is still possible that x_i^{unc} can be supported in equilibrium using trade credit. The borrowing limit of the final-good producer yields

$$p_i^s = [1 - \theta(1 - \delta(1 - \pi_i))] rev_i(x_i^{\text{unc}}). \tag{A.17}$$

To determine p_i^{tc}, note that it should satisfy

$$p_i^{tc} \leq N_i \beta \tilde{J} = N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1 - \pi_i))}{N_i} \right] rev_i(x_i^{unc}).$$

To support x_i^{unc} in equilibrium, the supplier's borrowing constraint must hold so that

$$W x_i^{unc} \leq \frac{p_i^s}{N_i} + (1 - \theta) \frac{p_i^{tc}}{N_i}.$$

Plugging in the upper limit for p_i^{tc} and the expression for p_i^s, we have the following condition

$$W N_i x_i^{unc} \leq [1 - \theta(1 - \delta(1 - \pi_i))] rev_i(x_i^{\text{unc}}) + (1 - \theta) N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1 - \pi_i))}{N_i} \right] rev_i(x_i^{unc}). \tag{A.18}$$

Note that the RHS is decreasing in θ. Let $\overline{\theta}_i$ be the level of θ such that the expression above holds with equality. If $\theta \in (\underline{\theta}_i, \overline{\theta}_i]$ then the supplier's borrowing constraint doesn't bind and $x = x^{unc}$ is the equilibrium allocation with a positive level of trade credit in the economy.

Finally, consider the case in which $\theta > \overline{\theta}_i$. In such case, x_i^{unc} cannot be supported in equilibrium and μ and ι are both strictly positive. In this case, the supplier's borrowing constraint binds, so that $x_i = x_i^{con}$ defined implicitly as

$$W N_i x_i^{\text{con}} = [1 - \theta(1 - \delta(1 - \pi_i))] rev_i(x_i^{\text{con}}) + (1 - \theta) N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1 - \pi_i))}{N_i} \right] rev_i(x_i^{\text{con}}),$$

with

$$p_i^s = [1 - \theta(1 - \delta(1 - \pi_i))] rev_i(x_i^{\text{con}}),$$

and

$$p_i^{tc} = N_i\beta\left[A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right] rev_i(x_i^{con}).$$

$\square$

Proof of Lemma 1

Proof. From proposition 4, we know that if $\theta > \underline{\theta}_i$

$$\frac{p_i^{tc}}{rev_i} = N_i\beta\left[A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right]$$

where

$$A_i \equiv 1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}$$

Then

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial\eta_i} = -\beta N_i\left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}\log\left(\frac{N_i-1}{N_i}\right) > 0 \text{ since } \frac{N_i-1}{N_i} < 1 \text{ and } \sigma > \gamma > 1$$

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial\pi_i} = \beta\delta > 0$$

Finally, provided that $\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma} < 1$

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial N_i} = \beta\left\{1-\left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} - \frac{1}{N_i}\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}\left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}\right\} < 0 \text{ for all } N_i > 1$$

$\square$

Proof of Proposition 5

Proof. This proof is very similar to the proof to Proposition 1. Note that the corporate subsidies do not alter the optimality conditions but only the constraints. In the steady state, $\lambda' = \lambda$ so that equation (16) implies $\kappa = 0$. Combining equations (14) and (15), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint boils down to

$$J = \frac{1}{1-\beta}\left(x^\eta - \tilde{p}^s - p^{tc} - T_f\right).$$

Since $\theta > \bar{\theta}$, we know that the monopolist borrowing constraints binds in the steady state when $T_f = T_s = 0$ (see proof to Proposition 1). We have that $p^s = (1-\tilde{\theta})x^\eta$ and the promise keeping constraint implies $(1-\beta)J = \theta x^\eta - p^{tc} - T_f$. The supplier's borrowing constraint is

$$(1-\theta)(1-\beta)J = \left[(1+\theta)(1-\theta)x^\eta - Wx + \theta T_f + T_s\right].$$

For the trade credit constraint to be satisfied, we must have $J \geq \theta x^\eta$. We restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\theta x^\eta - \beta T_f$. Plugging $J = \theta x^\eta - T_f$ into the supplier borrowing constraint we obtain:

$$(1-\theta)(1-\beta)\left(\theta x^\eta - T_f\right) = \left[(1+\theta)(1-\theta)x^\eta - Wx + \theta T_f + T_s\right].$$

Rearranging we obtain that

$$(1+\beta\theta)(1-\theta)x^\eta + (1-\beta(1-\theta))\,T_f + T_s = Wx$$

Totally differentiating with respect to x, T_f, and T_s, we have

$$\frac{1}{x}\eta(1+\beta\theta)(1-\theta)x^\eta dx + (1-\beta(1-\theta))\,dT_f + dT_s = Wdx,$$

rearranging we have

$$(1-\beta(1-\theta))\,dT_f + dT_s = \frac{1}{x}\left(Wx - \eta(1+\beta\theta)(1-\theta)x^\eta\right)dx.$$

Using the fact that when $T_s = T_f = 0$, we have that $(1+\beta\theta)(1-\theta)x^\eta = Wx$, we get that when $T_f = T_s = 0$, the equation above simplifies to

$$(1-\beta(1-\theta))\,dT_f + dT_s = (1-\eta)\,Wdx, \tag{A.19}$$

so that

$$\left.\frac{\partial x}{\partial T_f}\right|_{T_f=T_s=0} = \frac{1}{(1-\eta)W}, \qquad \left.\frac{\partial x}{\partial T_s}\right|_{T_f=T_s=0} = \frac{1-\beta(1-\theta)}{(1-\eta)W}$$

□

A.3 Data appendix

In this section, we detail the procedure we take to construct the balanced panel dataset we use in our analysis. We then present some descriptive statistics.

The raw unbalanced dataset between 2007–2015 contains 15,125,421 firm-year observations. We drop firms with consolidation codes "NA" (no financial data available), "LF" (limited financial data available), "C1" (consolidated statement with unconsolidated companion), and "C2" (consolidated statement with no unconsolidated companion). Leaving us with unconsolidated statements only - resulting in 8,418,833 firm-year observations.[24]

We then drop observations where total assets, accounts receivable, accounts payable, operating revenues, or sales are missing. Additionally, we drop observations with negative values for one of the following variables: total assets, employment, sales, wage bill, accounts receivable, accounts payable, short-term bank loans, long-term debt, depreciation, cash holdings, or total inventories. Finally, we drop observations where the year of operation is earlier than the date of incorporation of the firm. These steps leave us with 8,107,403 firm-year observations.

We drop financial firms (136,677 obs.), drop firms with less than 10,000 in total assets or annual sales (30,226 obs), and keep only firms with an "active" status (dropping 1,393,729 observations). We then keep firms with observations in all years so that the panel is balanced. The balanced panel contains 2,447,163 observations.

We construct a variable for intermediate inputs which subtracts the sum of operating profits, wage bill, and depraciation, from sales. For each observation we compute the ratio between accounts payable, accounts receivable, and intermediate inputs to sales. We winsorize the top and bottom 1%. Finally, we drop the top 1% and bottom 1% growing firms in terms of log sales in the data.[25]

The final dataset contains a balanced panel of 243,553 firms over 2007–2015, resulting in 2,191,977 observations. Table A-1 displays descriptive statistics using the final balanced sample of firms.

A.4 Numerical Algorithm

In this section, we lay out the numerical algorithm we use to solve the model. Our approach relies on policy function iteration combined with an approximate law-of-motion similar to

[24] We additionally drop 1,908 duplicate observations.

[25] We also drop 1,305 firm-year observations in the refinery sector as most purchases of refinery products are imported.

Table A-1: Descriptive statistics at the firm-level

Variable	Mean	P10	P25	P50	P75	P90	% of firms with data
Total assets (Millions)	8,022,889	139,528	329,347	903,717	2,683,331	7,910,160	100%
Sales	6,106,199	85,877	229,836	711,568	2,250,524	6,962,928	100%
Accounts payable	1,117,917	0	6,971	87,004	421,124	1,494,970	100%
Accounts receivable	1,380,116	0	6,227	120,835	571,589	2,010,485	100%
Short-term bank loans	677,022	0	0	6,767	160,416	779,963	100%
Intermediate inputs	6,332,176	111,181	265,464	734,417	2,280,475	7,030,696	78%

Krusell-Smith. We start by defining some notation. We then show how to we solve for the partial equilibrium, where aggregate levels are taken as given. Finally, we show how we solve for the full general equilibrium.

We show how to solve for the general case with corporate subsidies. The benchmark model is a special case when such subsidies are set to zero. To make it easy to see where these subsidies enter the equations, we color all such places in blue.

A.4.1 Notation

Before describing the numerical algorithm in details, it is useful to define some notation.The revenue of final-good producers in sector i is denoted by

$$rev_i(x) = C(\theta,\Omega)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta\frac{\gamma-1}{\gamma}}.$$

The first-best labor input in sector i is denoted by

$$x^{\text{fb}} = \left[\frac{\gamma-1}{\gamma}\eta\frac{C(\theta,\Omega)^{\frac{1}{\gamma}}}{w(\theta,\Omega)} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\right]^{\frac{\gamma}{(1-\eta)\gamma+\eta}}.$$

The promise keeping constraint is

$$\tilde{J}(\theta) = A_i rev_i - \frac{1}{N_i}p^s - \frac{1}{N_i}p^{tc} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')],$$

where

$$A_i \equiv 1 - \left(\frac{N_i-1}{N_i}\right)^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

Next, denote by $x^{\text{cons}}(\tilde{J}_\theta,\theta)$ the solution to $rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}}$. Namely,

$$x^{\text{cons}}(\tilde{J}_\theta,\theta) = \left[\frac{\tilde{J}_\theta + T_f(\theta)}{\left(A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right) C(\theta,\Omega)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\right]^{\frac{\gamma}{\eta(\gamma-1)}}$$

Finally, to ease notation, we introduce

$$\tilde{\theta}_i = \theta(1-\delta(1-\pi_i)),$$

where $\tilde{\theta}_i'$ is the equation above but with θ' instead of θ.

A.4.2 Partial Equilibrium

We start by solving the behavior of suppliers in a single sector taking the aggregate values $C(\theta,\Omega)$ and $w(\theta,\Omega)$ as given (simply denoted by C and w for current values, and C', and w' for next period's values). We assume that θ can take one of two values, θ_L and θ_H. The numerical algorithm uses policy function iteration to find the policy function in equilibrium. The policy function maps the current state of the economy, which includes both the aggregate state of financial frictions θ as well as the promise keeping value $\tilde{J}$, into two promise keeping values in the next period, one for each degree of financial frictions, θ'. The policy function is denoted by

$$\tilde{J}'(\theta' \mid \tilde{J}_\theta,\theta),$$

which denotes the promise keeping value the supplier must deliver in state θ', given that the current state is $\{\tilde{J}_\theta,\theta\}$.

Our policy function iteration proceeds in three steps:

1. Given a guess for the policy function, and the current state $\{\tilde{J}_\theta,\theta\}$, we compute future labor inputs x' for every θ' as a function of the current state $\{\tilde{J}_\theta,\theta\}$ and current Lagrange multipliers on the borrowing constraint of the supplier (ι) and the feasibility requirement (ρ).
2. Using the implied $x'(\theta';\tilde{J}_\theta,\theta,\iota,\rho)$ function, we update the policy function.
3. If the implied policy function is close to the guessed one, we have found the policy function in equilibrium. Otherwise, we update our guess for the policy function using the implied policy function, and repeat from step (1).

Below, we provide a detailed explanation for how each of the two steps are performed.

The state space grid is over $\{\tilde{J}_\theta, \theta\}$, where upper bound for $\tilde{J}_\theta$ is given by $\left(A_i - \frac{1-\theta'}{N_i}\right) rev_i^{\text{fb}}$ and the lower bound is close to zero.

A.4.2.1 Calculating x' given the policy function and current Lagrange multipliers

The optimality condition for x' (A.4) can be written as follows

$$\frac{\gamma-1}{\gamma}\eta\left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\left(x'\right)^{\eta\frac{\gamma-1}{\gamma}-1} = \frac{w'(1+\iota')}{\left(1-\tilde{\theta}_i'\right)\mu'+\rho'+\lambda'}. \tag{A.20}$$

In this section, we show how we can use the Lagrange multipliers ι and ρ, together with the state variables $\{\tilde{J}_\theta, \theta\}$, and the policy function $\tilde{J}'\left(\theta' \mid \tilde{J}_\theta, \theta\right)$, to compute x'. That is, we construct the function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$.

To find x', we will need to use the promise keeping in two periods, J''. To do so we apply the policy function $\tilde{J}'(\cdot)$ twice, starting from the state $\{\tilde{J}_\theta, \theta\}$. In particular, given the current state $\{\tilde{J}_\theta, \theta\}$, as well as θ', the expected promise keeping in two periods is given by

$$\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] \equiv \mathbb{E}_{\theta'}[\tilde{J}'(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta), \theta')]. \tag{A.21}$$

Note that when we apply the policy function twice, the second time we apply it can potentially be done for a point J' which is not a grid point. We use Chebyshev approximation to apply the policy function to points which lie between points on the grid.

We start by deriving an equation connecting the current Lagrange multipliers ι and ρ, with future ones (μ', ρ', ι'). From optimality condition (A.2) we have

$$\mu' + \rho' - \iota' = 1 - \lambda',$$

Using equation (A.7), together with optimality condition A.3 from the current period, we obtain

$$\rho - (1-\theta)\iota = \mu' + \rho' - \iota'. \tag{A.22}$$

Therefore, by knowing ι and ρ in the current period, we obtain the sum of $\mu' + \rho' - \iota'$. We split the derivation of x' into two cases, depending on the sign of $\rho - (1-\theta)\iota$.

Case A: $\rho - (1-\theta)\iota \geq 0$.

Suppose that the borrowing constraint of the supplier and the feasibility constraint do not bind ($\iota' = \rho' = 0$). In this case we have $\mu' = \rho - (1-\theta)\iota$. And from optimality condition

(A.2), we have $\lambda' = 1 - \mu'$. The optimality condition for x' (A.20) is then

$$\frac{\gamma-1}{\gamma}\eta\left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\left(x'\right)^{\eta\frac{\gamma-1}{\gamma}-1} = \frac{w'}{1-\tilde{\theta}'_i\mu'}.$$

Plugging in the value of μ' and the aggregate variables, we obtain x'. We now need to check whether the borrowing constraint of the supplier and the feasibility constraint are satisfied. [26]

$$w'x' - \frac{1-\tilde{\theta}'_i}{N_i}rev_i(x') \leq (1-\theta')\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta'';\tilde{J}_\theta,\theta)] + T_s(\theta') + T_f(\theta'), \tag{A.23}$$

$$\tilde{\theta}'_i rev_i(x') \geq N_i\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta'';\tilde{J}_\theta,\theta)]. \tag{A.24}$$

Note that final-good producers obtain T_f per supplier. So they receive N_iT_f in total.

In case both equations (A.23)–(A.24) above are satisfied, we have found $x'(\theta';\tilde{J}_\theta,\theta,\iota,\rho)$. Otherwise, we proceed as follows:

1. If both constraints (A.23)–(A.24) are violated then $\mu' > 0$, $\rho' > 0$ and $\iota' > 0$. In that case, both the borrowing constraint and the feasibility constraint binds so combining equations (A.23)–(A.24) with equality we obtain

$$w'x' = \frac{1-\theta'\tilde{\theta}'_i}{N_i}rev_i(x') + T_f(\theta') + T_s(\theta'), \tag{A.25}$$

   This equation pins down the level of x', and we obtain $x'(\theta';\tilde{J}_\theta,\theta,\iota,\rho)$. Note that when $T_f = T_s = 0$, we can derive an explicit formula for x.

2. If only the supplier's borrowing constraint (A.23) is violated, then $\mu' > 0$ and $\iota' > 0$. We guess that the feasibility constraint is not binding so that $\rho' = 0$. From equation (A.22) we have

$$\mu' - \iota' = \rho - (1-\theta)\iota \geq 0.$$

   From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$. So for $\mu' - \iota' \geq 0$, it must be that $\gamma' > 0$. So the borrowing constraints of both the supplier and the final-good producer are binding, as well as the trade-credit constraint. Combining all three equations we obtain

$$w'x' - \frac{1-\tilde{\theta}'_i}{N_i}rev_i(x') = (1-\theta')\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta'';\tilde{J}_\theta,\theta)] + T_f(\theta') + T_s(\theta'). \tag{A.26}$$

[26] When using $rev_i(x')$, aggregate consumption in the revenue function is C'.

This equation pins down x'. We now need to verify whether the feasibility constraint is indeed satisfied. We check whether

$$\tilde{\theta}_i' rev_i(x') \geq N_i \beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)].$$

If the equation above holds, then we found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, also $\rho' > 0$ and we obtain x' from step 1.

3. If only the feasibility constraint (A.24) is violated, then $\mu' > 0$ and $\rho' > 0$. We guess that the supplier borrowing constraint is not binding, $\iota' = 0$. From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$, so that $\gamma' = \mu' > 0$. So we have that the borrowing constraint of the final good producer is binding as well as the trade credit constraint and the feasibility constraint. We combine the three constraints to obtain

$$\left(A_i - \frac{1 - \tilde{\theta}_i'}{N_i}\right) rev_i(x') = \tilde{J}'\left(\theta'; \tilde{J}_\theta, \theta\right) + T_f(\theta'), \tag{A.27}$$

which pins down the value of x'. We need to check whether the borrowing constraint of the supplier is satisfied:

$$w'x' \leq \frac{1 - \theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

If the equation above holds, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, also $\iota' > 0$ and we obtain x' from step 1.

Case B: $\rho - (1 - \theta)\iota < 0$

From equation (A.22) it follows that $\iota' > 0$, as both μ' and ρ' are non-negative. From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$. Since $\gamma' \geq 0$, we have that $\mu' > 0$. We proceed in the following steps.

1. Suppose $\rho' = 0$. We solve for two cases. First, assuming that $\gamma' = 0$. And if the solution in that case violates the trade credit constraint, we move to the second case where $\gamma' > 0$.
   (a) Suppose $\gamma' = 0$. In this case, $\mu' = \theta'\iota'$. Plugging into (A.22), we have

$$\iota' = \frac{(1 - \theta)\iota - \rho}{1 - \theta'}.$$

   So we know both the values of ι' and $\mu' = \theta'\iota'$. From optimality condition (A.2), we obtain $\lambda' = 1 + \iota' - \mu'$. Plugging into equation (A.20), we then obtain x'. We need to check whether the trade credit constraint is satisfied. We use the supplier's borrowing constraint to obtain

$$p_j^{\mathrm{tc}\prime} = \frac{1}{1-\theta'}\left[w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta')\right].$$

So for the trade credit constraint to hold it must be that

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta') \leq (1-\theta')\beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)]. \qquad \text{(A.28)}$$

If the condition above holds, then we have a candidate for x' and only need to confirm $\rho' = 0$, which we do after the next subcase. Otherwise, we move to the next subcase where $\gamma' > 0$.

(b) In this subcase $\rho' = 0$, while $\iota' > 0$, $\gamma' > 0$, and $\mu' > 0$. That is, the borrowing constraints of both the supplier and final good producer hold with equality, as well as the trade credit constraint. Combining all three equations we obtain

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') = (1-\theta')\beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] + T_f(\theta') + T_s(\theta'). \qquad \text{(A.29)}$$

The equation above uniquely pins down x'. In this case, we have

$$p_j^{\mathrm{tc}\prime} = \beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)].$$

We move to check whether the feasibility constraint holds so that $\rho' = 0$.

After obtaining x' and $p_j^{tc\prime}$ from case (a) and (b), we need to verify the feasibility constraint is not violated. We check the following condition

$$p_j^{tc\prime} \leq \frac{\tilde{\theta}_i'}{N_i} rev_i(x'). \qquad \text{(A.30)}$$

If this condition holds, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, we conclude that $\rho' > 0$ and move to case 2 below.

2. In this case, we have that ι', μ' and ρ' are all strictly positive. That is, the borrowing constraints of both the supplier and final-good producer hold with equality as well

as the feasibility constraint. Combining all three equations we obtain equation (A.25):

$$w'x' = \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

This equation uniquely pins down x', and we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$.

A.4.2.2 Updating the guess of the policy function

In the previous section we have derived the function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$, which implicitly takes into account also the guess of the policy function. In this section, we use this function to find the equilibrium allocations in the current period, and to update the guess for the policy function.

Given x', the guess for the policy function is

$$\tilde{J}'(\theta') = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x') - T_f(\theta'), \tag{A.31}$$

which effectively assumes that the borrowing constraint of the final-good producer as well as the trade credit constraint hold with equality.

We proceed in cases, where each corresponds to a different set of Lagrange multipliers being strictly positive. Overall there are six cases. Case (I) considers the case in which all current multipliers are 0. Case (II) considers the case where μ and γ are strictly positive, while $\rho = \iota = 0$. Case (III) considers the case in which only $\iota = 0$. Case (IV) considers the case in which μ and ι are strictly positive, while $\gamma = \rho = 0$. Case (V) considers the case in which only $\rho = 0$. Finally, case (VI) assumes that μ, ι, and ρ are all strictly positive, while $\gamma \geq 0$. Note that in general there could be 16 options for which Lagrange multipliers are strictly positive, but using the optimality conditions, we narrow it down to six different combinations which can occur in equilibrium.

I) $\mu = \gamma = \rho = \iota = 0$. This case applies when the following two conditions hold:

$$wx^{\text{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta') \right] + T_s(\theta) + T_f(\theta),$$

and

$$\tilde{J}(\theta) + T_f(\theta) \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i^{\text{fb}}.$$

In this case the solution is:

$$
\begin{aligned}
x &= x^{fb}, \\
p_j^s &= \frac{1}{N_i}(1-\tilde{\theta}_i) rev_i^{fb} + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}
$$

II) $\mu = \gamma > 0$, and $\rho = \iota = 0$. This case applies when the following three conditions hold:

$$
\begin{aligned}
& w x^{\text{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta') \right] + T_s(\theta) + T_f(\theta), \\
& \tilde{J}(\theta) + T_f(\theta) < \left(A_i - \frac{1-\tilde{\theta}_i}{N_i} \right) rev_i^{\text{fb}}, \\
& \tilde{J}(\theta) + T_f(\theta) \geq \beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta') \right] \frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.
\end{aligned}
$$

The solution in this case is:

$$
\begin{aligned}
x &= x^{cons}(\tilde{J}_\theta, \theta), \\
p_j^s &= \frac{1}{N_i}(1-\tilde{\theta}_i) \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta), \\
p_j^{tc} &= \frac{1}{N_i} \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}
$$

III) $\mu = \gamma > 0$, $\rho > 0$, and $\iota = 0$. This case applies when the following three conditions hold:

$$
\begin{aligned}
wx^{\text{cons}} &\leq \frac{1-\tilde{\theta}_i}{N_i}\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\left[\frac{\tilde{\theta}_i\left(\tilde{J}(\theta)+T_f(\theta)\right)}{N_i A_i - (1-\tilde{\theta}_i)}\right] + T_s(\theta) + T_f(\theta),\\
\tilde{J}(\theta)+T_f(\theta) &< \beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,0,0)) - T_f(\theta')\right]\frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i},\\
\tilde{J}(\theta)+T_f(\theta) &\geq \beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,0,1)) - T_f(\theta')\right]\frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.
\end{aligned}
$$

The solution in this case is:

$$
\begin{aligned}
x &= x^{cons}(\tilde{J}_\theta, \theta),\\
p_j^s &= \frac{1}{N_i}(1-\tilde{\theta}_i)\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta),\\
p_j^{tc} &= \beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,0,\bar{\rho})) - T_f(\theta')\right],\\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,0,\bar{\rho})) - T_f(\theta'),
\end{aligned}
$$

where $\bar{\rho}$ is defined implicitly by solving the following equation:

$$
\beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,0,\bar{\rho})) - T_f(\theta')\right] = \tilde{\theta}_i \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.
$$

If $\{\tilde{J}_\theta, \theta\}$ is such that the conditions to cases (I)–(III) do not hold, it must be that $\iota > 0$. This implies $\mu > 0$. We first suppose the other two constraints are slack (case IV), then if these constraints are violated, we move to case (V) and to case (VI).

IV) $\mu > 0$, $\iota > 0$. We conjecture $\rho = \gamma = 0$. When $\gamma = 0$, we have that $\mu = \theta\iota$ so that the optimality condition for x can be written as follows:

$$
\left\{1 + \left[1 - \theta\tilde{\theta}_i\right]\iota\right\}\frac{\gamma-1}{\gamma}\eta C^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta\frac{\gamma-1}{\gamma}-1} = w(1+\iota).
$$

From the equation above, we can solve for x as a function of ι:

$$
x(\iota) = \left[\frac{1 + \left[1 - \theta\tilde{\theta}_i\right]\iota}{w(1+\iota)} \quad \frac{\eta C^{\frac{1}{\gamma}}}{N_i^{1-\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\frac{\gamma-1}{\gamma}\right]^{\frac{\gamma}{(1-\eta)\gamma+\eta}}.
$$

Since both the final-good producer's and supplier's borrowing constraints bind ($\iota > 0$

and $\mu > 0$), we can combine the two to obtain the level of trade credit as a function of x:

$$p_j^{tc} = \frac{wx}{1-\theta} - \frac{1}{N_i}\frac{1-\tilde{\theta}_i}{1-\theta} rev_i(x) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta}.$$

Then, using the promise keeping constraint, we have

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x) - \frac{wx}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')].$$

We then find $\bar{\iota}$ so that the promise keeping constraint holds with equality:

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x(\bar{\iota})) - \frac{wx(\bar{\iota})}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}'_i}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,\bar{\iota},0)) - T_f(\theta')\right].$$

The solution in this case is:

$$\begin{aligned} x &= x(\bar{\iota}), \\ p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(x(\bar{\iota})) + T_f(\theta), \\ p_j^{tc} &= \frac{wx(\bar{\iota})}{1-\theta} - \frac{1}{N_i}\frac{1-\tilde{\theta}_i}{1-\theta} rev_i(x(\bar{\iota})) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta}, \\ \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}'_i}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,\bar{\iota},0)) - T_f(\theta'). \end{aligned}$$

Since this case assumed two Lagrange multipliers are zero, to verify the solution there are two conditions we need to verify:

$$\tilde{J}_\theta \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i(x(\bar{\iota})) - T_f(\theta), \tag{A.32}$$

$$wx(\bar{\iota}) \leq \left(1-\theta+\theta(1-\tilde{\theta}_i)\right) rev_i(x(\bar{\iota})) + T_f(\theta) + T_s(\theta). \tag{A.33}$$

Equation (A.32) ensures that the trade credit constraint is satisfied, while equation (A.33) ensures the feasibility constraint is satisfied. If both conditions hold, then we have the solution above is valid. If condition (A.33) is satisfied but condition (A.32) is violated, then $\gamma > 0$ and we move to case (V). If condition (A.33) is violated, we move to case (VI).

V) $\mu > 0$, $\iota > 0$, $\gamma > 0$, and we conjecture $\rho = 0$. Since the borrowing constraint of the final-good producer binds and the trade credit constraint binds we have

$$x = x^{\text{cons}}(\tilde{J}_\theta, \theta),$$
$$rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

Since the supplier's borrowing constraint binds as well ($\iota > 0$), we also have that

$$wx = \frac{1-\theta}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta) + (1-\theta)\beta\mathbb{E}_\theta[\tilde{J}'(\theta')].$$

Therefore, we can find ι^* from the following equation

$$wx^{\text{cons}} = \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta) + T_s(\theta)$$
$$+ (1-\theta)\beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta')\right]$$

We need to verify that the feasibility constraint is satisfied, which boils down to the following condition:

$$\beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta')\right] \leq \frac{\tilde{\theta}_i}{N_i} \frac{\tilde{J}(\theta) + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

If this condition is violated, we move to case (VI). Otherwise, the solution is:

$$x = x^{\text{cons}}(\tilde{J}_\theta, \theta),$$
$$p_j^s = \frac{1}{N_i}(1-\tilde{\theta}_i)\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta),$$
$$p_j^{tc} = \beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta')\right],$$
$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta').$$

VI) $\mu > 0$, $\iota > 0$, $\rho > 0$, and $\gamma \geq 0$. When the borrowing constraints of both the final-good producer and the supplier bind, and the feasibility constraint is also binding, we obtain:

$$
\begin{aligned}
p_j^s &= \frac{(1-\tilde{\theta}_i)}{N_i} rev_i(x) + T_f(\theta), \\
p^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(x), \\
wx &= \frac{1-\theta\tilde{\theta}_i}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta).
\end{aligned}
$$

The final equation pins down implicitly the level of x, which we denote by $\bar{x}$. The promise keeping constraint is then

$$
\tilde{J}(\theta) = \left(A_i - \frac{1}{N_i}\right) rev_i(\bar{x}) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\tilde{J}'(\theta')\right].
$$

Finally, we take advantage of the fact that the function $x'(\cdot)$ depends on the value of $\rho - (1-\theta)\iota$, regardless of the individual values of ρ and ι. Let $\zeta \equiv \rho - (1-\theta)\iota$. Then, the value of ζ is given by the solution to the following equation:

$$
\tilde{J}(\theta) = \left(A_i - \frac{1}{N_i}\right) rev_i(x) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \zeta)) - T_f(\theta')\right].
$$

The solution in this case is:

$$
\begin{aligned}
x &= \bar{x}, \\
p_j^s &= \frac{(1-\tilde{\theta}_i)}{N_i} rev_i(x) + T_f(\theta), \\
p_j^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(x), \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \zeta)) - T_f(\theta').
\end{aligned}
$$

A.4.3 General equilibrium

The previous section detailed our algorithm for solving the policy function of suppliers in an industry given current and future aggregate variables, C_t and w_t. In this section, we explain how we solve the full general equilibrium model with aggregate fluctuations.

Firms in the model need to form beliefs regarding future levels of consumption and the real wage. The full state variable in the economy contains the degree of financial frictions θ_t as well as the distribution of promise keeping values across all sectors $\{J_i t\}_i$. As our model contains 58 sectors, the curse of dimensionality prevents us from solving the model as a function of all J's. Instead, we conjecture that the level of consumption as well as the real wage follow an AR(1) process in logs, where the AR(1) coefficients depend on the degree of financial frictions. That is,

$$\ln C_t = (1 - \rho_c(\theta_t))\mu_c(\theta_t) + \rho_c(\theta_t) \ln C_{t-1},$$
$$\ln w_t = (1 - \rho_w(\theta_t))\mu_w(\theta_t) + \rho_w(\theta_t) \ln w_{t-1},$$

where $\rho_c(\theta_t)$, $\mu_c(\theta_t)$, $\rho_w(\theta_t)$, and $\mu_w(\theta_t)$ are a function of θ. Since we have two levels of θ, this leads to 8 unknowns. We denote the vector of these 8 state variables as $\vec{\zeta}$. We show below that this formulation yields an accurate approximation for the law of motion of both C_t and w_t.

Given the laws of motion for aggregate consumption and the real wage, suppliers only need to know the level of consumption and real wage in order to form beliefs on the future. Thus, the policy function can be written as $\tilde{J}'(\theta'; \tilde{J}_\theta, \theta, \ln C^L, \ln w^L)$, where the value denotes the promised surplus in the next period if the degree of financial frictions is θ', given the current promised surplus ($\tilde{J}_\theta$), the current degree of financial frictions (θ), lagged log aggregate consumption ($\ln C^L$), and the lagged log-level of the real wage ($\ln w^L$).

We adapt the partial equilibrium algorithm to solve for the policy function in general equilibrium, given the laws of motion ($\vec{\zeta}$). Instead of the state space being $\{\tilde{J}_\theta, \theta\}$, the state space is now given by $\{\tilde{J}_\theta, \theta, \ln C^L, \ln w^L\}$. Solving the policy function then follows exactly the same steps as described in the previous section, with two adjustments. First, the future levels of aggregate consumption and real wage vary with the future level of θ according to their laws of motion. Second, the expected future promise keeping value in two periods (A.21) takes into account the laws of motion:

$$\begin{aligned}
&\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta, \ln C^L, \ln w^L)] \equiv \\
&\quad \mathbb{E}_{\theta'}[\tilde{J}'(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta, \ln C^L, \ln w^L), \theta', (1 - \rho_c(\theta))\mu_c(\theta) + \rho_c(\theta) \ln C^L, (1 - \rho_w(\theta))\mu_w(\theta) + \rho_w(\theta) \ln w^L)].
\end{aligned}$$

We proceed as follows:

1. Start with a guess for $\vec{\zeta}$.
2. Given $\vec{\zeta}$, solve the policy function for each of the 58 sectors in the economy independently.
3. Simulate θ_t for T periods using the transition matrix.[27] Use $\vec{\zeta}$ to obtain C_t and w_t,

[27] We set $T = 5,000$ when solving the model.

starting with the first value being the steady state one when $\theta = \theta_L$.

4. Using the policy functions together with the sequence $\{\theta_t, C_t, w_t\}$, find the employment of suppliers to each sector and the level of output produced by each sector. Denote the implied output levels by $y_{it}(\vec{\zeta})$ and total employment of suppliers to sector i by $x_{it}(\vec{\zeta})$.

5. Use the definition of aggregate consumption as well as the optimality condition of households to obtain the implied levels of aggregate consumption and the real wage:

$$C_t(\vec{\zeta}) = \left[\frac{1}{58}\sum_i y_{it}(\vec{\zeta})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}},$$

$$w_t(\vec{\zeta}) = \chi\left[\frac{1}{58}\sum_i N_i x_{it}(\vec{\zeta})\right]^{\frac{1}{\psi}}.$$

6. Regress $\ln C_t(\vec{\zeta})$ on $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_L)$, $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_H)$, $\mathbb{1}(\theta_t = \theta_L)$, and $\mathbb{1}(\theta_t = \theta_H)$ (no constant). Run a similar regression for $\ln w_t(\vec{\zeta})$. Denote the regression coefficients for these implied laws-of-motion as $\bar{\zeta}$.

7. If $\bar{\zeta}$ is sufficiently close to $\vec{\zeta}$, we have solved the model. Otherwise, update the law-of-motion coefficients $\vec{\zeta}$ to be a convex combination of the current guess $\vec{\zeta}$ and the implied ones $\bar{\zeta}$. In particular, set the new guess to be $0.5\vec{\zeta} + 0.5\bar{\zeta}$. Then repeat from step (2).

For our benchmark specification, the $\mathbb{R}^2$ for the regression of the law-of-motion for log-consumption is 0.99999975 and the $\mathbb{R}^2$ for the regression of the law-of-motion for log-wage is 0.99999974. That is, the approximation is very accurate.